Photon Transfer Analysis

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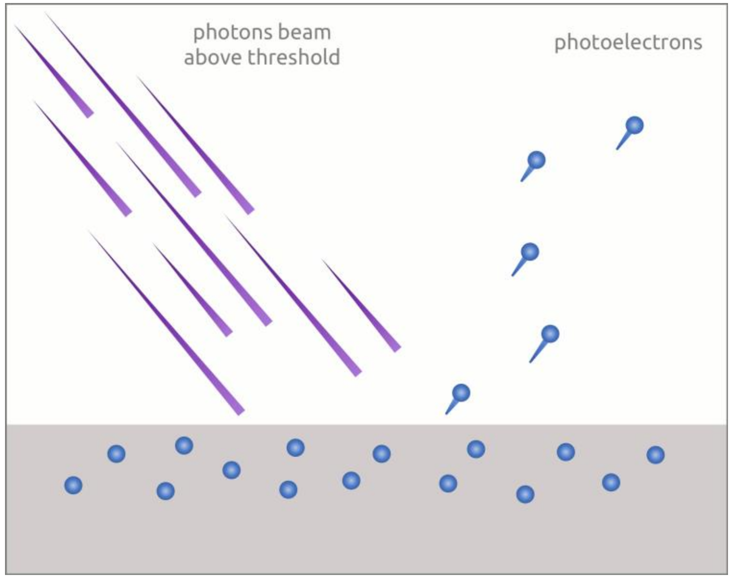
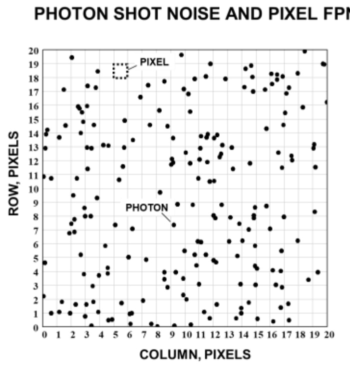
Outline

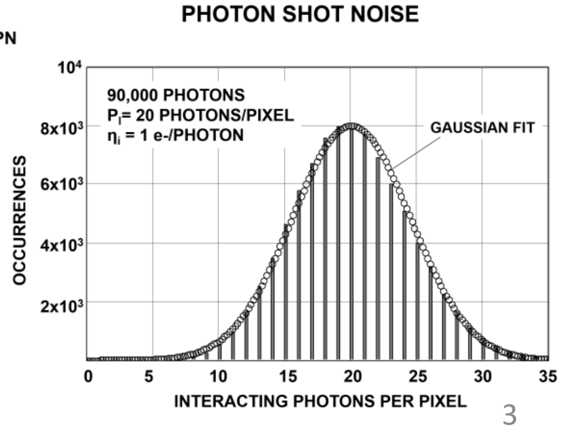
- Tech Backgrounder
- Noise components
- Photon Transfer Basics
- Photon Transfer Application Examples
    1) Flat Fielding & analysis
    2) RBI analysis
    3) Assessing System Noise Impact on # exposures, length of exposures and SNR
    4) Analyzing images with modulation/ optimizing SNR

Brief Technology Backgrounder

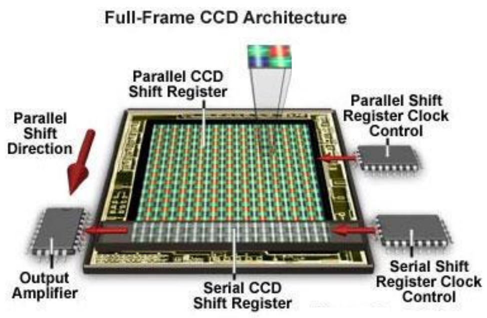


- What are we imaging?
  - Light is modeled as a stream of discrete photons
  - The camera takes exposures during a finite exposure length
- How do we detect the photons?
  - Photoelectric effect (Einstein 1921 Nobel Prize, Physics)
    - Light shining on conductor ejects electrons (charge)
    - The energy of the incident photons determines the # electrons liberated per Interacting Photon (“$\mathbf{P_i}$”), called **Quantum Yield** (“$\mathbf{n_i}$”)
      - For silicon:
        - Visible Light: Quantum Yield = 1 $\mathbf{n_i} = 1$
        - Xrays: Quantum Yield > 1 (Quantum Yield is variable and dependent on energy of Xray photon) $\mathbf{n_i}$ = E(xray)/3.65 [energy in electron-volts]
    - **Quantum** Efficiency (“**QE**”) is the measure of the number of interacting photons per incident photon
    - Some incident photons fail to interact with the silicon
      - Scattering
      - Optically dead structure in pixel
  - Photons arrive at random times, within the exposure
    - How many photons interact per pixel during the exposure?
      - This variability / uncertainty follows a Poisson Distribution
      - Called **Shot Noise** (“σ_s”)



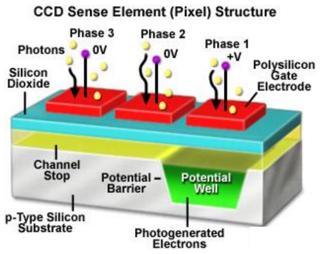


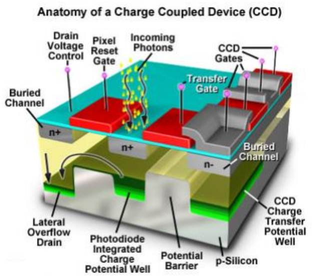
Brief Technology Backgrounder

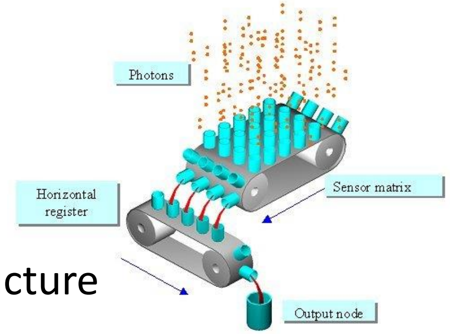


- Pixel Properties
  - The silicon chip has an array of electrically isolated regions used for collecting the charge liberated by light (Pixels)
    - The **Full Well** charge capacity of a pixel is related to the area of the pixel
    - Each pixel may have slightly different sensitivity, the variation of sensitivity leads to **Fixed Pattern Noise** (“**FPN**”)
  - Real pixels also have charge leakage into the pixels: with no light applied, thermally generated charge accumulates in the pixel
    - This adds **Dark Shot Noise** (“σ_D”)
    - It also adds **Dark Fixed Pattern Noise** (“**DFPN**”)

Brief Technology Backgrounder







- Measuring charge
  - The charge from each pixel is measured using an amplifier
  - In a CCD there are only a small number of amplifiers and the charge must be moved from the pixel to the amplifier
    - Moving charge is done using the CCD structure and clocking through the serial structure
    - Moving the charge is a NOISELESS process
  - The charge measurement amplifier is a MOSFET transistor connected as a source-follower
    - The charge is applied to the gate (looks like a capacitor: “converts” charge to a voltage on the gate)
    - The output voltage of the source-follower amplifier “tracks” the gate voltage of the MOSFET
    - The smaller the gate area (capacitance) the larger the voltage for a given amount of charge
    - This plays a role setting the **GAIN** of the camera
  - CMOS image sensors have an amplifier per pixel or for a small cluster of pixels so there is no charge transfer through a long serial structure
    - This is the major difference in a CMOS vs CCD image sensor
  - The source follower amplifier is also a noise source for both types of sensors
    - It’s the key contributor to **Read Noise** (“σ_R”)

Photon Transfer Basics

Noise

- Image noise sources
- Noise Equation
- Graphical Representation

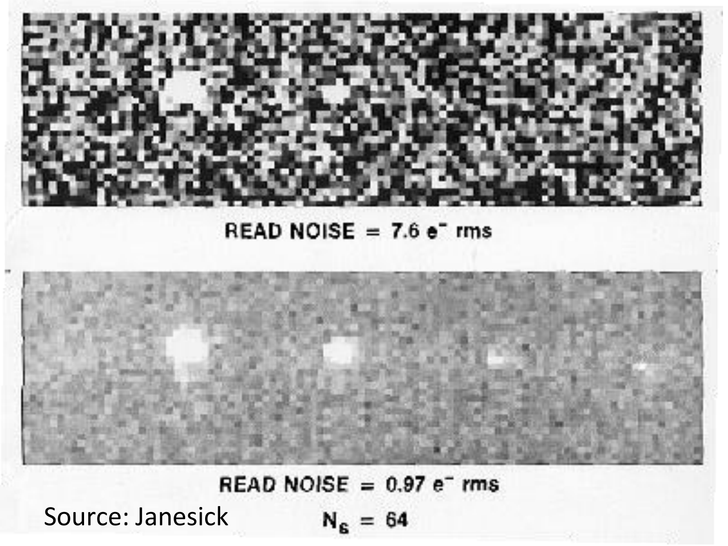
Image Noise Sources

- For an unmodulated image (flat field image), the key noise sources* are
  - Read noise
  - Signal Shot Noise (aka: photon noise, photon shot noise: it will be called Shot Noise or Signal Shot Noise for the remainder of this document)
  - Fixed Pattern Noise
- Depending on signal level any of them can dominate the noise in a single image frame

*neglecting dark signal noise sources which can be managed by cooling

Read Noise

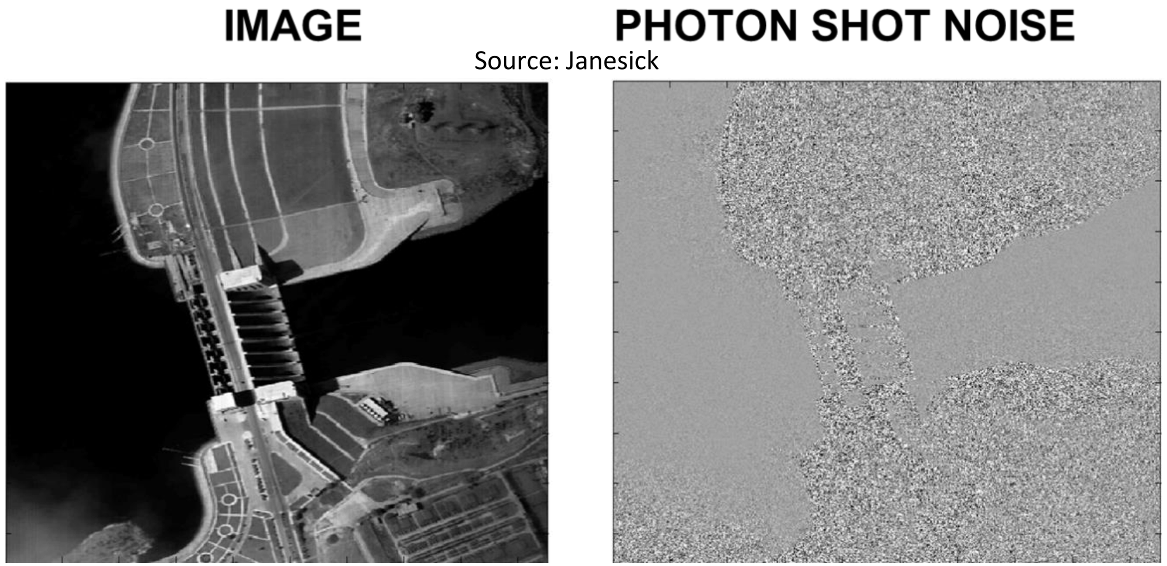
- The read noise is the noise observed in an image frame when no signal is present
- The noise in a bias frame approximates the read noise
  - zero length exposure
  - no light applied
  - bias frame noise is always greater than read noise due to dark signal accumulation during finite readout time
- Read noise can obliterate faint signals



Source: Janesick

Shot Noise

- The discrete nature of photons and their random arrival time causes a variation in the number of interacting photons from pixel to pixel and frame to frame.
- The variation is the cause of photon shot noise or shot noise as it is also known
- The more intense the image, the greater is the shot noise
- Shot noise is inherent in the image and cannot be avoided and represents the noise floor
- Shot noise in a final image can be eliminated by combining multiple images

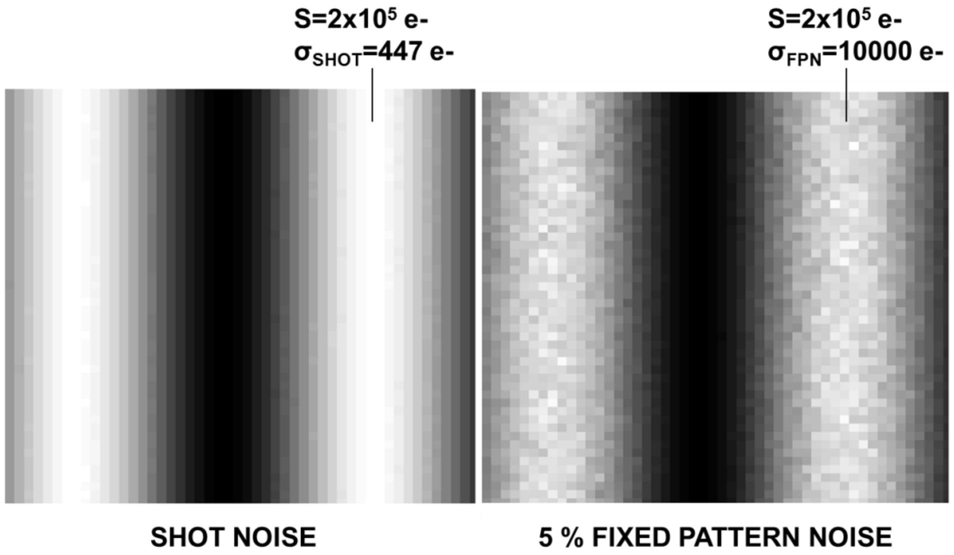


Source: Janesick

Fixed Pattern Noise

- For a flat field exposure, any modulation observed that remains constant from frame to frame is Fixed Pattern Noise (FPN)
- For perfect flat-field illumination of the sensor the FPN observed is caused by variations in the photoresponse of each pixel. This represents the floor of the FPN of the system
- For the camera installed on a practical optical imaging system, variations of light intensity are generally observed
  - Non-uniform light intensity across the frame (ie, “hot centers” or vignetting)
  - Dust motes
  - Filter transmission variations
- These Optical FPN components add to the FPN inherent in the sensor and frequently dominate the overall FPN of the system
- Once FPN dominates the noise of the image, collecting additional signal does not improve the Signal to Noise Ratio (SNR). FPN places an upper limit on the SNR of the system unless removed
- FPN is removed via Flat Fielding (explained later)

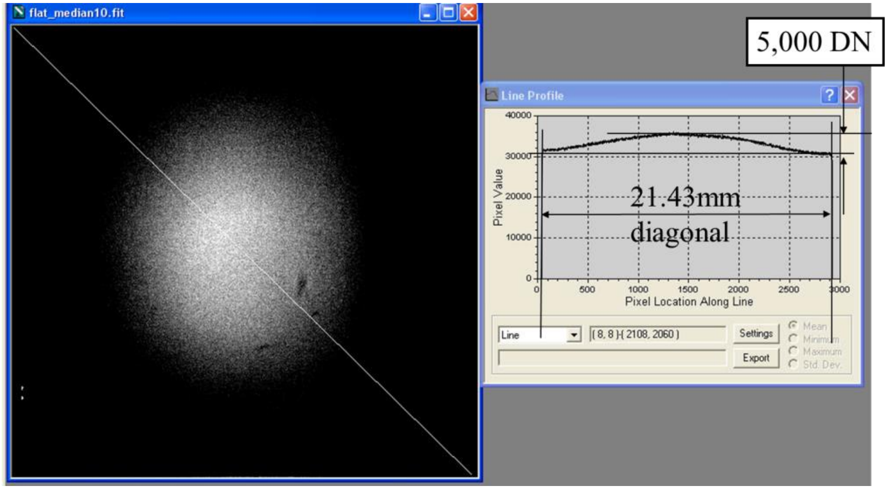
Examples of Fixed Pattern Noise



Source: Janesick

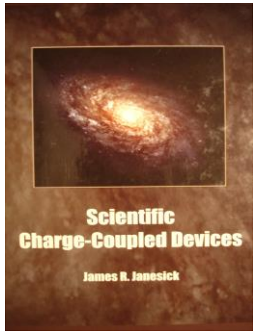
Sensor FPN

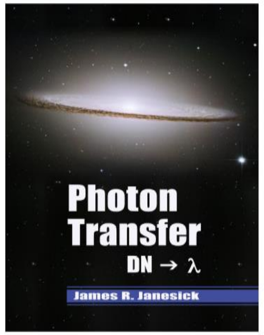
Optical FPN



Photon Transfer Characterization

- Developed at JPL by Jim Janesick & Tom Elliott for quantitatively measuring the performance of electronic imaging systems
- Graphical Technique: Plot noise components against signal for basic chart
- Can use with Empirical or Modeled datasets
- Basic Technique can accurately measure: Read Noise, Full Well, Gain, Linearity, Photo Response Non-Uniformity & Dark Signal Non-Uniformity





Buy these books (SPIE Press)

Noise Equation

- To quantitatively analyze noise it must be described mathematically
- When combining the effects of multiple noise sources that are uncorrelated, quadrature summation is used (square root of the sum of the squares of the noise from each separate source)

Noise Equation*

$$Total\_Noise = \sqrt{(read\_noise)^2 + (signal\_shot\_noise)^2 + (fixed\_pattern\_noise)^2}$$

- Assumptions:
  - Flat field target: no modulation
  - Dark signal sources are negligible (cooling)

Noise Equation Cont'd

$$Total\_Noise = \sqrt{read\_noise^2 + signal\_shot\_noise^2 + fixed\_pattern\_noise^2}$$

recognizing:

$$Signal\_Shot\_Noise = \sqrt{signal}$$

$$Fixed\_pattern\_noise = Signal \times PRNU$$

PRNU is Photo-Response-Non-Uniformity
(this will be covered in depth later)

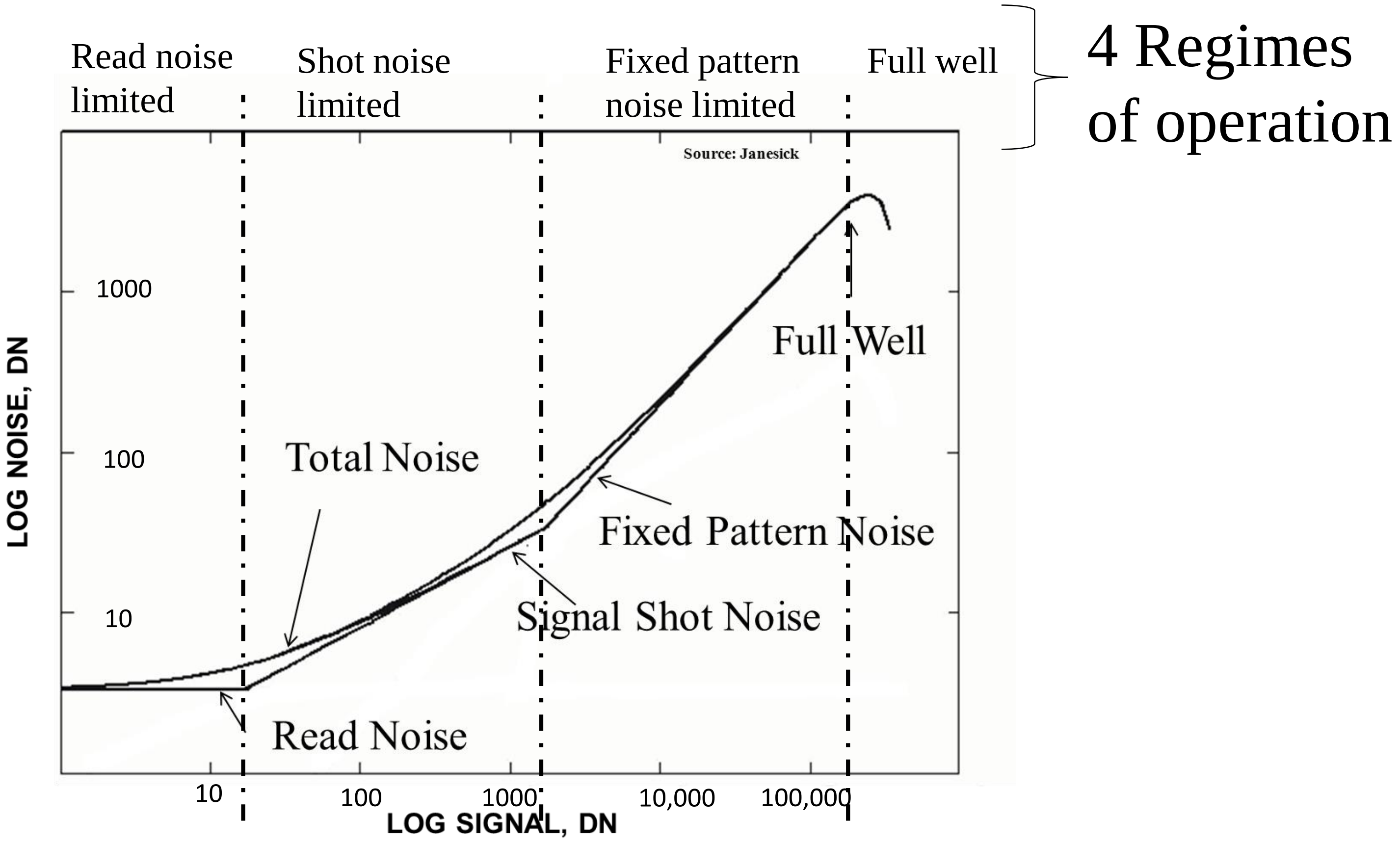
we get:

$$\underline{Total\_Noise = \sqrt{read\_noise^2 + signal + (signal \times PRNU)^2}}$$

Graphical Representations

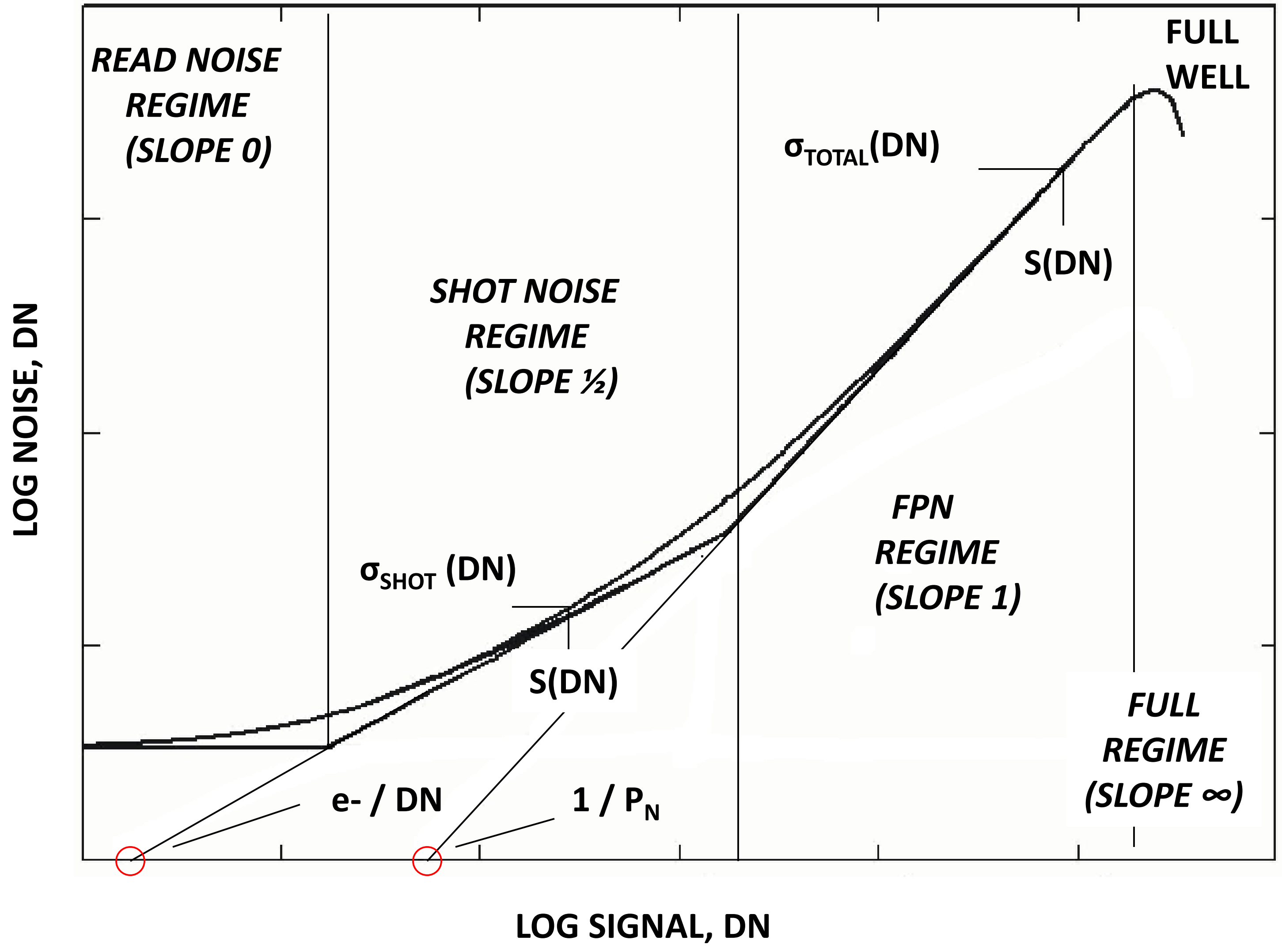
- The noise performance of electronic imaging systems is commonly analyzed using graphical techniques
- Noise is plotted on the Y axis with Signal level plotted on the X axis
- Because the noise and signal may range over several orders of magnitude, it is convenient to use logarithmic axes for the plots to accommodate the large range of data extent

Noise Versus Signal



$$Total\_Noise = \sqrt{read\_noise^2 + signal + (signal \times PRNU)^2}$$

PHOTON TRANSFER CURVE
(TOTAL NOISE)



e-/DN = S(DN) / (σ_{SHOT}(DN))^2

P_N= σ_{FPN} / S(DN)

PHOTON TRANSFER NOISE REGIMES

Total noise (e^-) = ($\sigma_{READ}{}^2 + \sigma_{SHOT}{}^2 + \sigma_{FPN}{}^2$)$^{1/2}$

σ_{READ} = Read noise (rms e^-)

σ_{SHOT} = Shot noise (rms e^-)

= (Signal (e^-))$^{1/2}$

All Gaussian Distributed

σ_{FPN} = Fixed Pattern Noise (rms e^-)

= P_N x Signal (e-)

P_N = Pixel nonuniformity factor

= Noise / Signal

SIGNAL-TO-NOISE REGIMES

READ NOISE REGIME

S/N= SIGNAL / READ NOISE

SHOT NOISE REGIME

S/N = SIGNAL / SHOT NOISE = SIGNAL / SIGNAL$^{1/2}$ = SIGNAL$^{1/2}$

FIXED PATTERN NOISE (FPN) REGIME

S/N = SIGNAL / FPN = 1 / P_N

ONSET OF FPN (e^-) = 1 / P_N^2 CONSTANT !

(APPROXIMATELY 10,000 e-)

TOTAL SIGNAL TO NOISE

$$S/N = S / (\sigma_{READ}^2 + \sigma_{SHOT} + \sigma_{FPN}^2)^{1/2}$$

$$= S(e\text{-}) / (\sigma_{READ}^2 + S(e\text{-}) + (S(e\text{-})P_N)^2)^{1/2}$$

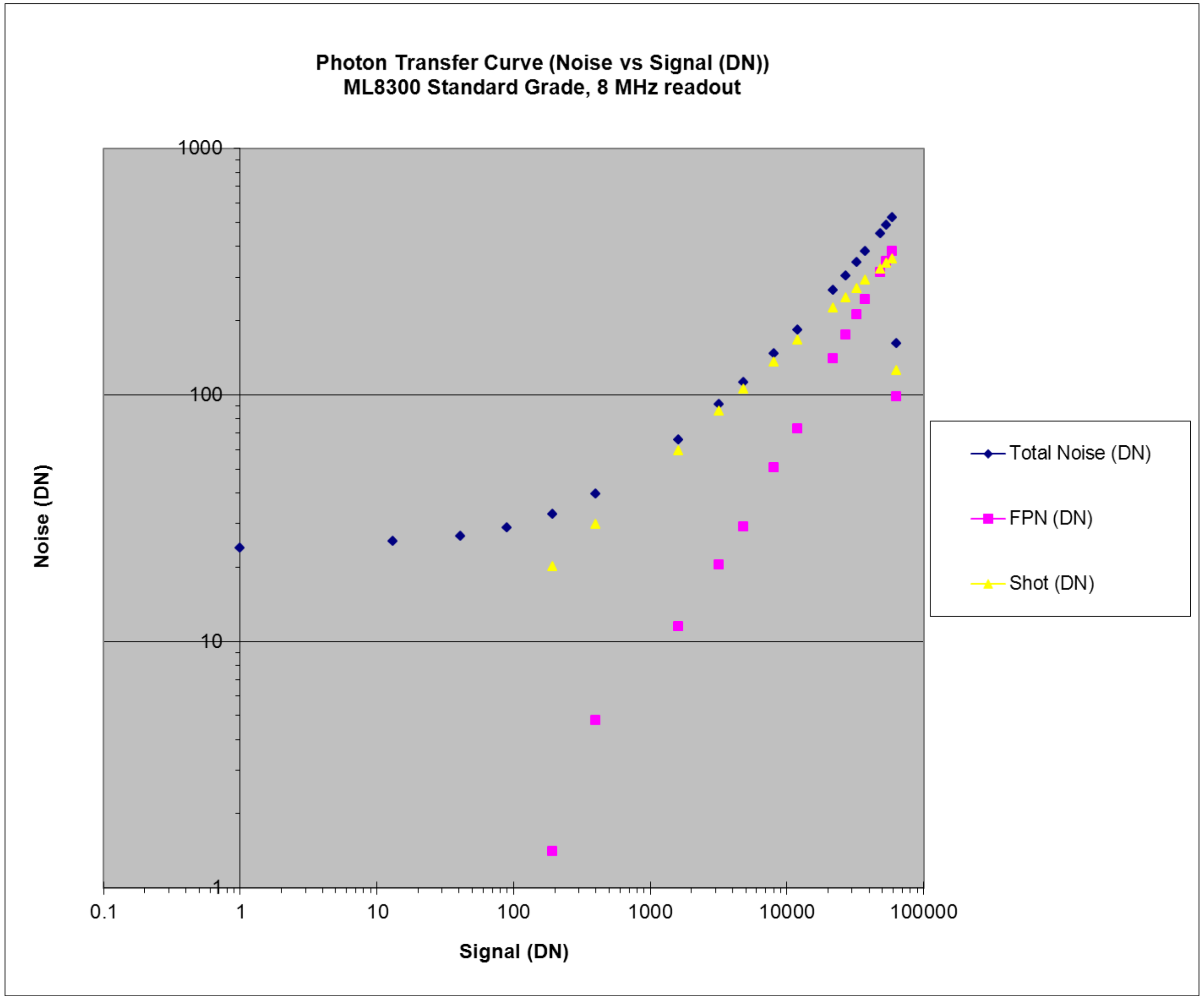
Photon Transfer Analysis Basic Concepts (tools we will use)

- Basic PTC and what we learn from it
- How to make a PTC
- Common PTC errors and how to diagnose and fix them
- Other types of PTCs

Basic PTC and what we learn from it

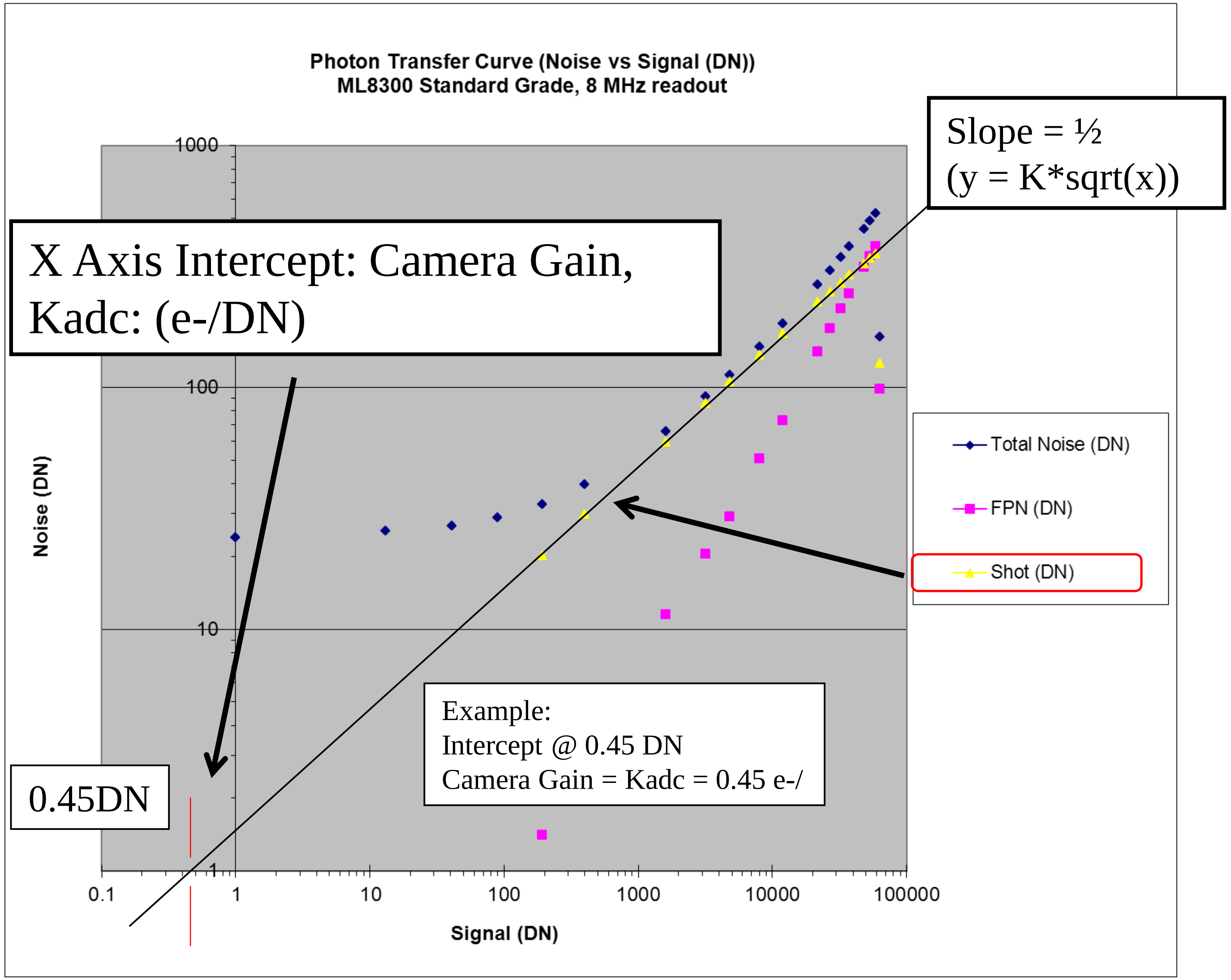
- A basic Photon Transfer Curve ("PTC") is a graph of Noise versus Signal measured from a collection of identical pairs of flat-field images of varying signal levels. Typically the noise parameters plotted are Total Noise, Fixed Pattern Noise and Shot Noise
- The signal level of the source images used to make the graph span the range from very low signal level to full well
- Each signal level captured is used as a data point for making the graph
- Once the data is plotted in graphical form we can graphically measure
  - Full well
  - Read Noise
  - Camera Gain ("Kadc")
  - PRNU
- We can supplement the PTC with dark frame data and learn the DSNU (Dark Signal Non Uniformity: a measure of how noisy the chip is to assist you in establishing a proper operating temperature: ie using an engineering grade sensor without suffering from excessive noise
- We can also make a separate chart to track linearity by plotting Kadc vs Signal using the same dataset

Example PTC

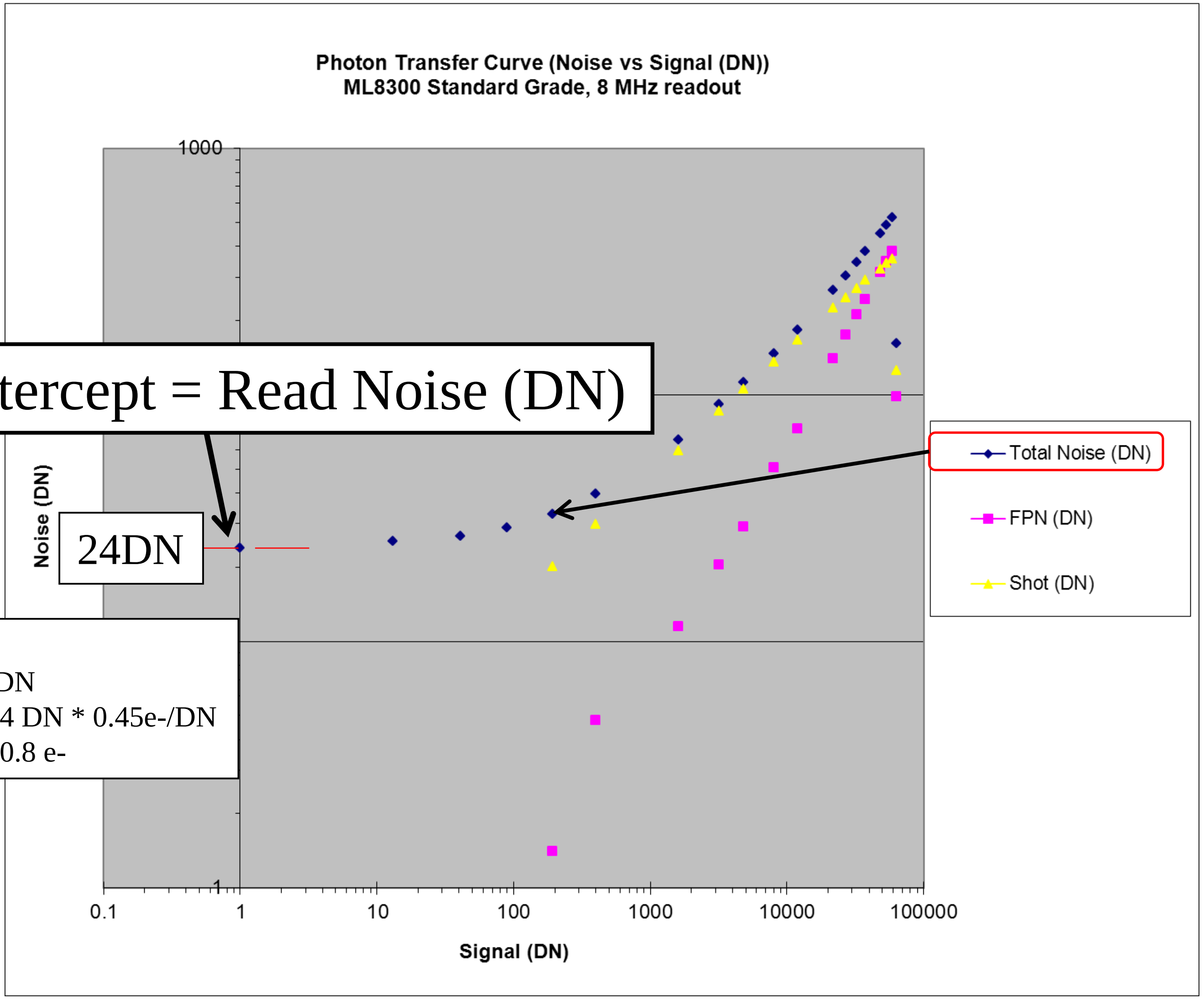


Photon Transfer Curve (Noise vs Signal (DN))
ML8300 Standard Grade, 8 MHz readout
1000
100
10
1
Noise (DN)
0.1
1
10
100
1000
10000
100000
Signal (DN)
Total Noise (DN)
FPN (DN)
Shot (DN)

Example PTC: Measuring Gain



Example PTC: Measuring Read Noise

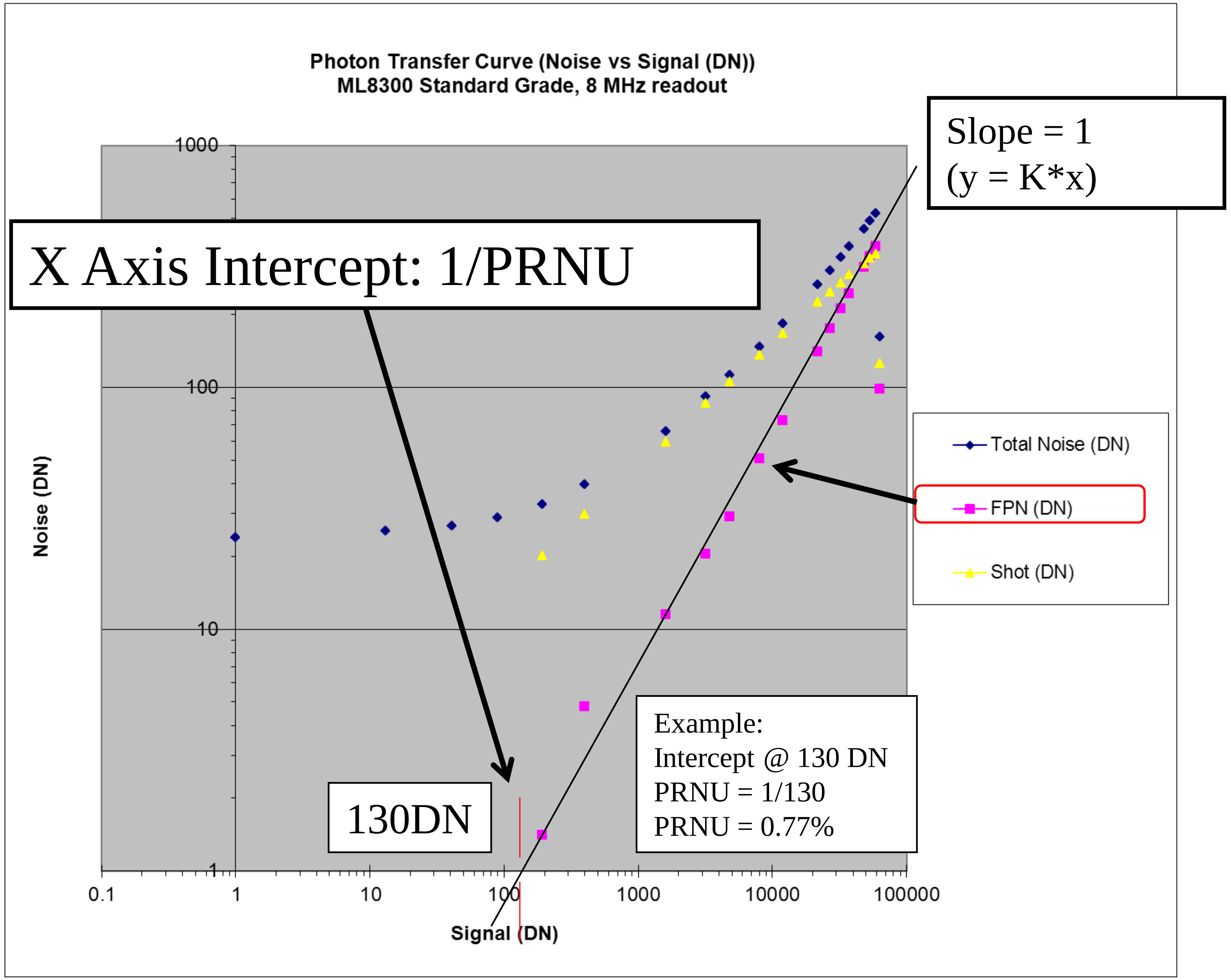


Y axis intercept = Read Noise (DN)

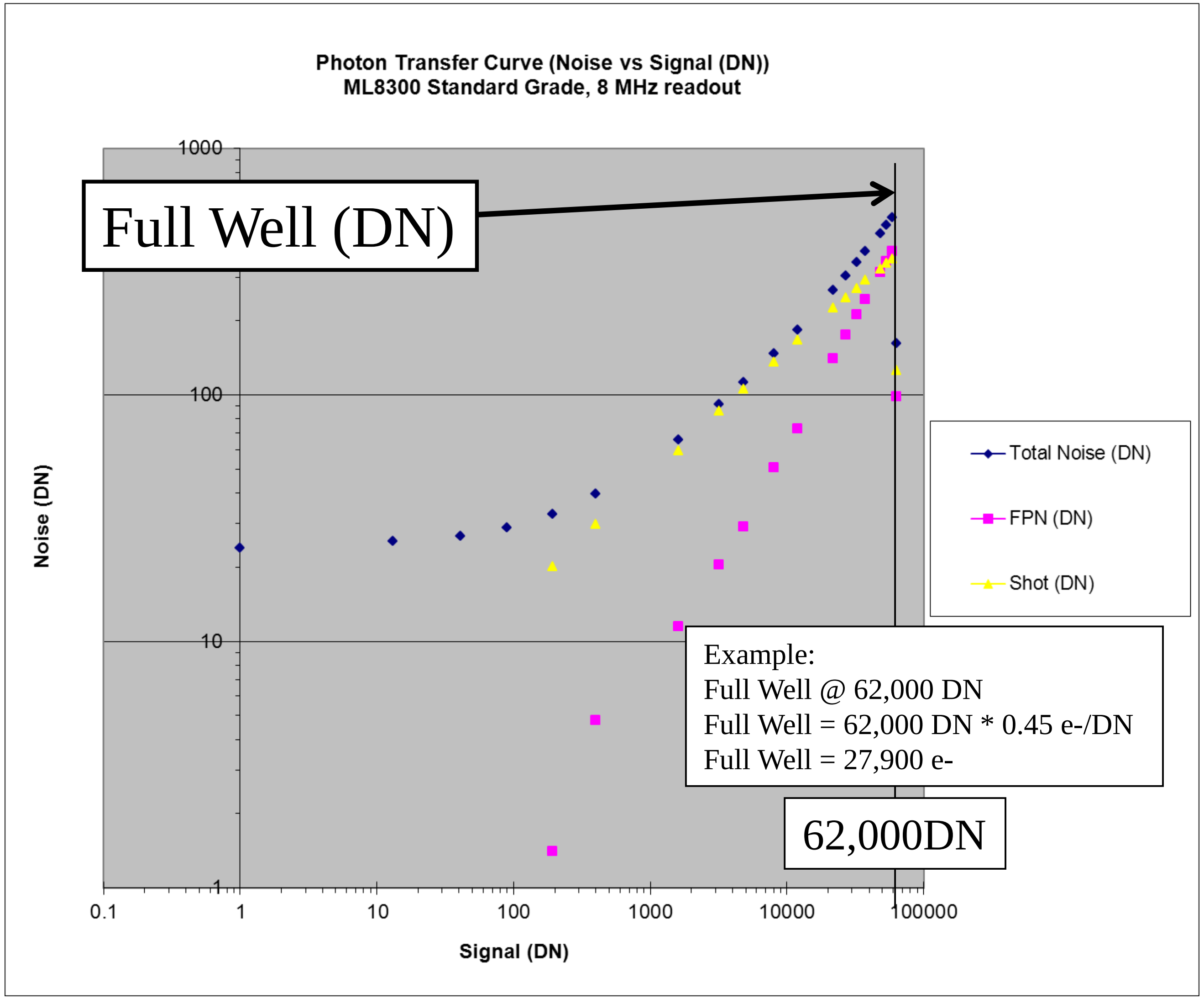
24DN

Example:
Intercept @ 24DN
Read Noise = 24 DN * 0.45e-/DN
Read Noise = 10.8 e-

Example PTC: Measuring PRNU



Example PTC: Measuring Full Well



How to make a PTC

- Data Collection:
  - Operate the camera at -25C for the collecting the data below so that dark signal is negligible
  - Collect pairs of identical flat field exposures of varying intensity: ranging from nearly dark to fully saturated. You can do this inside a house with the bare camera looking at a ceiling in a dimly lit room (preferably adjustable light intensity level)
  - Take one bias frame
  - Usually about 16-20 flat pairs is sufficient

Reducing the data

- Using Excel or some other spreadsheet program label several columns for recording data
  - Raw Signal, Offset, Standard Deviation, Delta Standard Deviation, Signal – Offset, Average Signal –Offset, Total Noise, Shot + Read, Read(DN), FPN, Shot, Offset Correction

- Recording measurements from the collected data (this is where it gets tedious)
  - Pick analysis region to use for all data: 100 x 100 yields accuracy of 1% (1/sqrt(#pixels) = accuracy)
  - Crop the bias frame using the size/location chosen above and measure the average signal level and record that into the OFFSET column in the spreadsheet
  - Crop each image within a flat field pair to the analysis region size/location
  - Using the spreadsheet program record the average value of each cropped frame into the Average Signal column and the standard deviation into the Standard Deviation column (measure this using Maxim DL's "Information" window in "Area" mode)
  - Using Pixel Math in Maxim, subtract one cropped region from the other while adding a fixed value of 5000DN to the minuend (to prevent negative numbers).
    - Record the standard deviation of the difference into the Delta Standard Deviation column in the spreadsheet
  - Repeat for each pair of flats

Spreadsheet View

signal	offset	std_dev	delta-std	signal-offset	average_signal-offset	total noise	shot+rd	read(DN)	FPN	sig_shot	offset_correction
2088	2088	24.147	33.853								
2087	2087	23.922									
2097	2088	24.924	35.366								
2097	2088	24.775									
2111	2088	25.521	36.174								
2111	2088	25.613									
2139	2088	26.506	37.678								
2139	2088	26.884									
2188	2088	28.88	40.866								
2188	2088	28.91									
2292	2088	32.776	46.31								
2292	2088	32.777									
2496	2088	39.729	55.851								
2495	2088	39.833									
3703	2088	65.913	91.78								
3703	2088	65.9									
5305	2088	91.551	126.564								
5304	2088	92.031									
6908	2088	112.26	153.468								
6907	2088	112.476									
10111	2088	147.108	195.242								
10109	2088	147.151									
14107	2088	184.69	239.094								
14102	2088	183.669									
23754	2088	266.283	319.866								
23752	2088	266.137									
29119	2088	304.443	352.58								
29110	2088	304.917									
34466	2088	343.677	383.58								
34465	2088	345.222									
39792	2088	384.786	416.567								
39795	2088	381.22									
50381	2088	454.617	461.335								
50382	2088	450.876									
55626	2088	486.514	485.382								
55621	2088	490.762									
60840	2088	522.294	505.087								
60854	2088	526.425									
65477	2088	160.53	181.48								
65476	2088	162.704									

Filling in the Equations

- The next task is to create equations for the remaining columns
- First fill in the value zero into the Offset Correction column. This value may need to be changed after plotting the data
- Use the following equations for the remaining columns (see the next page)

Equations

- Signal – Offset:

  = (signal – offset – offset correction)

signal	offset	std_dev	delta-std	signal-offset	average_signal-offset	total noise	shot+rd	read(DN)	FPN	sig_shot	offset_correction
2088	2088	24.147	33.853								0
2087	2087	23.922									0
2097	2088	24.924	35.366								0
2097	2088	24.775									0
2111	2088	25.621	38.174								

Equations

- Average Signal – Offset:

  = AVG(signal – offset (n), signal – offset (n+1))

signal	offset	std_dev	delta-std	signal-offset	average_signal-offset	total noise	shot+rd	read(DN)	FPN	sig_shot	offset_correction
2088	2088	24.147	33.853								0
2087	2087	23.922									0
2097	2088	24.924	35.366								0
2097	2088	24.775									0
2111	2088	25.521	38.174								

Equations

- Total Noise:

  = AVG(std_dev(n), std_dev(n+1))

signal	offset	std_dev	delta-std	signal-offset	average_signal-offset	total noise	shot+rd	read(DN)	FPN	sig_shot	offset_correction
2088	2088	24.147	33.853								0
2087	2087	23.922									0
2097	2088	24.924	35.366								0
2097	2088	24.775									0
[illegible]	[illegible]	[illegible]	[illegible]								

Equations

- Shot + Rd:

  = delta-std / sqrt(2)

signal	offset	std_dev	delta-std	signal-offset	average_signal-offset	total noise	shot+rd	read(DN)	FPN	sig_shot	offset_correction
2088	2088	24.147	33.853								0
2087	2087	23.922									0
2097	2088	24.924	35.366								0
2097	2088	24.775									0
2111	2088	25.621	36.174								

Equations

- read:

  Write in the standard deviation of the cropped bias frame

signal	offset	std_dev	delta-std	signal-offset	average_signal-offset	total noise	shot+rd	read(DN)	FPN	sig_shot	offset_correction
2088	2088	24.147	33.853								0
2087	2087	23.922									0
2097	2088	24.924	35.366								0
2097	2088	24.775									0

= Read (n-1) value
(recursive from previous row)

The value of the read noise will likely be adjusted later to correct a common PTC error

Equations

- FPN:

  =SQRT(Total Noise^2 – Read+Shot^2)

signal	offset	std_dev	delta-std	signal-offset	average_signal-offset	total noise	shot+rd	read(DN)	FPN	sig_shot	offset_correction
2088	2088	24.147	33.853								0
2087	2087	23.922									0
2097	2088	24.924	35.366								0
2097	2088	24.775									0
[illegible]	[illegible]	[illegible]	[illegible]								

Equations

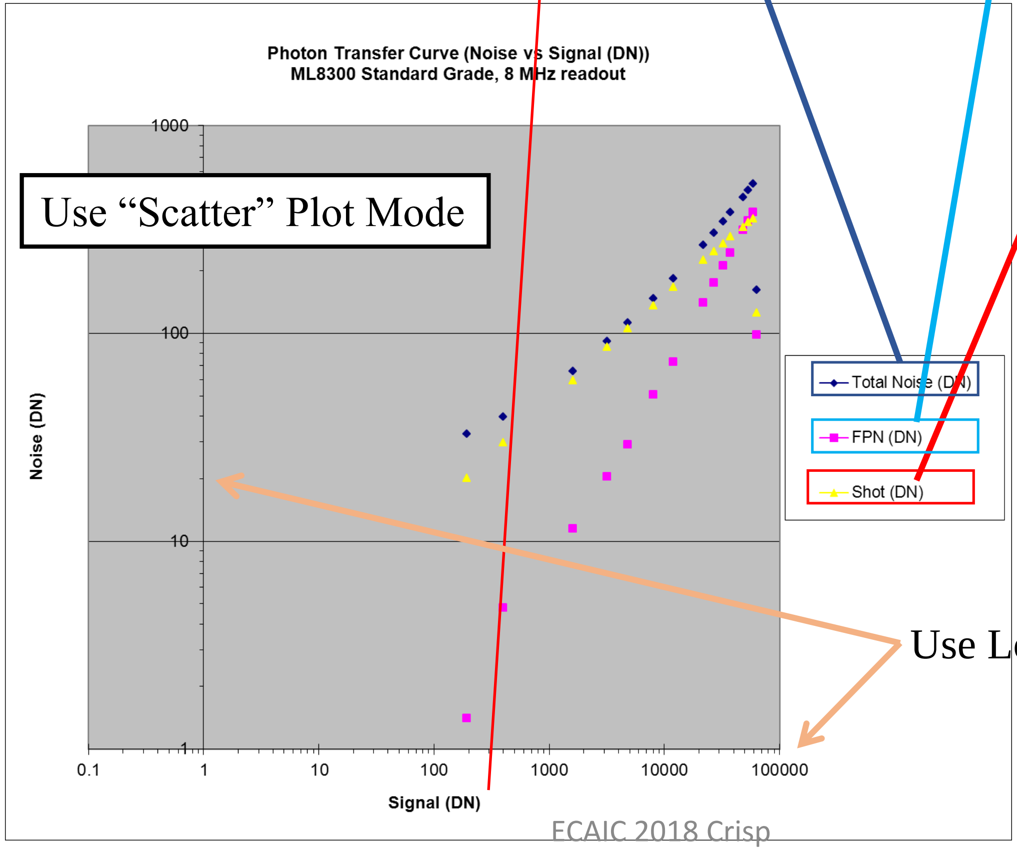
- Sig Shot:

  =SQRT(Shot+Read^2 – Read^2)

signal	offset	std_dev	delta-std	signal-offset	average_signal-offset	total noise	shot+rd	read(DN)	FPN	sig_shot	offset_correction
2088	2088	24.147	33.853								0
2087	2087	23.922									0
2097	2088	24.924	35.366								0
2097	2088	24.775									0
2111	2088	25.521	36.174								

Result

signal	offset	std_dev	delta-std	signal-offset	average_signal-offset	total noise	shot+rd	read(DN)	FPN	sig_shot	offset_correction
2292	2088	32.776	46.31	194	194	32.7765	32.74612	25.8	1.410993	20.16601	10
2292	2088	32.777		194							10
2496	2088	39.729	55.851	398	397.5	39.781	39.49262	25.8	4.781303	29.90029	10
2495	2088	39.833		397							10
3703	2088	65.913	91.78	1605	1605	65.9065	64.89826	25.8	11.48401	59.54951	10
3703	2088	65.9		1605							10

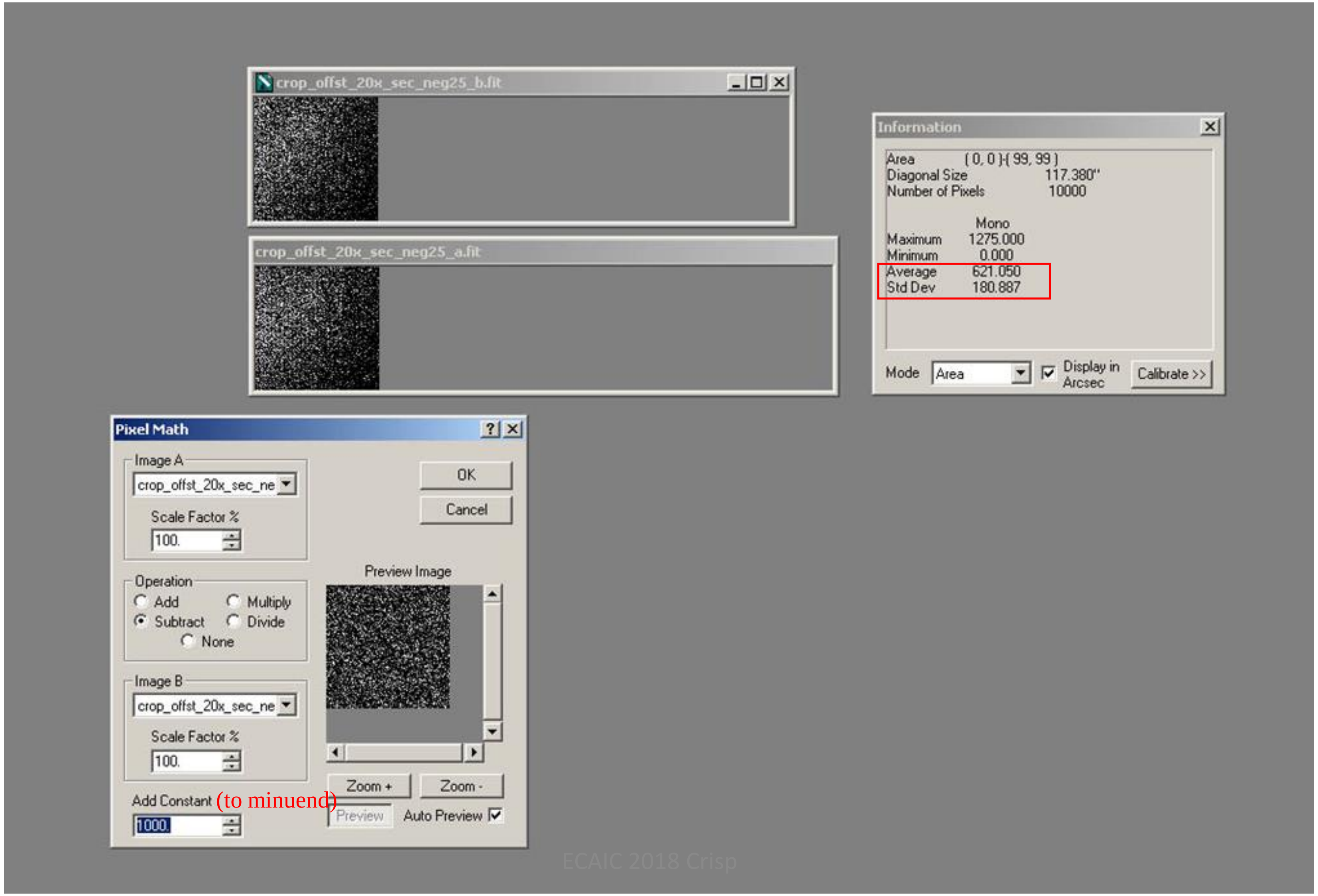


Use Logarithmic Axes

Practical matters: measuring the image data

- After cropping the pair of identical flat field exposures, the average and standard deviation are recorded. Then a fixed offset (recommend 5000DN) is added to the minuend and then one is subtracted from another. The standard deviation of the result is recorded

Practical matters: measuring the image data

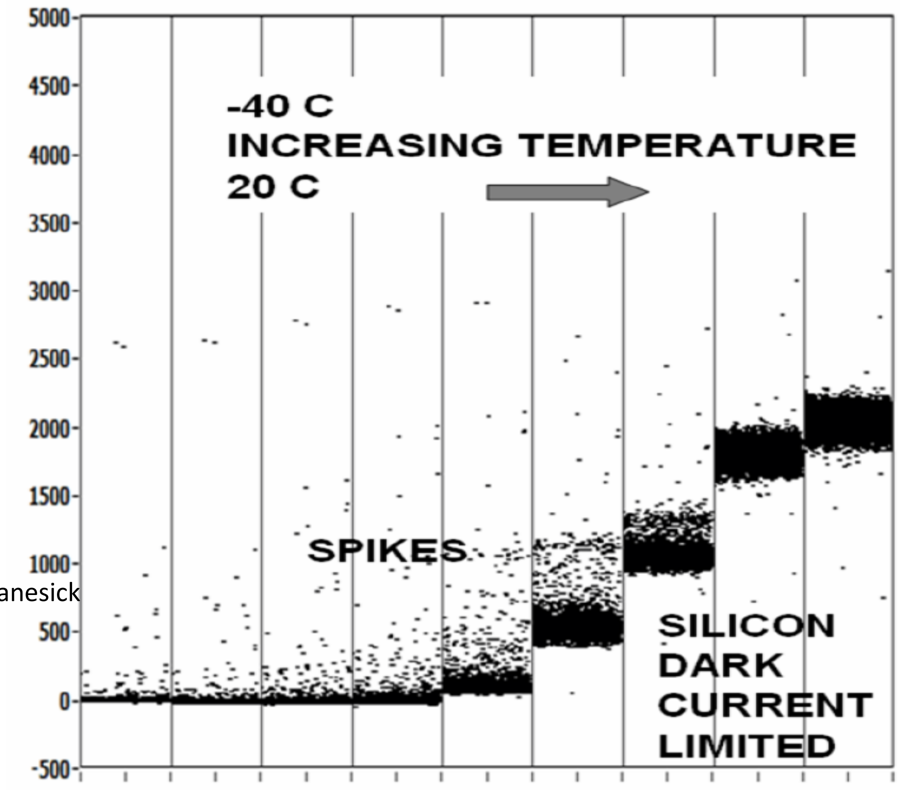


Adding Dark Signal to the PTC

- You can add data taken from dark frames to the same PTC and learn the value of the Dark Signal Non Uniformity (“DSNU”). That is usually called a Dark Transfer Curve (“DTC”)
- Additionally you will pick up lower values for the shot noise curves. The dark signal portion should simply extend the portion derived using flat fields
- From a spreadsheet perspective: simply copy the one you built and place below: then enter data from your dark frames to replace the light-on data. The pairs of identical darks are cropped using the same selection box as for the light-on data
- It is important to take the dark data at elevated temperature:
  - Dark signal accumulates faster
  - The DSNU is measured by the manufacturer at elevated temperatures
  - DSNU tends to increase as temperature is reduced: the low end of the histogram dark current histogram is truncated by thermally generated charge as the temperature is increased. That reduces the variability of the data hence the value of the DSNU.
- Whatever temperature picked, it needs to remain constant so it should be colder than room temperature so that the cooler can maintain a constant temperature

Dark Spikes vs Temperature

Reduced temperature: less thermal current, more variance of dark spikes: increases DSNU



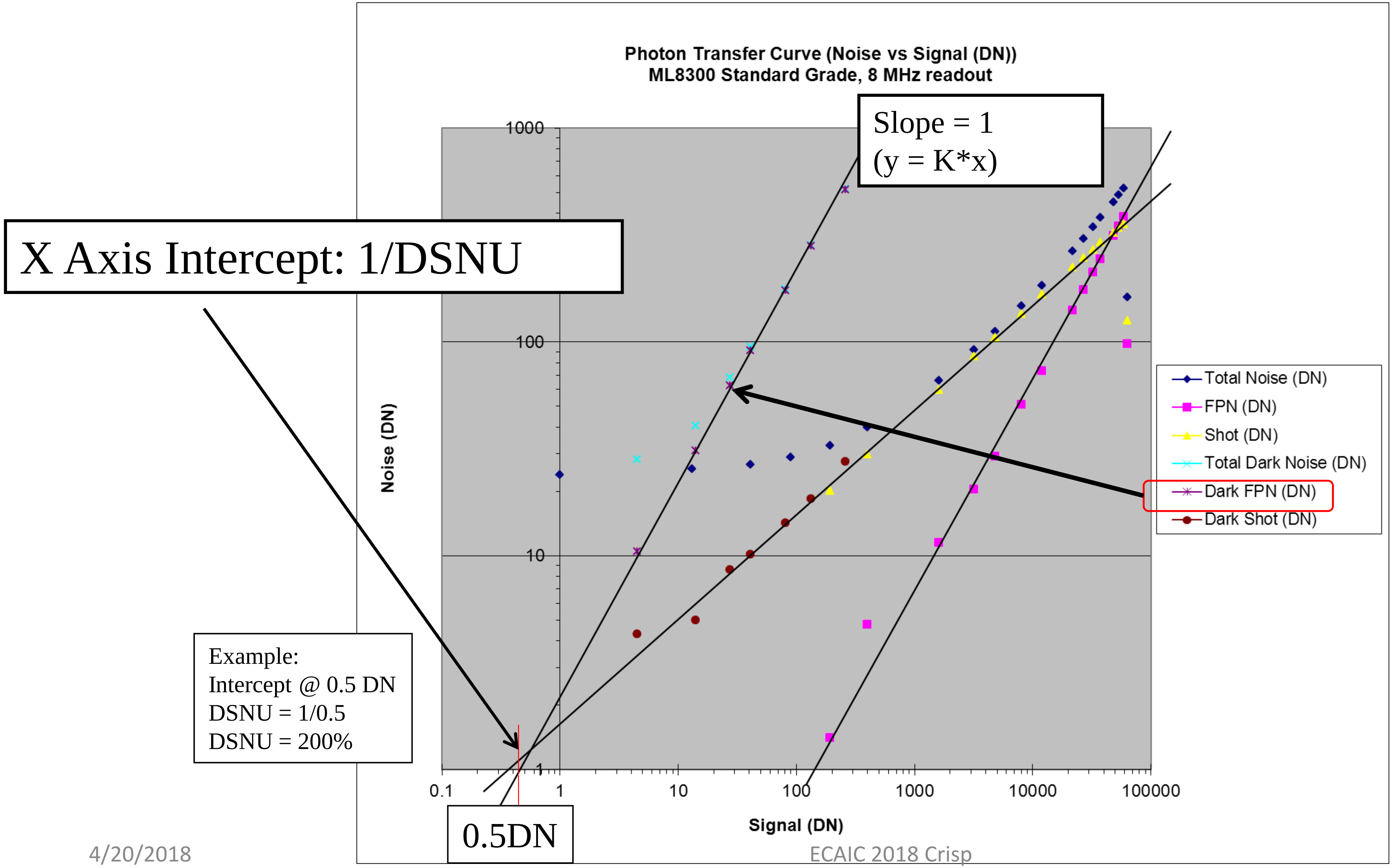
Source: Janesick

Increasing DSNU

Dark Current Limited: Reduces DSNU

KAF series DSNU is specified at room temperature (25C)

DTC/PTC

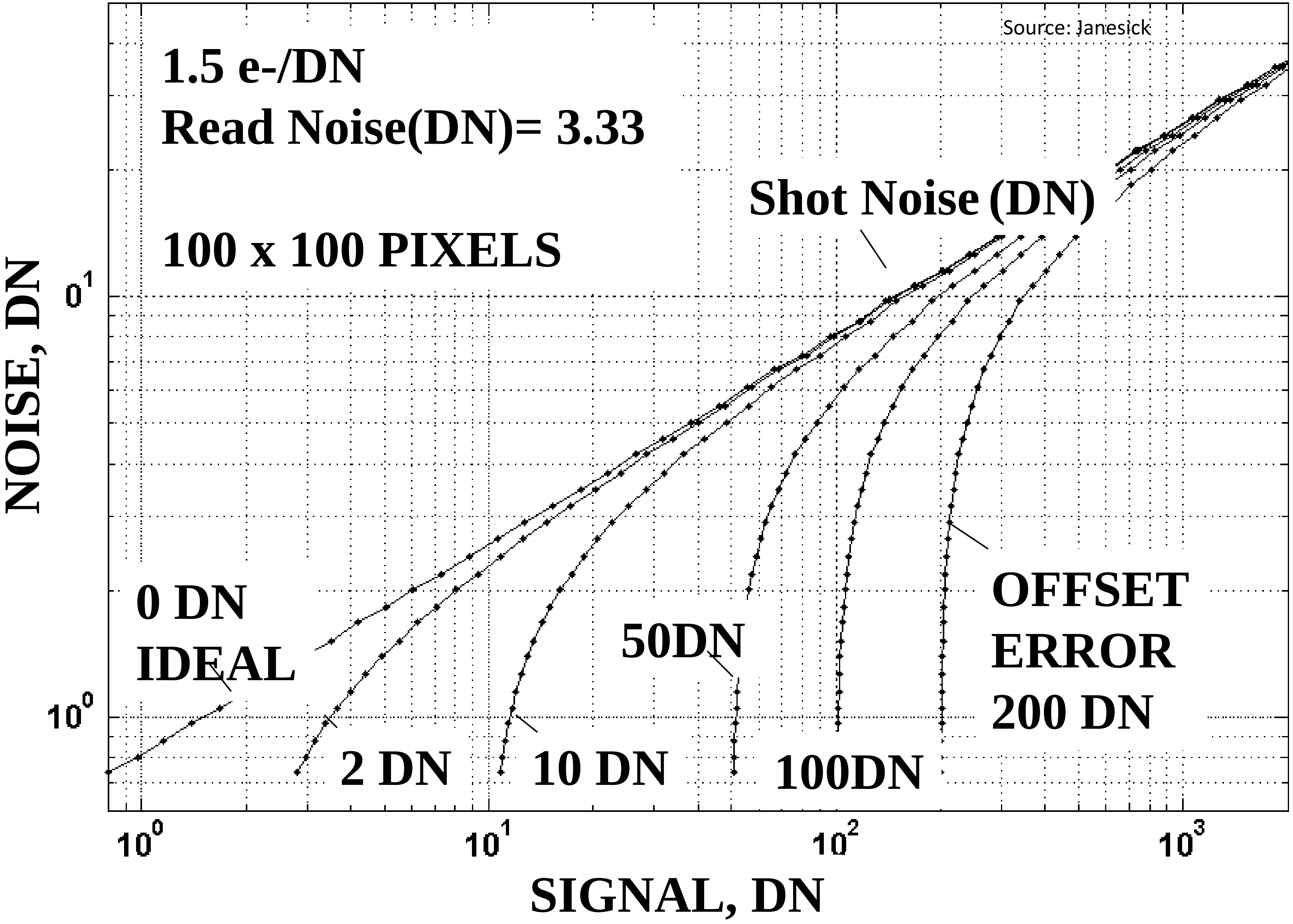


Photon Transfer Curve (Noise vs Signal (DN))
ML8300 Standard Grade, 8 MHz readout
Slope = 1
(y = K*x)
X Axis Intercept: 1/DSNU
Example:
Intercept @ 0.5 DN
DSNU = 1/0.5
DSNU = 200%
0.5DN
Noise (DN)
Signal (DN)
1000
100
10
1
0.1
1
10
100
1000
10000
100000
Total Noise (DN)
FPN (DN)
Shot (DN)
Total Dark Noise (DN)
Dark FPN (DN)
Dark Shot (DN)

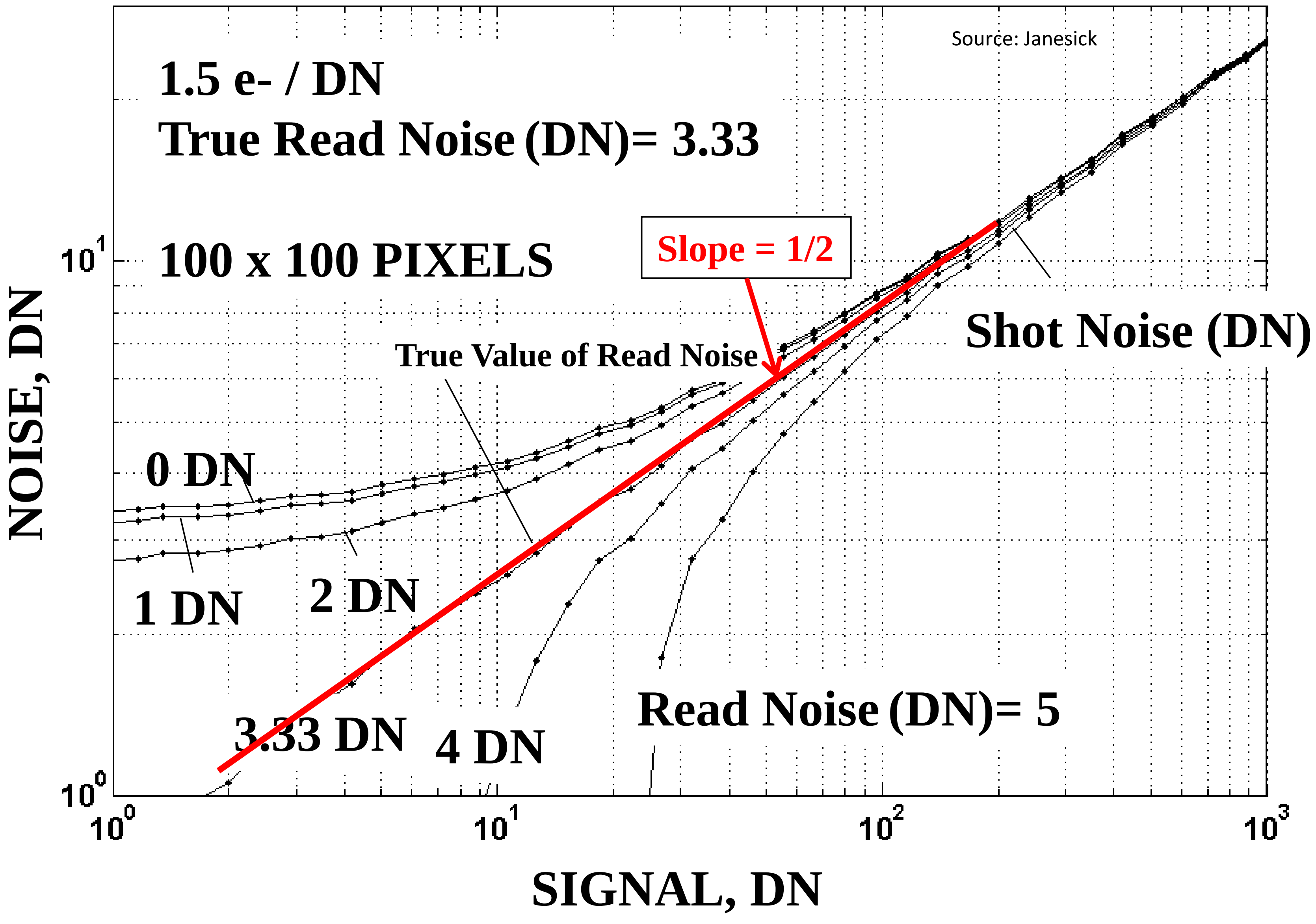
Common PTC Errors

- Common errors for the PTCs are using the wrong value for the read noise and for the offset
- The offset cannot be directly measured, the bias value is close but includes a bit of dark signal and may have other error sources.
- For the read noise a good initial value is the standard deviation of the bias frame. Or you can look at where the total noise crosses the Y axis and use that value.
- The symptoms of error are easy to spot:
    - when the Shot noise and FPN curves doesn't show a straight line to the X axis intercept, that usually means the offset and or read noise is off
- Using the Offset Correction column, the offset can be adjusted to straighten out the FPN curve.
- Then the Read Noise value can be adjusted to better straighten out the shot noise curve. It is common to iterate back and forth a bit.
    - The read noise can be very accurately measured: to 3 significant digits if desired
- For the low-valued flat field pairs, it is helpful to take a bit more exposure pairs because one unfortunately placed noise hit can foul up the low valued data making the offset and read noise adjustments difficult to judge (you may see wild data or negative numbers under a square root)
    - You will possibly have to toss some "flyer" data points as a result.
- It is beneficial to take linearly spaced flats for the low valued ones such as 1, 1.1, 1.2, 1.3, 1.4 etc to not lose data coverage if a few of the pairs are ruined by anomalous pixel events

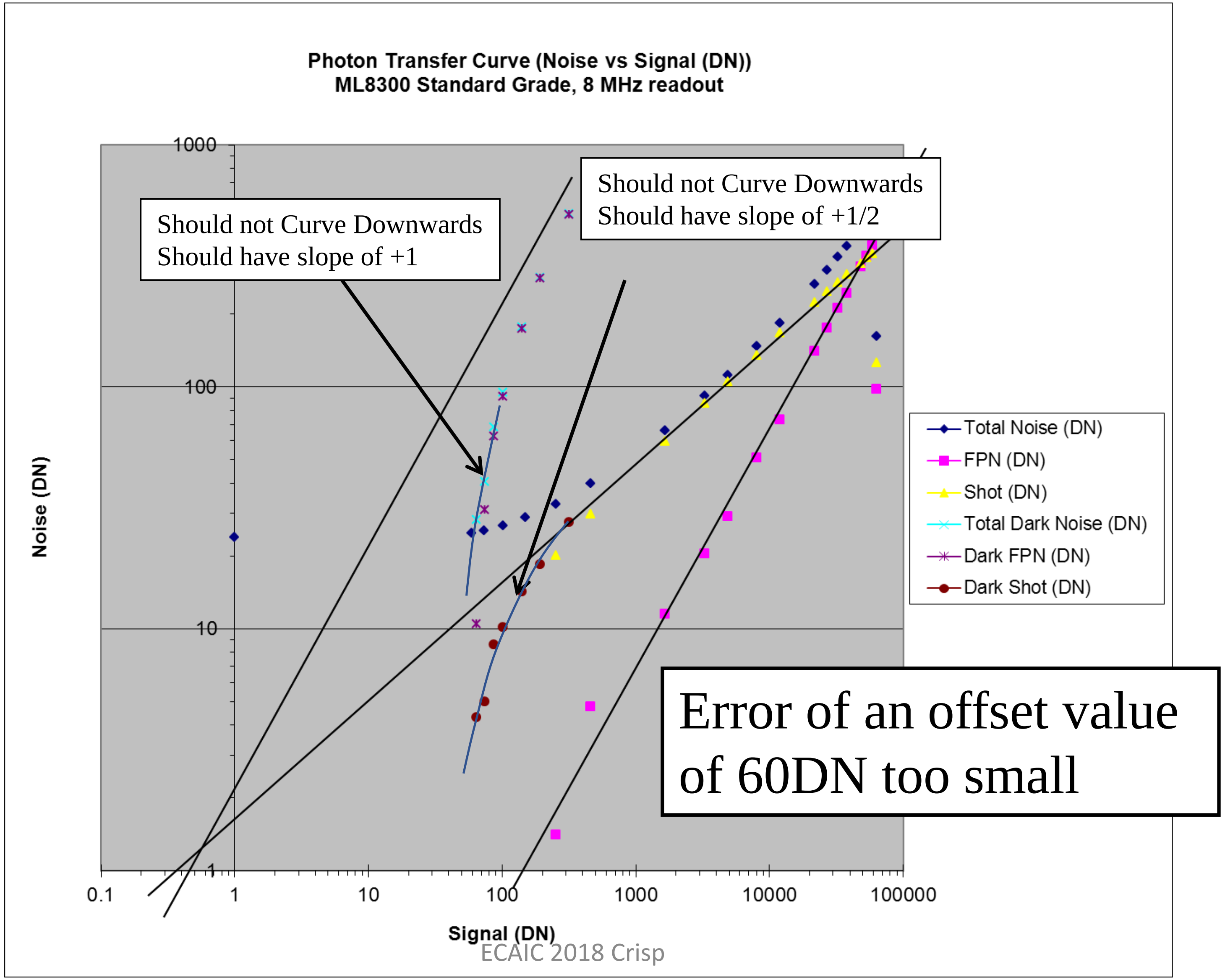
PTC OFFSET ERROR



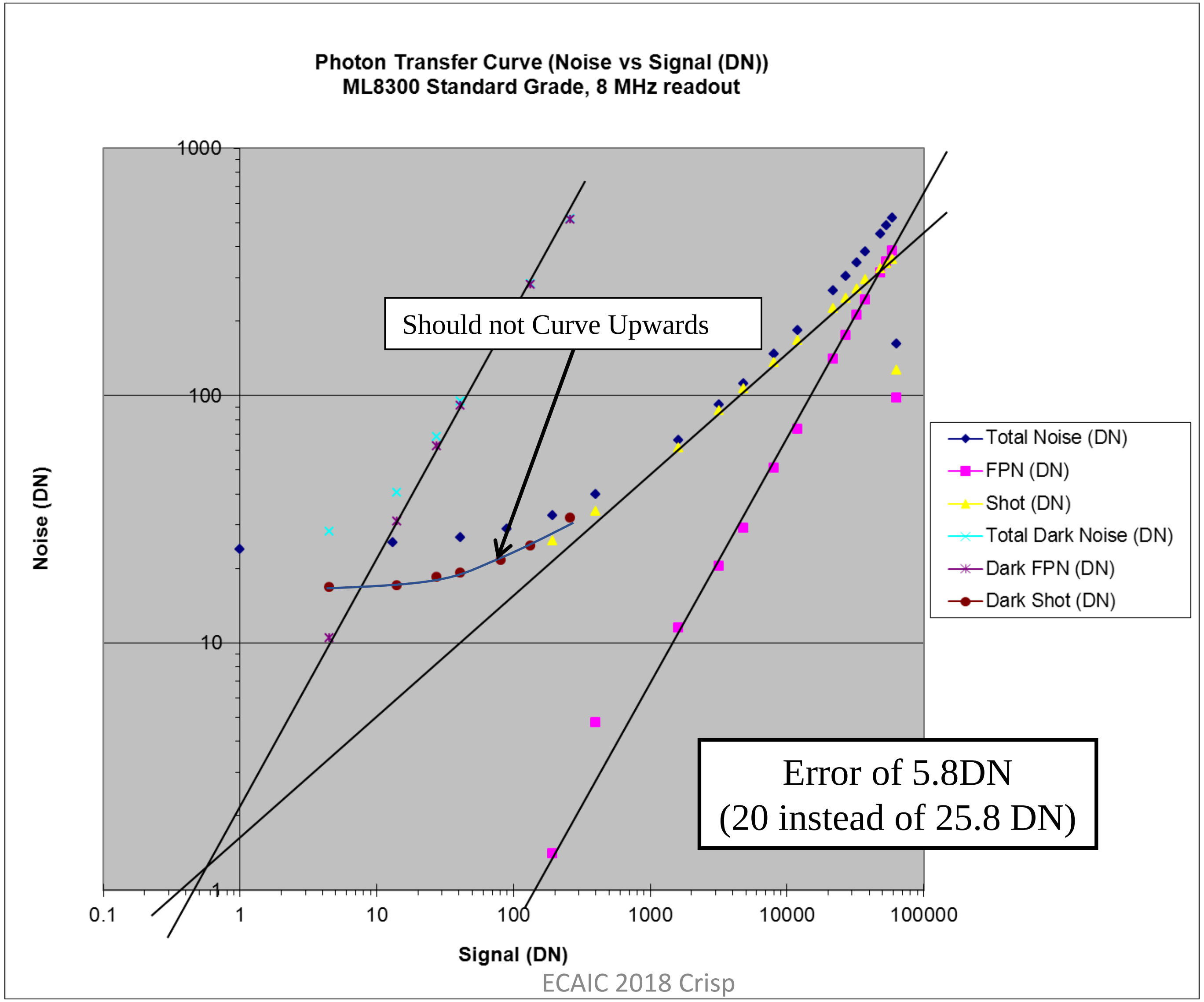
PTC READ NOISE ERROR



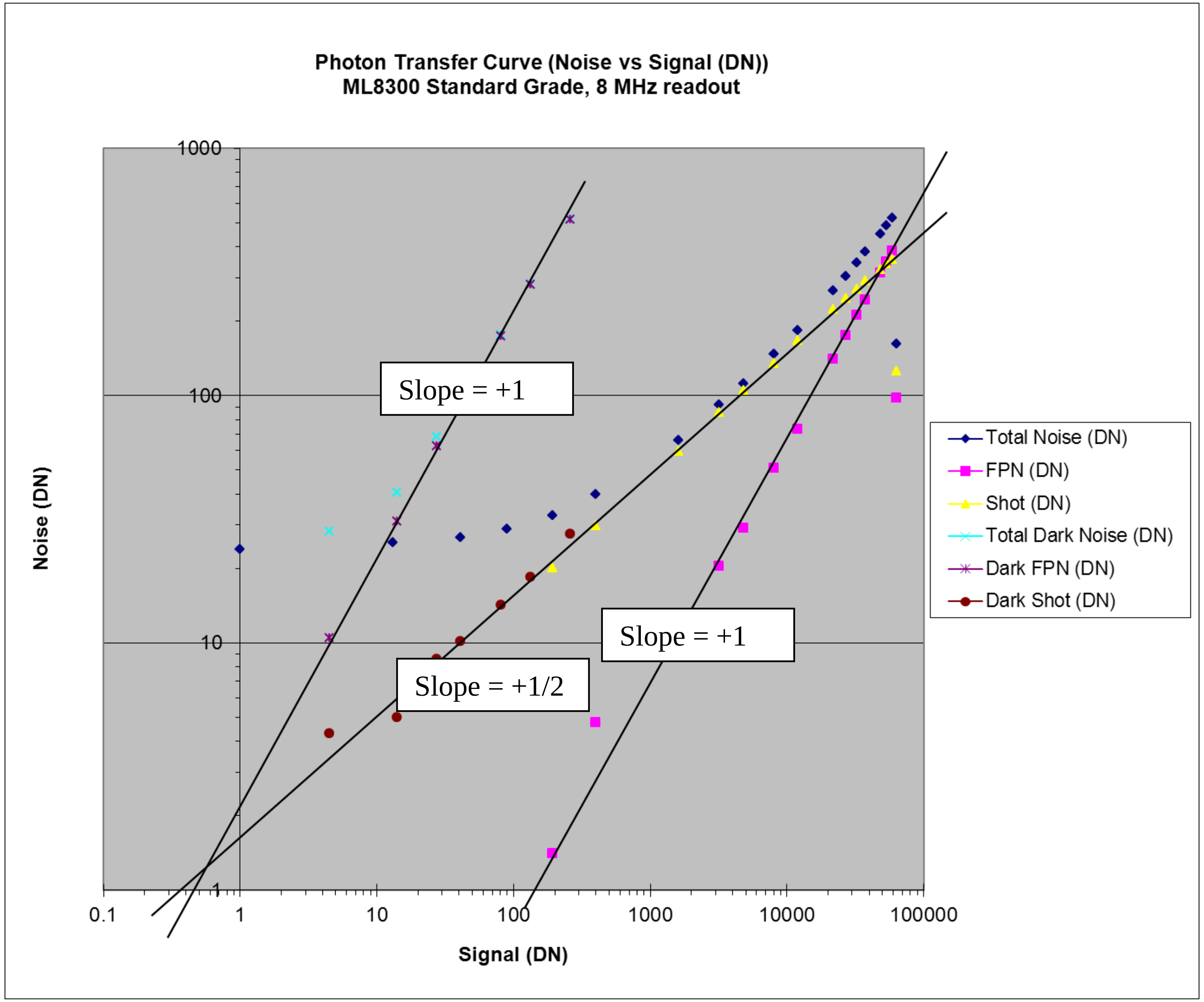
PTC Offset Error



PTC Read Noise Error



Correct Values of Offset, Read Noise



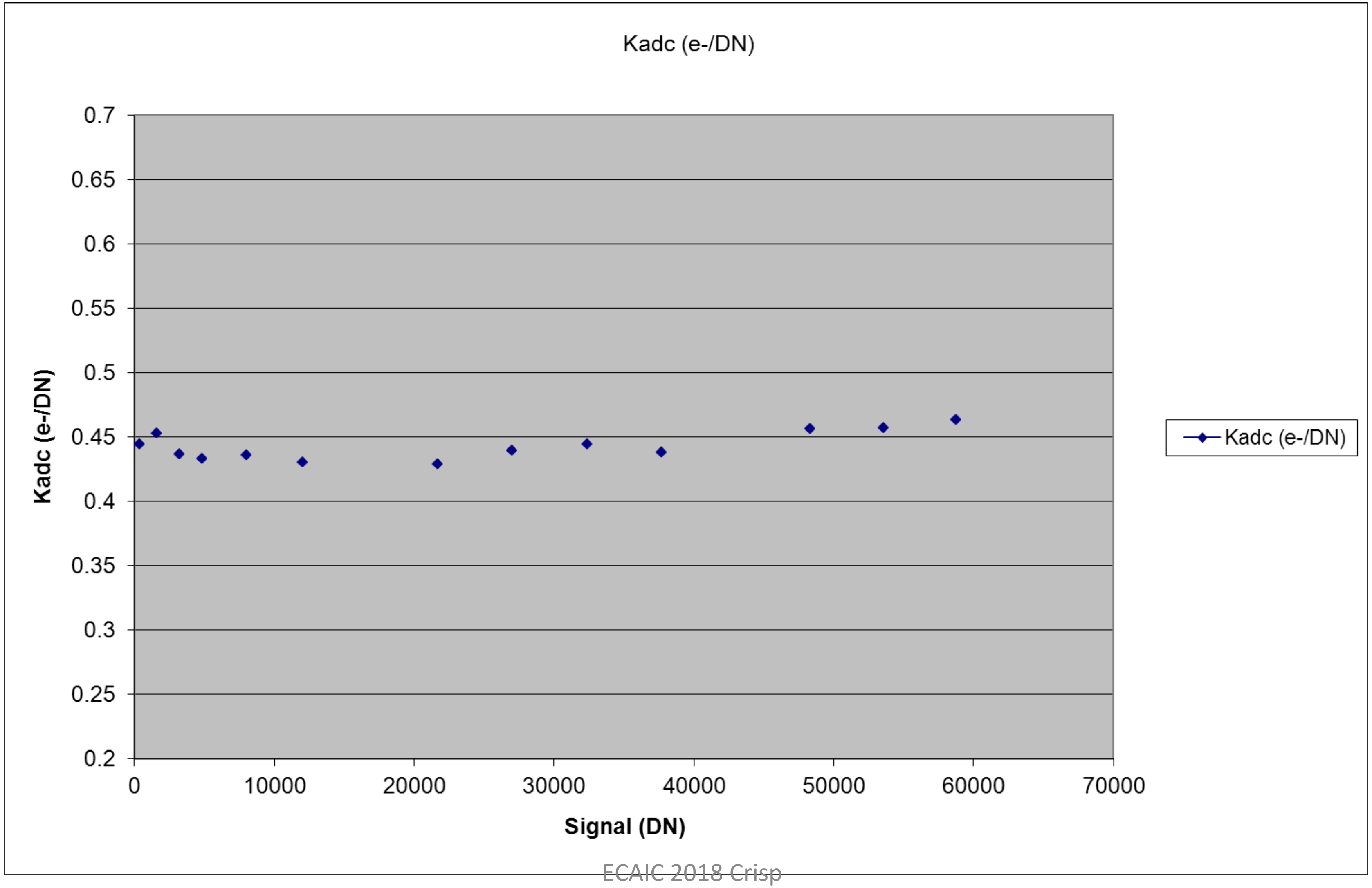
Adding in Linearity

- Kadc can be calculated point by point by the following relation

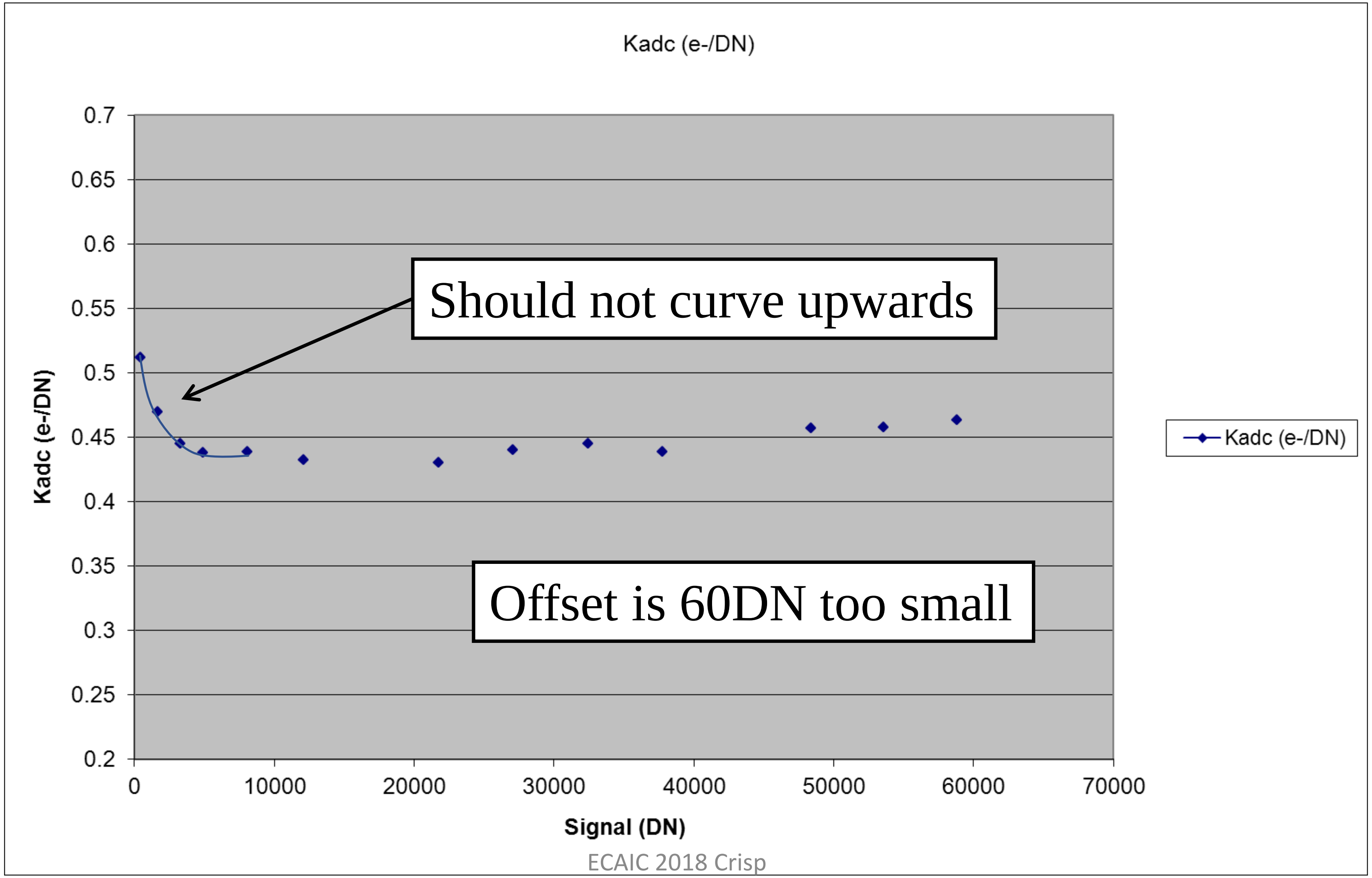
Kadc = signal/signal_shot^2

- Kadc is very sensitive to offset and read noise errors

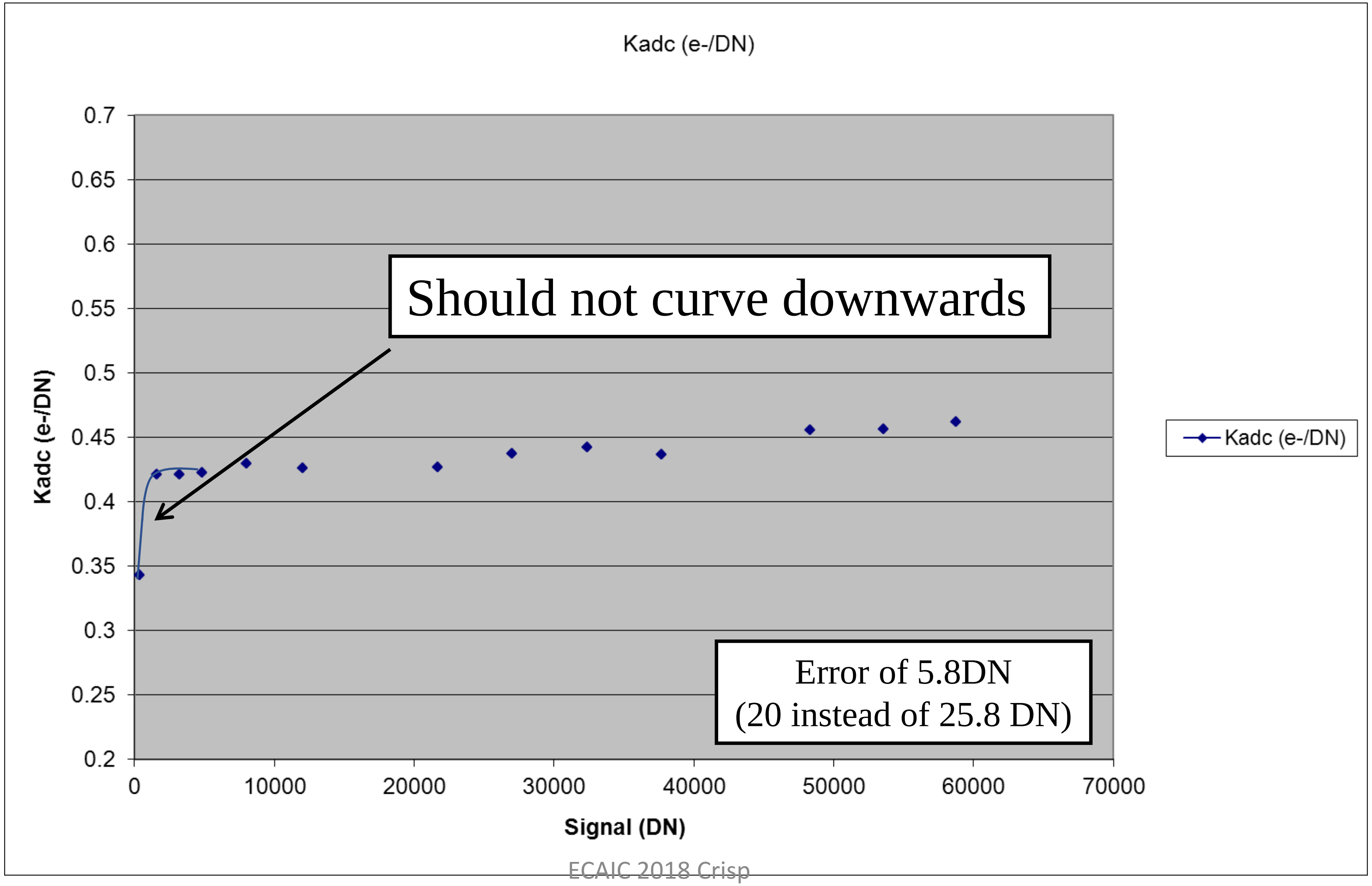
Linearity Plot (correct offset, read noise)



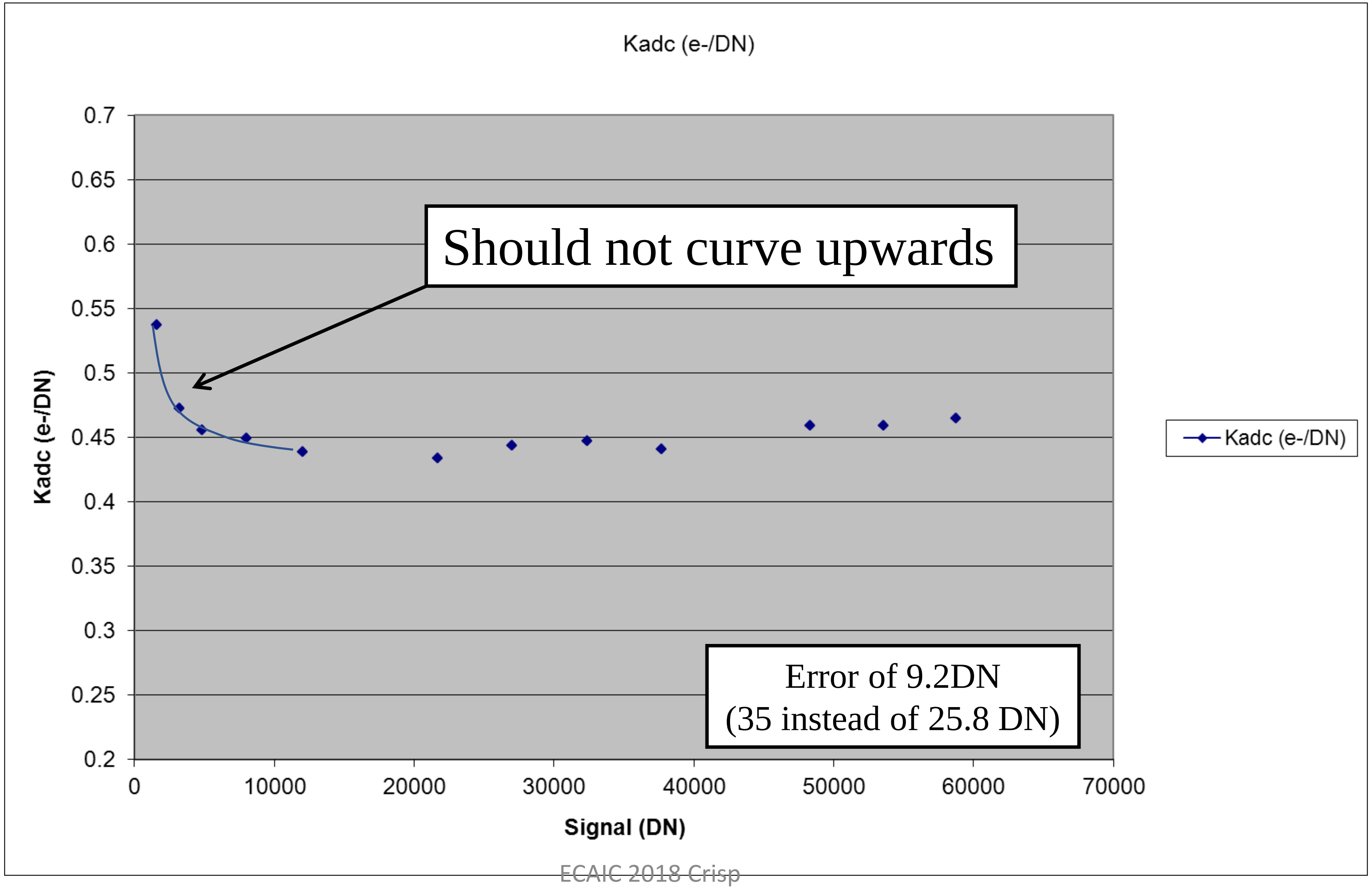
Linearity plot with offset error



Linearity plot with read noise error



Linearity plot with read noise error



Summary

- PTC: Noise versus Signal
- Noise Equation
- Making a PTC
- Common PTC Errors

PTC Application Example #1 Flat-Fielding Optimization

PTC Application #1 Flat Fielding Optimization

- What is Flat Fielding and how does it work?
- What will it correct and what will it not correct?
- How do you know what level to shoot for a flat?
- How do you know how many flats to shoot?
- How can you measure how well the flats are working?
- How can you determine the total signal needed in the set of flats (signal level * number of flats)?

What is flat fielding?

- Flat-fielding is a process used to remove FPN from an image
- Unless the FPN is removed from the image, it will place an upper limit on the maximum possible Signal to Noise Ratio ("SNR")
- Flat fielding is performed by dividing an image by a "flat-field" image on a pixel by pixel basis
- The flat-field image is an image of a featureless background. This image reveals the non-uniformities of the combination of the optics and camera's sensor which appear in every image taken by the system.

How does FPN limit the SNR?

Neglecting dark current sources, the noise in an image is expressed by the familiar noise equation:

$$\text{Noise}_{\text{IMAGE}} = \sqrt{\text{Signal\_shot\_noise}^2 + \text{Fixed\_pattern\_noise}^2 + \text{Read\_noise}^2}$$

recall:

$$\text{Signal\_shot\_noise} = \sqrt{\text{Signal}}$$

$$\text{Fixed\_pattern\_noise} = \text{PRNU} * \text{Signal}$$

Substituting, we get:

$$\text{Noise}_{\text{IMAGE}} = \sqrt{\text{Signal} + (\text{Signal} * \text{PRNU})^2 + \text{Read\_Noise}^2}$$

How does FPN limit the SNR cont'd

$$\text{Signal/Noise}_{\text{IMAGE}} = \text{Signal} / \sqrt{\text{Signal} + (\text{Signal} * \text{PRNU})^2 + \text{Read\_Noise}^2}$$

When: $\text{Signal} \geq \dfrac{1}{\text{PRNU}^2}$

$$\text{Signal/Noise}_{\text{IMAGE}} \leq 1/\text{PRNU}$$

This establishes a ceiling on our Signal/Noise ratio which is bad

So we will use flat-fielding to eliminate the FPN term in the noise equation

The flat fielding operation

The flat fielding operation consists of dividing, pixel by pixel, the raw image by a flat field image. The corrected ith pixel of an image that has been flat-fielded is expressed as:

$$S_{COR\,i} = \mu_{FF} \frac{S_{RAW\,i}}{S_{FF\,i}} \qquad (1)$$

$$S_{COR_i} = \text{corrected signal}$$

$$S_{FF_i} = \text{signal in flat field}$$

$$S_{RAW_i} = \text{signal in raw image}$$

$$\mu_{FF} = \text{average signal level in flat field}$$

Noise in a flat-fielded image

In order to test the efficacy of the flat-fielding operation, we need to know the noise of this corrected image. We now seek the equation for the noise of the corrected image in terms of the signal level in the flat field and the raw images.

Since the noise of the corrected image is simply the square root of the variance of the image we can calculate the variance.

The corrected image is a function of two variables, the raw signal and the flat field signal. To calculate the variance of a function of two variables where the variables are uncorrelated, we use the simplified propagation of errors formula:

$$\sigma_{\mathrm{Q}}^2 = \sigma_{\mathrm{x}}^2\left(\frac{\partial \mathrm{Q}}{\partial \mathrm{x}}\right)^2 + \sigma_{\mathrm{y}}^2\left(\frac{\partial \mathrm{Q}}{\partial \mathrm{y}}\right)^2 \qquad (2)$$

Noise in a flat-fielded image cont'd

Applying (2) to (1) and including a read noise term for a practical system we get

$$\sigma_{\mathrm{COR}}^2 = \sigma_{\mathrm{FF-shot}}^2 \left(\frac{\partial \mathrm{S_{COR}}}{\partial \mathrm{S_{FF}}} \right)^2 + \sigma_{\mathrm{RAW-shot}}^2 \left(\frac{\partial \mathrm{S_{COR}}}{\partial \mathrm{S_{RAW}}} \right)^2 + \sigma_{\mathrm{READ}}^2 \qquad (3)$$

Performing the differentiation and doing a lot of manipulation while substituting

$$\sigma^2{}_{\text{FF-shot}} = \mathrm{S_{FF}}$$

$$\sigma^2{}_{\text{RAW-shot}} = \mathrm{S_{RAW}}$$

equation (3) simplifies to

$$\sigma_{\mathrm{COR}}^2 = \mathrm{S_{RAW}} \left(1 + \frac{\mathrm{S_{RAW}}}{\mathrm{S_{FF}}} \right) + \sigma_{\mathrm{READ}}^2 \qquad (4)$$

How it works

$$\sigma^2_{\mathrm{COR}} = \mathrm{S}_{\mathrm{RAW}}\left(1 + \frac{\mathrm{S}_{\mathrm{RAW}}}{\mathrm{S}_{\mathrm{FF}}}\right) + \sigma^2_{\mathrm{READ}} \qquad (4)$$

so long as $\mathrm{S}_{\mathrm{FF}} >> \mathrm{S}_{\mathrm{RAW}}$ equation (4) reduces to

$$\sigma^2_{\mathrm{COR}} = \mathrm{S}_{\mathrm{RAW}} + \sigma^2_{\mathrm{READ}}$$

which is shot noise limited when $\mathrm{S}_{\mathrm{RAW}} > \sigma^2_{\mathrm{READ}}$

indicating the Fixed Pattern Noise is completely removed thereby meeting our goal

One remaining issue related to finite well depth

Unfortunately with a finite well depth the inequality $S_{FF} >> S_{RAW}$ cannot always be guaranteed when using a single flat field frame to calibrate a raw image containing a high signal level. A solution can be found by averaging N_{FF} frames of signal level S_{FF}

$$\sigma^2_{COR} = S_{RAW}\left(1 + \frac{S_{RAW}}{N_{FF} S_{FF}}\right) + \sigma^2_{READ} \qquad (5)$$

Solving the finite well depth issue

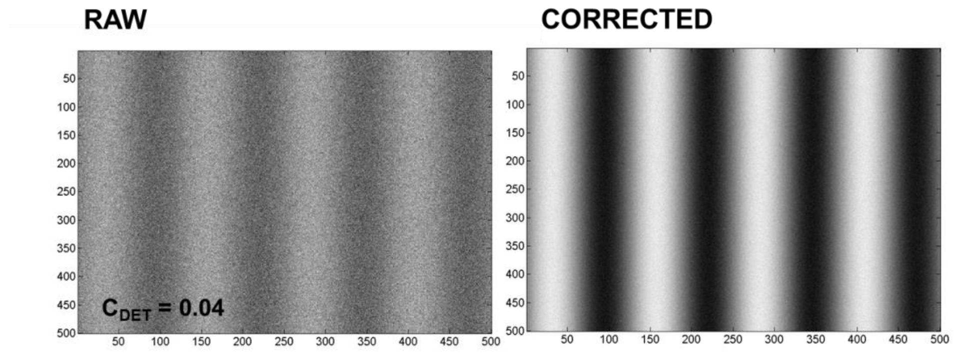
Since any arbitrary number of flat field images can be combined together, it is a simple matter to guarantee $N_{FF}S_{FF} >> S_{RAW}$ by selecting an appropriate value of N_{FF} and S_{FF} such that (5) simplifies to

$$\sigma^2_{COR} = S_{RAW} + \sigma^2_{READ} \qquad (6)$$

Taking the square root of each side and substituting descriptive names for the variables (6) transforms into our desired noise equation, which is free of the SNR-limiting FPN term

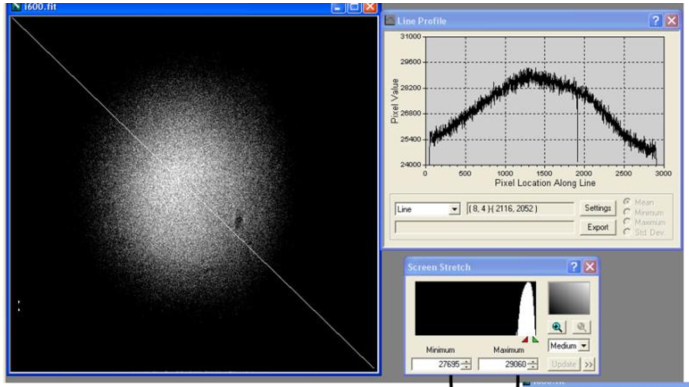
$$Noise_{IMAGE} = \sqrt{Signal + Read\_noise^{2}}$$

Flat Fielding for FPN Removal



Source: Janesick

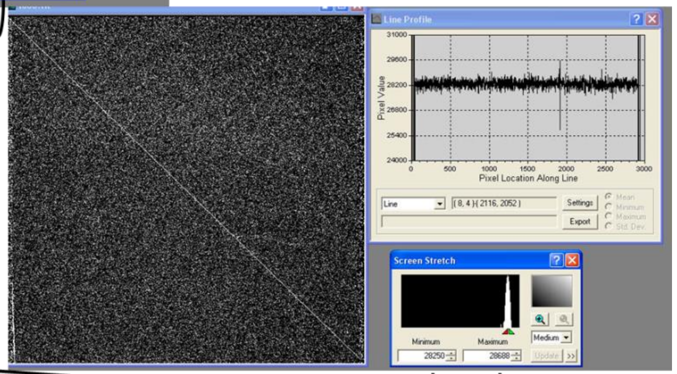
Sensor FPN removal



More uniform light distribution than measured lens before Flat field: less noise at outer parts of image post/flat field

Optical FPN removal

Tighter range of data values than measured lens
in image DN histogram prior to flat field operation



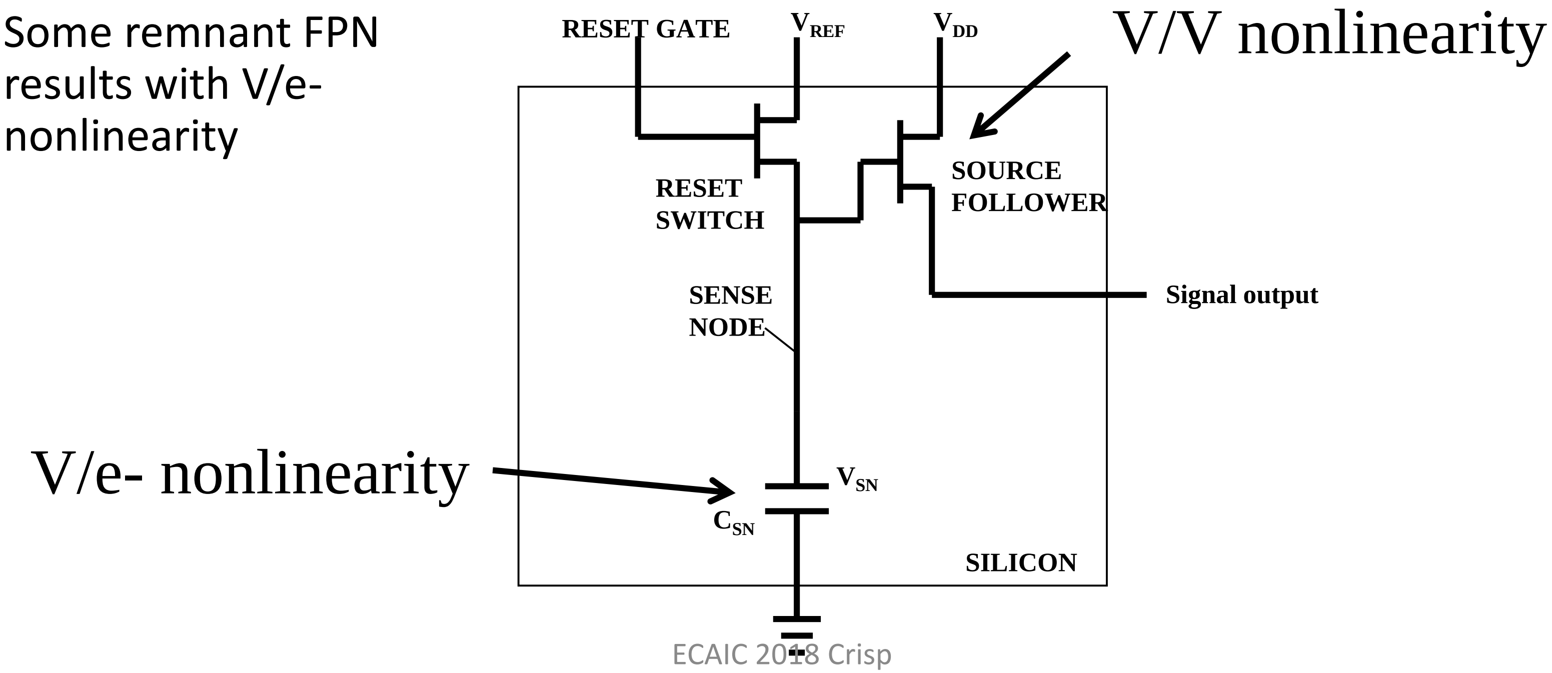
Very tight range of data values
in image DN histogram after flat field operation

What will Flat-Fielding correct and not correct?

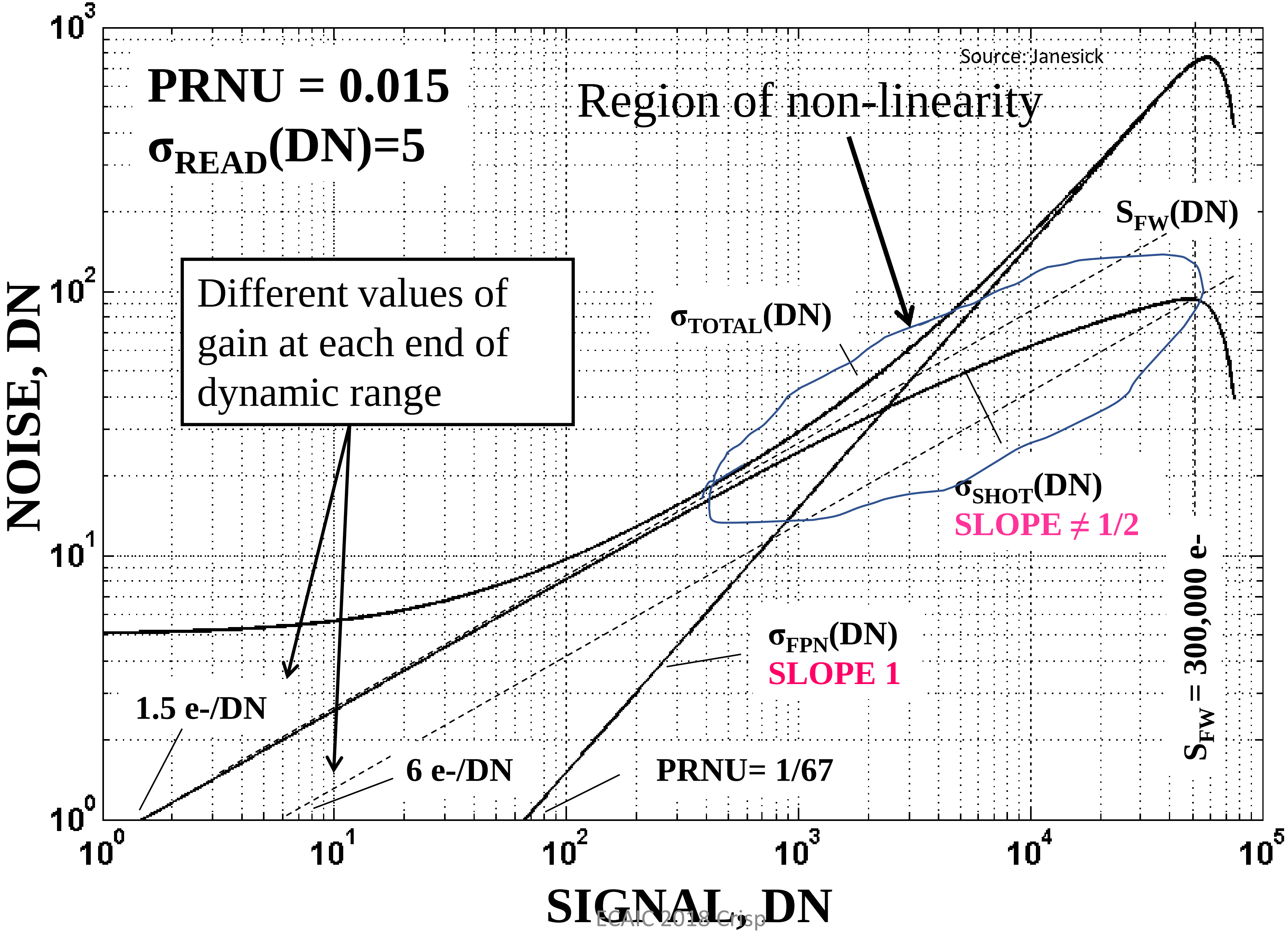
- Flat-fielding will correct fixed pattern noise
- Fixed pattern noise is always proportional to signal level: FPN = PRNU*Signal
- Flat-fielding will not correct noise that is not proportional to signal level such as saturated pixels, RBI trap non-uniformity, dark spikes, charge “skim” traps, bad columns, dead pixels etc

What about non-linearity and flats?

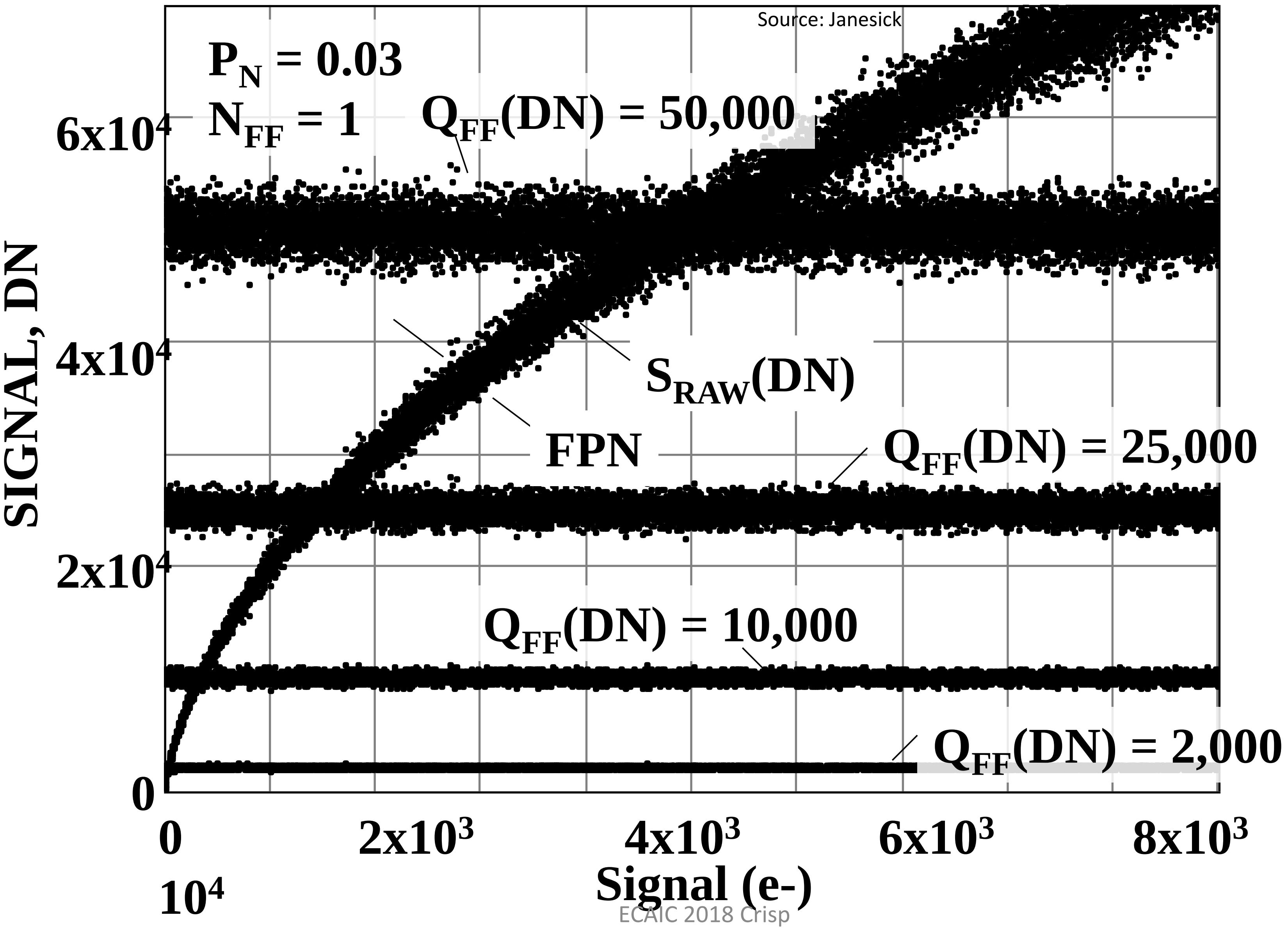
- There are two types of non-linearity in CCDs and CMOS image sensors:
    - V/V nonlinearity (variation of linearity of output source-follower amplifier)
    - V/e- nonlinearity (variation of capacitance versus voltage of sense node capacitor)
        - For CCDs the V/e- nonlinearity isn't much of a concern due to the comparatively high reverse bias voltage on the sense node capacitor
            - The dC/dV is small with high bias voltages as found in CCDs
- Flat-fielding is applicable in the presence of V/V nonlinearity: no ill effect is observed
- Some remnant FPN results with V/e- nonlinearity



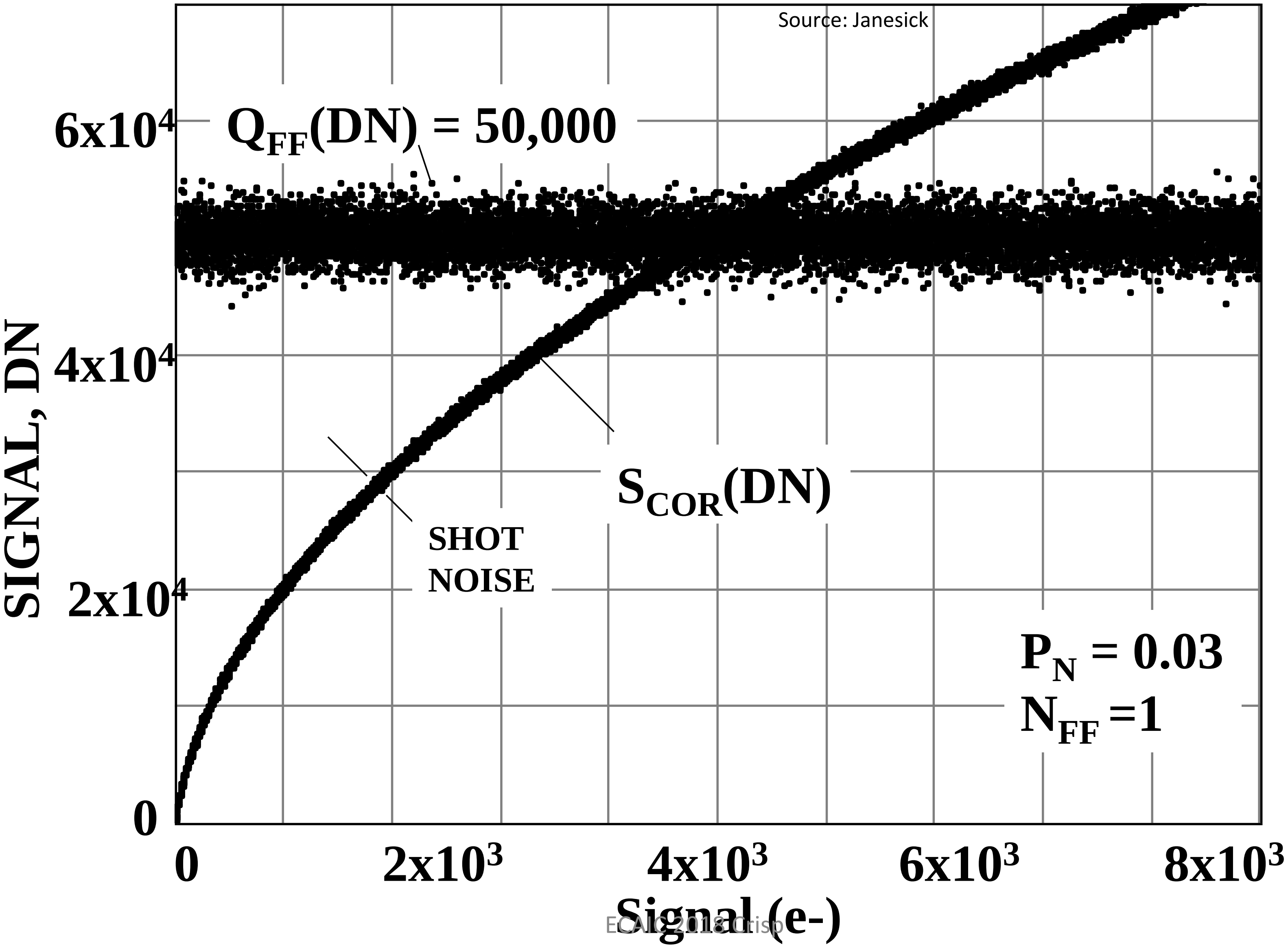
V/V Nonlinearity in a PTC



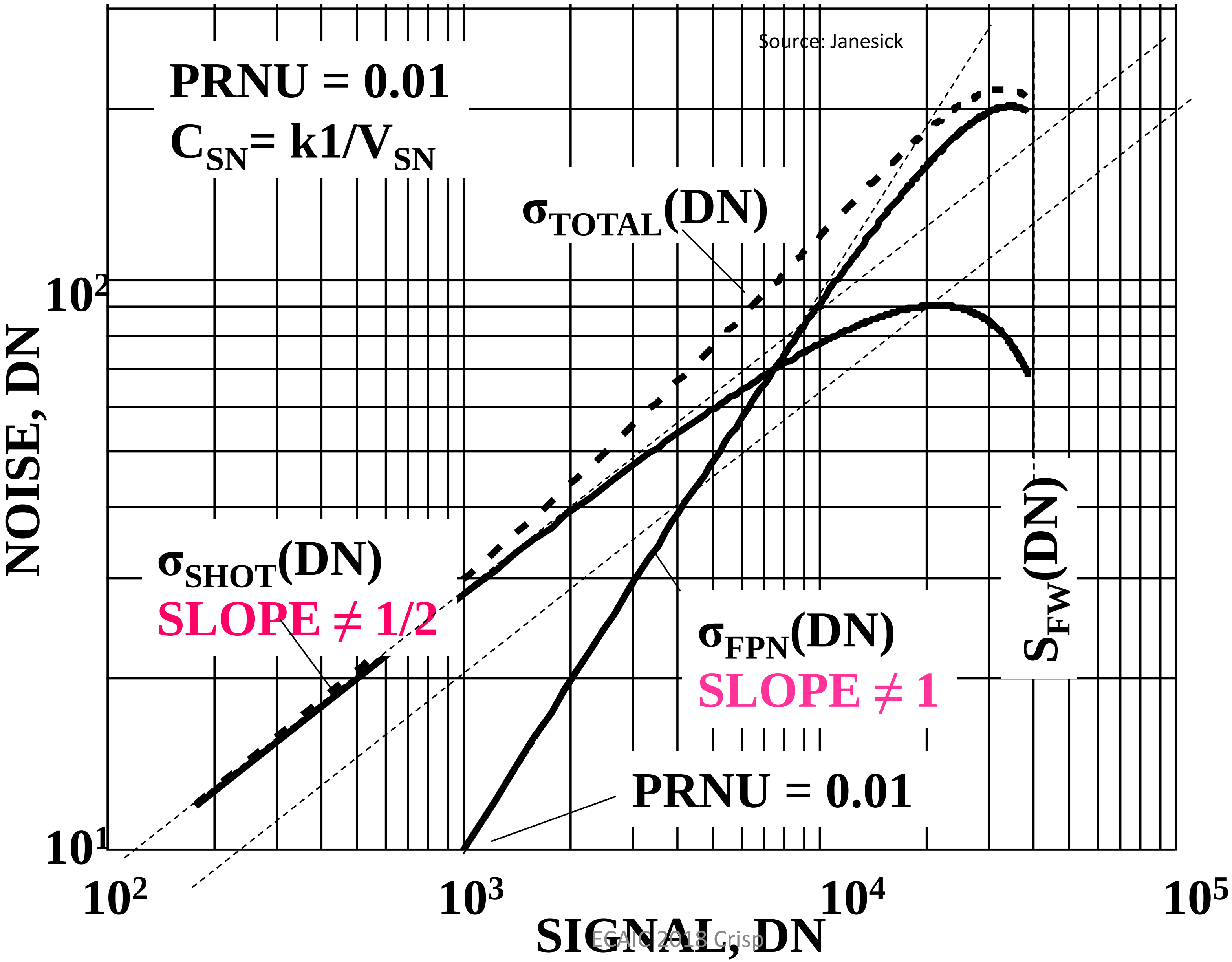
Raw Signal with V/V nonlinearity



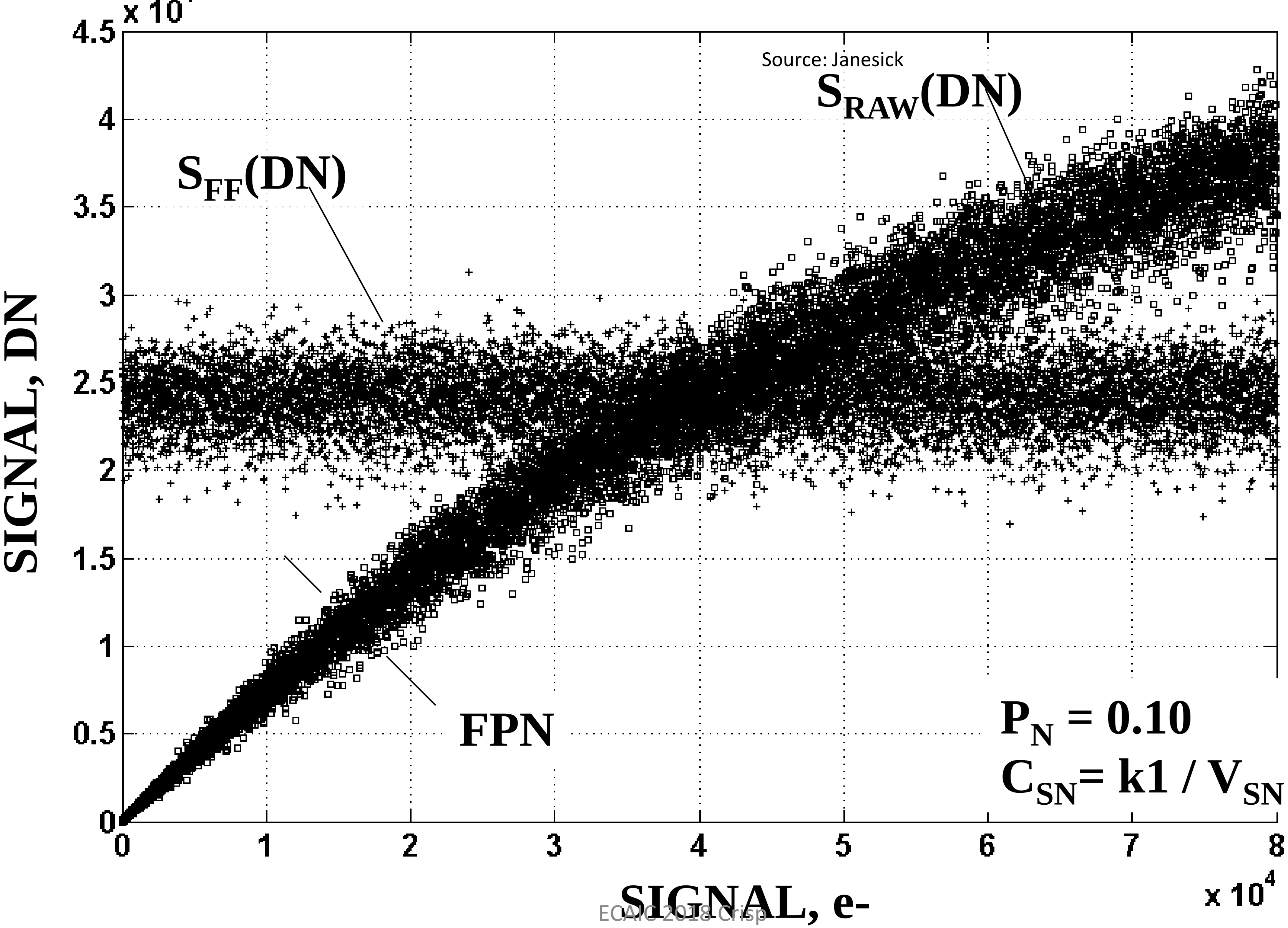
Flat fielding with V/V nonlinearity

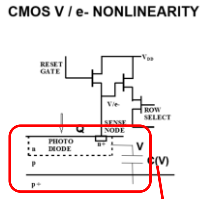


V/e- nonlinearity in a PTC CMOS issue!



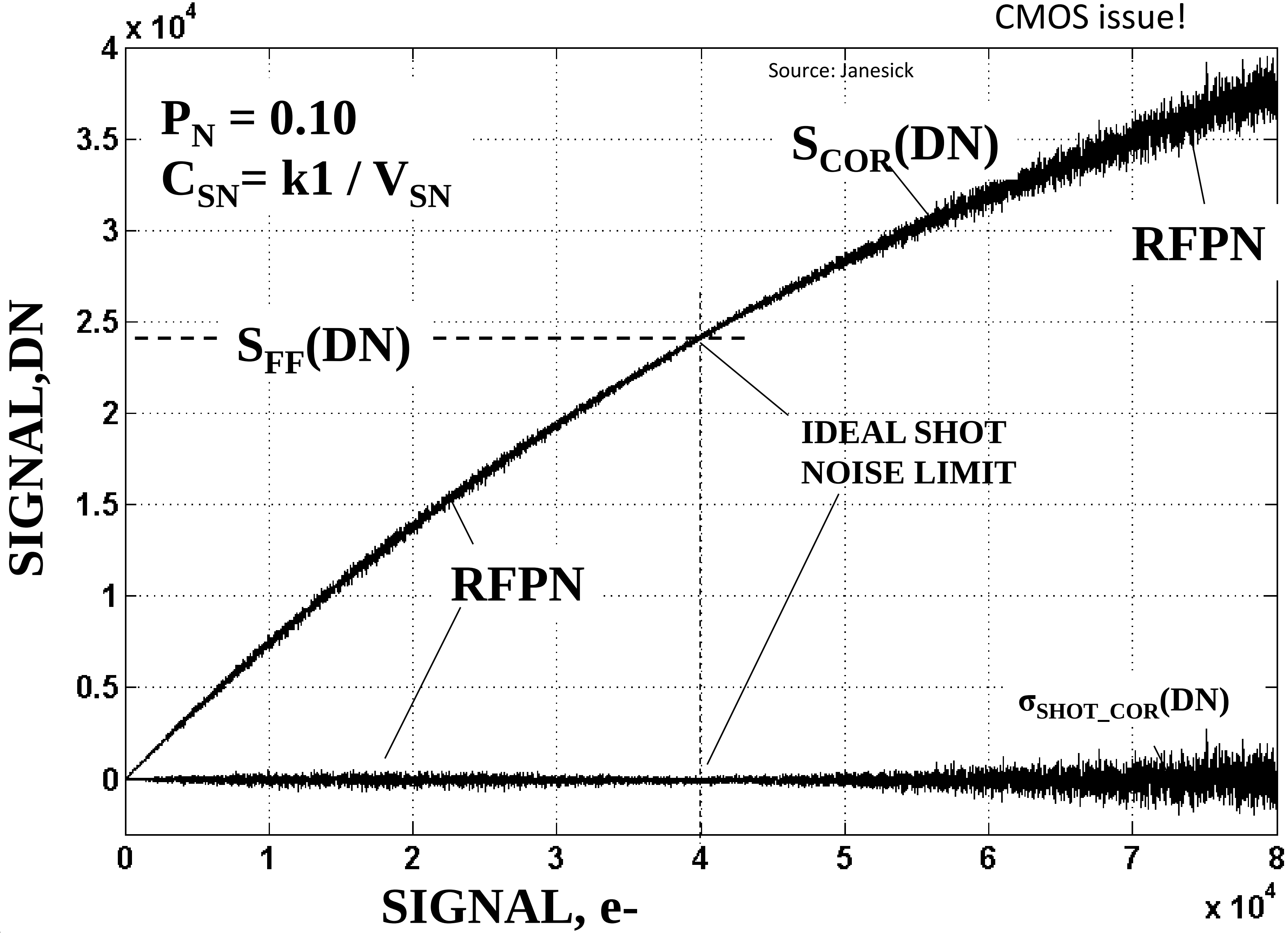
Flat-Fielding with V/e- nonlinearity CMOS issue!



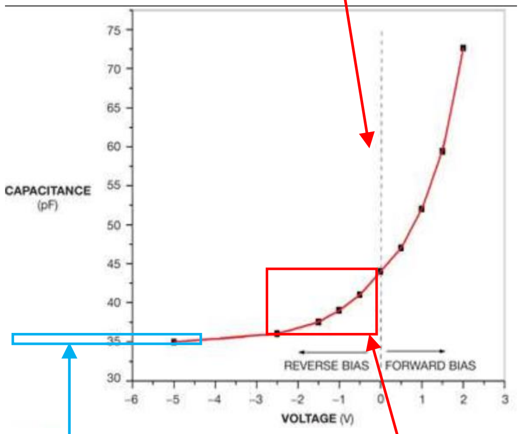


Flat Fielding with V/e- nonlinearity

CMOS issue!



Reverse biased diode Capacitance vs Voltage (like sense node floating diffusion)

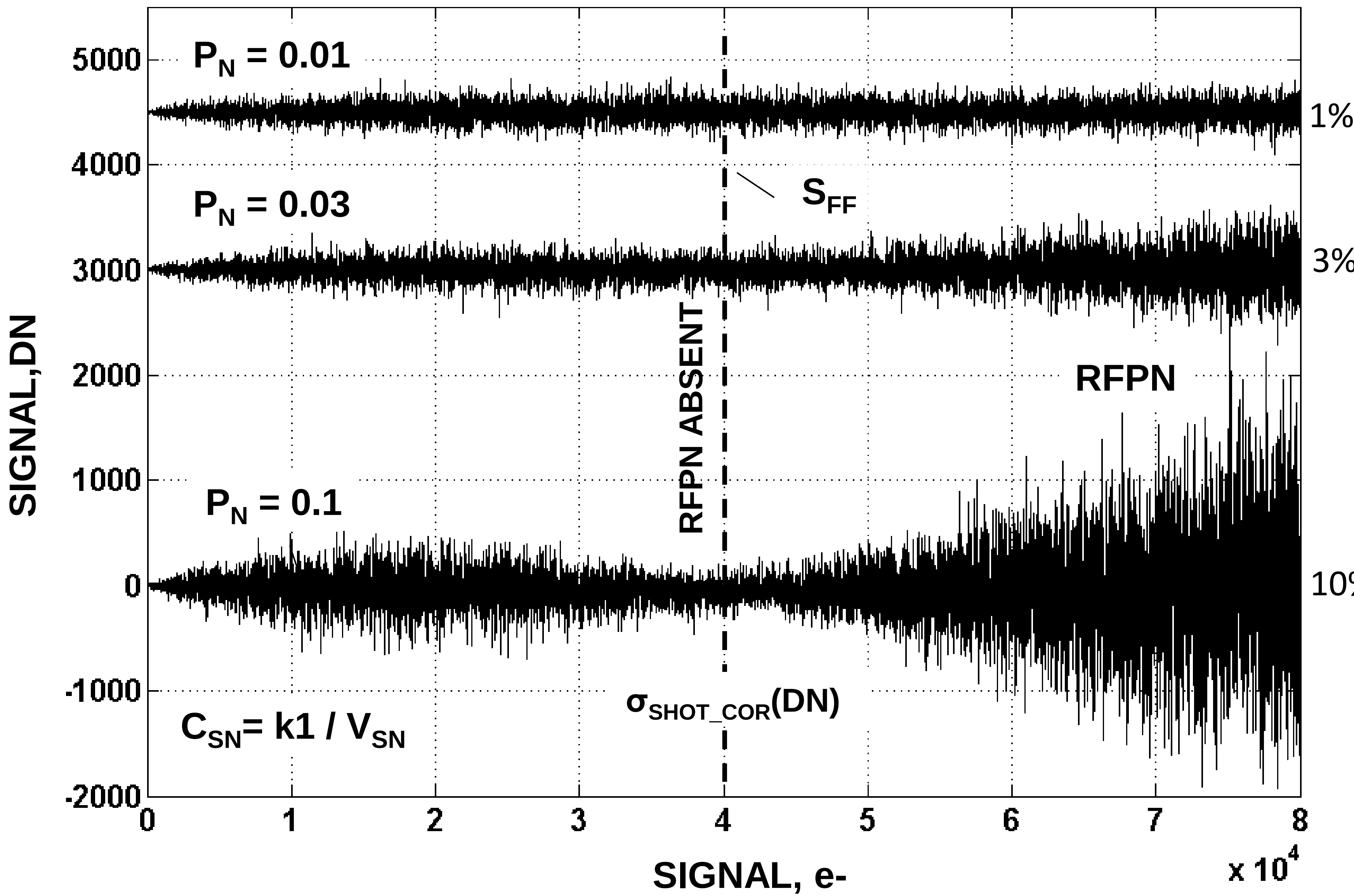


Typical CCD sense node voltage swings

Typical CMOS sense node voltage swings

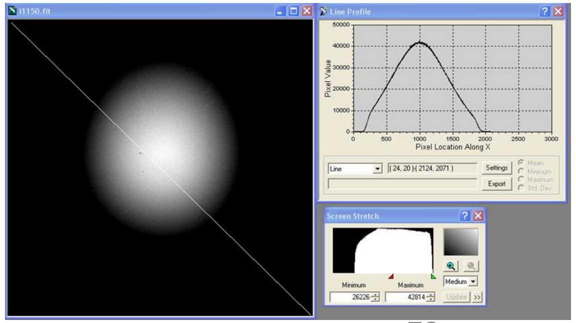
FLAT FIELDING
(V/e- NONLINEARITY)

CMOS issue!



IF PRNU is 1% or less flat fielding is effective in presence of V/e- nonlinearity

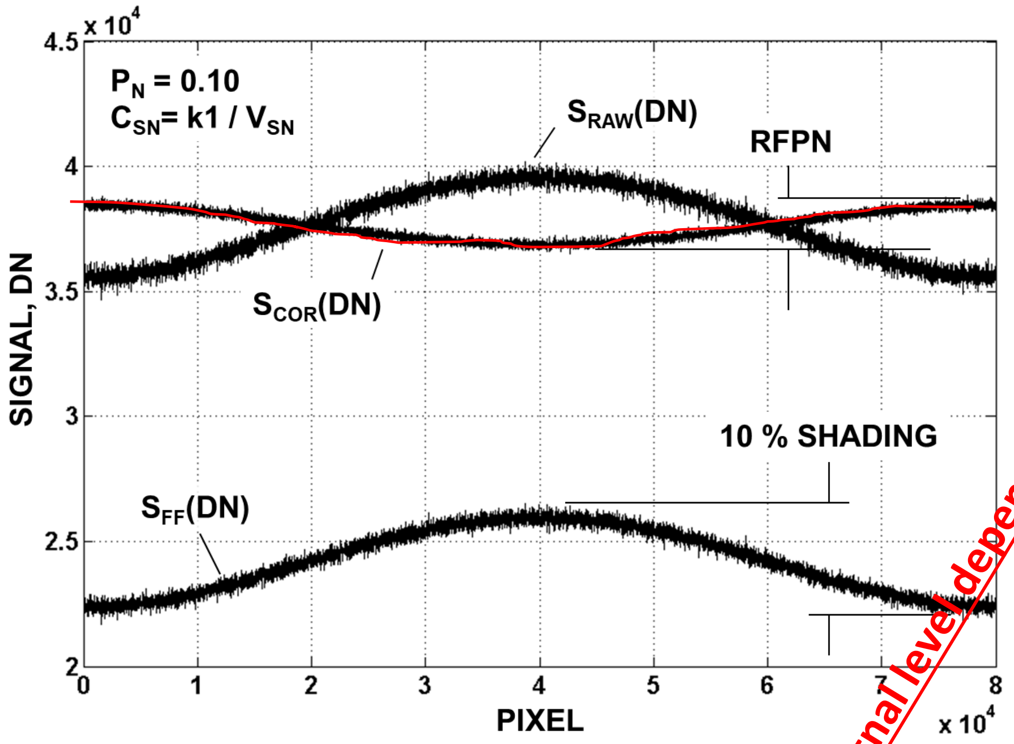
For larger values of PRNU (ie lens cos^4 rolloff), flat fielding may not be 100% effective



FLAT FIELDING
(V/e- NONLINEARITY)

CMOS issue!

Corrected Image's remnant FPN error is sensitive to signal level in Flat vs Raw image

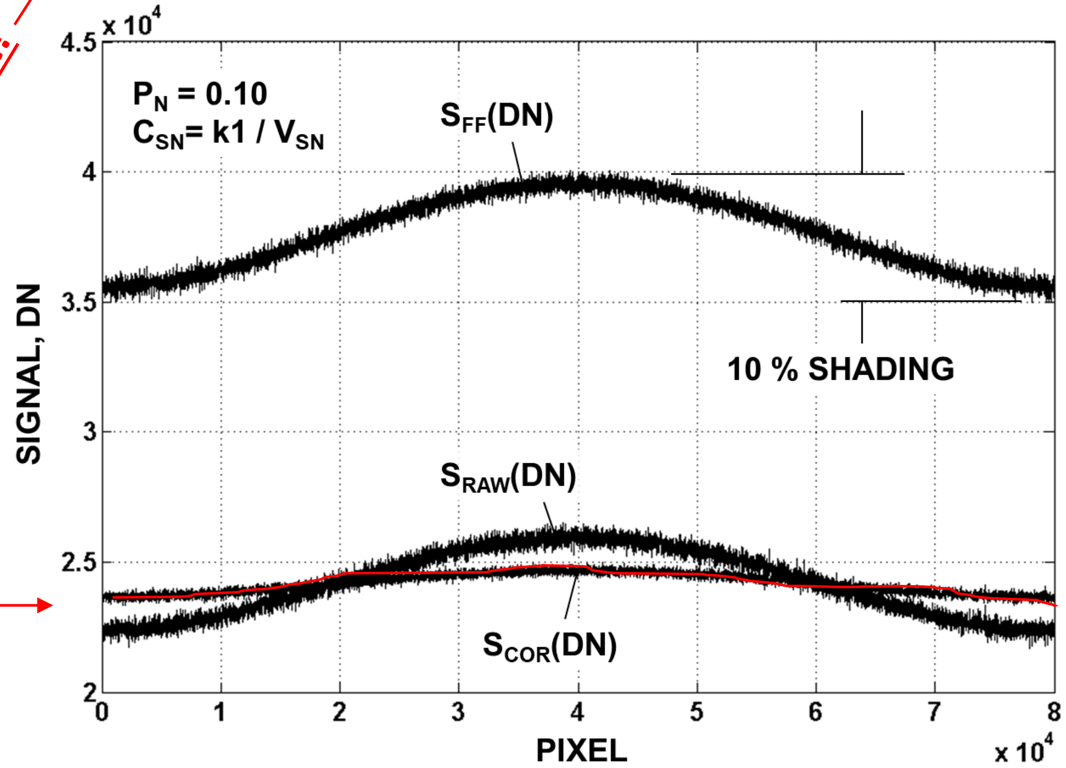


Lower level flat

Corrected (dark center)

Signal level dependent!

Corrected (hot center)



Higher level flat

FFPTC

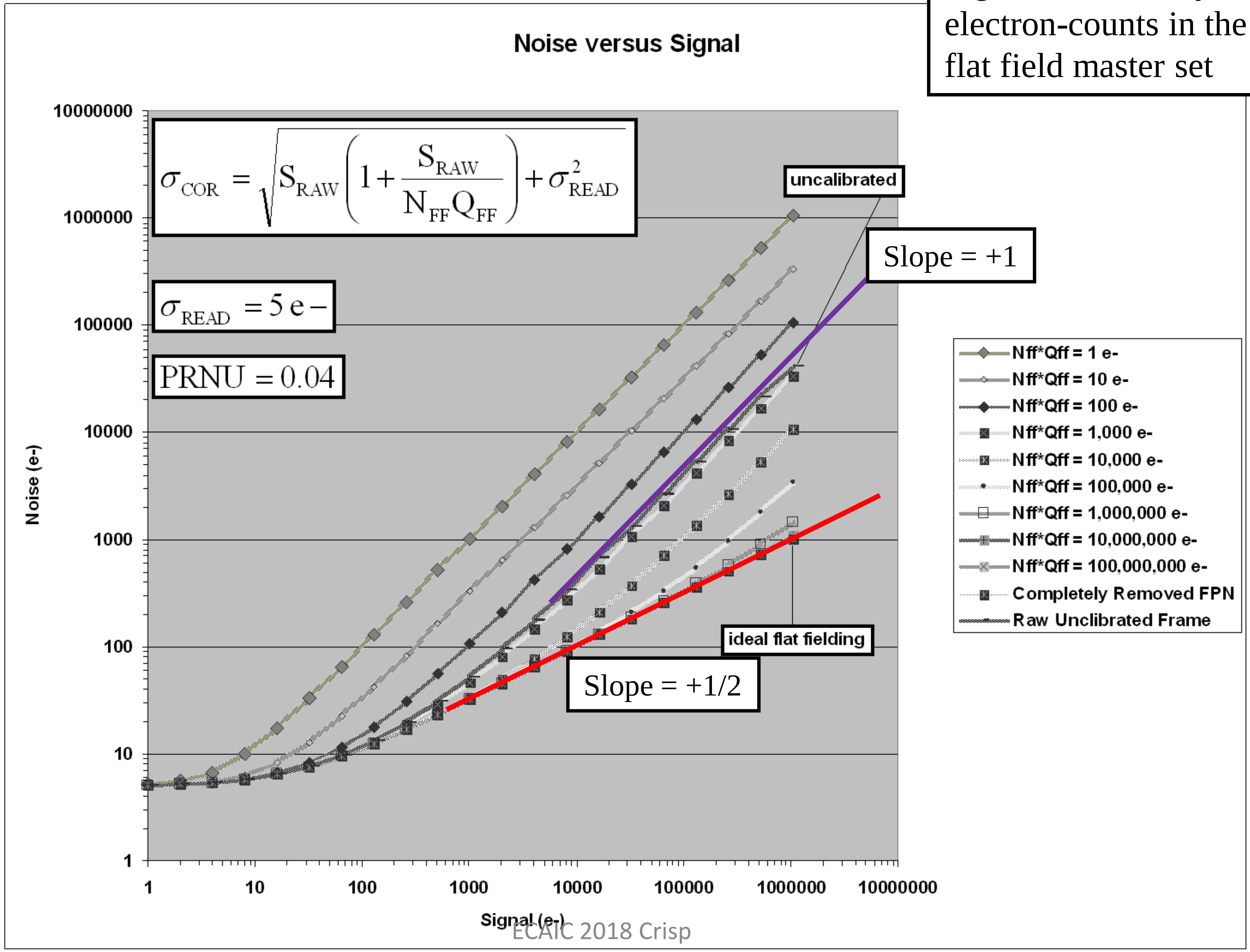
- The FFPTC plots noise against signal for flat field data that has been flat-fielded by a master flat. These are useful to gauge the efficacy of a flat-fielding protocol under development
- For example you can test a proposed set of flats before using them. You can measure the difference in combining 10 versus 25 flats for example

Making a FFPTC

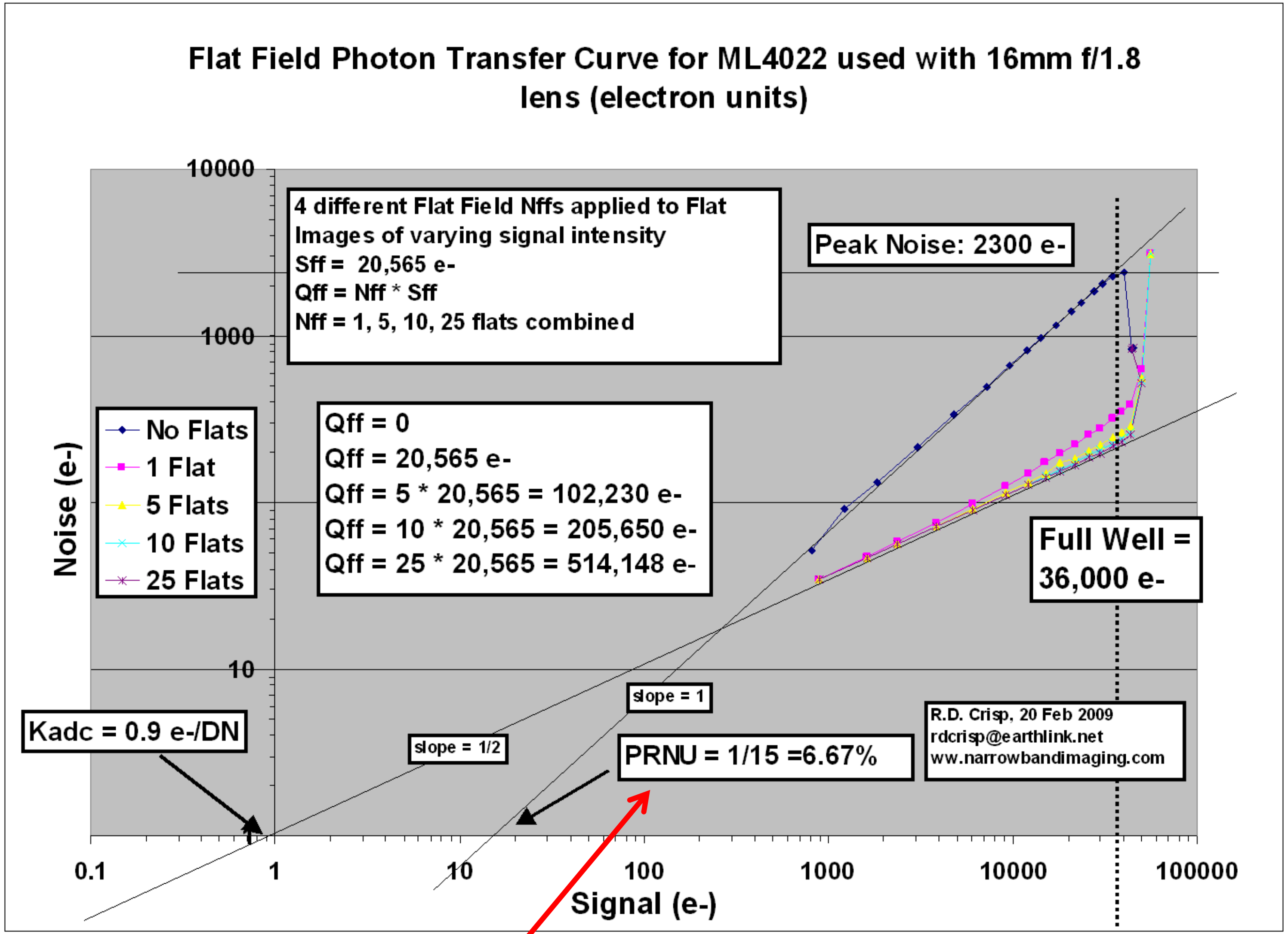
- Slight difference in data collection:
  - Take this data set using your focused optics
  - Take set of flats as you would use for normal calibrations
  - Then shoot a set of exposures (flat field) starting from minimum and keep doubling exposure time until full well is reached
  - Iterate around full well if desired or skip this part
  - No need to take pairs of exposures, single exposures at each point are all that are needed
- Data reduction:
  - Measure offset and subtract from calibration flats
  - Make multiple calibration flat masters: zero frames, 1 frame, 5 frames, 10 frames, 25 frames combined for example
  - Measure and subtract offset from each exposure frame then apply flat field to each
  - Measure average value and standard deviation of each calibrated exposure and plot

FFPTC

Shows Noise versus Signal for a variety of electron-counts in the flat field master set



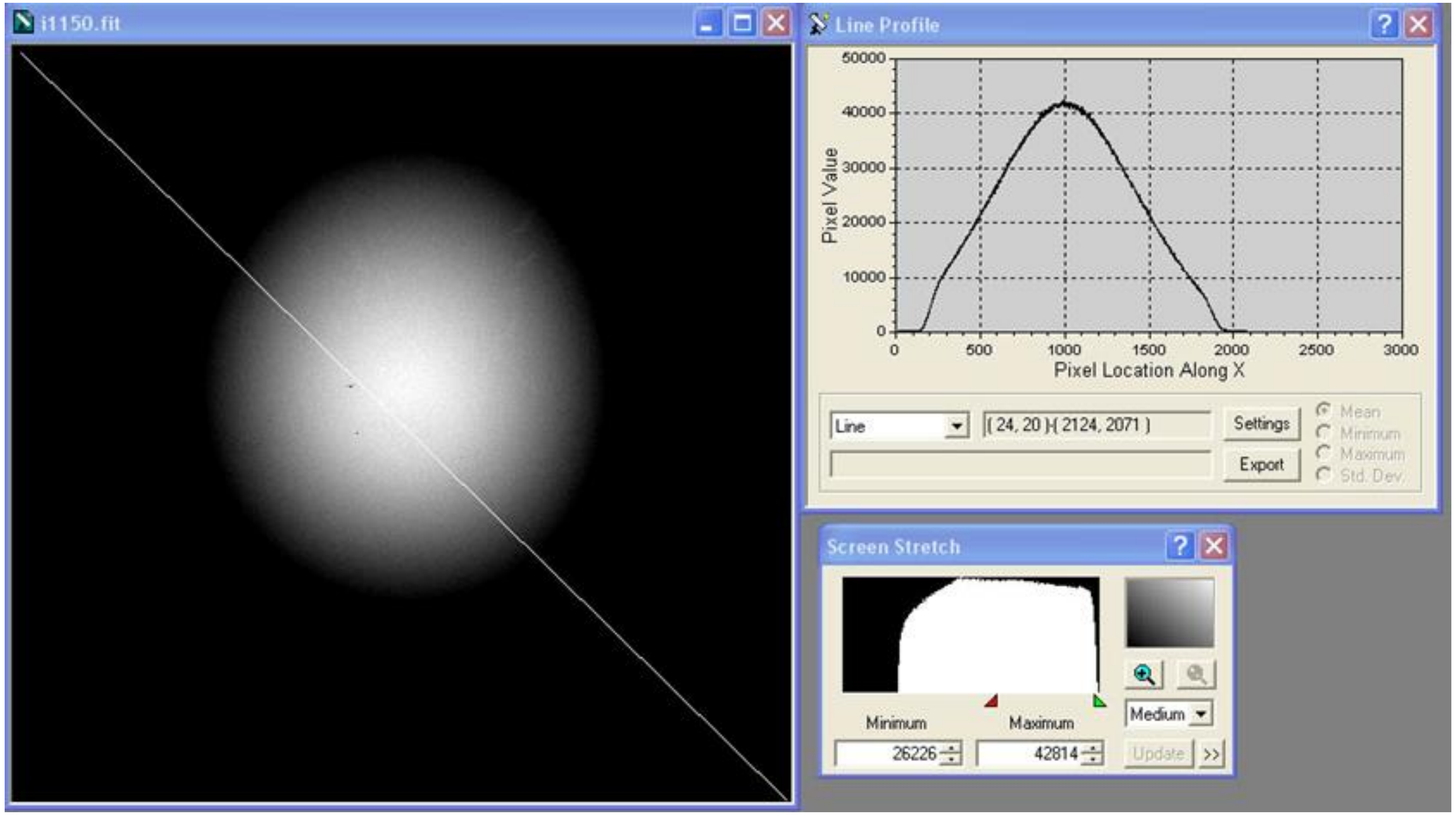
FFPTC used with heavily vignetted optics



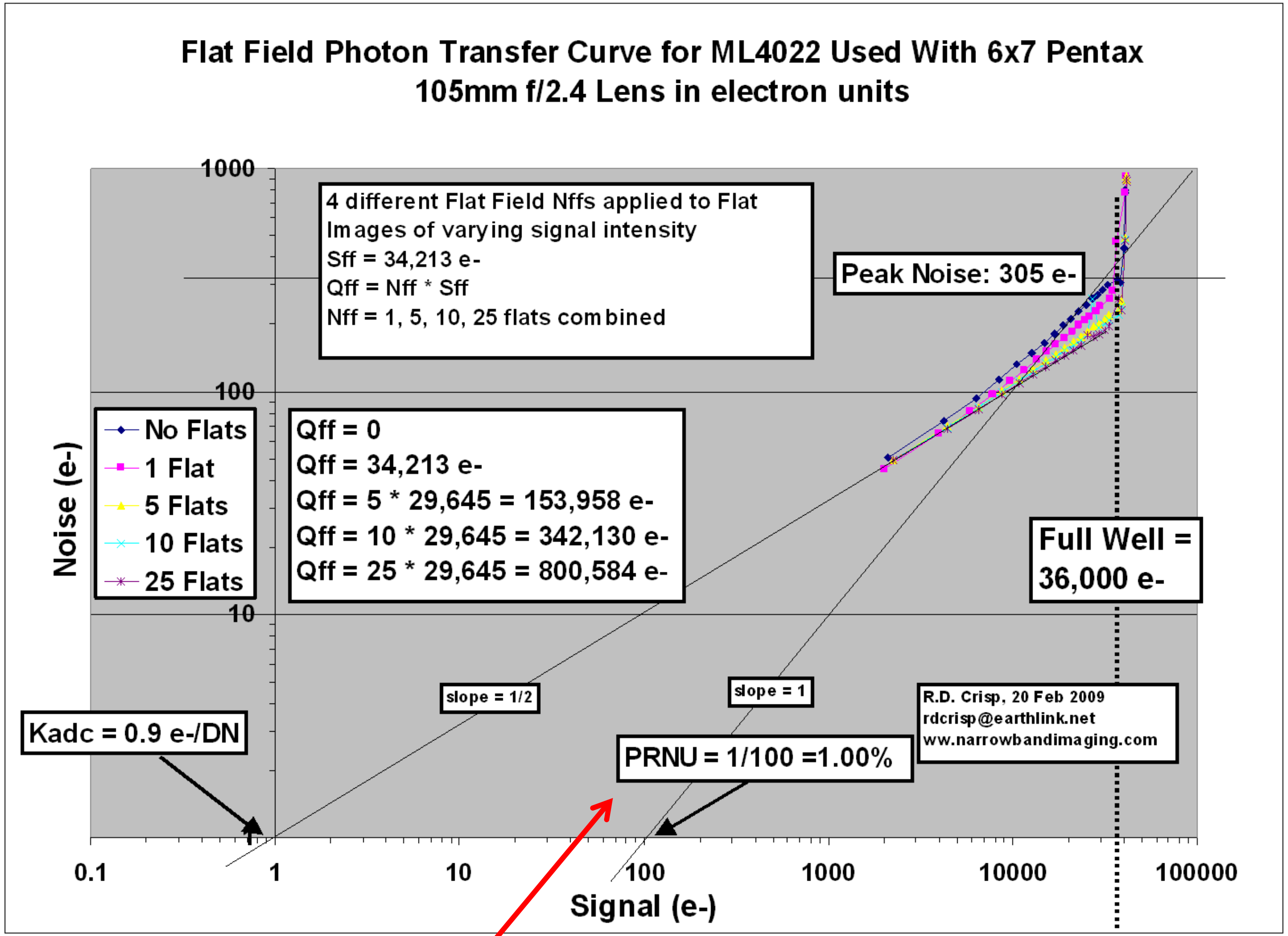
PRNU is very high due to optical vignetting

Heavy vignetting: High system-level PRNU

16mm f/1.8



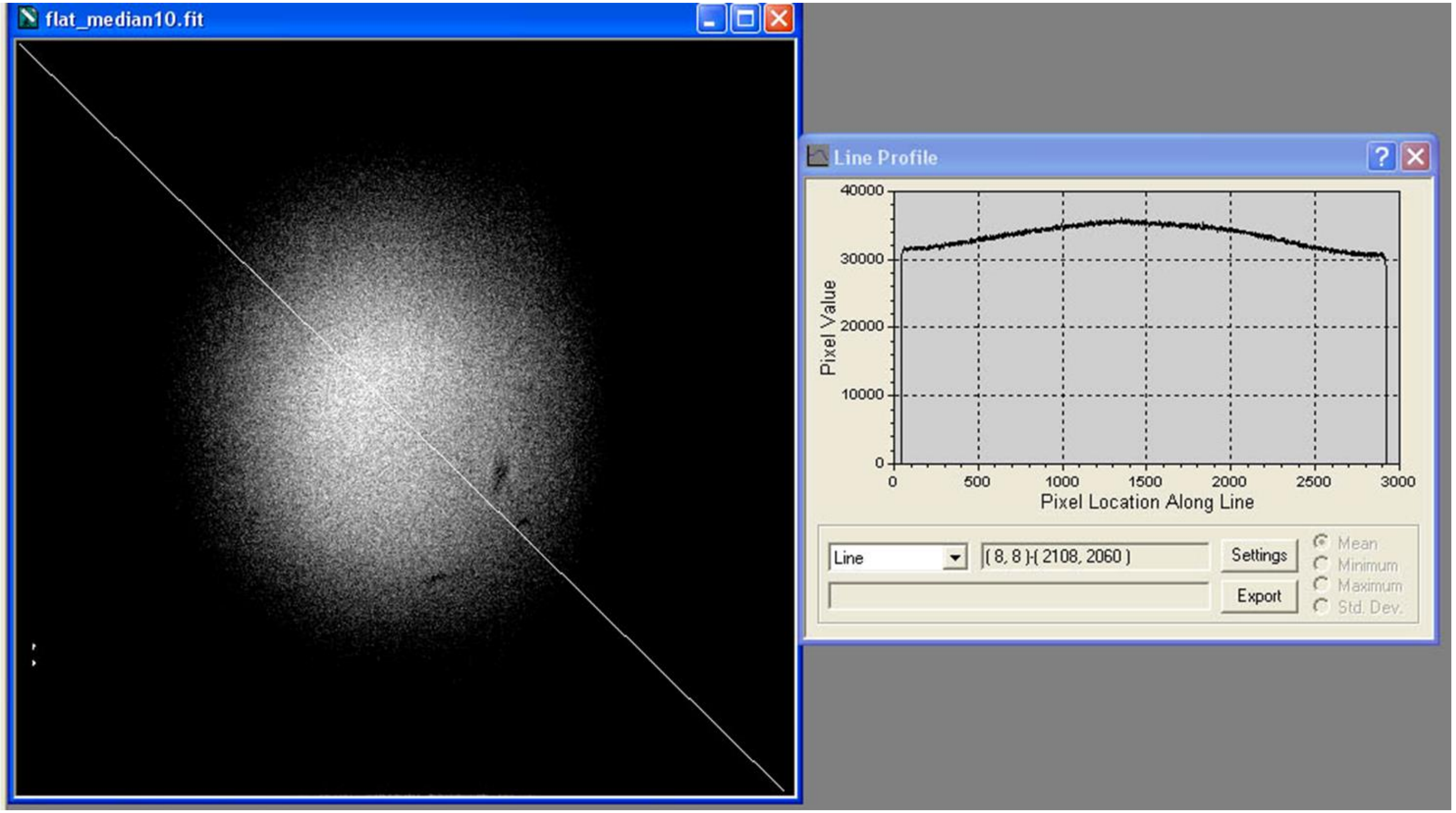
FFPTC used with non-vignetted optics



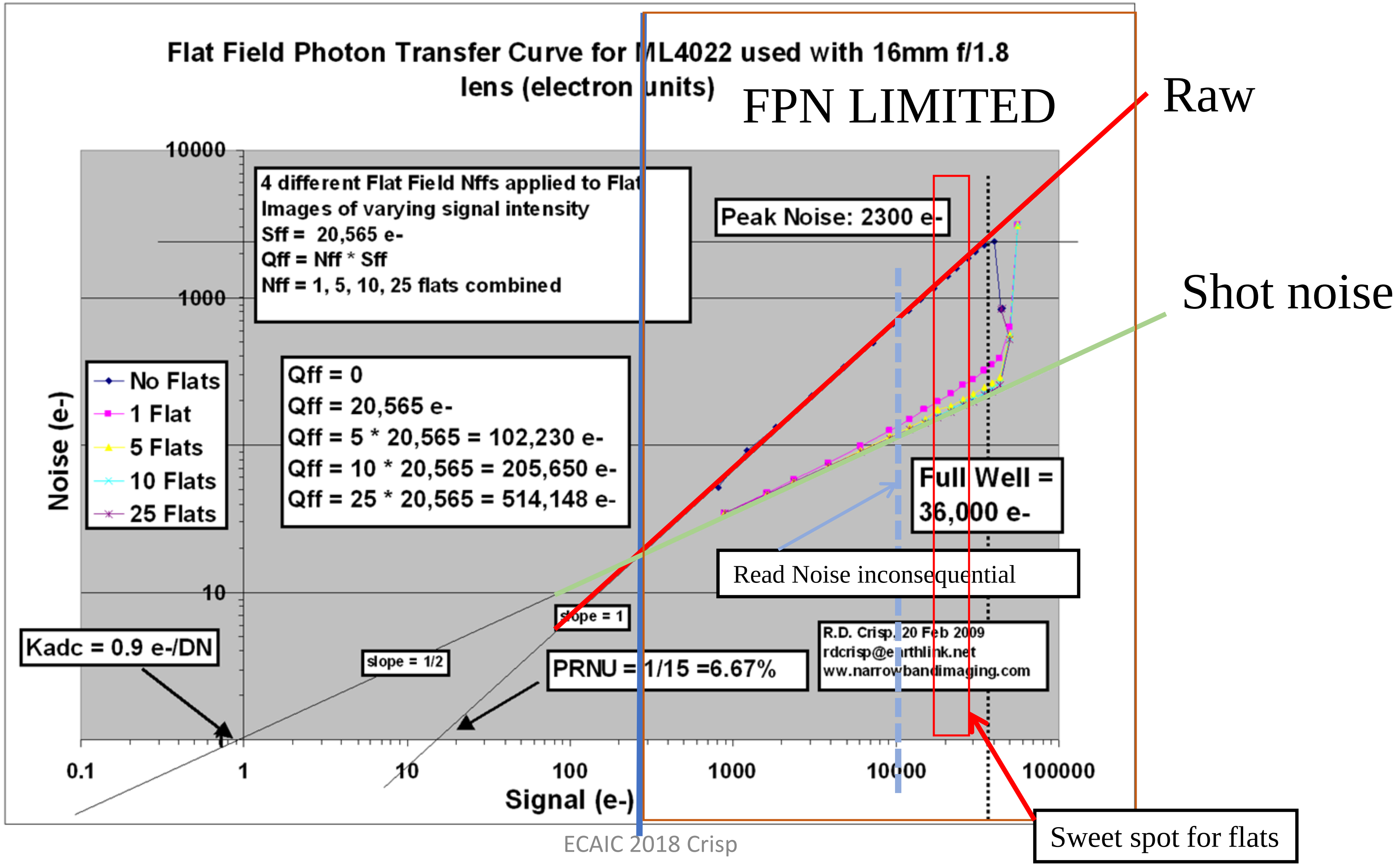
PRNU is in normal range of sensor due to non-vignetted optics

Low vignetting: Low system-level PRNU

Pentax 6x7 105mm f/2.4

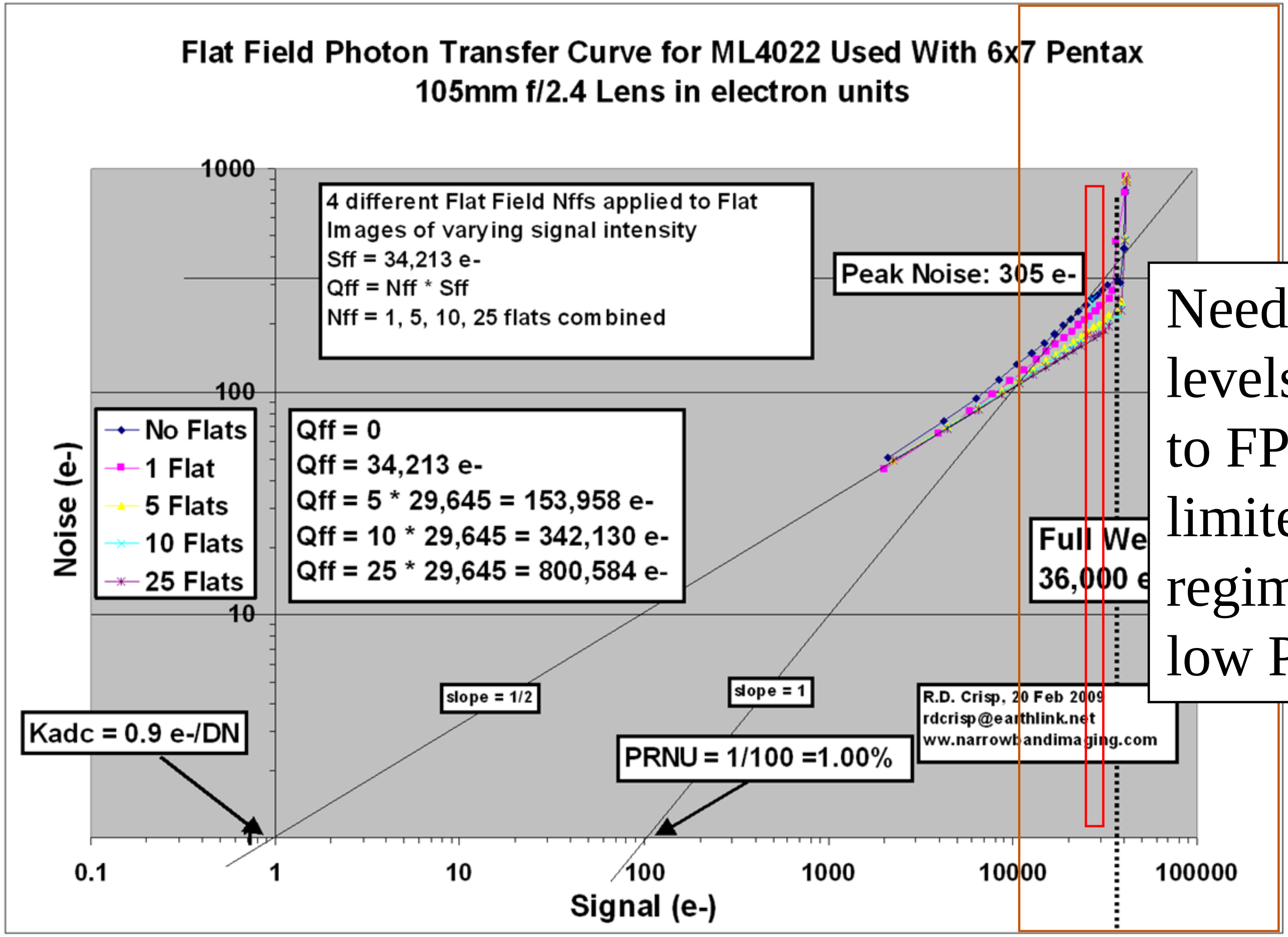


Signal level in flats cont'd (high PRNU case)

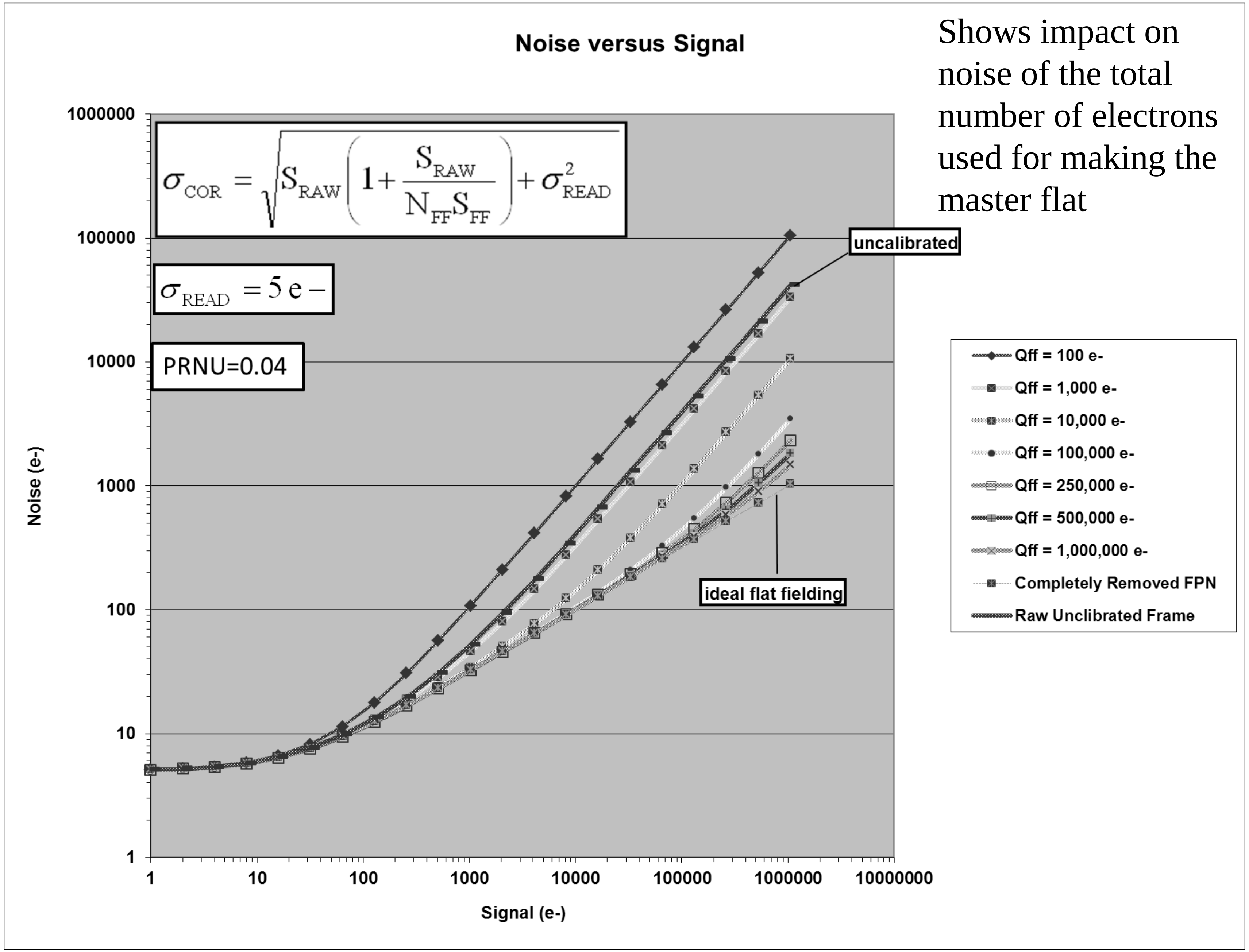


Signal level in flats cont'd (low PRNU case)

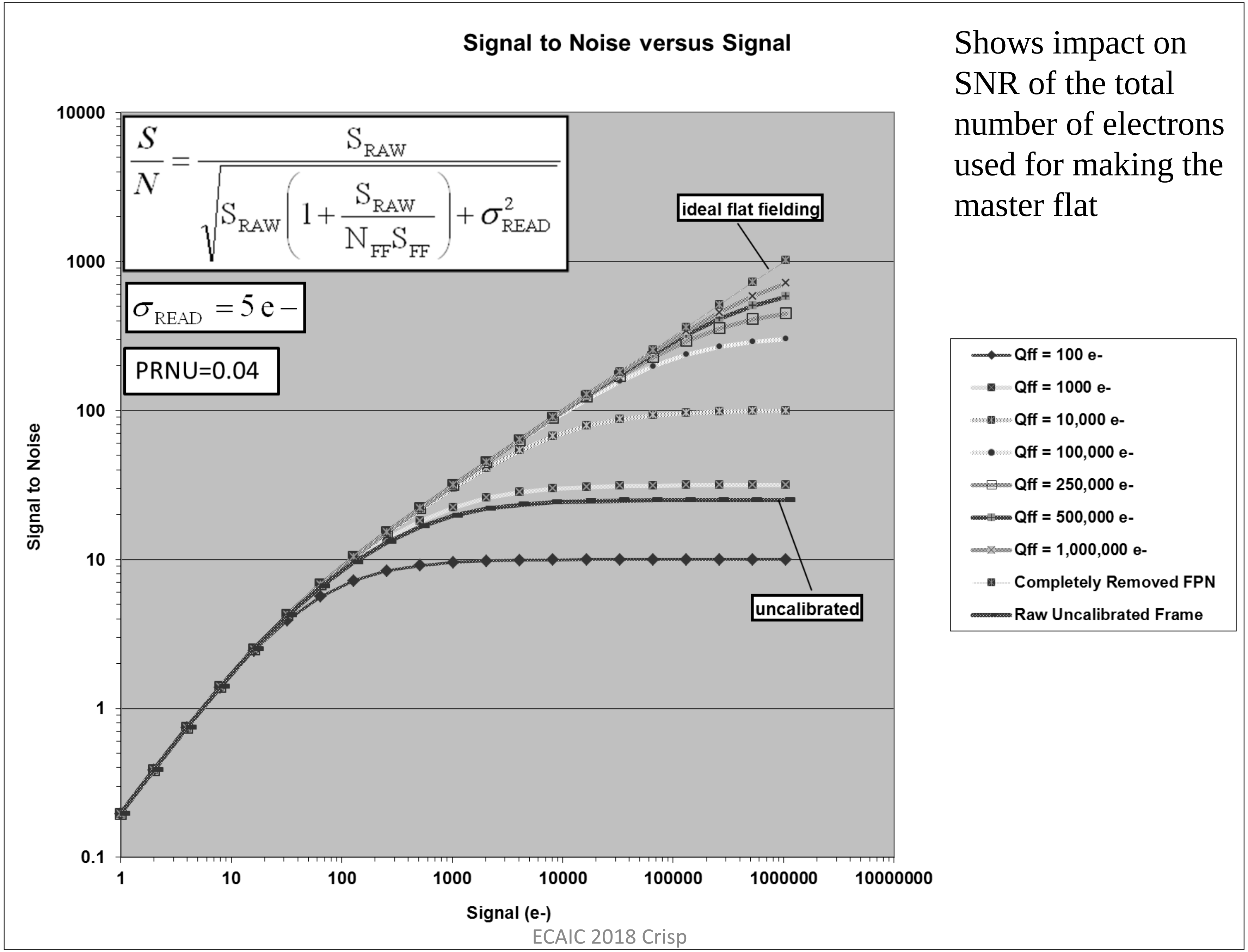
FPN LIMITED



Need higher levels to get to FPN limited regime with low PRNU



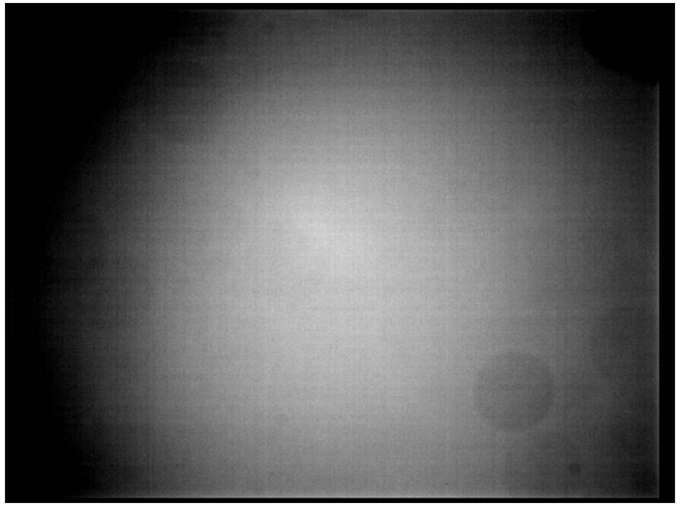
Shows impact on noise of the total number of electrons used for making the master flat

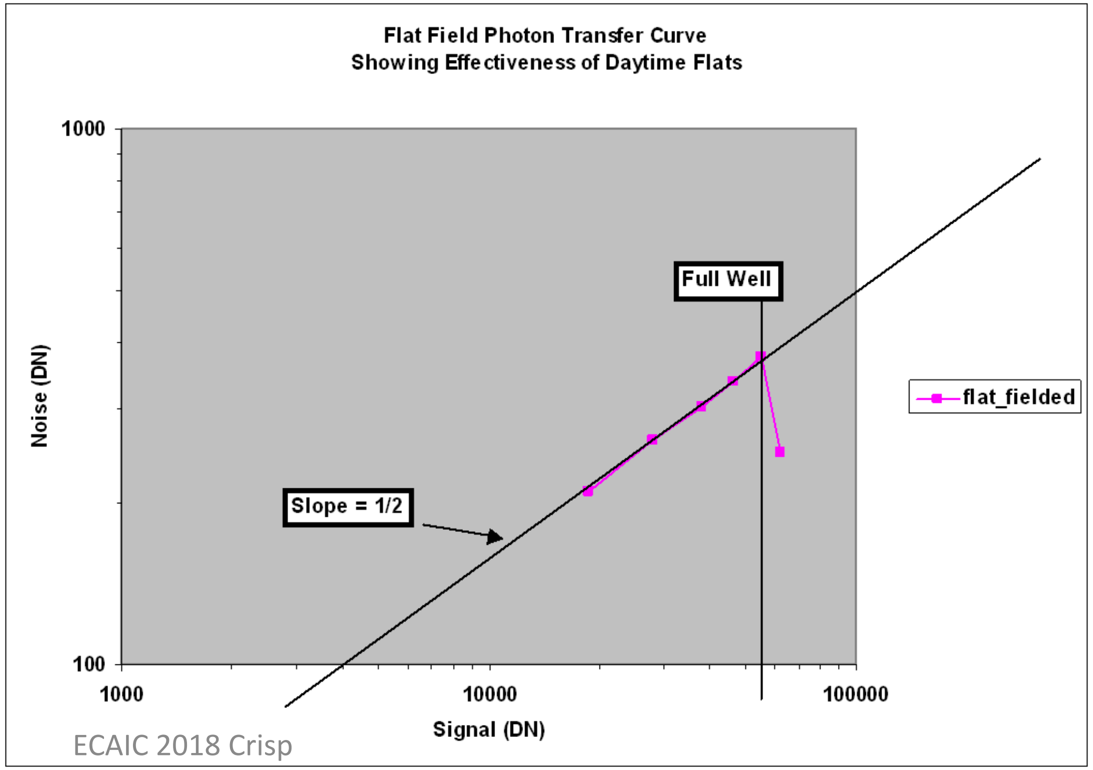


Shows impact on SNR of the total number of electrons used for making the master flat

Testing Master Flat

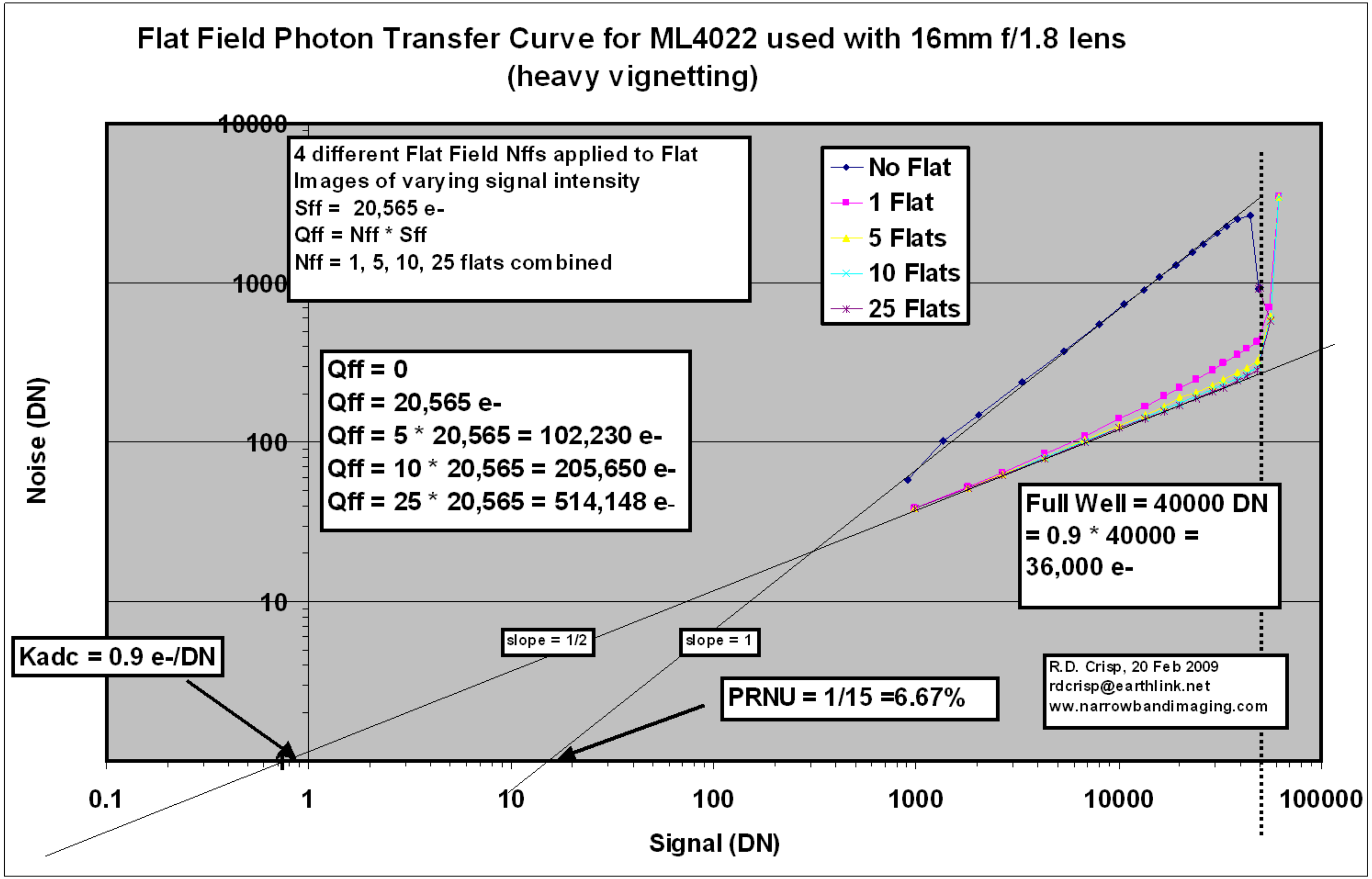
- Once you have prepared a master flat, you can test it by making a FFPTC (flat field photon transfer curve: explained in part 2)
- If the slope of the FFPTC is +1/2 then the FPN is completely removed





FFPTC: How many flats is enough?

- You can also combine varying numbers of the flat frames to make several master flats and test each on the same set of FFPTC raw data to see how many flats are actually necessary to reach a given performance level



For modest signal levels, 5-10 frames seems sufficient for this system

Total Signal Level for “breakeven”

- When a flat is used to flat-field another flat of equal signal level, the shot noise in the resulting image is increased by SQRT(2)

- If the noise (electron units) of the flat-fielded image is set to be equal to the noise of the non-flat-fielded image the “breakeven” level of signal (electrons) in the flat field dataset is determined in terms of PRNU (see next page):
  - For the dataset used for the master flat, we can determine the minimum amount of signal needed to prevent increasing the noise after flat-fielding
  - If the signal is less than this minimum, flat-fielding will increase the noise in the image: this is counter to the purpose of flat-fielding

Breakeven signal level

$$\text{Noise}_{\text{IMAGE}} = \sqrt{\text{Signal} + (\text{Signal} * \text{PRNU})^2 + \text{Read\_Noise}^2} = \sqrt{\text{Signal}\left(1 + \frac{\text{Signal}}{Q_{\text{FF}}}\right) + \text{Read\_Noise}^2}$$

Solving for Q_{FF}

$$Q_{\text{FF}_{\text{Breakeven}}} = \frac{1}{\text{PRNU}^2}$$

This says that as PRNU increases, the minimum number of electrons needed in the flat field set to avoid increasing noise, drops.

Ie: the noisier is the raw image, so the noisier can be the flat without increasing the noise in the calibrated image

The relationship between well capacity and optimum electron-count in flat-field set

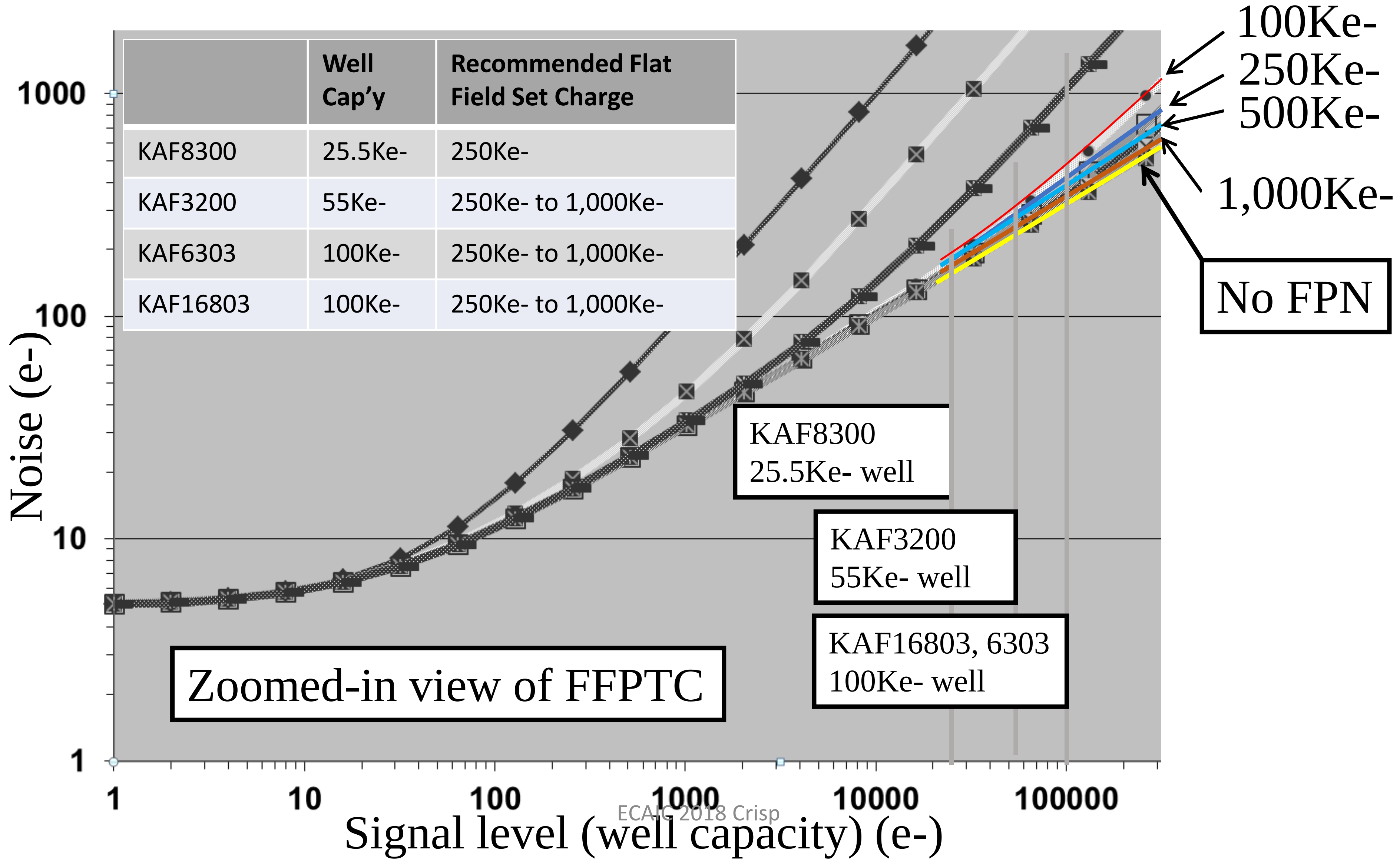
- For good flat-fielding, the signal level in the flats needs to be higher if the signal level in the raw image is higher
  - This is shown in each of the FFPTCs we have seen
  - As signal increases, more electrons are needed in the flat set to remove the FPN
  - The limit to the signal is the well capacity

How do you know what signal level to use for the flats?

- As has been shown, signal levels for the flats should be as high as practical to minimize the FPN and to reduce the total number of flats needed
- The signal level should be high enough so that read noise is inconsequential;
  - ex: 10e- read noise = shot noise of 100e- signal
    - To be inconsequential, should be less than 1-2%: signal levels
    - Signal level at least 10,000 e- or more for 10e- read noise camera
- The signal level should be low enough so that no pixels saturate
- SNR of Flat is proportional to SQRT(#Frames) and proportional to SQRT(signal level in frame).
- Easier to get a good “image” of the FPN if the camera is operated in the FPN-Limited region: fewer flats, high signal level

Finding recommended electron-count for flat-field set for selected sensors

	Well Cap'y	Recommended Flat Field Set Charge
KAF8300	25.5Ke-	250Ke-
KAF3200	55Ke-	250Ke- to 1,000Ke-
KAF6303	100Ke-	250Ke- to 1,000Ke-
KAF16803	100Ke-	250Ke- to 1,000Ke-



Summary

- What is flat fielding and how does it work?
- Flat Field Photon Transfer Charts (FFPTC)
- Testing master flats
- Designing a flat field protocol

PTC Application Example #2 RBI Studies

Key Points

- Determine RBI Trap Capacity
- Determine maximum practical exposure time (vs temperature) with and without RBI Light Flood Mitigation
- Avoid PTC Characterization issues taking low level dark dataset

Data Collection Procedure

- Collect non-RBI mitigated dark data
  - Start camera from power-off regime with sensor at room temperature
  - Leave cooler off: take 100 bias frames and discard
  - Enable cooler: let temperature stabilize
  - Collect pairs of darks: two each of bias and various timed dark frames (60s, 300s, 600s, 900s, 1200s, 1800s) with out using Light Flood RBI Mitigation Protocol
  - Reduce sensor temperature and let stabilize (data collected at -15C to -40C in 5C steps
  - Repeat the collection of pairs of darks

Data Collection Procedure

- Collect RBI mitigated dark data
  - Start camera from power-off regime with sensor at room temperature
  - Enable cooler: let sensor temperature stabilize at target
  - Collect set of pairs of darks: two each of bias and various timed dark frames (60s, 300s, 600s, 900s, 1200s, 1800s) using Light Flood RBI Mitigation Protocol
  - Reduce sensor temperature and let stabilize (data collected at -15C to -40C in 5C steps
  - Repeat the collection of pairs of darks

Data Reduction: Measuring Total Noise

- Select a pair of identical exposures, add 10,000DN to one frame and subtract the other identical frame from it (you add the 10,000DN offset to prevent clipping the histogram) and save result
- Repeat for set of data
- Using 100 x 100 selection window, record the standard deviation of a low noise portion of each difference frame
- The Standard Deviation = Sqrt 2 * Total Noise

PTC Equations

$$Total\_noise = \sqrt{Read\_noise^2 + Dark\_shot\_noise^2} \quad (1)$$

$$Dark\_shot\_noise = \sqrt{Total\_noise^2 - Read\_noise^2} \quad (2)$$

$$Dark\_shot\_noise = \sqrt{Total\_dark\_signal} \quad (3)$$

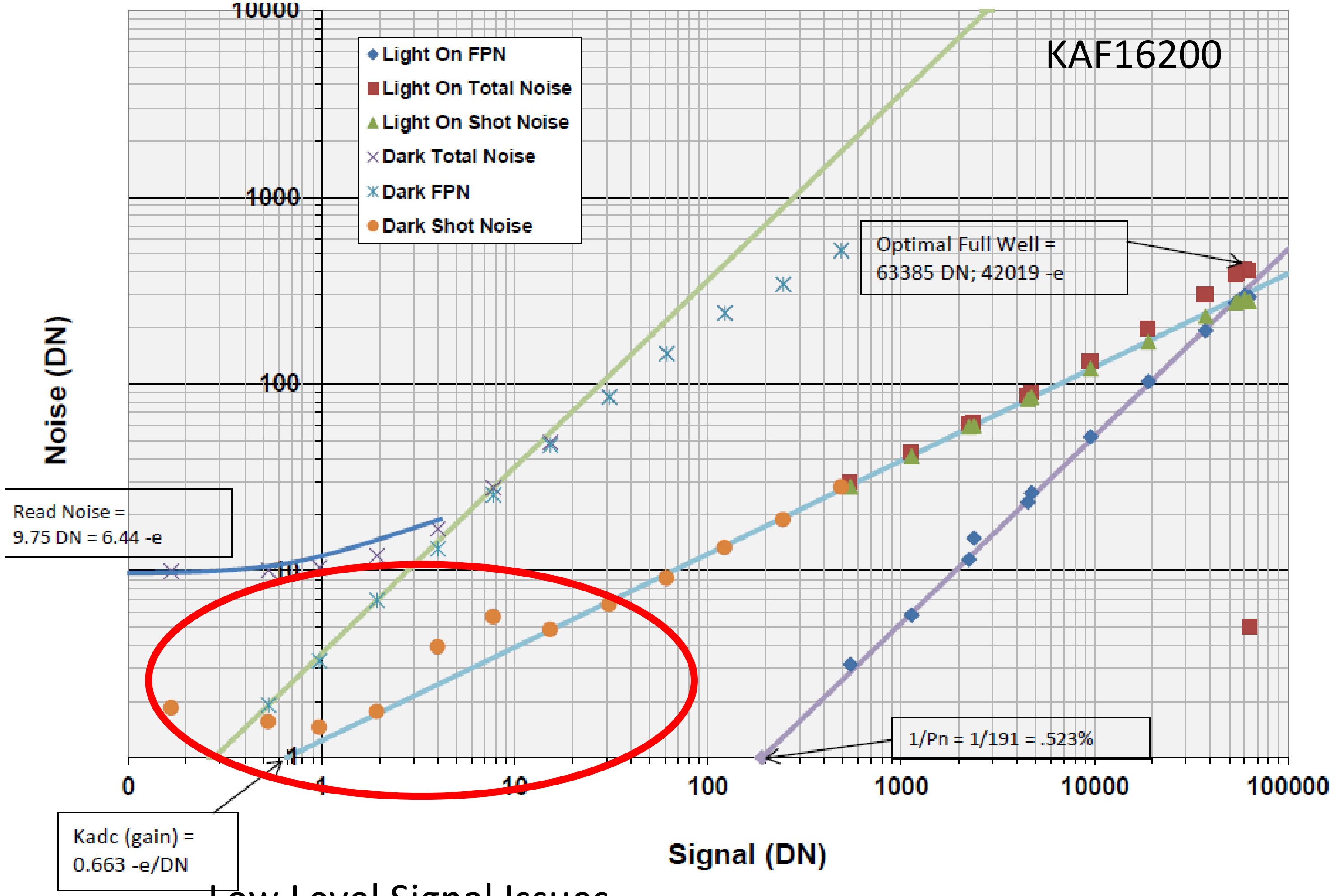
$$Total\_dark\_signal = Thermal\_dark\_signal + Trap\_leakage \quad (4)$$

For no-light flood case, Trap_leakage is zero:

$$Total\_dark\_signal = Thermal\_dark\_signal \quad (5)$$

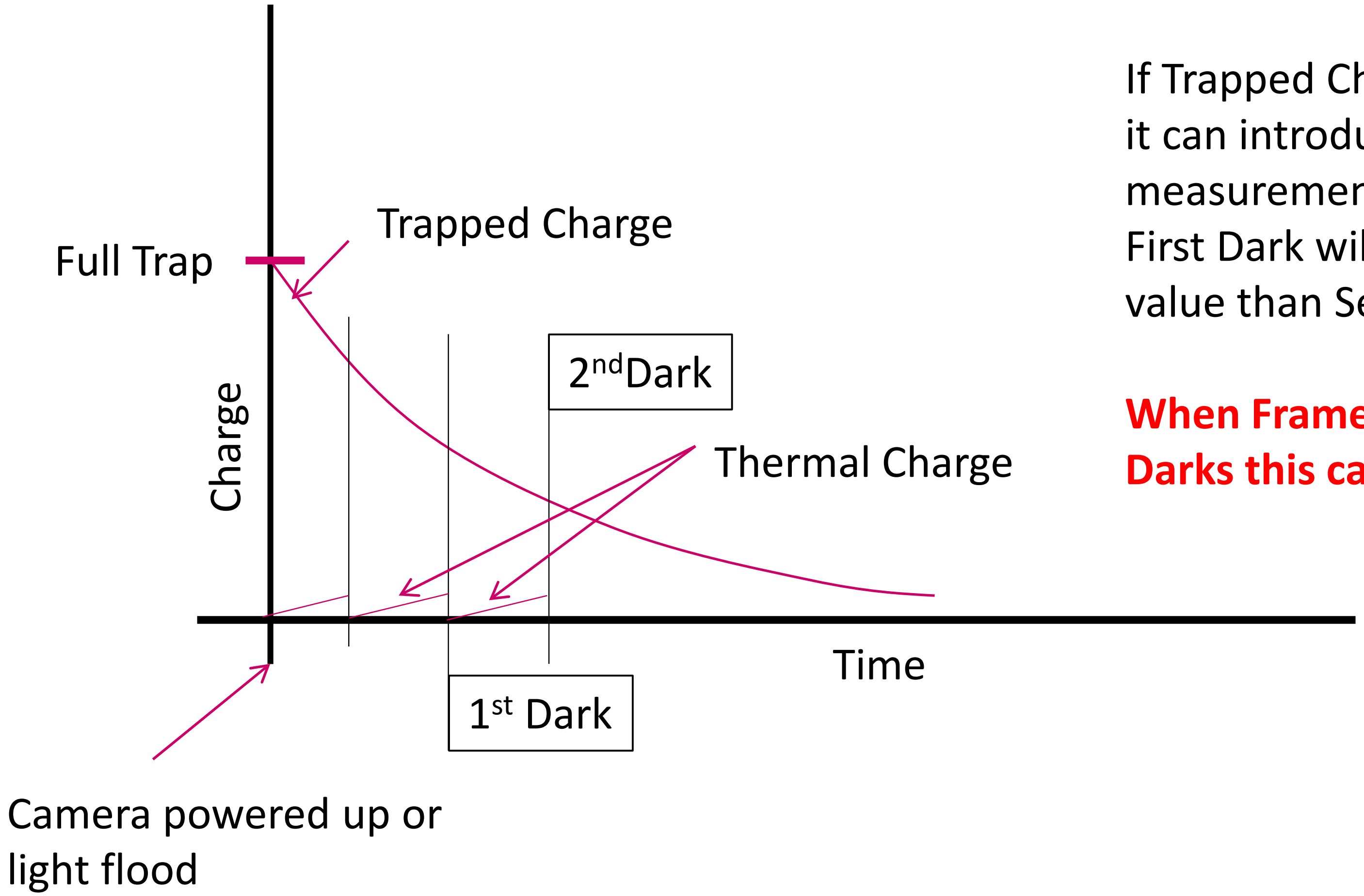
$$Trap\_leakage = Total\_noise^2 - Read\_noise^2 - Thermal\_dark\_signal \quad (6)$$

Trapped Charge: Hazard for PTCs, No flushing before cooling after power-up



Low Level Signal Issues

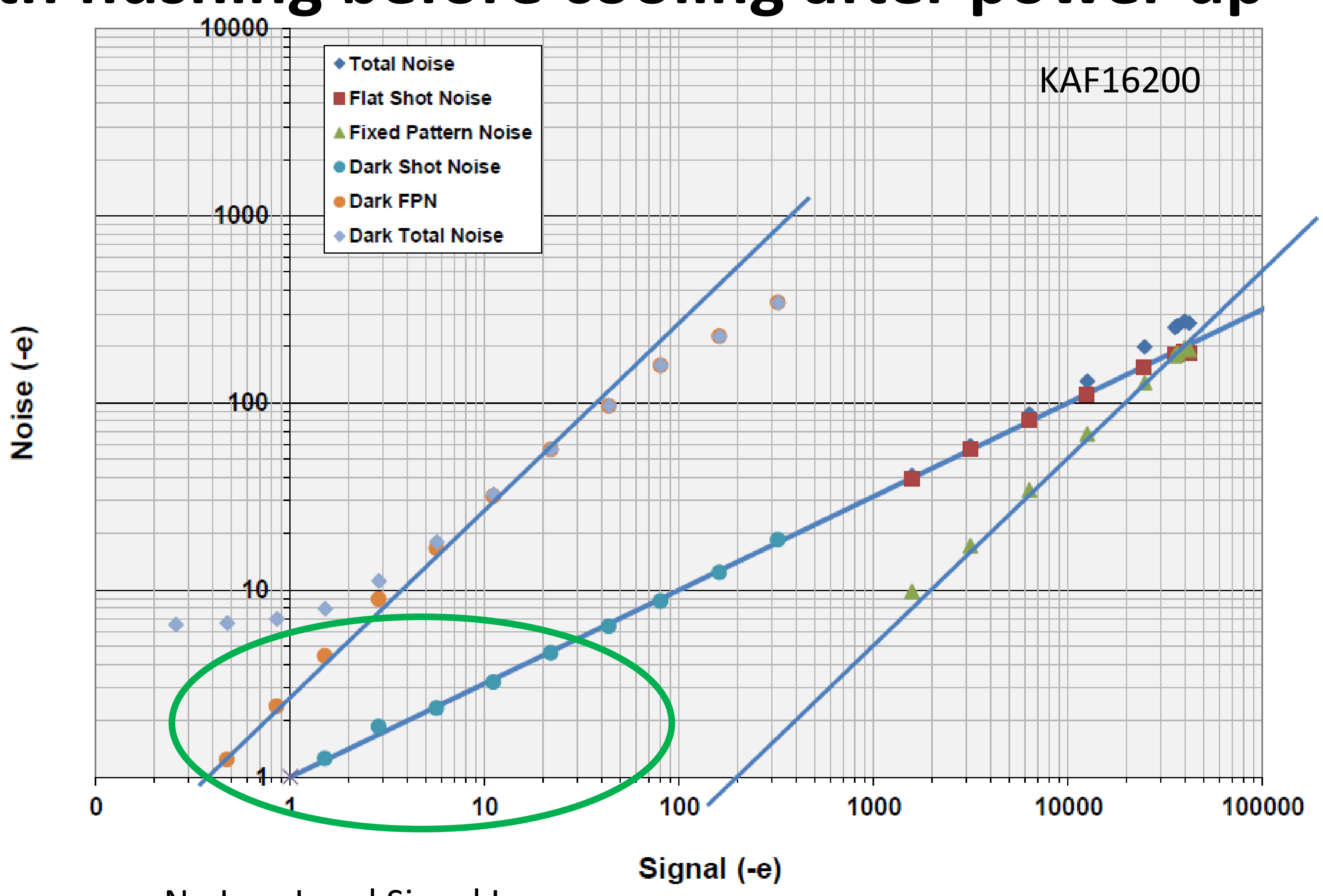
Low Level Signal Issues: differencing exposures in presence of trapped charge



If Trapped Charge is leaking, it can introduce measurement errors: First Dark will have higher value than Second Dark

When Frame-Differencing Darks this can be a problem

With flushing before cooling after power up



No Low Level Signal Issues

Trapped Charge PTC Investigation Methodology

Use Photon Transfer Methods

- Use PTC characterization data for Read Noise and Camera Gain measurement
- Measure Dark Signal Noise versus Time
- Take pairs of identical dark exposures and difference them to eliminate DFPN (leaves read noise and dark shot noise in remaining difference)
- Two major cases: with and without light flood
- Examine at -15, -20, -25, -30, -35 & -40C operating temperature

$$Total\_noise = \sqrt{Read\_noise^2 + Dark\_shot\_noise^2} \quad (1)$$

$$Dark\_shot\_noise = \sqrt{Total\_noise^2 - Read\_noise^2} \quad (2)$$

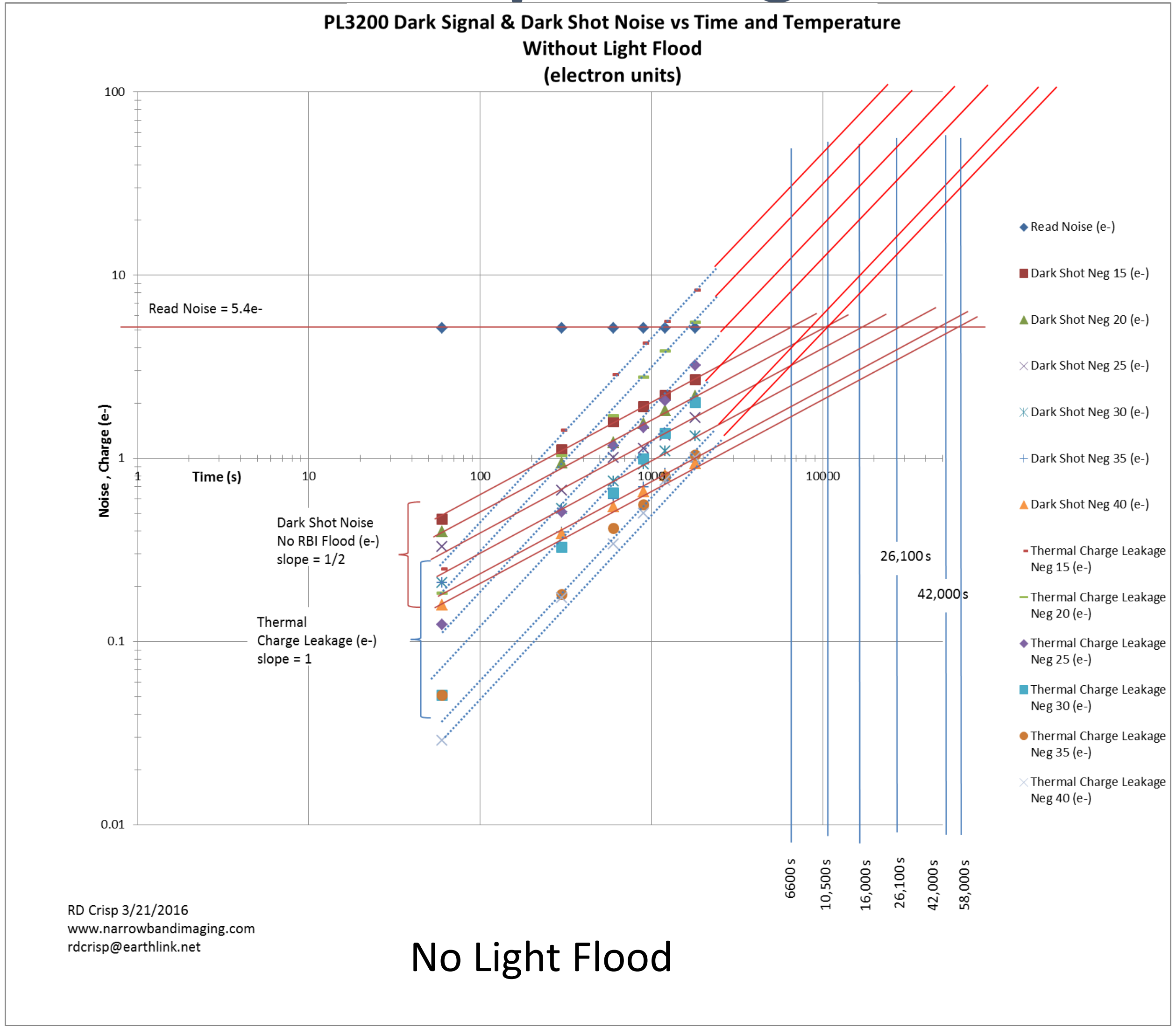
$$Dark\_shot\_noise = \sqrt{Total\_dark\_signal} \quad (3)$$

$$Total\_dark\_signal = Thermal\_dark\_signal + Trap\_leakage \quad (4)$$

For no-light flood case, Trap_leakage is zero:

$$Total\_dark\_signal = Thermal\_dark\_signal \quad (5)$$

Baseline Case: no trap leakage

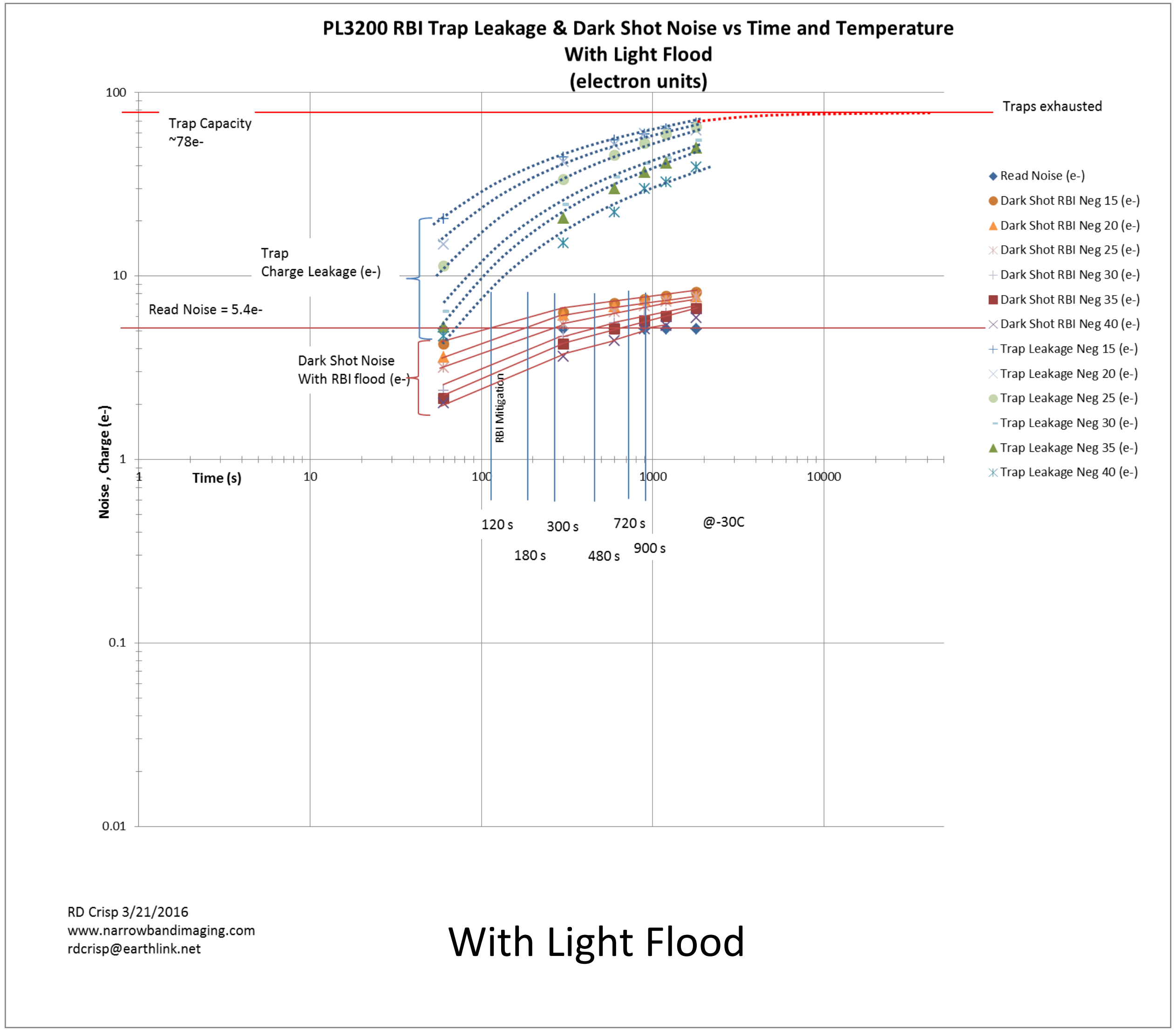


Calculating Trap Leakage

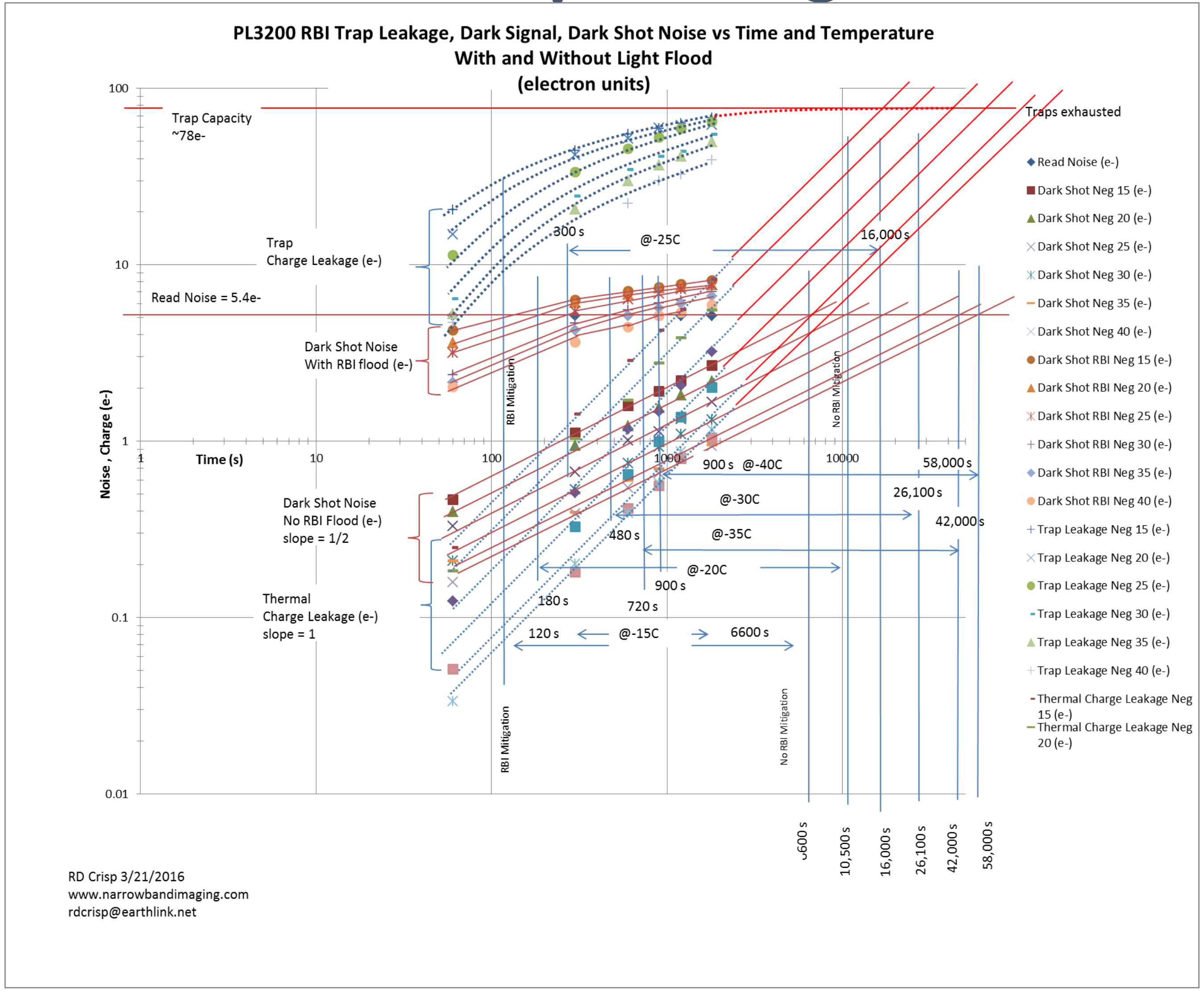
To determine the trap leakage you use the thermal dark signal data from the non light-flooded case and the Total Noise from the light-flooded case

$$Trap\_leakage = Total\_noise^2 - Read\_noise^2 - Thermal\_dark\_signal \tag{6}$$

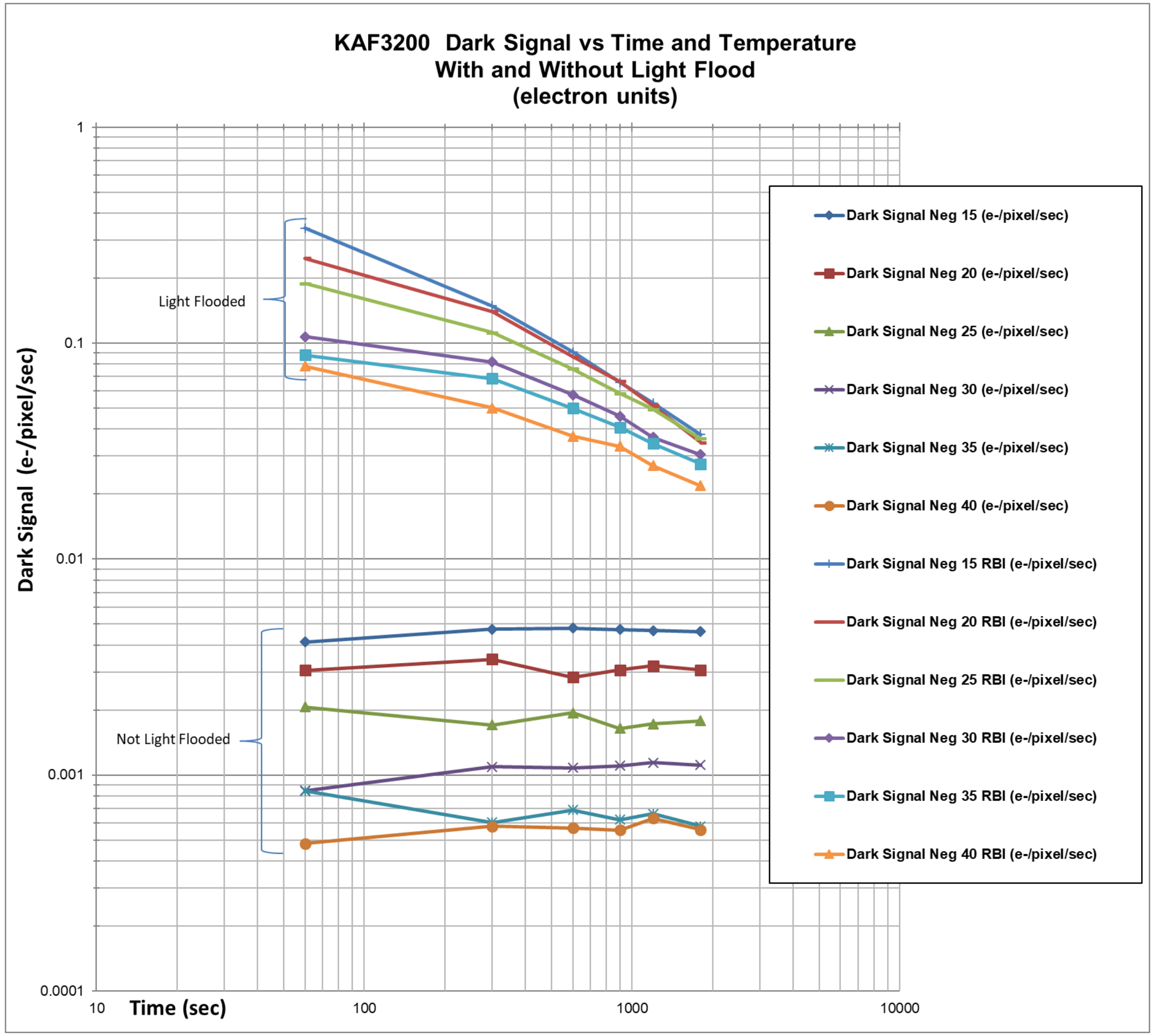
With Trap Leakage



With and Without Trap Leakage



Dark Signal With and Without Trap Leakage



Summary of Results (FLI Proline 3200)

Operating Temperature (Celsius)	Max Practical Exposure* W/O RBI Mitigation (seconds)	Max Practical Exposure with RBI Mitigation (seconds)
-15	6,600	120
-20	10,500	180
-25	16,000	300
-30	26,100	480
-35	42,000	720
-40	58,000	900

Read Noise = 5.4 e-
Kadc = 0.8668 e-/DN

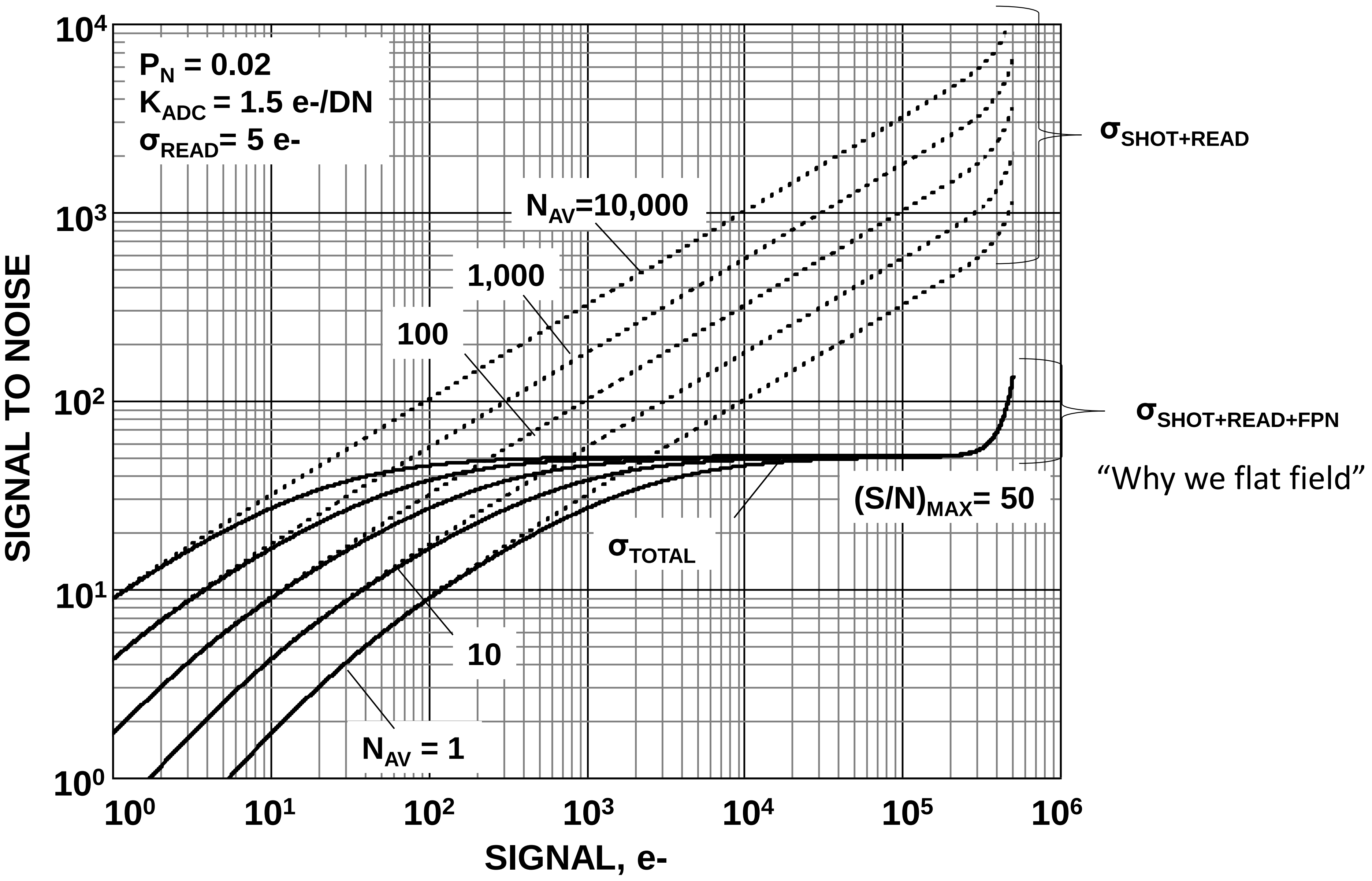
*Maximum Practical Exposure Time
Defined as that exposure time when the Dark Shot noise matches the Read Noise

Summary

- Measuring Critical RBI Parameters via PTC
  - Trap Capacity
  - Trap Decay
  - Maximum Exposure Time vs Temperature
- Avoiding Hazards involving trapped charge when making PTCs

Application #3 Random Noise impact on # exposures needed for Target SNR Goal

IMAGE COMBINING



Goals, Method

- Determine # exposures needed to stack to attain an arbitrary SNR for a given system noise* and signal level
- Use noise equation to solve analytically
- Plot results for specific values

system noise* (quadrature sum of read noise and dark shot noise)

Equations

$$Noise = \sqrt{Signal + SystemNoise^2} \tag{1}$$

$$SNR = \frac{Signal}{\sqrt{Signal + SystemNoise^2}} \tag{2}$$

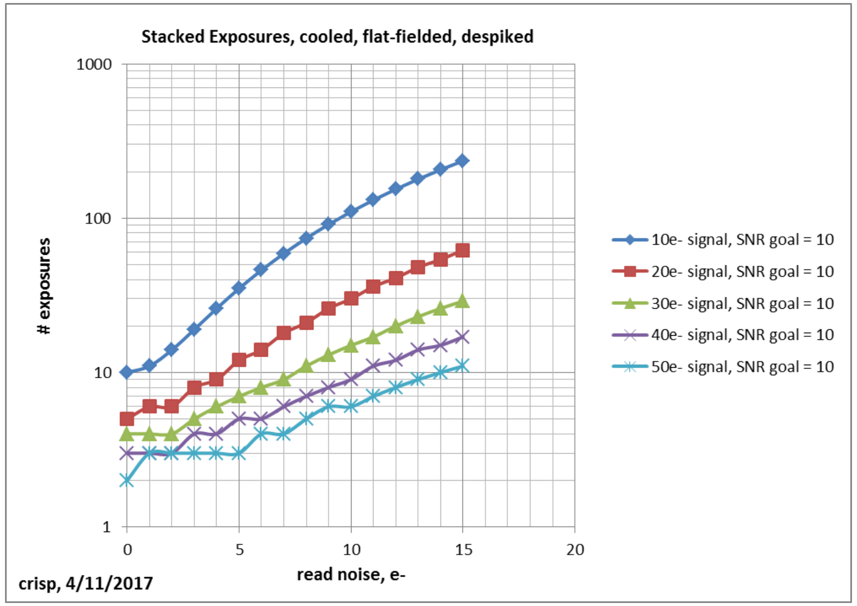
Results for Specific Cases

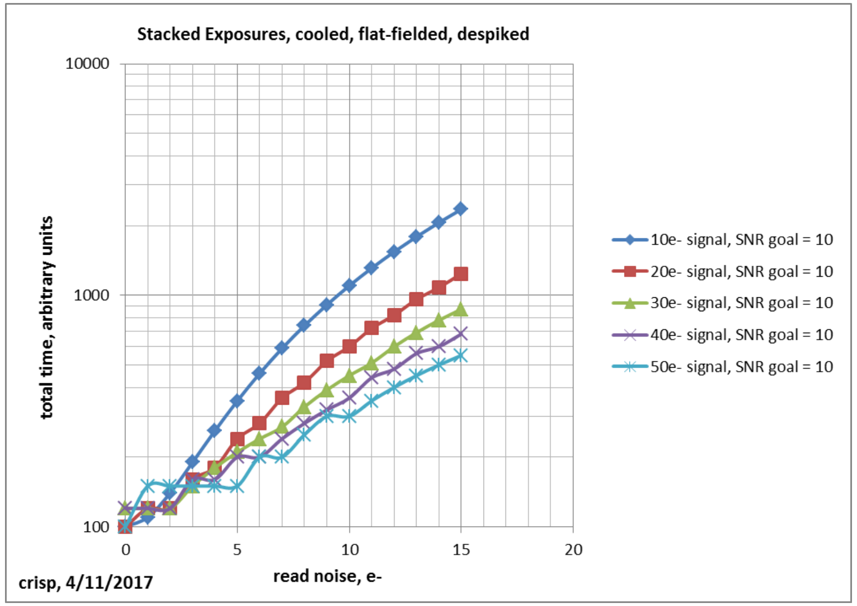
- Case 1 Narrowband
  - Signal levels range from 10 e- to 50 e-
  - System noise: 0 to 15 electrons
  - SNR goal for stacked result: 10
- Case 2 broadband
  - Signal levels range from 100 e- to 500 e-
  - System noise: 0 to 15 electrons
  - SNR goal for stacked result: 50

Can cool such that Dark Shot noise is negligible in most cases

Comment on Time Units

- Arbitrary units are chosen for time
- One arbitrary time unit is that amount of exposure time that results in 1 electron of signal
- Example: 10 e- takes 10 arbitrary time units





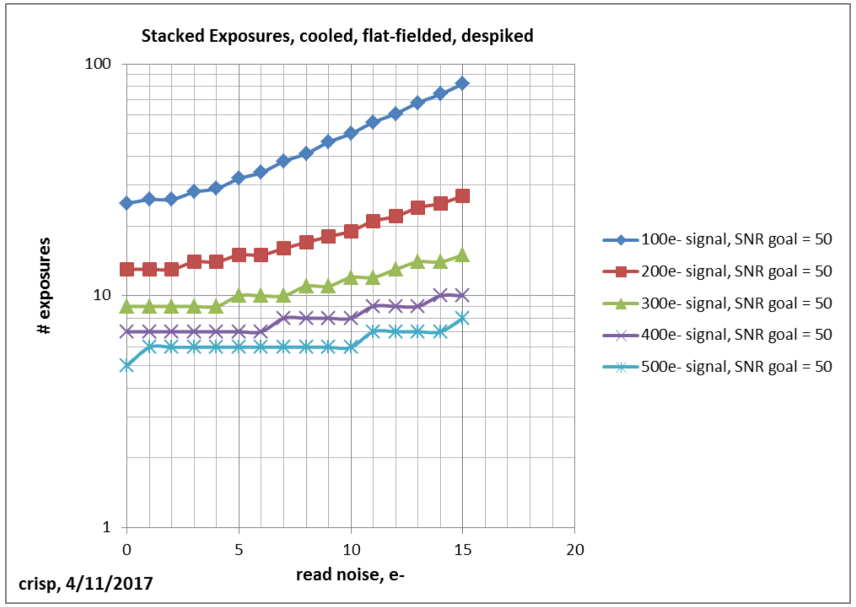
Side note:

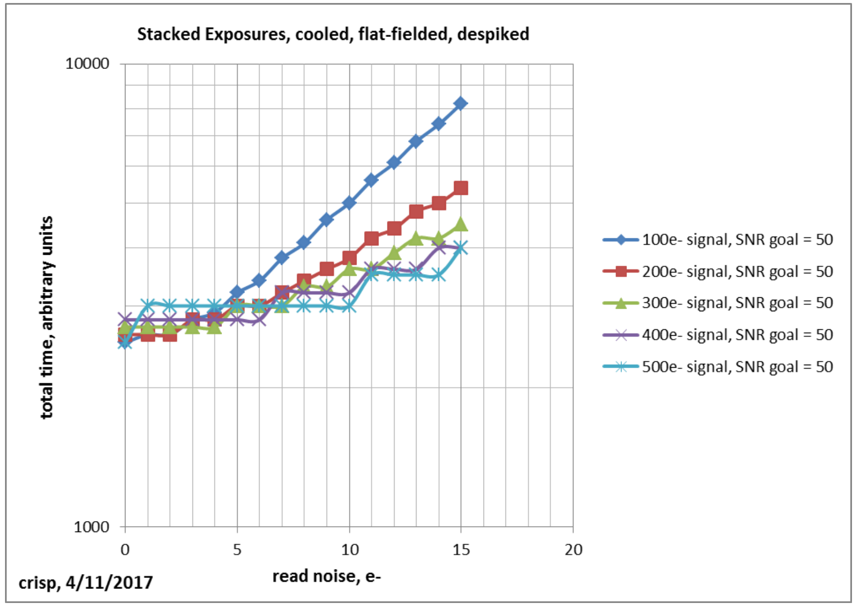
To include "sky noise"

Redefine "read noise" as quadrature sum of read noise and sky noise. Both are uncorrelated so they can be added in quadrature

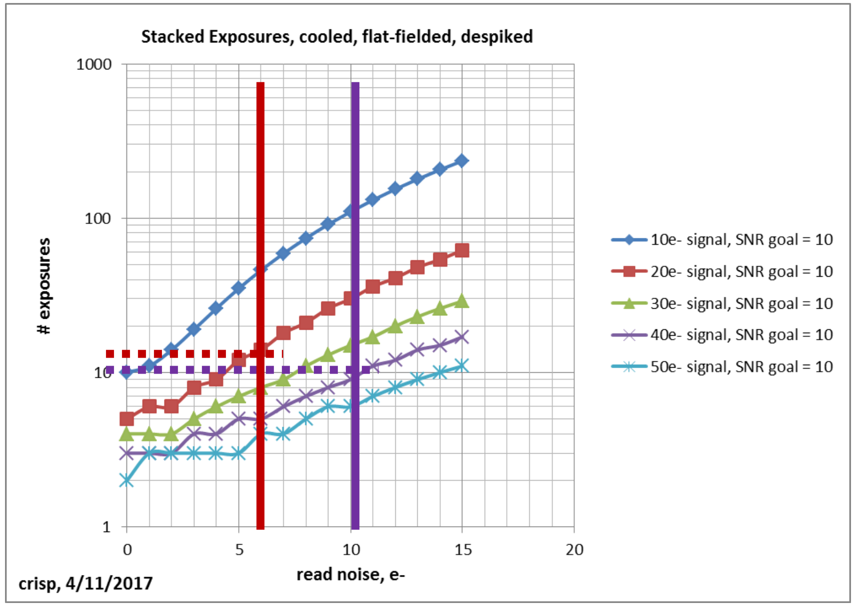
Sqrt(read_noise^2 + sky_noise^2)

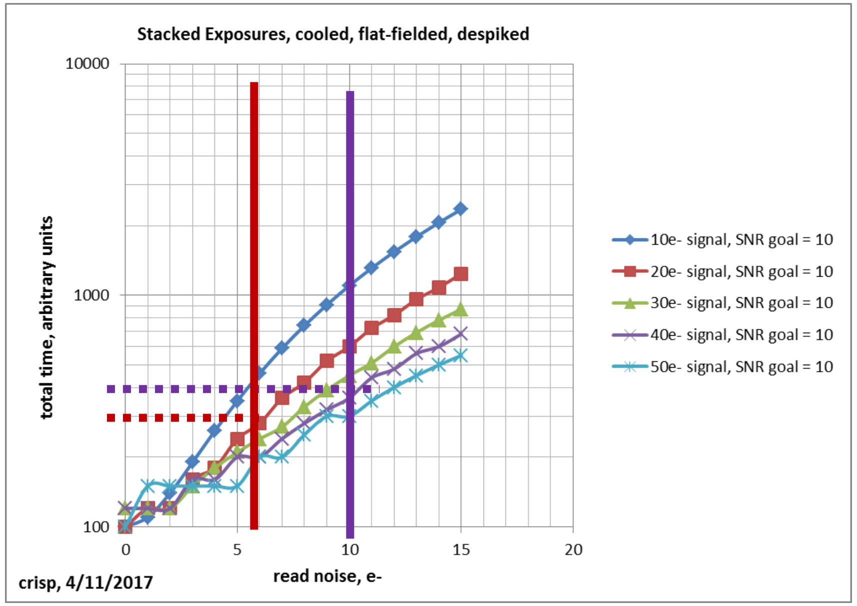
Narrowband Typical





Broadband Typical

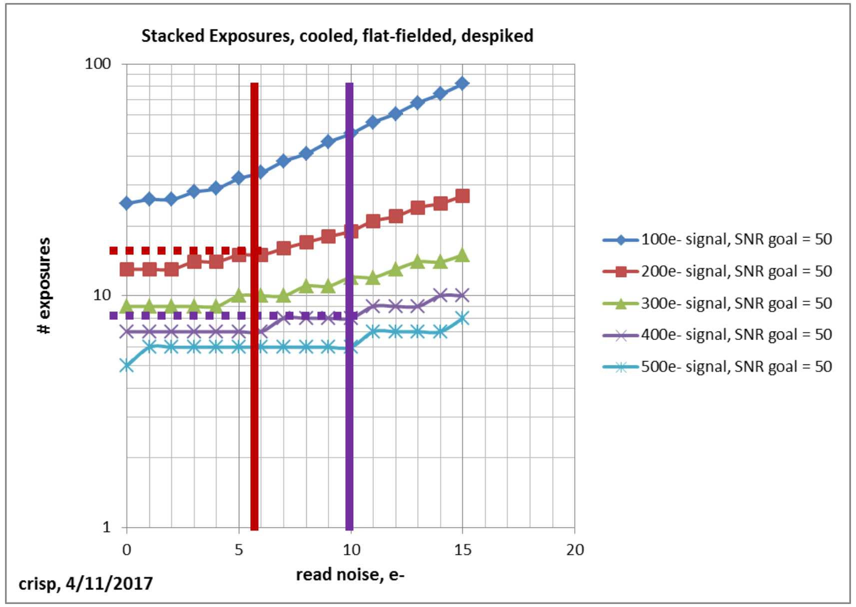


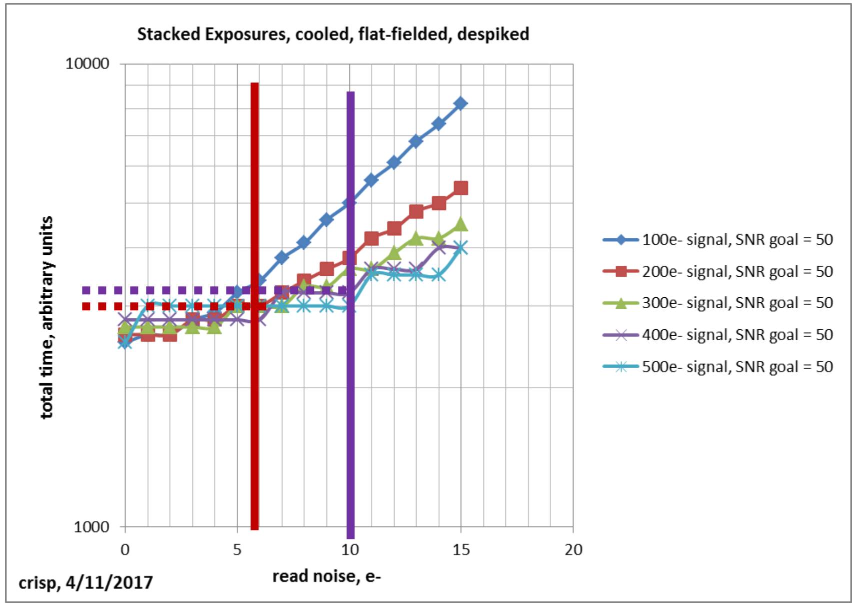


Narrowband Typical

Assume 10 e- read noise Camera 1 and 40 e- signal levels
Assume 6 e- read noise Camera 2 and 20 e- signal levels
Compare total exposure time for SNR 10

camera	Read Noise (e-)	Signal level (e-)	# exp	Total Time (arb units)
Camera 1	10	40	9	360
Camera 2	6	20	14	280



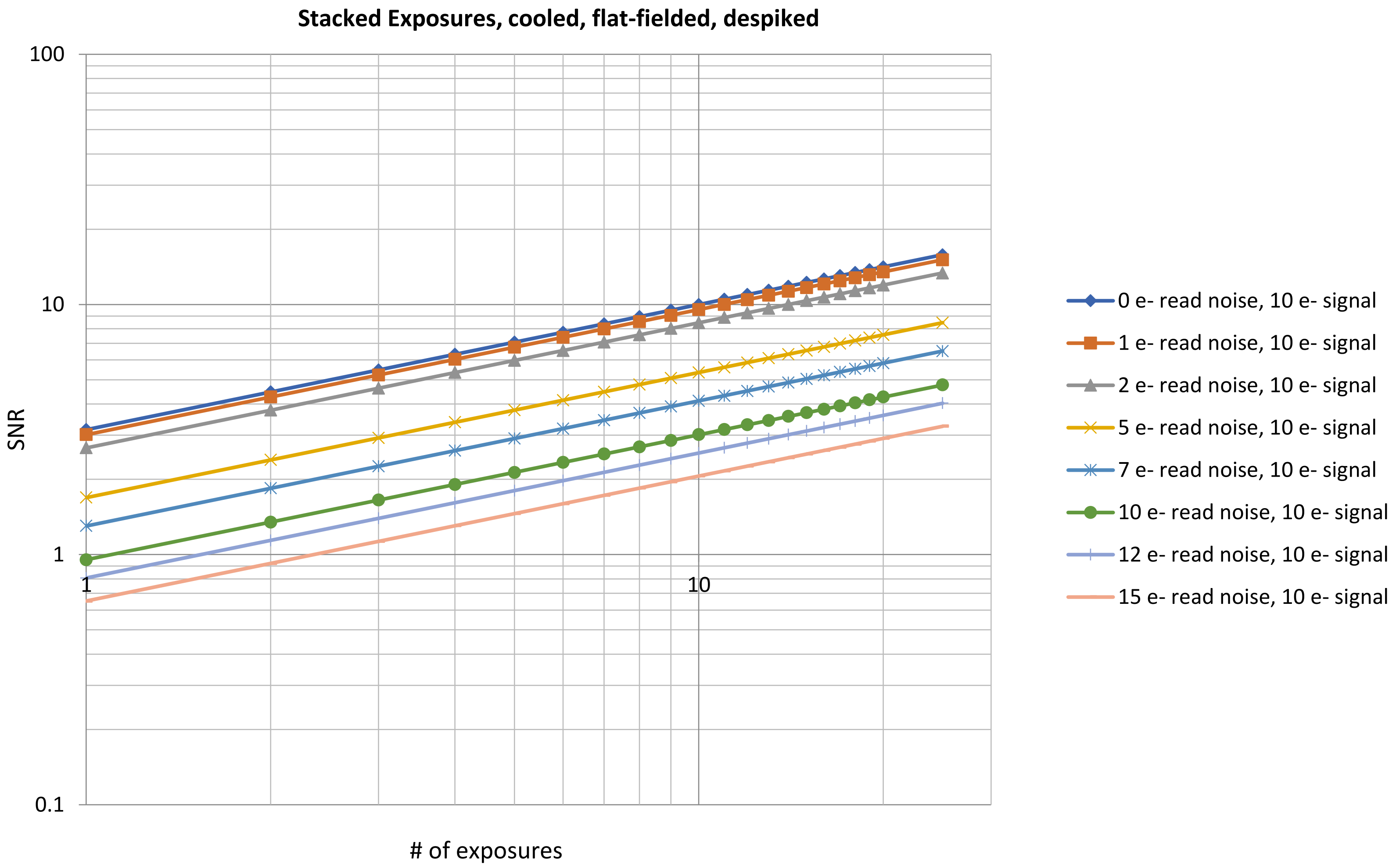


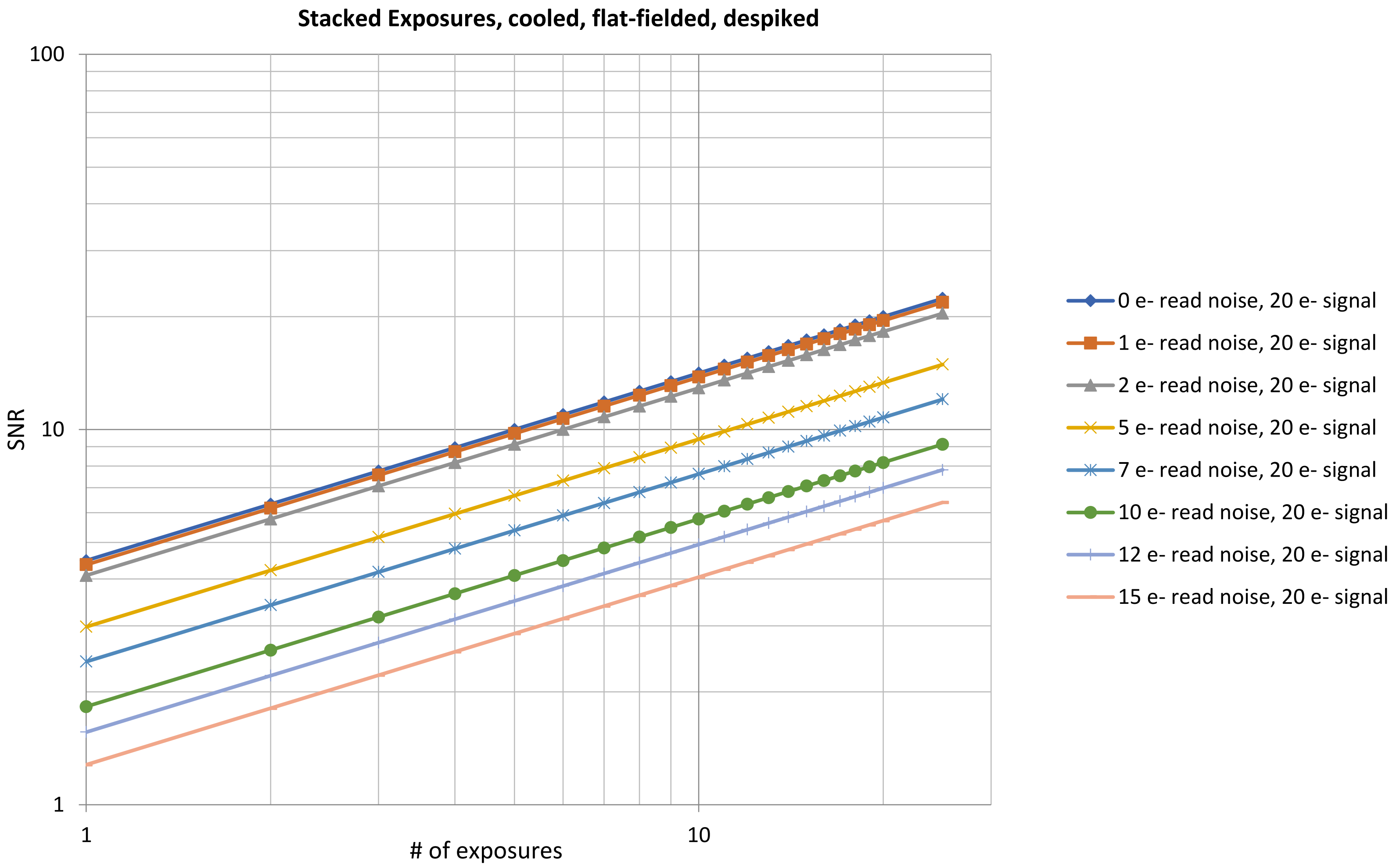
Broadband Typical

Assume 10 e- read noise Camera 1 and 400 e- signal levels
Assume 6 e- read noise Camera 2 and 200 e- signal levels
Compare total exposure time for SNR 50

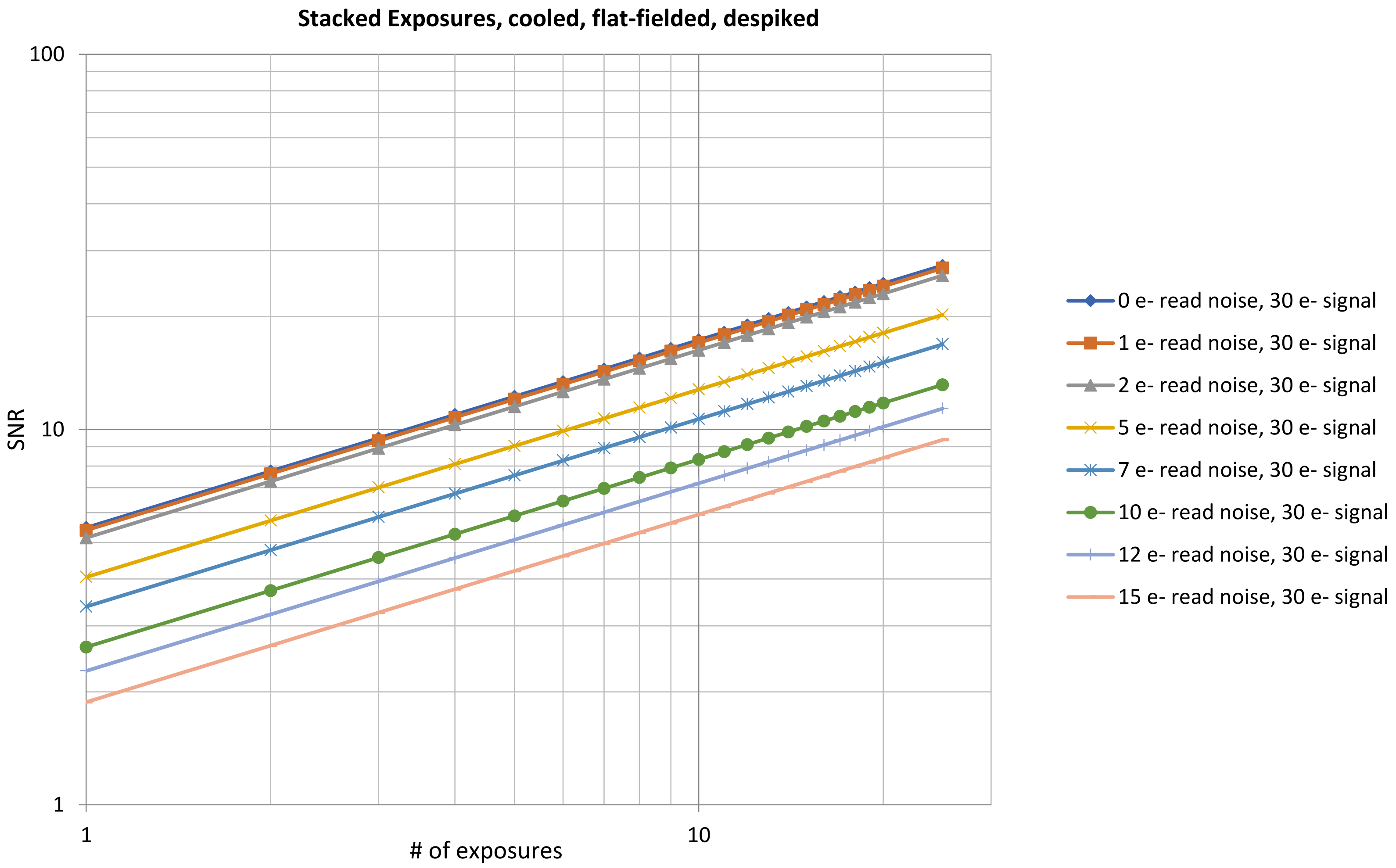
camera	Read Noise (e-)	Signal level (e-)	# exp	Total Time (arb units)
Camera 1	10	40	8	3200
Camera 2	6	20	15	3000

Low Signal Levels
Like in Narrowband Imaging

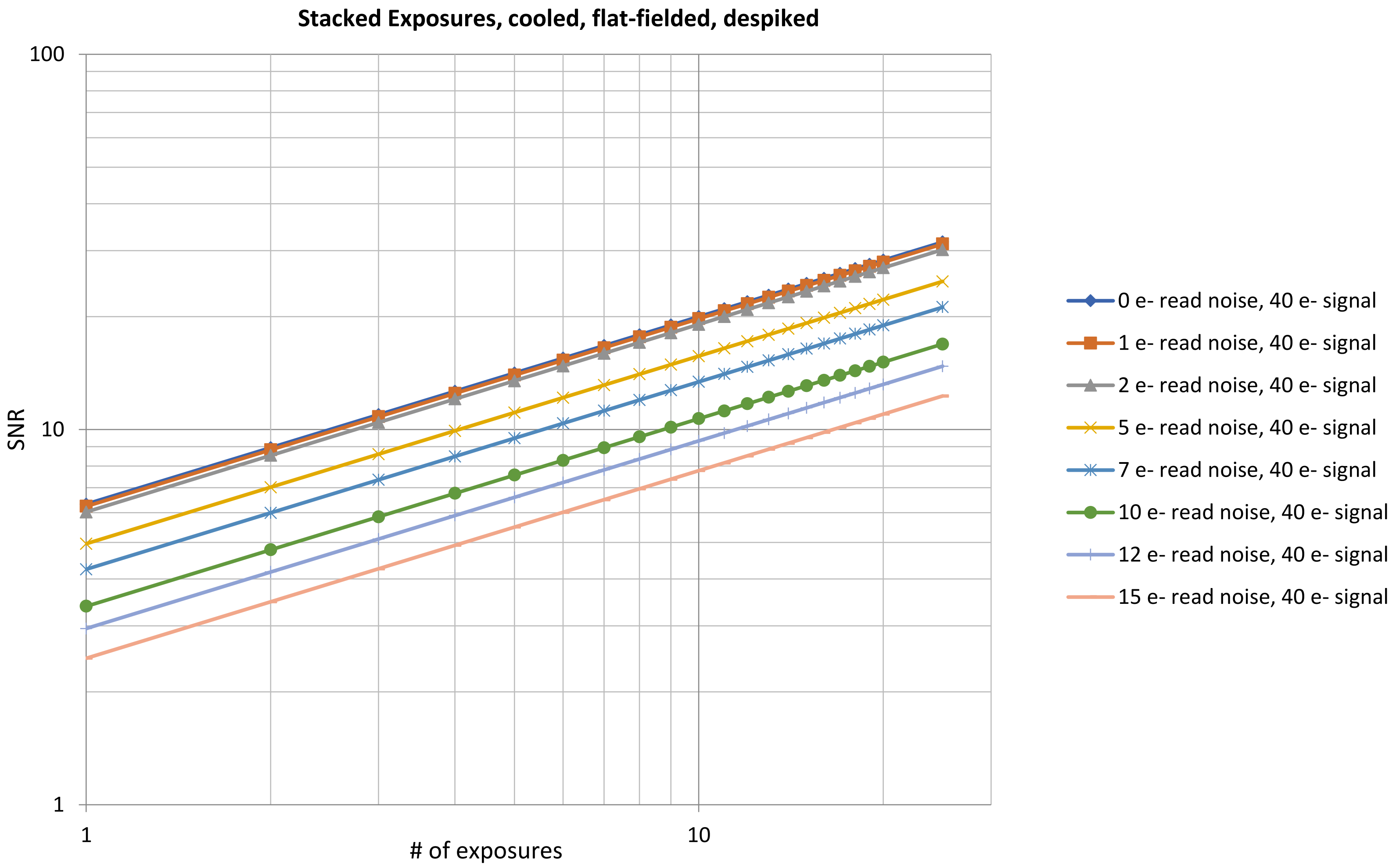

Stacked Exposures, cooled, flat-fielded, despiked
100
10
1
0.1
SNR
1
10
# of exposures
0 e- read noise, 10 e- signal
1 e- read noise, 10 e- signal
2 e- read noise, 10 e- signal
5 e- read noise, 10 e- signal
7 e- read noise, 10 e- signal
10 e- read noise, 10 e- signal
12 e- read noise, 10 e- signal
15 e- read noise, 10 e- signal



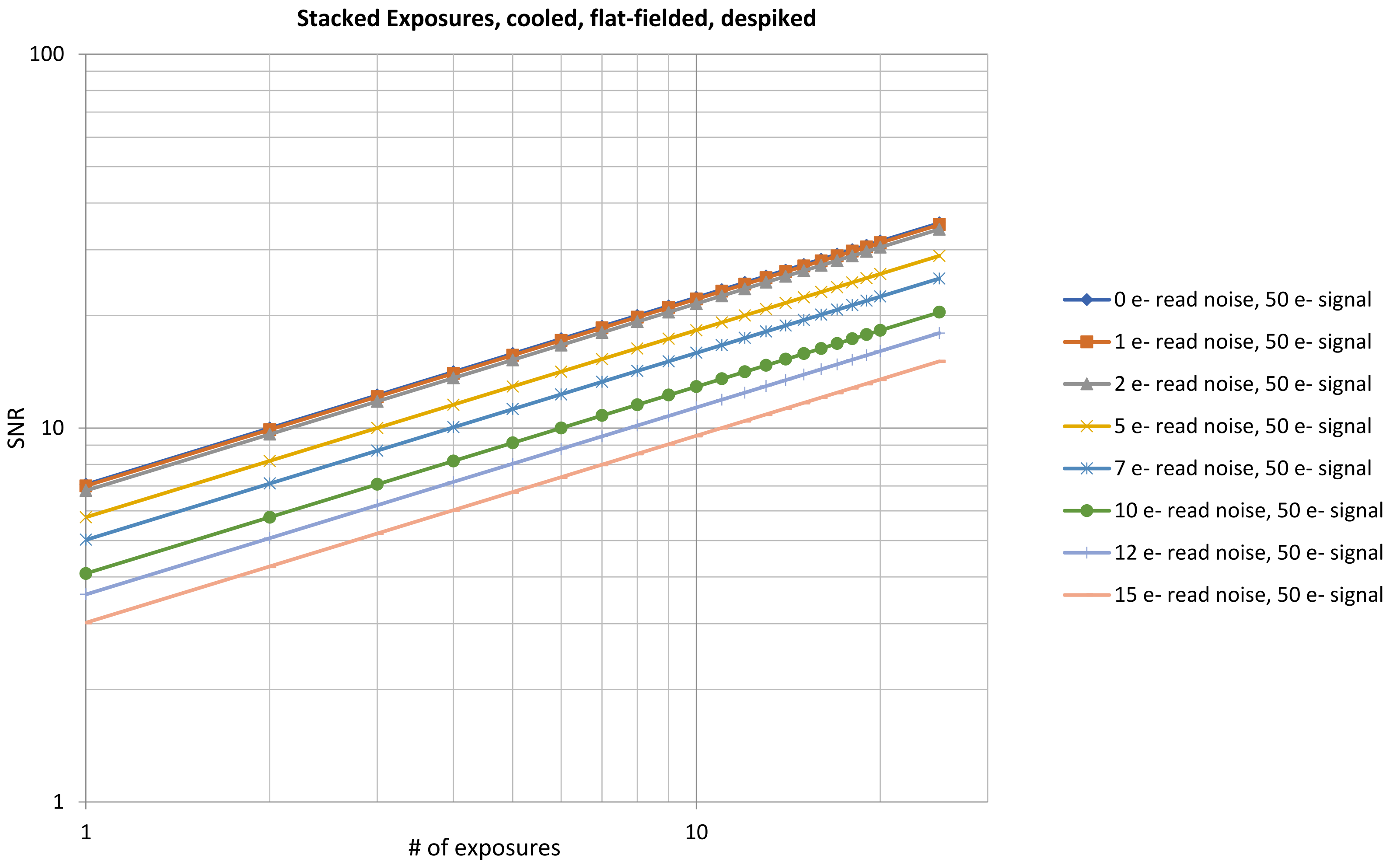
Stacked Exposures, cooled, flat-fielded, despiked
100
10
1
SNR
1
10
# of exposures
0 e- read noise, 20 e- signal
1 e- read noise, 20 e- signal
2 e- read noise, 20 e- signal
5 e- read noise, 20 e- signal
7 e- read noise, 20 e- signal
10 e- read noise, 20 e- signal
12 e- read noise, 20 e- signal
15 e- read noise, 20 e- signal



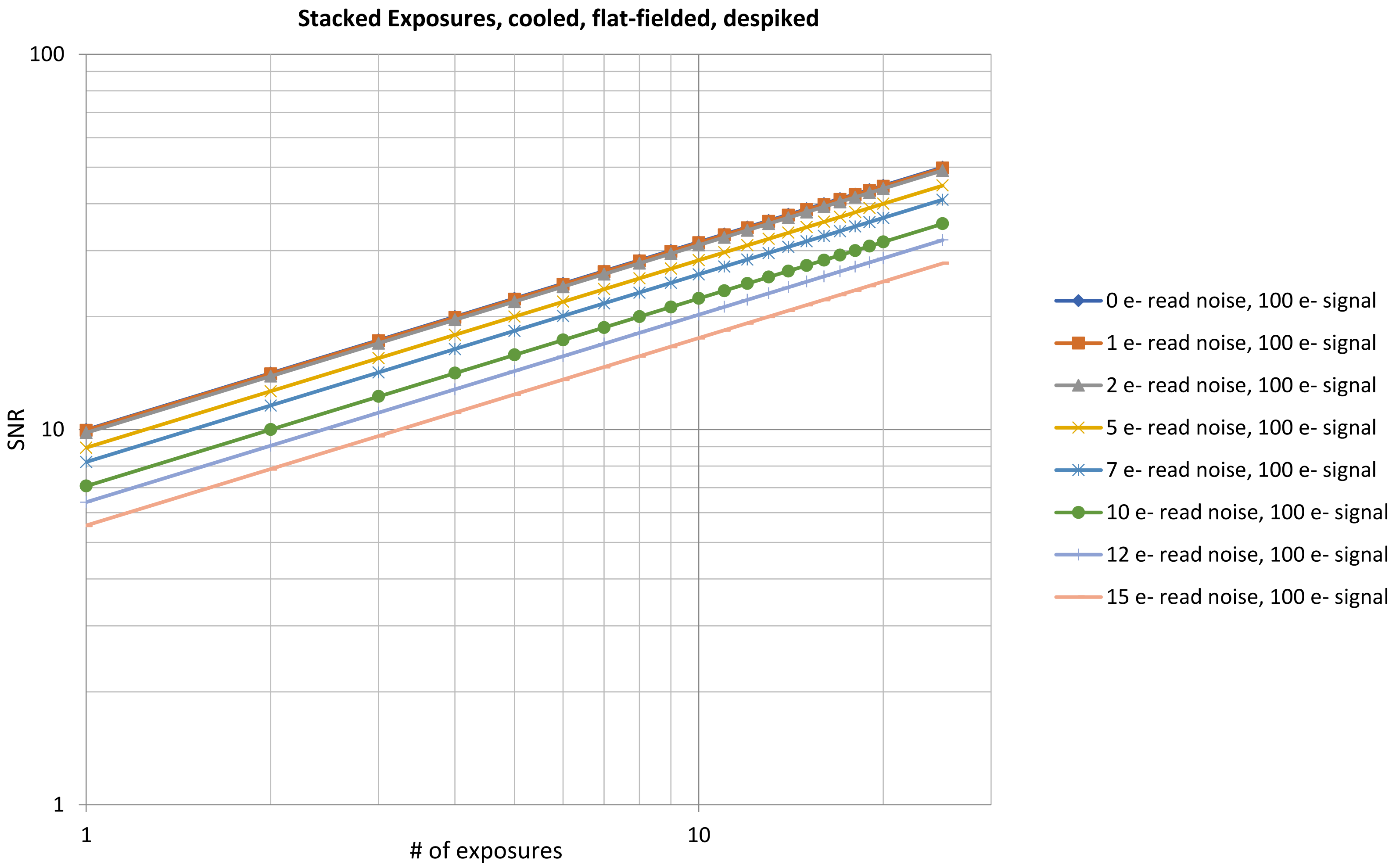
Stacked Exposures, cooled, flat-fielded, despiked
100
10
1
SNR
1
10
# of exposures
0 e- read noise, 30 e- signal
1 e- read noise, 30 e- signal
2 e- read noise, 30 e- signal
5 e- read noise, 30 e- signal
7 e- read noise, 30 e- signal
10 e- read noise, 30 e- signal
12 e- read noise, 30 e- signal
15 e- read noise, 30 e- signal



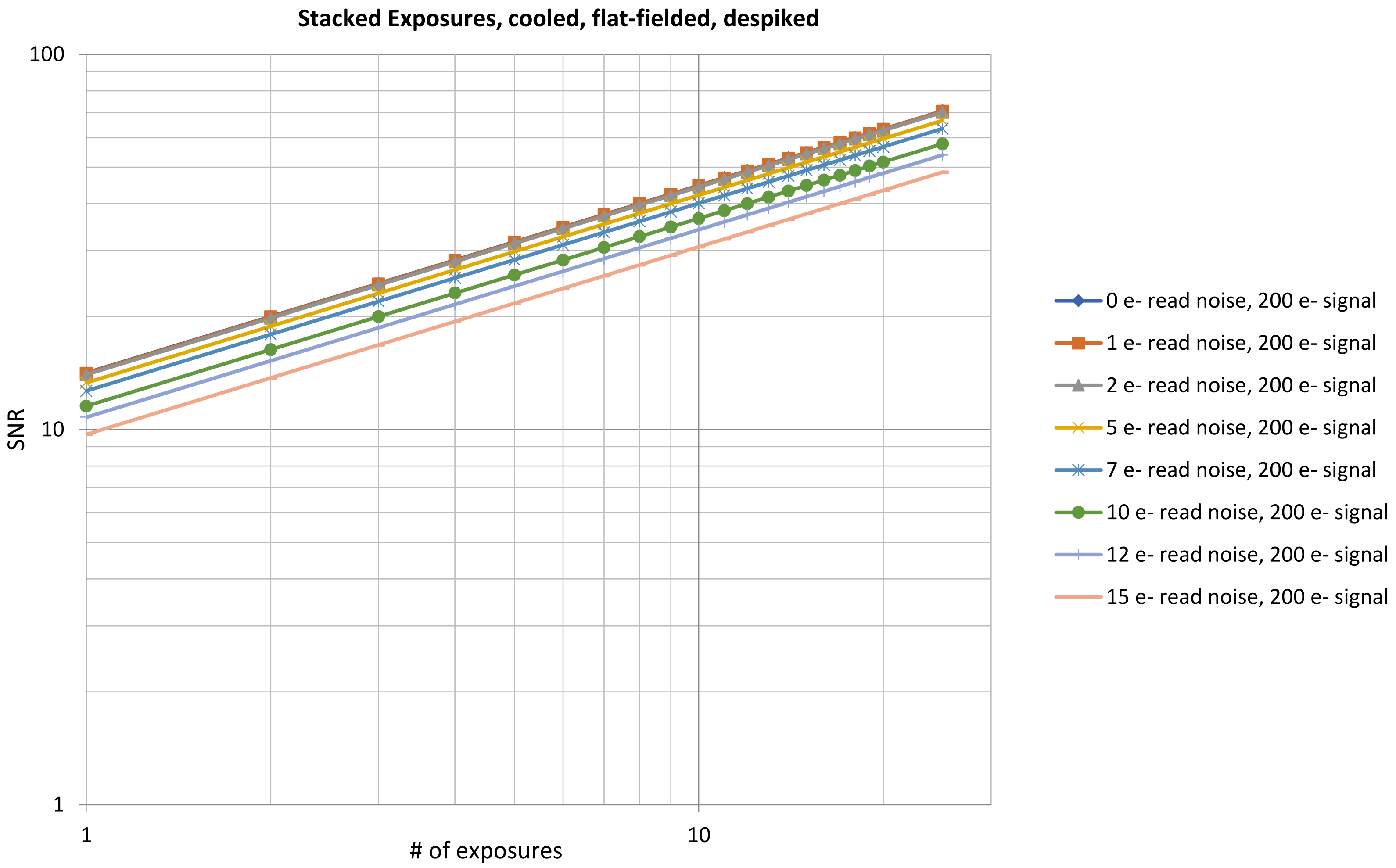
Stacked Exposures, cooled, flat-fielded, despiked
100
10
1
SNR
1
10
# of exposures
0 e- read noise, 40 e- signal
1 e- read noise, 40 e- signal
2 e- read noise, 40 e- signal
5 e- read noise, 40 e- signal
7 e- read noise, 40 e- signal
10 e- read noise, 40 e- signal
12 e- read noise, 40 e- signal
15 e- read noise, 40 e- signal

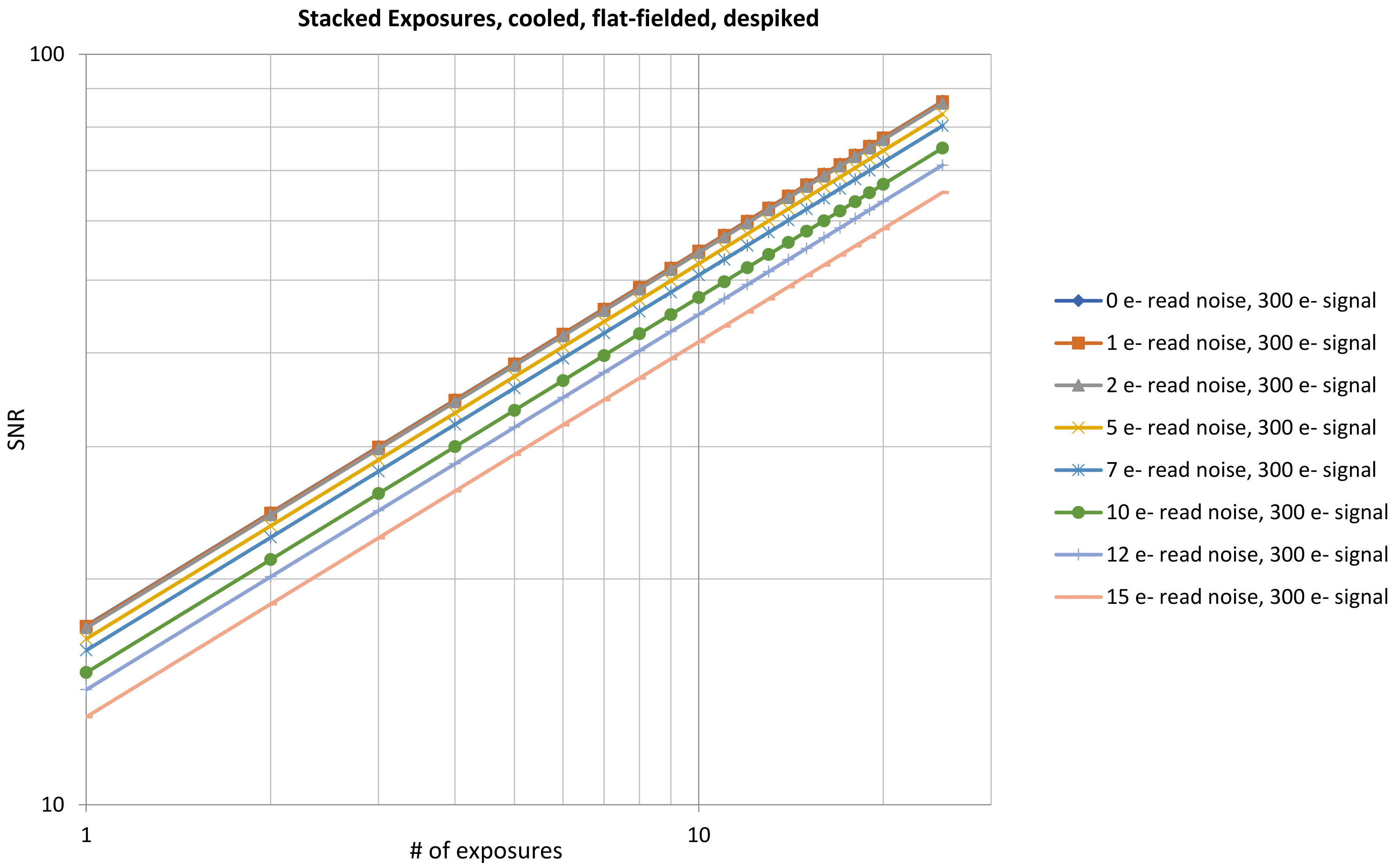

Stacked Exposures, cooled, flat-fielded, despiked
100
10
1
SNR
1
10
# of exposures
0 e- read noise, 50 e- signal
1 e- read noise, 50 e- signal
2 e- read noise, 50 e- signal
5 e- read noise, 50 e- signal
7 e- read noise, 50 e- signal
10 e- read noise, 50 e- signal
12 e- read noise, 50 e- signal
15 e- read noise, 50 e- signal

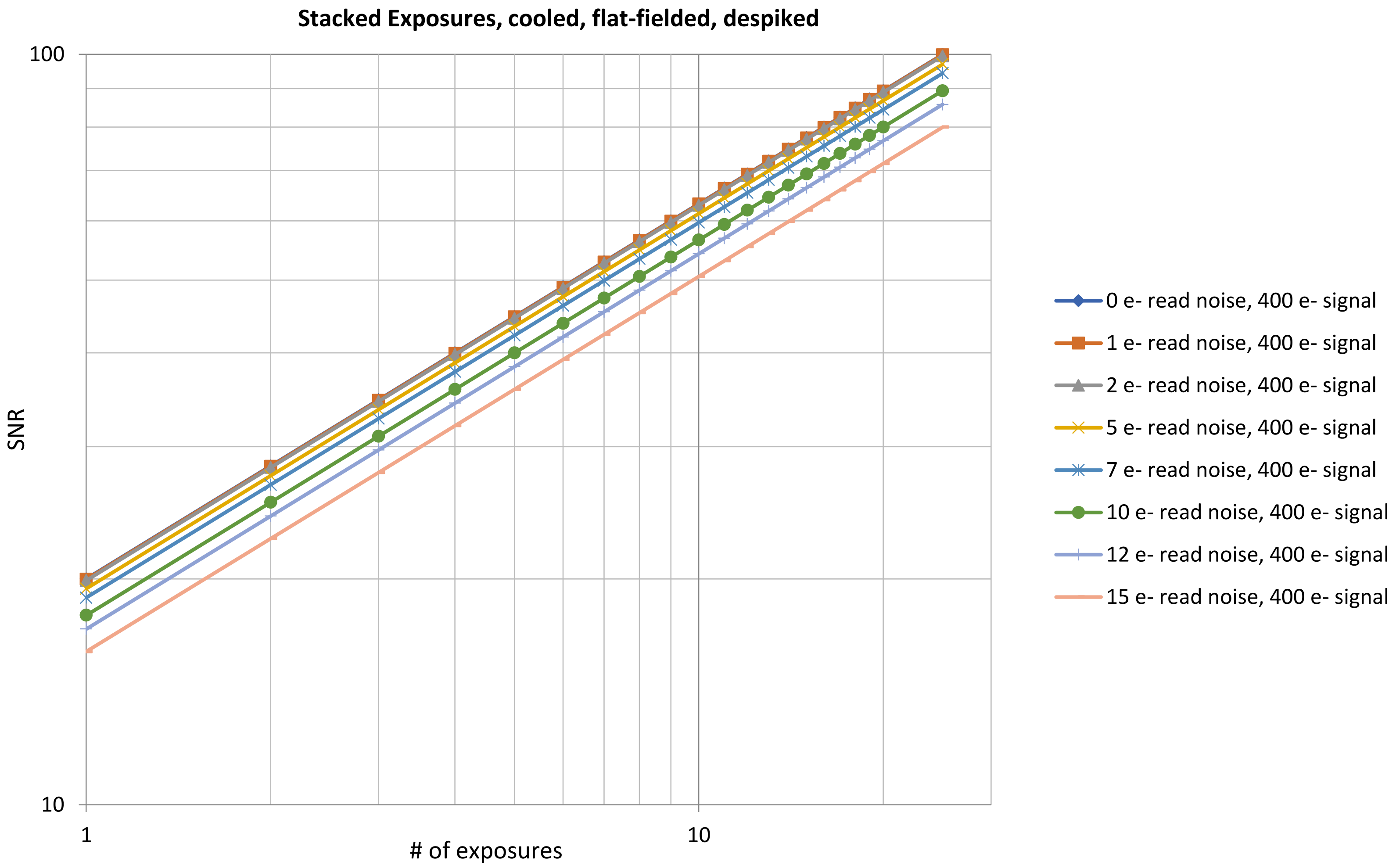
High Signal Levels
Like in Broadband / Terrestrial Imaging



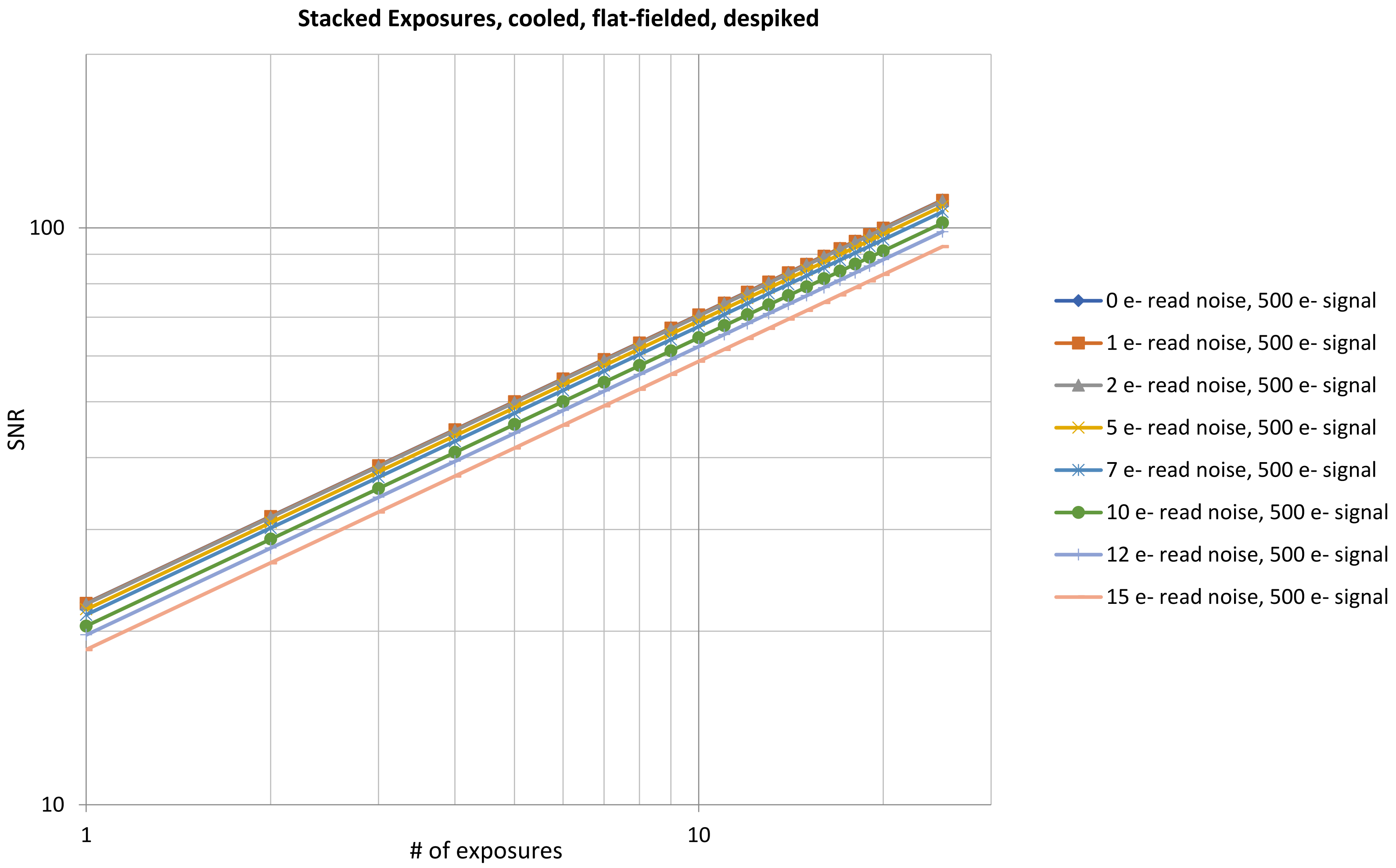
Stacked Exposures, cooled, flat-fielded, despiked
100
10
1
SNR
1
10
# of exposures
0 e- read noise, 100 e- signal
1 e- read noise, 100 e- signal
2 e- read noise, 100 e- signal
5 e- read noise, 100 e- signal
7 e- read noise, 100 e- signal
10 e- read noise, 100 e- signal
12 e- read noise, 100 e- signal
15 e- read noise, 100 e- signal


Stacked Exposures, cooled, flat-fielded, despiked
100
10
1
SNR
1
10
# of exposures
0 e- read noise, 200 e- signal
1 e- read noise, 200 e- signal
2 e- read noise, 200 e- signal
5 e- read noise, 200 e- signal
7 e- read noise, 200 e- signal
10 e- read noise, 200 e- signal
12 e- read noise, 200 e- signal
15 e- read noise, 200 e- signal


Stacked Exposures, cooled, flat-fielded, despiked
100
10
SNR
1
10
# of exposures
0 e- read noise, 300 e- signal
1 e- read noise, 300 e- signal
2 e- read noise, 300 e- signal
5 e- read noise, 300 e- signal
7 e- read noise, 300 e- signal
10 e- read noise, 300 e- signal
12 e- read noise, 300 e- signal
15 e- read noise, 300 e- signal



Stacked Exposures, cooled, flat-fielded, despiked
100
10
SNR
1
10
# of exposures
0 e- read noise, 400 e- signal
1 e- read noise, 400 e- signal
2 e- read noise, 400 e- signal
5 e- read noise, 400 e- signal
7 e- read noise, 400 e- signal
10 e- read noise, 400 e- signal
12 e- read noise, 400 e- signal
15 e- read noise, 400 e- signal


Stacked Exposures, cooled, flat-fielded, despiked
SNR
100
10
1
10
# of exposures
0 e- read noise, 500 e- signal
1 e- read noise, 500 e- signal
2 e- read noise, 500 e- signal
5 e- read noise, 500 e- signal
7 e- read noise, 500 e- signal
10 e- read noise, 500 e- signal
12 e- read noise, 500 e- signal
15 e- read noise, 500 e- signal

Summary

- Showed way to analyze reaching a target SNR
- How many exposures
- What target signal level
- Showed impact of random noise

Application #4 Analyzing / Optimizing Images with Modulation

Analysis of Images with Modulation

An image with modulation can be analyzed using the noise equation, by treating the RMS modulation as another noise term:

$$\delta_{total} = \mathrm{SQRT}({\delta_{modulation}}^2 + Read\_noise^2 + Shot\_noise^2 + fixed\_pattern\_noise^2 + Dark\_fixed\_pattern\_noise^2 + Dark\_shot\_noise^2)$$

If the image to be analyzed has been flat-fielded and the operating temperature is low enough, then the dark and fixed pattern noise terms can be ignored simplifying the equation to:

$$\delta_{total} = \mathrm{SQRT}({\delta_{modulation}}^2 + Read\_noise^2 + Shot\_noise^2)$$

Images with Modulation

Mathematically the modulation, $\delta_{modulation}$, is modeled the same as fixed pattern noise, hence the value of the modulation is proportional to the average signal level. In this case instead of PRNU for FPN analysis a different constant, M_I is used.

For the analysis, the Modulation, $\delta_{modulation}$, is decomposed into a modulation constant, M_I, and an equivalent flat-field image with a signal level equal to the average value of the modulated image. The modulation constant, M_I , can be thought of like PRNU: the resulting modulation is proportional to signal level and this constant

$$\delta_{modulation} = M_I * \text{Equivalent\_flatfield\_average\_signal}$$

In general each feature in the image will have a different M_I and it is the variation of the M_I across the image that gives the image its appearance.

Measuring Signal to Noise from Images

To calculate the signal to noise ratio of a modulated image you begin with the noise equation and solve for M_I where the total modulation, δ_{total}, is what is measured by using the Standard Deviation measuring tool on sampling unit (100 x 100 pixel in this analysis) measurement box placed over a region of interest in the image. The image must have the offset removed. If the image is fully calibrated (flat-fielded and despiked), the analysis is simplest because the FPN and dark signal terms can be ignored and the offset is removed.

<u>To solve the equation you need to know a few parameters:</u>

<u>Read noise & Offset:</u>

Using Photon Transfer analysis, the read noise & offset of the camera can be accurately measured.

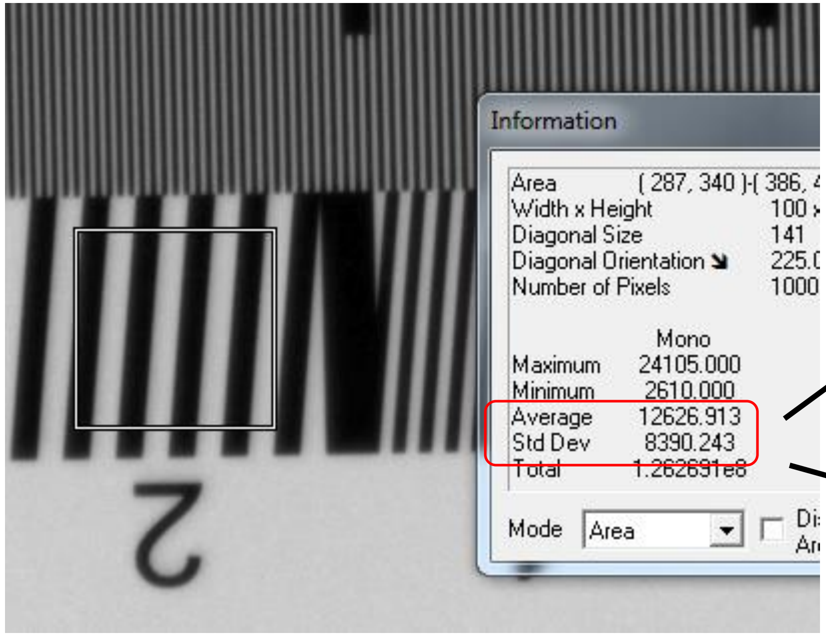
<u>Total RMS modulation ($\delta_{modulation}$):</u>

This is the standard deviation measured in the selection box when placed over a region of interest in the image

<u>Average Signal Level:</u>

This is measured at the same time as the Standard Deviation. This will be the signal level for the equivalent flat-field image and will be used to calculate the shot noise of the equivalent flat-field image

Example of Image Measurements needed to calculate SNR



Average = Flat-Field average signal value = 12626.913

Signal_shot_noise =
SQRT(Flat-Field average signal - offset)

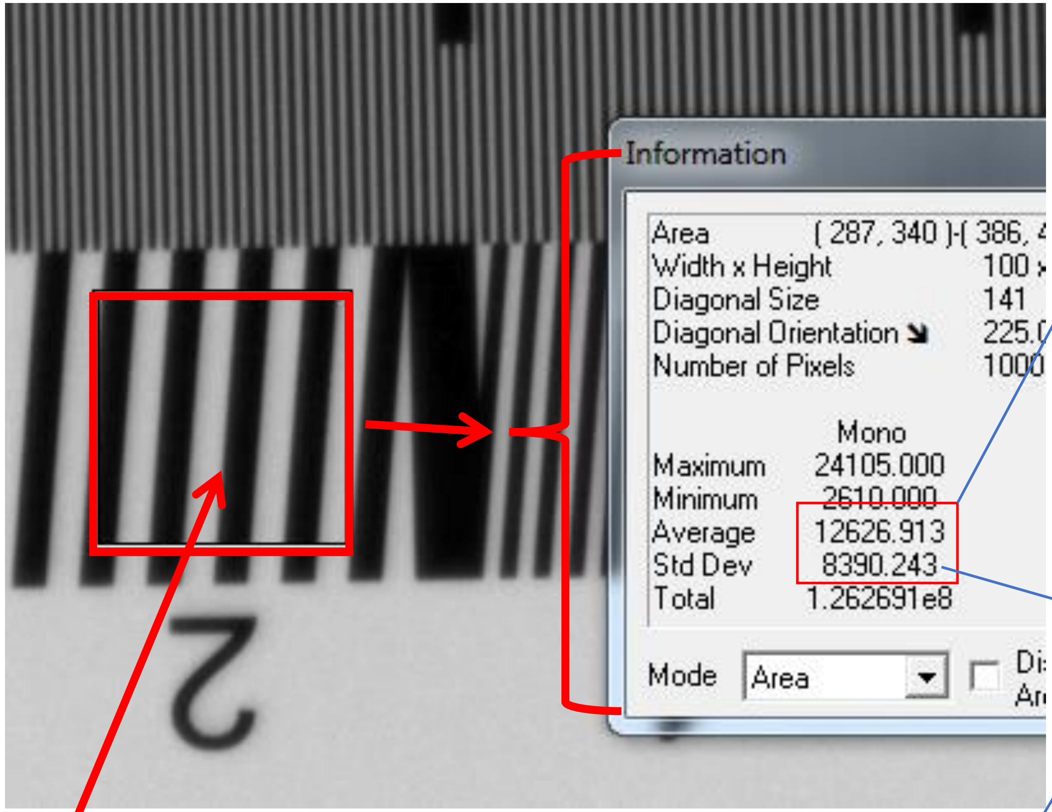
Standard Deviation = δ_{total} = 8390.243

Example:
Assume Read Noise = 10.1DN, Offset = 1950DN from PTC characterization

δ_{total}= SQRT((M_I*Flat-Field_average_signal)^2+ read_noise^2+ Flat-field_average_signal)

Solving for M_I we get: M_I= 78.5%

Parameter Sources and Interrelationships



Equivalent Flat Field Signal Level

Offset from PTC

SNR of Flat= 103.32 SQRT(Avg-offset) (assumed shot limited)

δ_{total}

Read_noise from PTC

Noise Equation

SNR= 81.19

M_I

78.5%

Calculating the SNR

The SNR of a modulated image is calculated by using the M_I factor multiplied by the SNR of a flat field image of the same average signal level:

$$SNR_{modulated\ image} = M_I * SNR_{Equivalent\ FF}$$

The total noise of the equivalent flat-field is equal to the shot noise of the flat field of that signal level: Noise = SQRT(Flat-Field Signal level)
SNR = (Flat-Field Signal level)/SQRT(Flat-Field Signal level)

SNR = SQRT(Flat-Field Signal level)

So in our example, the Signal Level for the Equivalent Flat-Field was 12,626.913DN – 1950DN of offset, making the SNR of the equivalent flat-field:

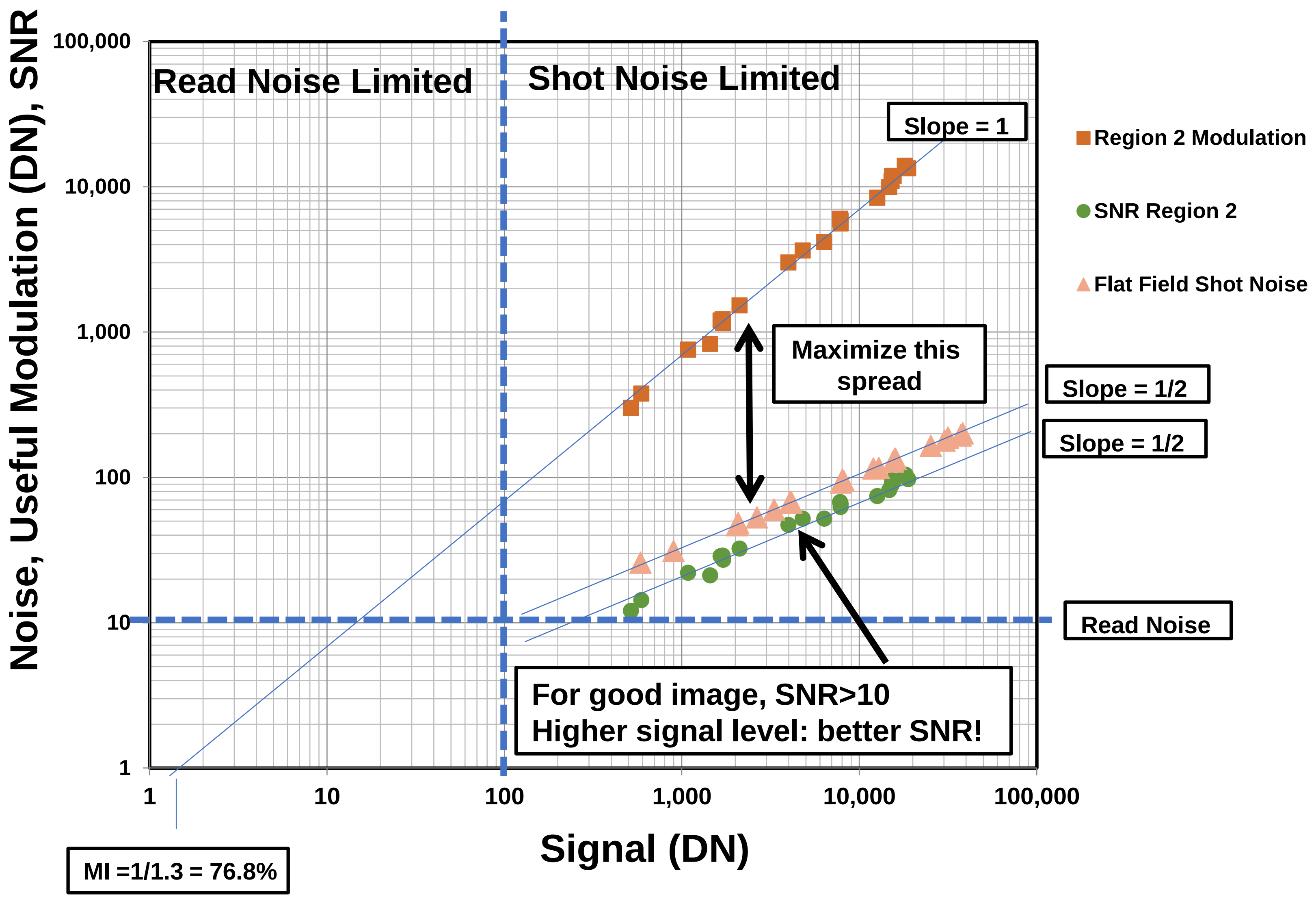
SQRT(12,626.913-1950) = 103.32

So with an M_I = 78.5% we get a final SNR for the modulated image region of:

$SNR_{image-region} = 0.785 * 103.32 = 81.19$

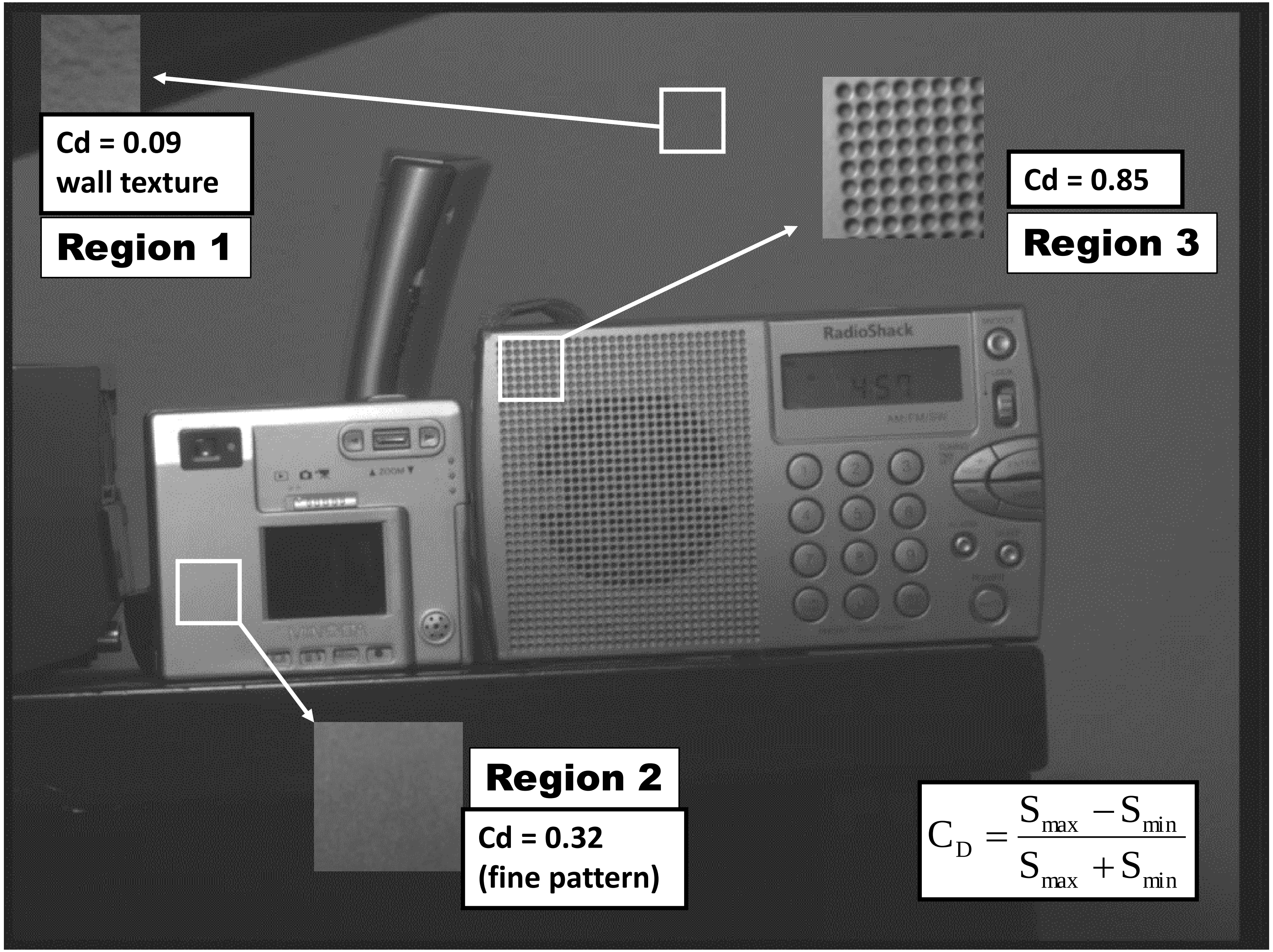
This is the calculation method that will be used for creating the curves that follow

Modulation, Noise and SNR vs Signal Level

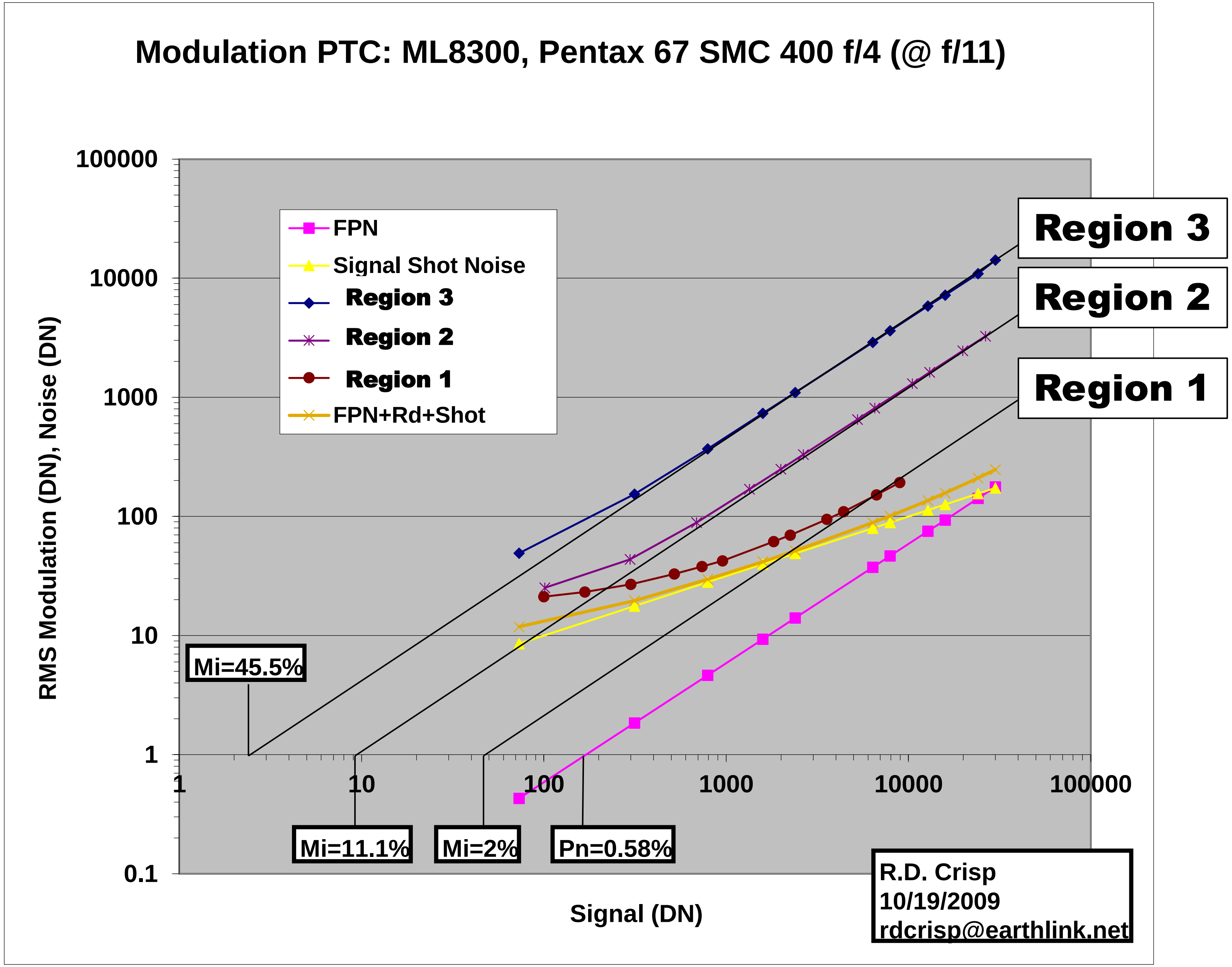


What does that Chart Mean?

- The higher the average signal level the:
  - Higher the SNR
  - Higher the useful modulation
- With a read noise of 10DN, you need at least 100DN of signal to have the shot noise exceed the read noise: more signal is better….
- To have a decent image, SNR should be greater than 10
- You need to have at least 500DN worth of signal to reach that point in Region 2 of the image. More signal: better SNR….



$$C_D = \frac{S_{max} - S_{min}}{S_{max} + S_{min}}$$



Modulation PTC: ML8300, Pentax 67 SMC 400 f/4 (@ f/11)
RMS Modulation (DN), Noise (DN)
100000
10000
1000
100
10
1
0.1
1
10
100
1000
10000
100000
Signal (DN)
FPN
Signal Shot Noise
Region 3
Region 2
Region 1
FPN+Rd+Shot
Region 3
Region 2
Region 1
Mi=45.5%
Mi=11.1%
Mi=2%
Pn=0.58%
R.D. Crisp
10/19/2009
rdcrisp@earthlink.net

IMAGE SIGNAL TO NOISE

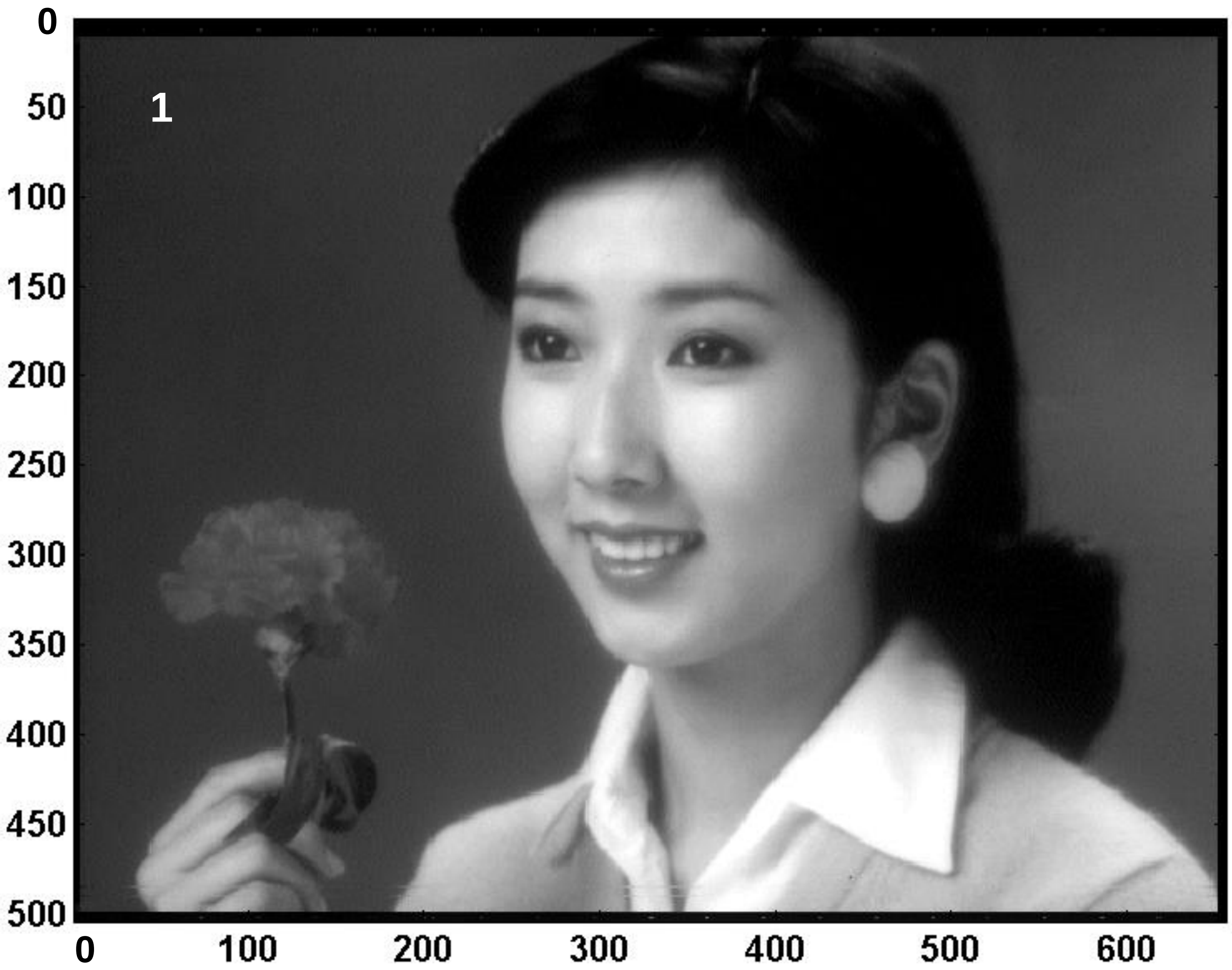
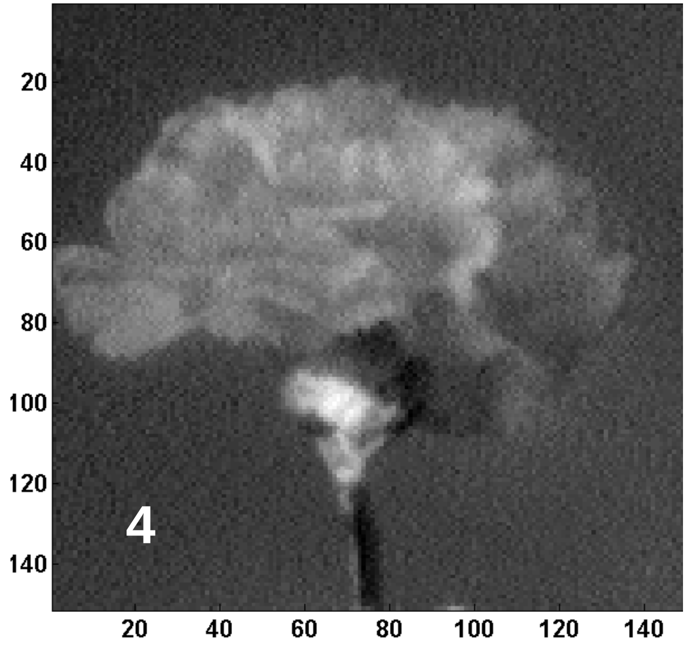
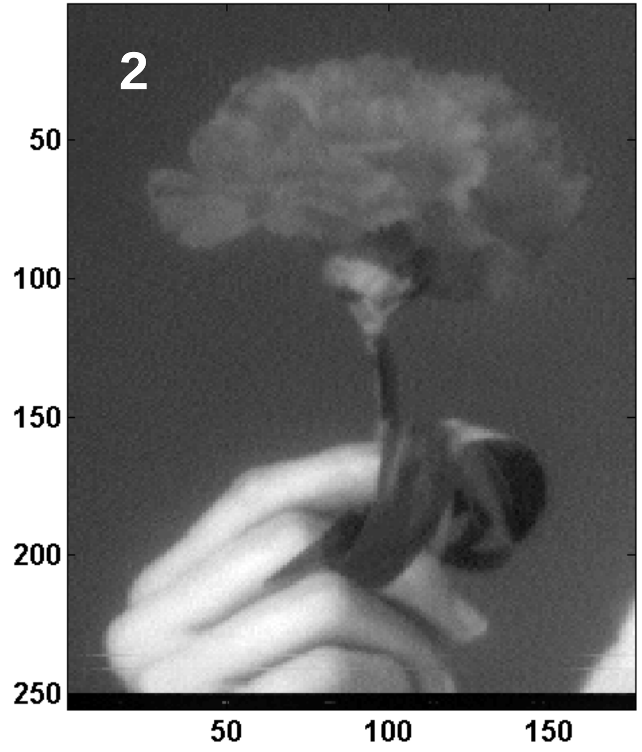
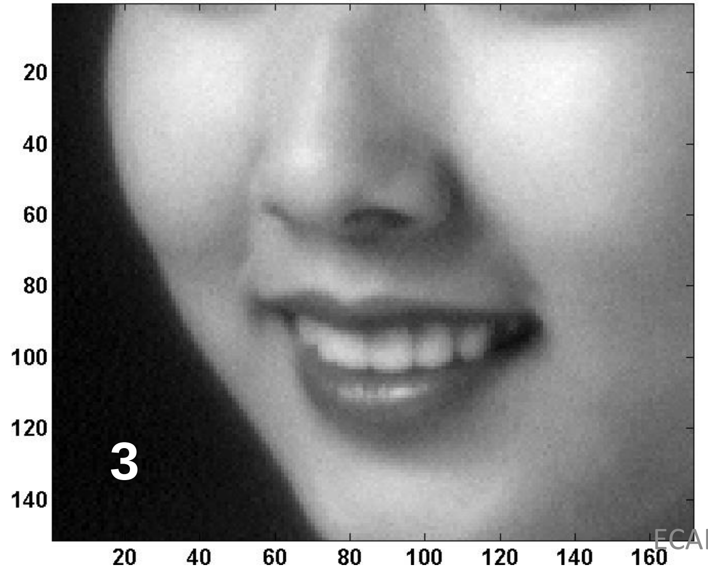


IMAGE SIGNAL TO NOISE







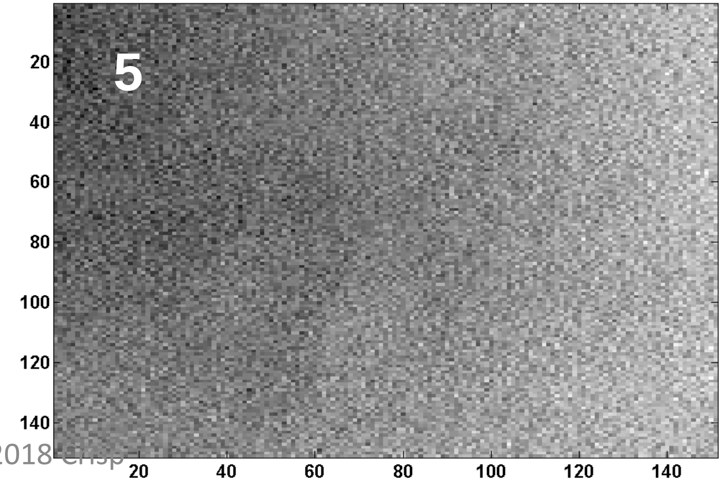
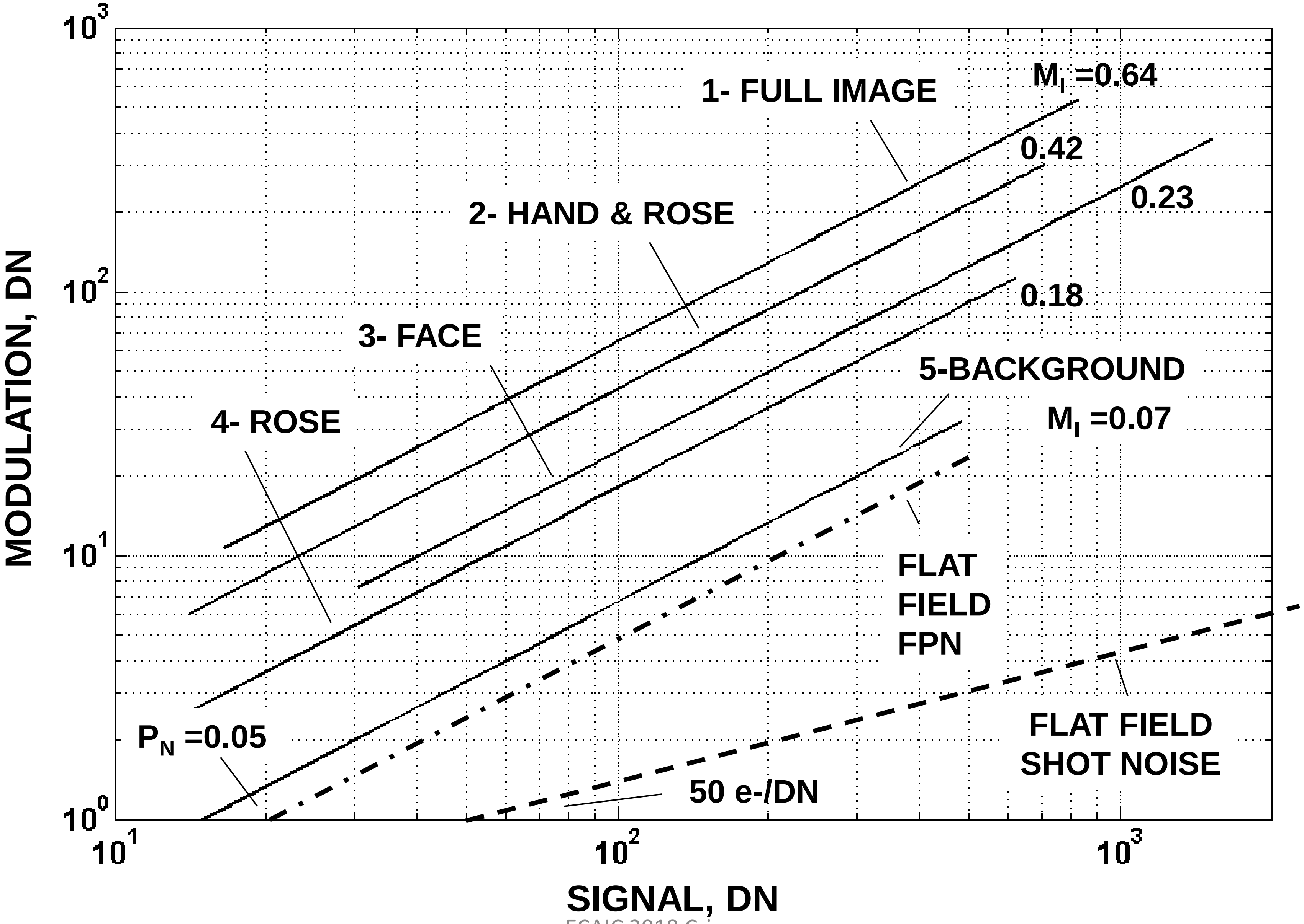
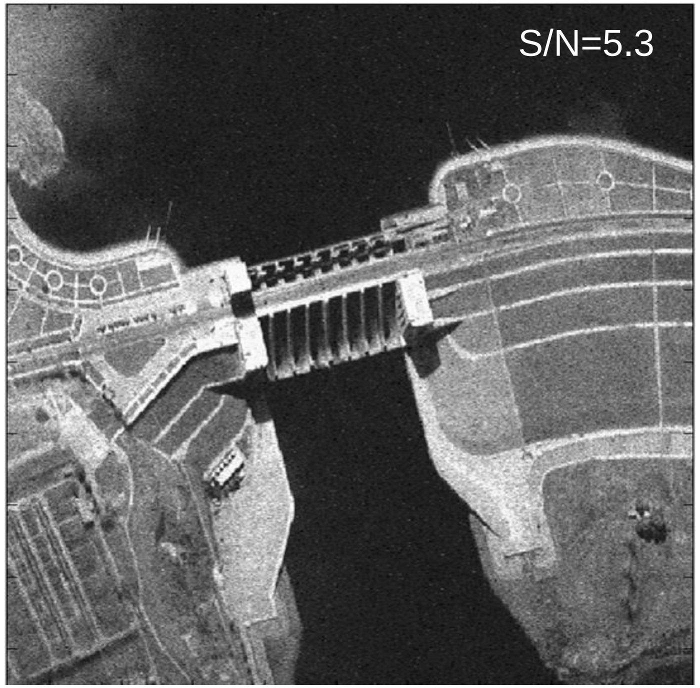


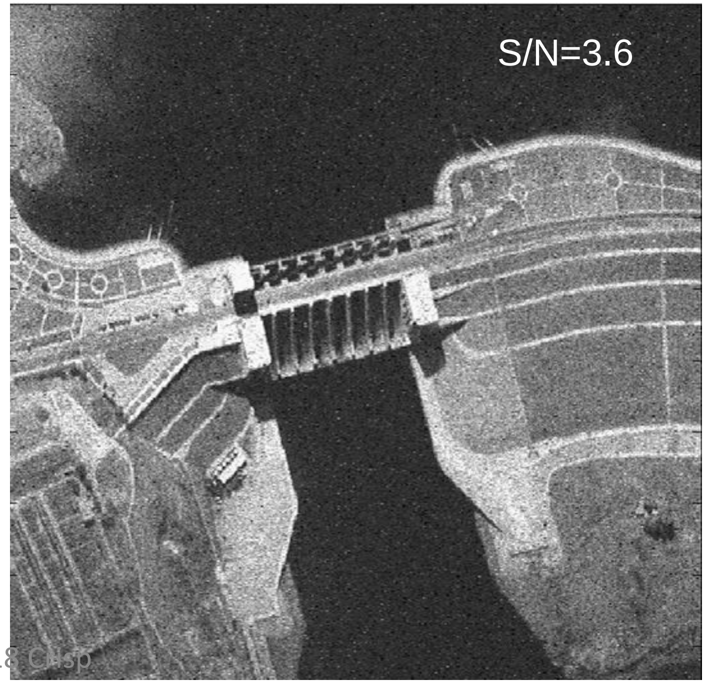
IMAGE SIGNAL TO NOISE

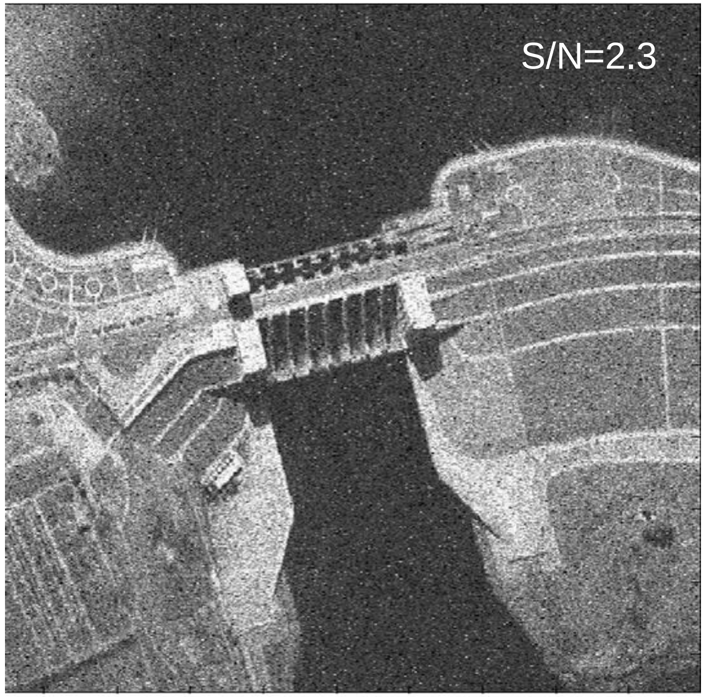


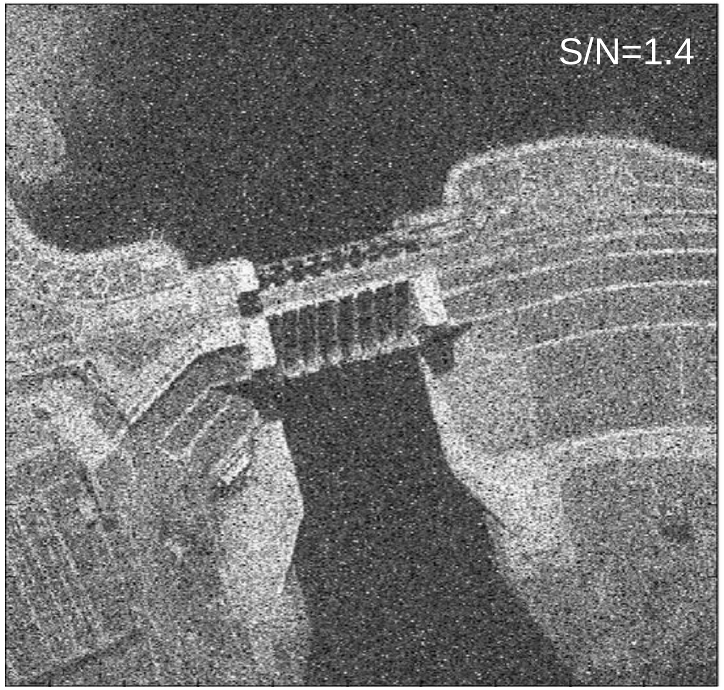

S/N=28

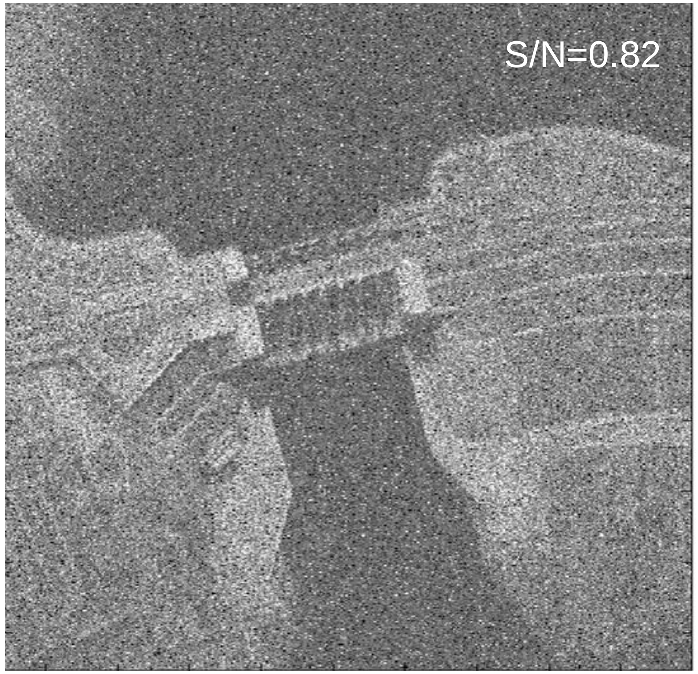

S/N=12.5

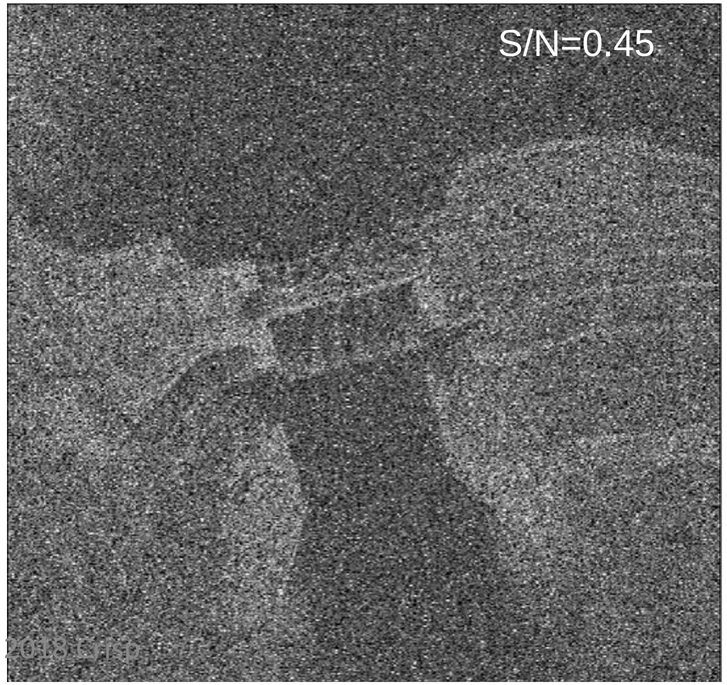

S/N=5.3


S/N=3.6

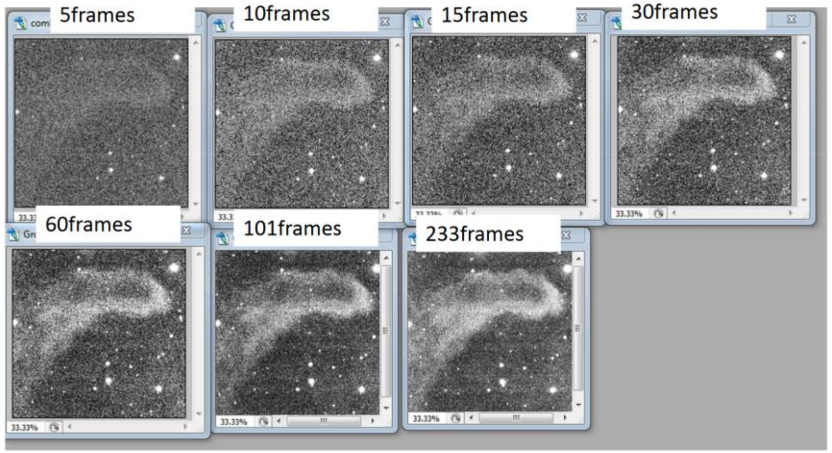

S/N=2.3

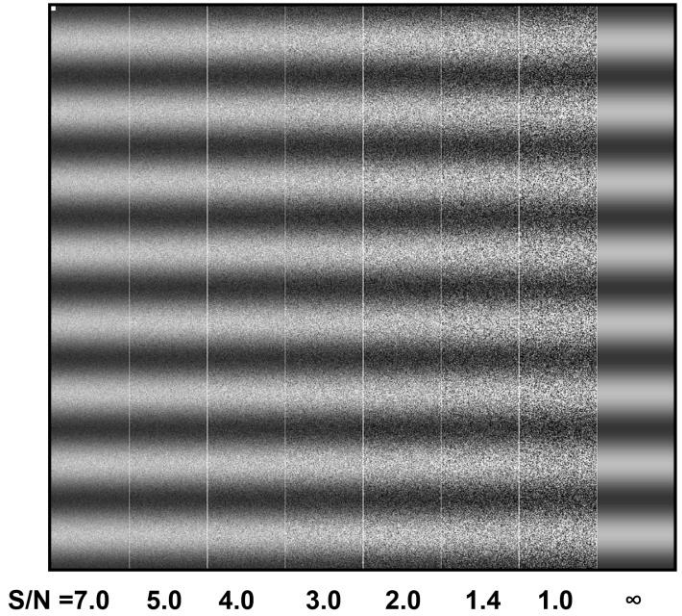

S/N=1.4


S/N=0.82

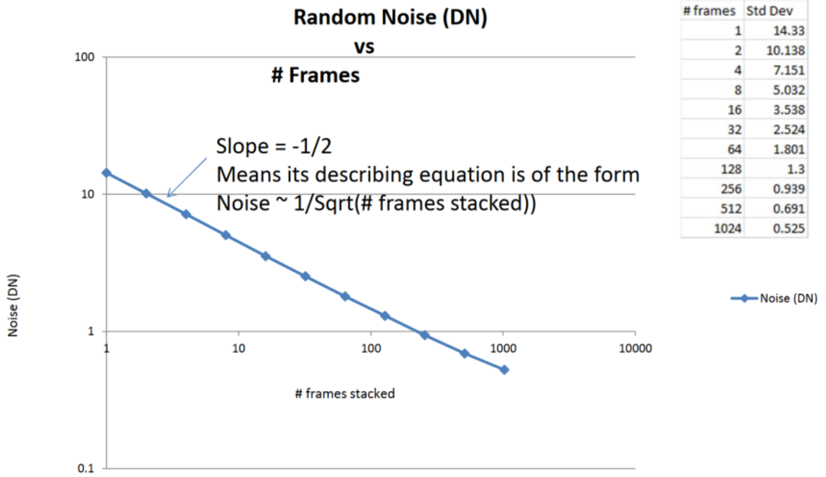

S/N=0.45

Combining Frames to improve SNR





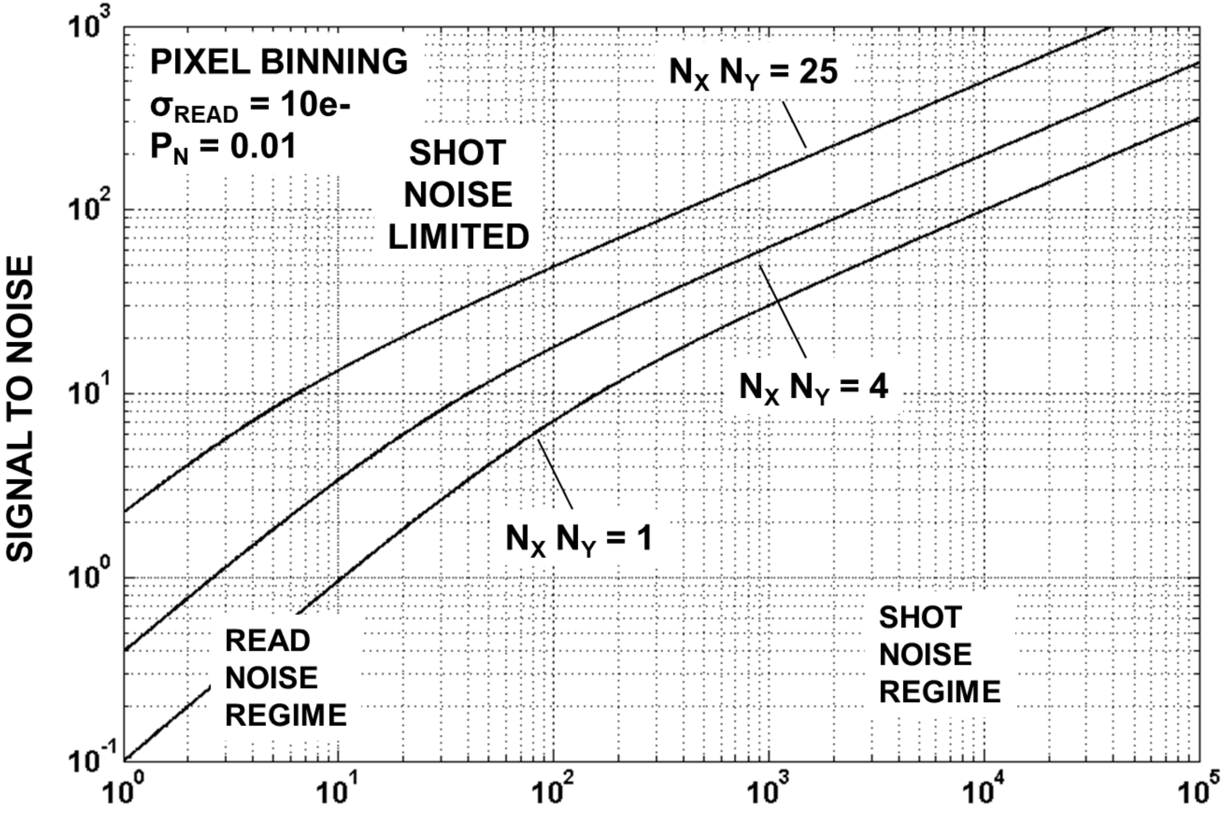
Noise Reduction by Stacking



# frames	Std Dev
1	14.33
2	10.138
4	7.151
8	5.032
16	3.538
32	2.524
64	1.801
128	1.3
256	0.939
512	0.691
1024	0.525

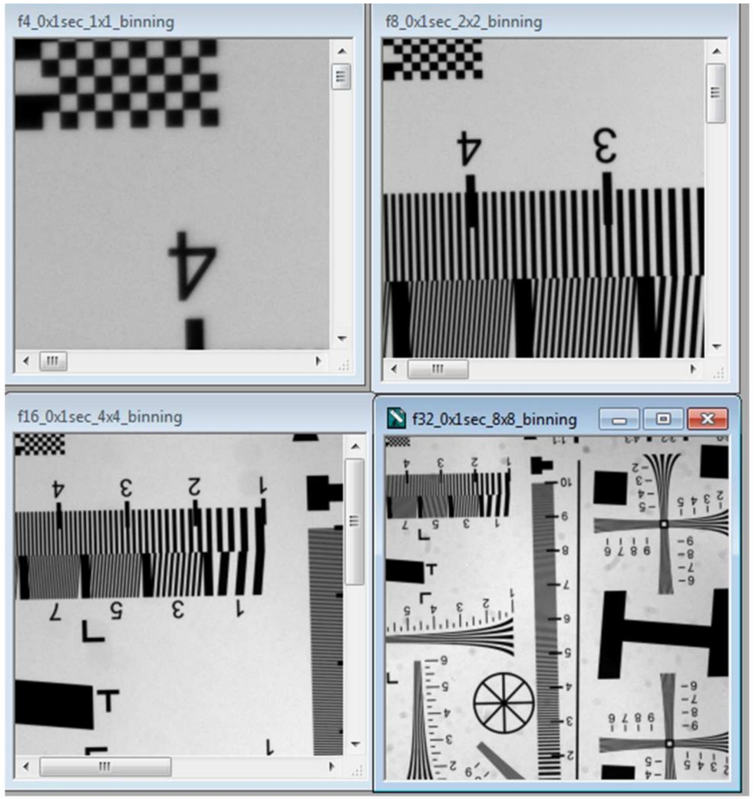
Can get **arbitrarily low noise** by combining sufficient number of frames

ON-CHIP BINNING TO IMPROVE SNR
Trades SNR for Resolution



Single Unbinned Pixel SIGNAL, e-

1x1 2x2



4x4 8x8

Resolution

Summary

- Showed how to decompose modulated image into flat field and modulation terms
- Use ptc methods to optimize SNR of flat field
  - Signal level
  - Number of frames to combine
- Show how to reach any SNR target by combining sufficient # of frames
- Showed effect of on-chip binning on SNR and Resolution