Parallel Merge-Sort Algorithm

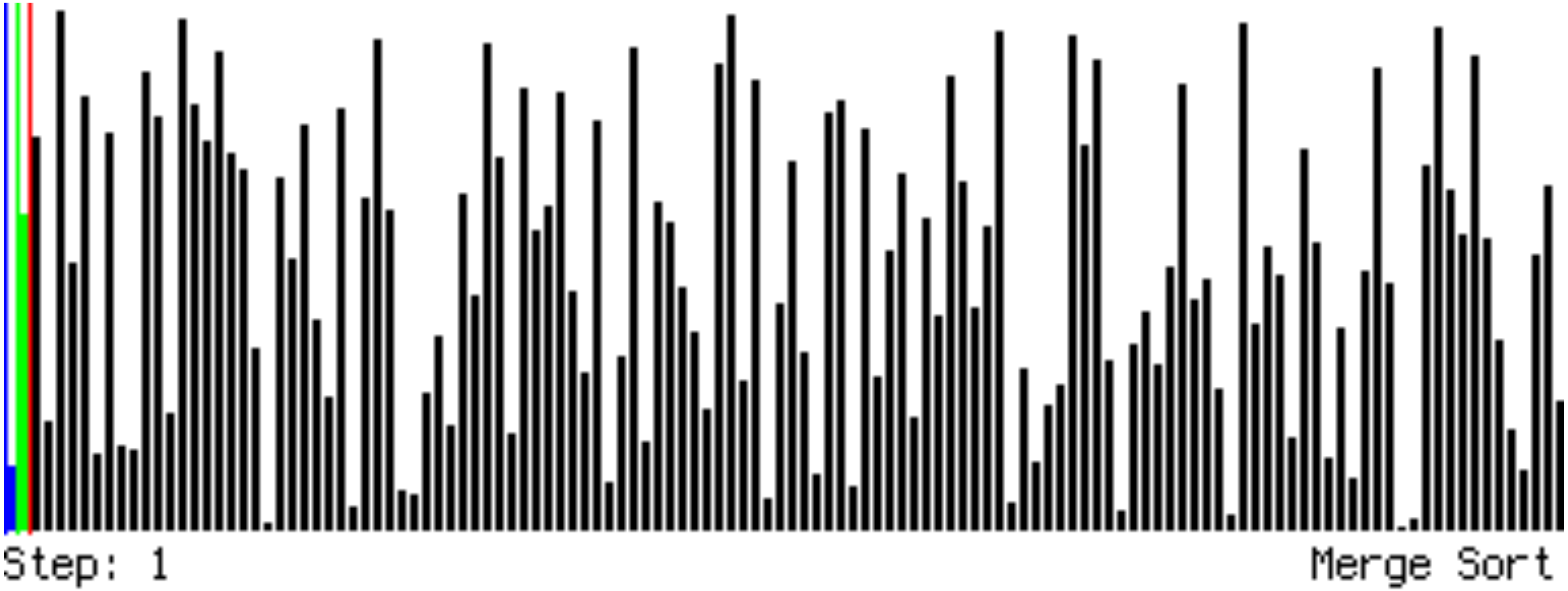
Parallel Programming DS295

Ghanshyam Chandra, 11 Feb, 2024

Agenda

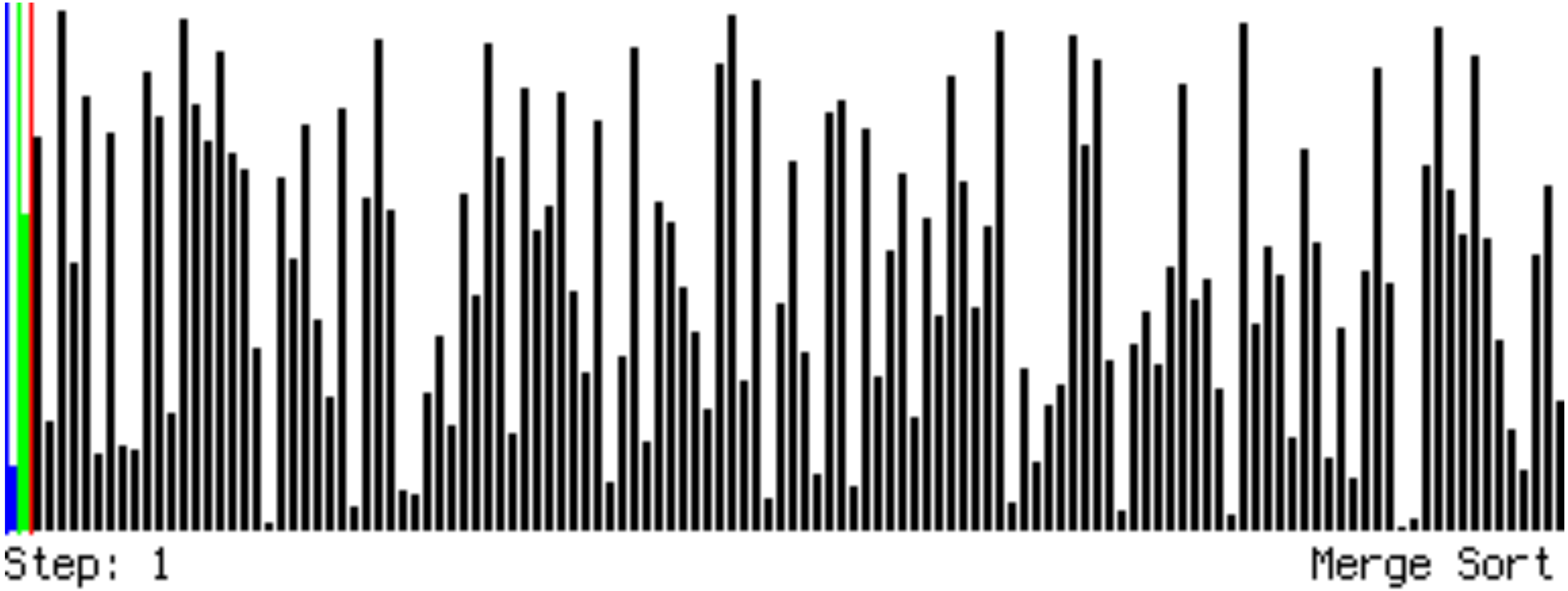
- Sorting Algorithms (Complexity Overview)
- Merge-Sort Algorithm
- Parallel Merge-Sort Algorithm
- Distributed Implementation of Merge-Sort Algorithm (MPI)
- Theoretical Analysis

Sorting Animation



Credit: Lane Wagner

Sorting Animation



Credit: Lane Wagner

Sorting Algorithms
(Complexity Overview)

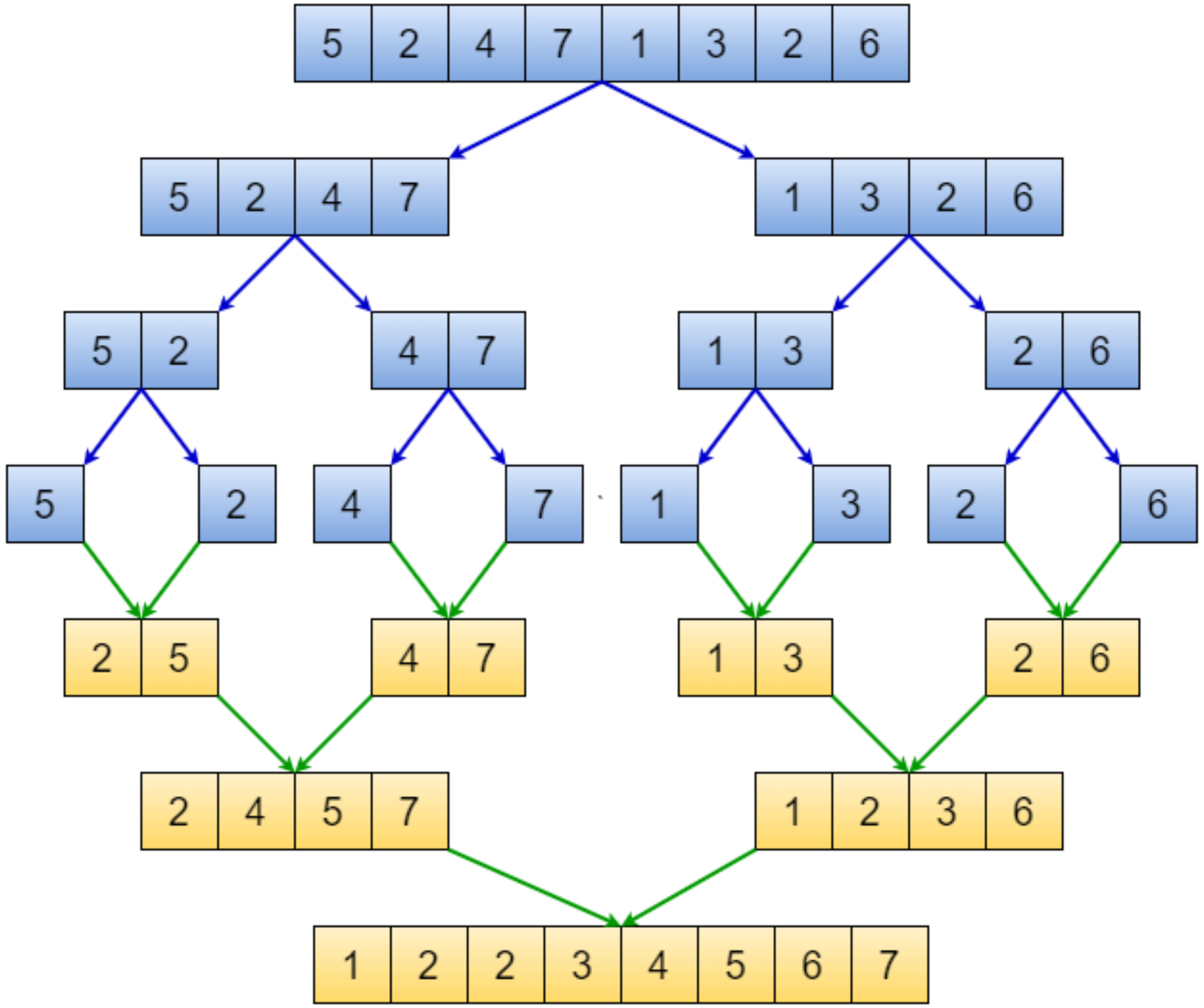
N: Input Size

Algorithm	Best	Average	Worst	Memory	Stable	Method	Notes
Quick-Sort	$N \log N$	$N \log N$	N^2	$\log N$	*No*	Partitioning	Easy to Implement
Heap-Sort	$N \log N$	$N \log N$	$N \log N$	In-place	*No*	Selection	
Intro-sort	$N \log N$	$N \log N$	$N \log N$	$\log N$	*No*	Partitioning & Selection	Used in C++ STL Containers
Merge-Sort	$N \log N$	$N \log N$	$N \log N$	N	*Yes*	Merging	Highly Parallelizable

Ref: https://en.wikipedia.org/wiki/Sorting_algorithm

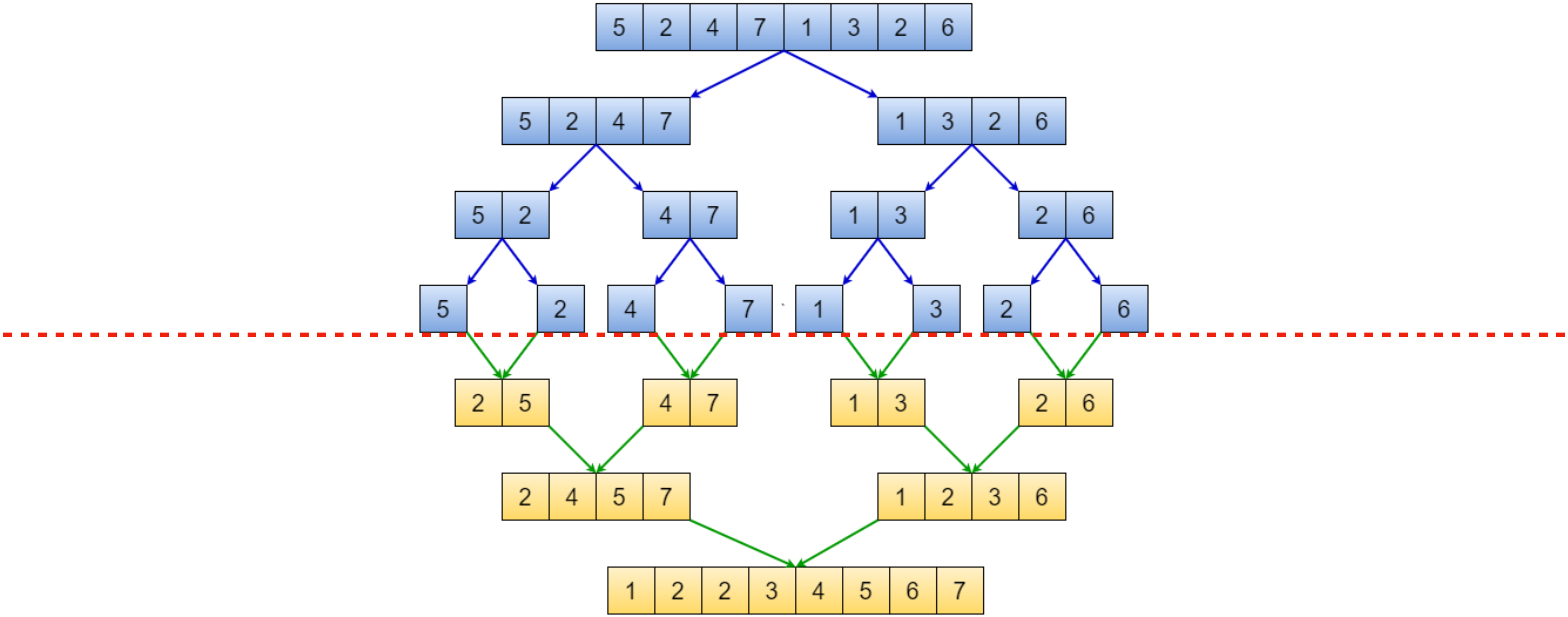
Merge-Sort Algorithm

Overview



Merge-Sort Algorithm

Overview



Merge-Sort Algorithm

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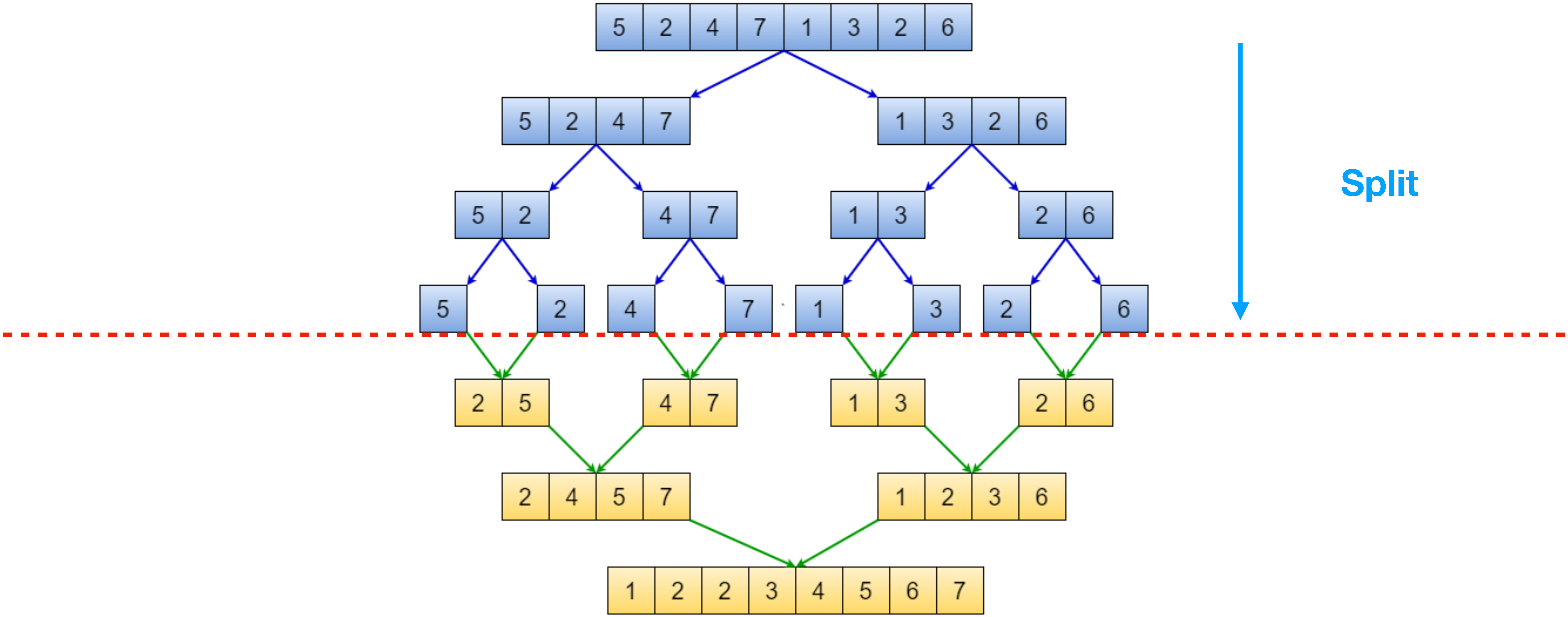

Merge-Sort Algorithm

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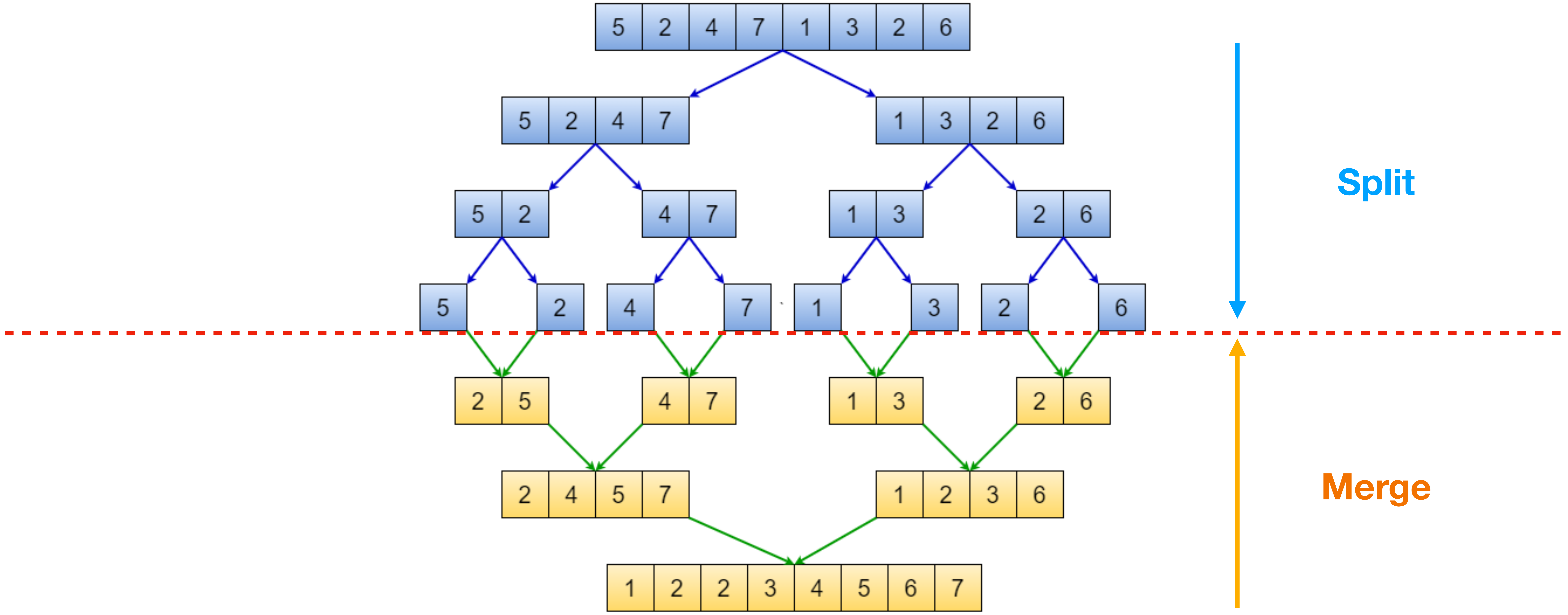

Merge-Sort Algorithm

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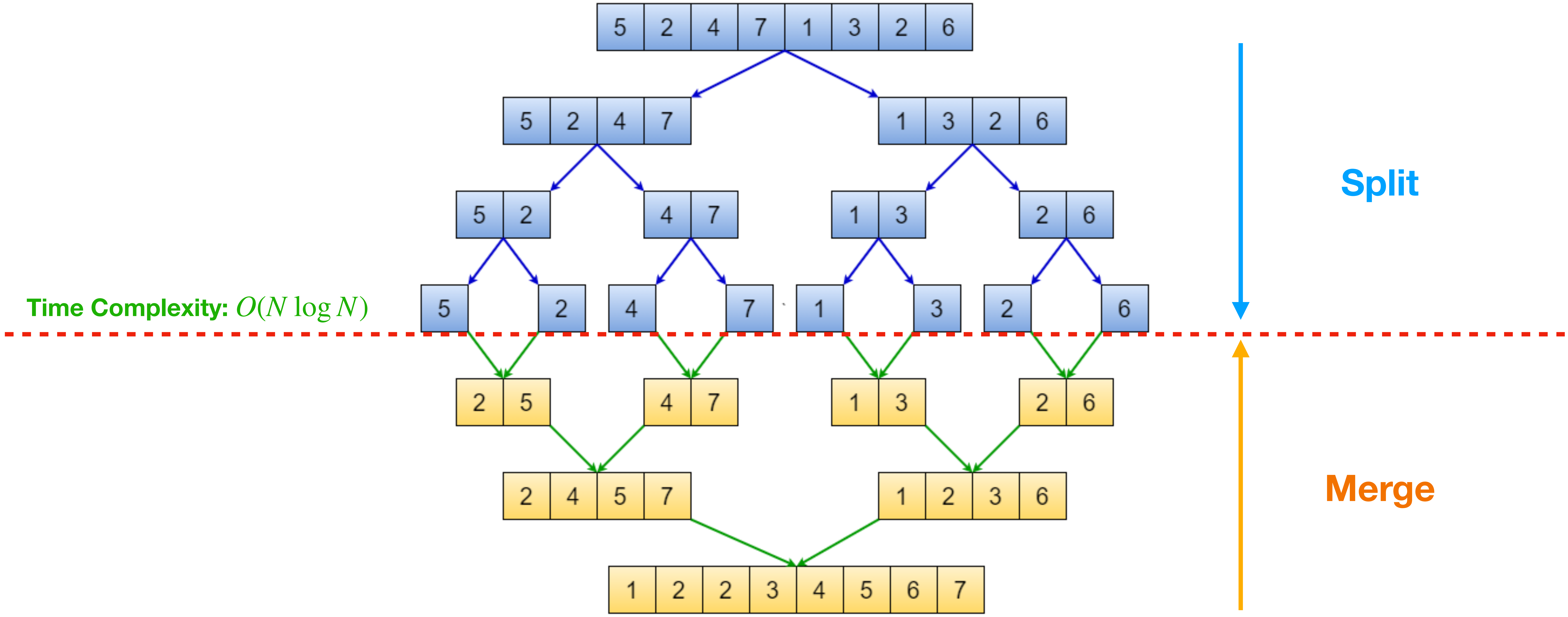
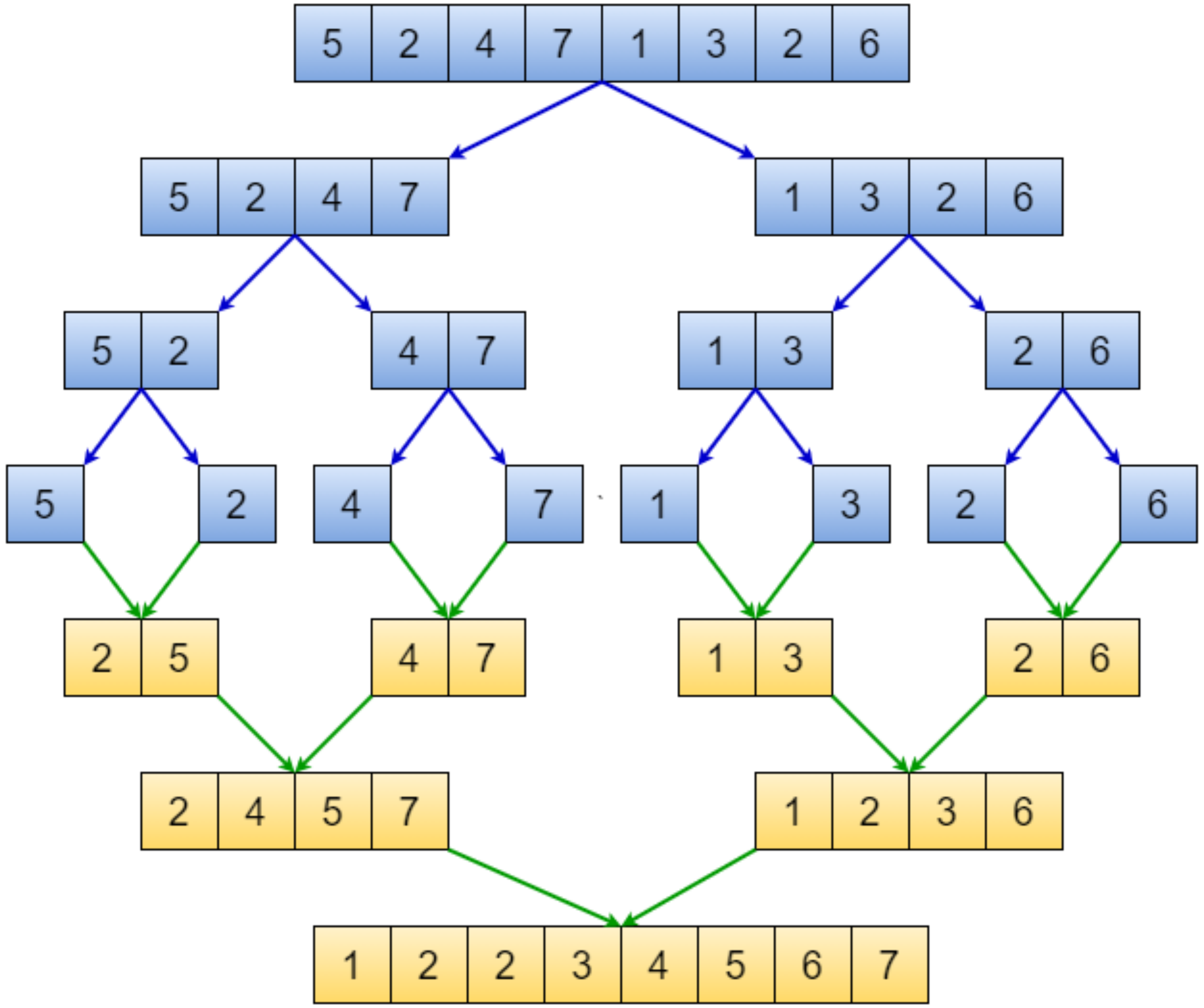


Parallel Merge-Sort Algorithm

Overview



Parallel Merge-Sort Algorithm

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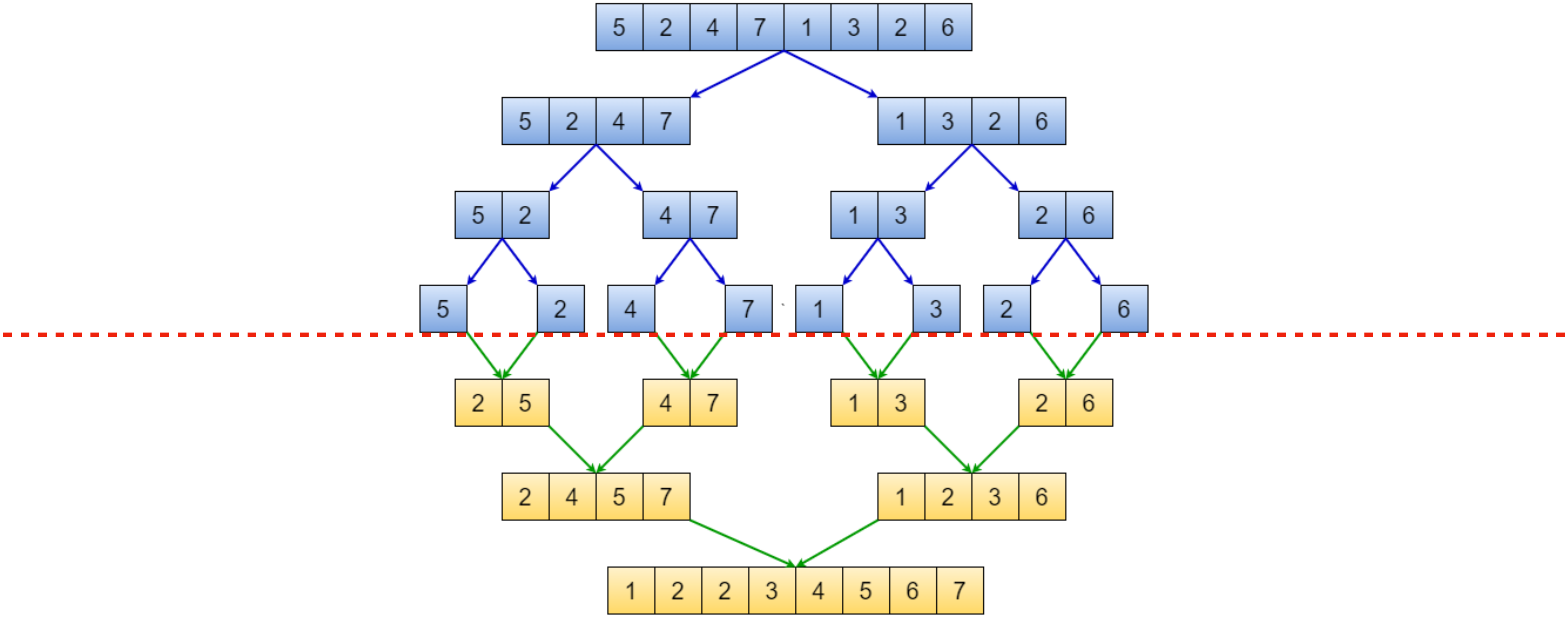

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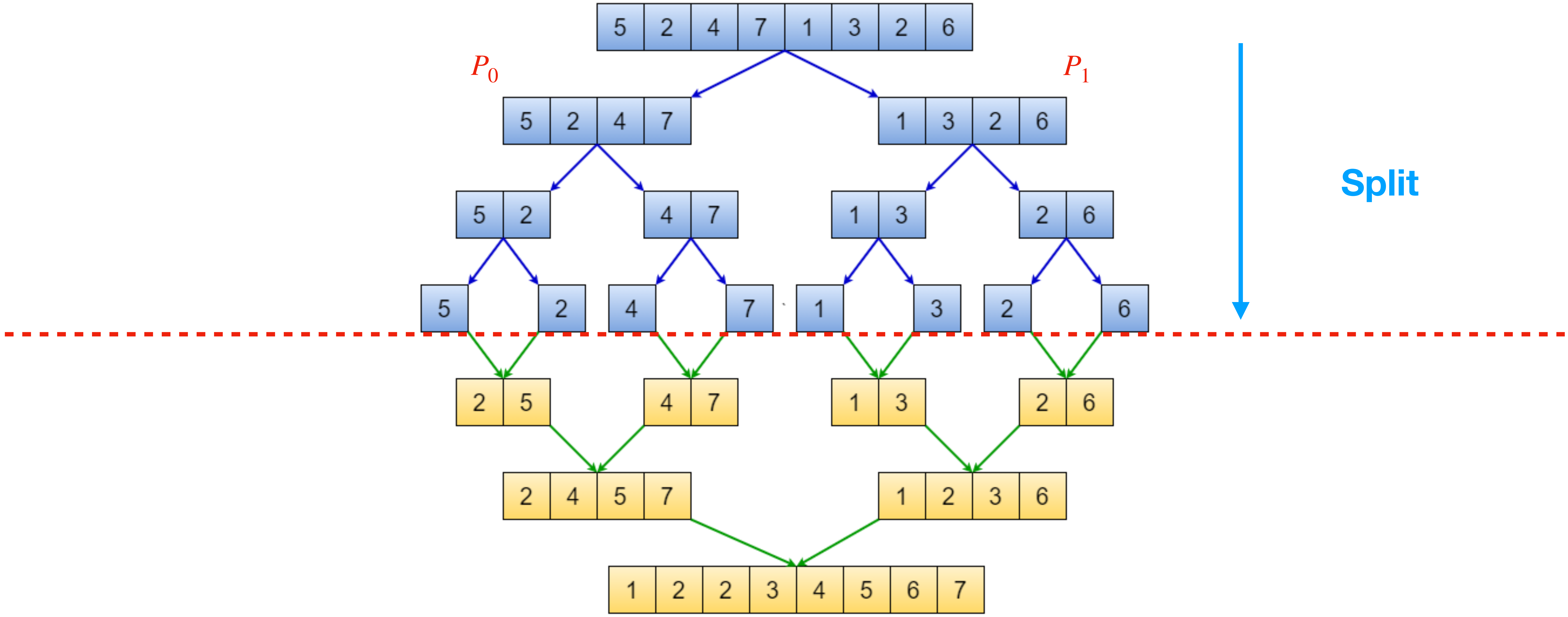

Parallel Merge-Sort Algorithm

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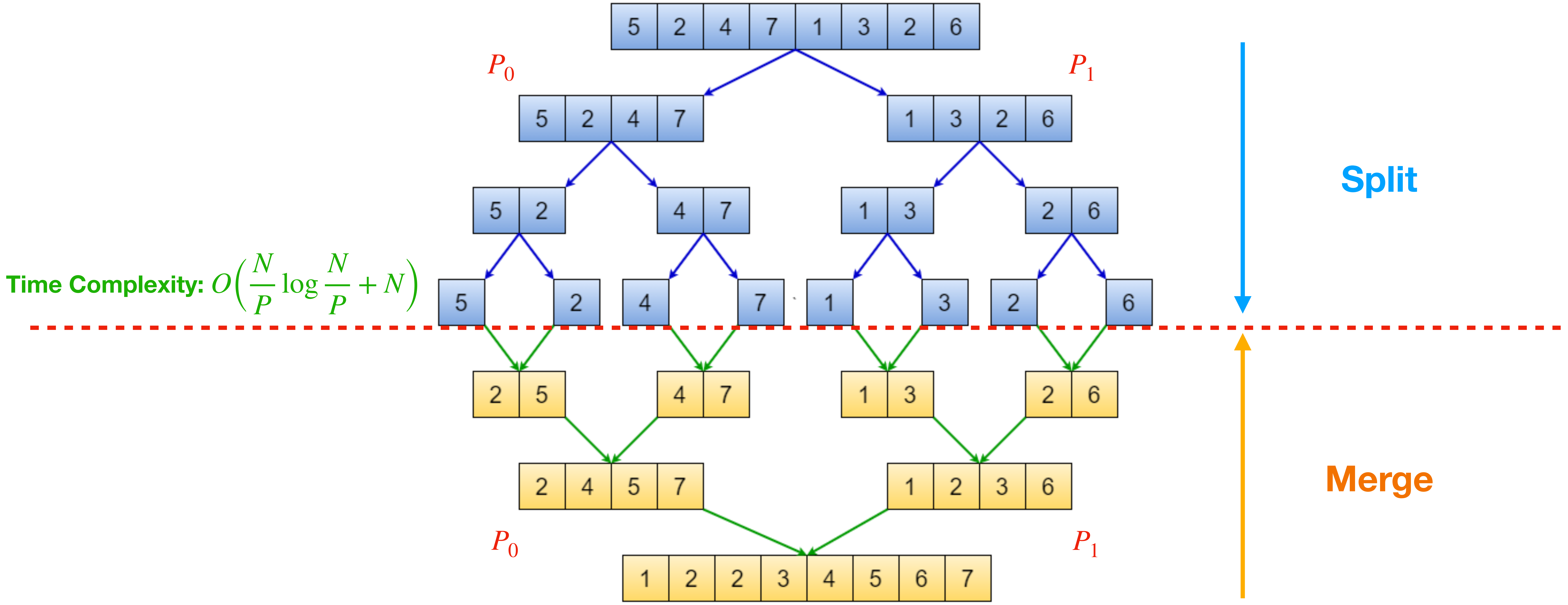


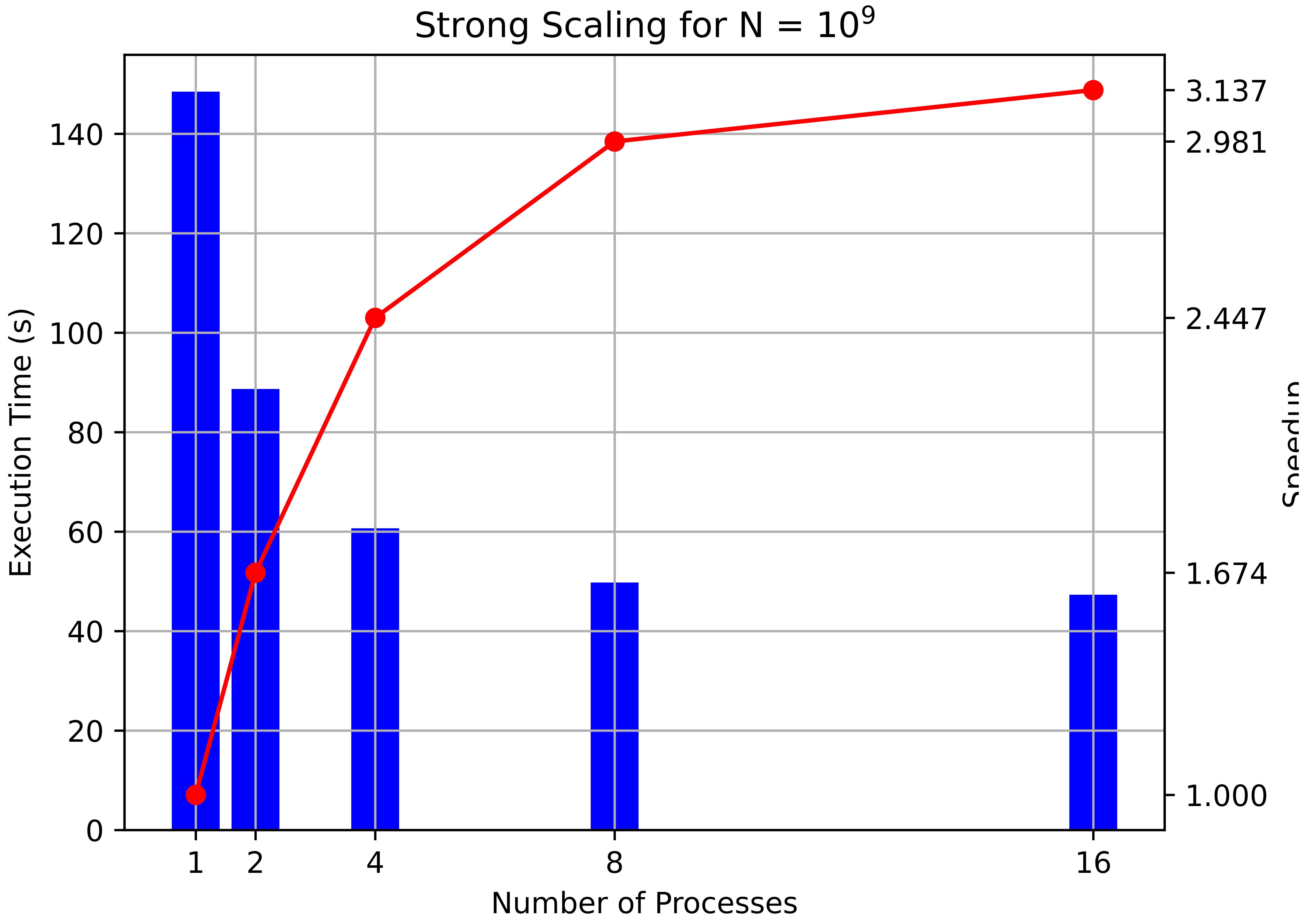
Merge-Sort Algorithm
Distributed Implementation

Live Demo!

Ref: https://github.com/racorretjer/Parallel-Merge-Sort-with-MPI

Merge-Sort Algorithm

Distributed Implementation



Parallel Merge-Sort Algorithm

Theoretical Analysis

Sequential Runtime: $$T(n) = 2T(\frac{n}{2}) + O(n) = O(n \log n)$$

Parallel Runtime:
(with sequential merge) $$T(n,p) = 2T(\frac{n}{2}, \frac{p}{2}) + O(n) = O(\frac{n}{p} \log \frac{n}{p} + (n + \frac{n}{2} + \frac{n}{4} \cdots \frac{n}{p}))$$

$$\implies T(n,p) = O(\frac{n}{p} \log \frac{n}{p} + n)$$

Upper bound on S (speedup): $$\lim_{p \mapsto \infty} \frac{O(n \log n)}{O(\frac{n}{p} \log \frac{n}{p}) + O(n)} = O(\log n)$$

Thank you!
धन्यवाद!

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