Gravitational Wave Astronomy Problem Set

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The twenty-first century has seen the dawn of a totally new type of astronomy. For millennia, people across the world have been trying to understand the stars by looking at the visible light they emit. Scientists in the past century have realized that the stars communicate in many ways, devising techniques for looking at all types of electromagnetic waves—radio, microwave, infrared, ultraviolet, x-ray, and gamma—and at “cosmic ray” particles like protons. But the strangest messages that astronomers have learned to read are carried by spacetime itself: gravitational waves.

How do gravitational waves work? How do we observe them? This course will give an intuitive introduction to the concepts and methods for listening to the words of gravity.

Problem 1: Waves

Just like visible light, gravitational waves are waves. A **wave** is a message that travels along a stationary chain, the **medium**.

Think about a line of people holding hands. Everyone in this chain has agreed that when they feel the person next to them raise or lower their linked hands for a second, they will do the same on the other side. What happens when the person on the end raises and then lowers their left hand?

[animation. A chain of people hold hands facing the camera. The person on the left has nobody on their left. There is a “Raise hands” button; when pressed, the person on the left raises their hands, starting a wave that travels down the chain]

A “message” of raising hands travels down the chain of stationary people.

In this example, what’s the wave? What’s the medium? (Drag your finger to match.)

In the example	**Concept**
The raising of hands	Wave
The air	Medium
The chain of people	Neither

Problem 2: Spacetime

Gravitational waves are waves that travel on the medium of spacetime. **Spacetime** is the stretchy background of the universe that determines how far apart everything is, both distance and time. “Everything” includes all matter and energy, like people, planets, and light.

Spacetime tells everything how to fall. Spacetime is part of Einstein’s theory of **gravity**. When we talk about gravity “pulling” something down, that’s really spacetime guiding it effortlessly forwards. That’s why waves in spacetime are called gravitational waves.

Everything else also tells spacetime how to distort (bend and stretch and compress). When spacetime is empty, it tells things to move in nice straight lines. But very large objects like the Earth distort spacetime. When spacetime gets distorted, those nice straight lines look curved. That’s why objects fall toward the Earth, and it’s why the planets orbit the Sun.

[diagram. a circle on the left with “spacetime” written in it, and another on the right with “matter and energy” written in it. A semicircular arrow pointing from left to right is labeled “guides”, and another pointing from right to left is labeled “shapes”.]

It’s not a perfect metaphor, but you can get a sense for spacetime by imagining it as a rubber sheet. Notice that the rubber sheet is like a chain, but in two dimensions. What happens when you stretch a bit of the fabric apart and let it go?

[animation with a play button. A white-gloved hand hovers above a purple fabric with a square pattern. It’s above a large marble indenting the fabric. The indentation distorts the square pattern. In slow motion, the hand reaches down to grab the marble with two fingers, moves it quickly one centimeter north and then back one centimeter south, and lets go. Transverse waves travel off to the east and west.]

The distortion is carried along the fabric as a wave.

Explore the animation to answer this question. Why would a spacetime distortion travel as a gravitational wave?

- Very large objects carry the distortion from one place to the next.
- When one part of spacetime is distorted, it causes the next part to distort.
- Spacetime causes the distortion to fall.

Problem 3: Wineglass modes

Most spacetime distortions that travel as gravitational waves have a special shape. It’s the same shape that vibrating wineglasses make.

Explore the animation below. Notice that there are two separate **modes** that can be added together, the **plus (+) mode** and the **cross (×) mode**.

[Animation of three wineglasses. The left one shows the selected plus mode, the right one shows the selected cross mode, and the center one shows them added together. Both modes have a period of 1 second.

Slider 1: Plus mode amplitude, -1 to 1

Slider 2: Cross mode amplitude, -1 to 1

Slider 3: Cross mode delay, -0.5 sec to +0.5 sec]

Try to combine the plus and the cross modes to make the shapes below.

[animation A: a plus mode rotated by 22.5 degrees]

[animation B: a combination of 1 plus mode with .5 cross mode, slightly delayed]

[animation C: clockwise rotation]

Which ones can you make by combining the plus and cross modes? (Check all that apply.)

- A
- B
- C

Problem 4: Sensing gravitational waves

Scientists have figured out a way to listen to gravitational waves. Since spacetime determines distances and times, distortions in spacetime cause distances and times to change. When a gravitational wave passes, it can temporarily make the distance between two points increase (or decrease) as if there were "more space" in between (or less).

A gravitational wave distorts spacetime "sideways" (compared to the direction it's traveling). For example, a gravitational wave traveling north would cause distortions east-west and up-down—not north-south.

[3d animation of a plus-mode gravitational wave traveling to the right. The perspective allows a cross-section of the right end to be seen. A square grid deforms along with the wineglass mode shape.]

In the image below, a gravitational wave is traveling towards three pairs of stars.

[3d animation with a "show/hide wave" button. Two red stars are separated horizontally by a dashed red arrow labeled "A". Same for two yellow stars except separated in the into/out-of-the-phone direction and labeled "B", and two blue stars except vertically and labeled "C". At first, a large blue arrow labeled "wave" points straight down at the stars. When the button is pressed, an animation of the wave, like the one above but traveling downwards, is layered transparently over the image.]

When the gravitational wave passes, which pairs of stars might get closer or farther apart? (Select all that apply.)

- A
- B
- C

Problem 5: Gravitational wave observatories

Scientists have built observatories to look for gravitational waves. As of 2020, there are five observatories in the world that could directly detect gravitational waves.

- LIGO-Hanford and LIGO-Livingston in the U.S. (LIGO stands for Laser Interferometer Gravitational-Wave observatory)
- Virgo in Italy
- GEO600 in Germany
- KAGRA in Japan (KAGRA stands for Kamioka Gravitational Wave Detector)

All of them work by looking for "more space" (or less) between two points. They use lasers and a technique called interferometry to measure tiny changes in a huge distance. Each observatory has two arms pointing at 90-degree angles, which it measures at the same time and compares.

This is what LIGO-Hanford looks like.

[Satellite image of LIGO-Hanford]

When the two arms of LIGO-Hanford are changing lengths differently, that's a sign that spacetime is stretching and compressing like a wineglass. That means there might be gravitational wave passing through. Scientists would call such an observation a **signal**, an observation that they are hoping to see. Scientists need multiple observatories pointing in different directions in order to get a strong signal from all incoming gravitational waves.

A few possible gravitational waves coming from directly above LIGO-Hanford are illustrated below.

A. [Plus mode animation on above image. A grid aligned with the arms is superimposed, stretching and compressing like the plus mode.]
B. [Cross mode animation on above image. As above.]
C. [CW mode animation on above image. As above.]
D. [CCW mode animation on above image. As above.]

Which of them would LIGO-Hanford be able to detect? (Select all that apply.)

- A
- B
- C
- D

Problem 6: Interpreting gravitational wave signals

Once scientists see a gravitational wave, they need to work backwards to figure out what created the wave. The gravitational waves we can see are usually produced by enormous, dense stars moving very fast. Examples are:

- a **supernova**, the explosion of a very large star
- a **neutron star**, the extremely dense remains of a star that went supernova
- a **black hole**, the remains of a star that are so dense that spacetime prevents light from escaping

Sometimes the signal speeds up or slows down, or gets stronger or weaker. Patterns like this can be used to guess what made the signal.

Here's an illustration of one signal that scientists have seen before.

[animation of 25 April 2019 plot. it starts out slow and gets faster and faster before stopping suddenly]

Here are a few situations that scientists think create gravitational waves. One of these might have caused the signal above.

A. An oval-shaped neutron star spinning very fast [animation. the star wobbles at a steady frequency]
B. Two neutron stars orbiting each other and colliding [animation. the stars rotate around each other faster and faster before merging]
C. A star's core collapsing in on itself during a supernova [animation. the core's outer surface accelerates inward before suddenly squishing outward along the equator, then experiencing wineglass-shaped oscillations that decay in amplitude]

Which of the above most likely caused this signal?

- A
- B
- C