ECON 6090
Microeconomics I Notes

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Introduction

I am creating this set of unified notes for ECON 6090: Microeconomics I, as taught at Cornell University in the Fall 2024 semester. Due to unforseen departmental circumstances, this course was taught by six different professors (David Easley, Philipp Kircher, Adam Harris, Larry Blume, Levon Barseghyan, and Marco Battaglini). This structure necessarily created some confusion in notation and material, so these notes function as my attempt to create a universe of the material we learned.

I rely heavily on the notes created from Prof. Easley's course, which were originally compiled by Julien Manuel Neves and subsequently updated by Ruqing Xu and Patrick Ferguson. I additionally rely on notes and slides provided by Prof. Harris, slides provided by Prof. Blume, slides from Ted O'Donoghue provided by Prof. Barseghyan, and notes provided by Prof. Battaglini. These notes are supplemented with the canonical Microeconomic Theory textbook by Andreu Mas-Colell, Michael Whinston, and Jerry Green (hereafter, MWG); my preferred analysis textbook, Foundations of Mathematical Analysis by Richard Johnsonbaugh and W.E. Pfaffenberger; and the excellent Mathematics notes provided by Takuma Habu. All mistakes are my own.

I will occasionally make reference to the Stanford ECON 202 notes, created by Jonathan Levin, Ilya Segal, Paul Milgrom, and Ravi Jagadeesan. This will mainly be if there exists intuition that I believe is helpful.

Notation. A large part of this project is an attempt to unify the notation used by our separate professors. I default to the notation used in the Easley notes, then to MWG, and then use my own judgement. New definitions will have a word highlighted in *blue*, and theorems will be named in *red*.

Structure. The course (and these notes) are organized as follows. Prof. Easley taught an introduction to choice theory, Section 1. Prof. Kircher taught consumer theory, Section 2. Prof. Harris taught producer theory, and some concepts of market failures, Section 3. Prof. Blume introduced the theory of choice under uncertainty, Section 4, and Prof. Barseghyan continued with theoretical applications for uncertainty and expected utility maximization,

Section 5. Prof. Battaglini taught on information theory, Section 6. We also include here exercises with solutions, divided into the various sections and sources. This is Section 7.

1 Choice (Easley)

1.1 Preference Theory

Assumption 1.1. *Let X be a finite set of objects.*

Definition. Define $\succsim$, a *preference relation* on X, as $x \succsim y \iff x$ is *at least as good as y*, for $x, y \in X$. $\succsim$ is a binary relation.

Definition. x is *strictly preferred* to y, denoted as $x \succ y$, if $x \succsim y$ and $y \not\succsim x$.

Definition. x is *indifferent* to y, denoted as $x \sim y$, if $x \succsim y$ and $y \succsim x$.

Definition. A preference relation $\succsim$ is *complete* if $\forall\, x, y \in X$, either $x \succsim y$, $y \succsim x$, or both.

Definition. A preference relation $\succsim$ is *transitive* if, $\forall\, x, y, z \in X$ where $x \succsim y$ and $y \succsim z$, $x \succsim z$.

Definition. A preference relation $\succsim$ is *rational* if it is complete and transitive.

Remark. Prof. Easley takes some issues with this definition. The main issue is that there is an English word 'rational' that has absolutely nothing to do with it. Hereafter, always read rational as 'complete and transitive'.

Remark. These are all of the abstract concepts in choice theory! From here, we will apply them, and see what we can get.

Definition. (Informal) Define a *choice structure $C^\star$* over subsets $B \subseteq X$ as $C^\star(B, \succsim) := \{x \in B : x \succsim y \,\forall\, y \in B\}$.

Remark. Some direct implications:

  (i) If $x \in C^\star(B, \succsim)$ and $y \in C^\star(B, \succsim)$, then $x \sim y$.

  (ii) Suppose that $x \in B$, $x \notin C^\star(B, \succsim)$, and $C^\star(B, \succsim) \neq \varnothing$. Then there exists $y \in B$ such that $y \succ x$.

We will now formalize the above.

Definition. Let the *power set* of X, denoted $\mathcal{P}(X)$, be the set of all subsets of X. Note that since X is finite, $\mathcal{P}(X)$ is finite.

Definition. (Formal) A correspondence $C^\star : \mathcal{P}(X) \rightrightarrows X$ is a *choice correspondence* for some (not necessarily complete; not necessarily transitive) preference relation $\succsim$ if $C^\star(B) \subseteq B$ for all $B \subseteq X$.

Remark. This definition is from the Stanford notes – I find it more intuitive than defining it the other way, but it requires divorcing the choice structure from the preference relation. Some intuition that's helpful for me: Easley's definition starts with the preference relation and then defines the choice correspondence, while Segal's definition starts with the choice correspondence and then applies it to a preference relation. They will (as we will see below) often be equivalent, but it's a subtle distinction. I will denote an arbitrary choice correspondence by $C^\star(\cdot)$ and one connected with a preference relation $\succsim$ by $C^\star(\cdot, \succsim)$.

Proposition 1.1. *If $\succsim$ is a rational preference relation on X, then*

$$C^\star : \mathcal{P}(X) \setminus \varnothing \to \mathcal{P}(X) \setminus \varnothing$$

In words, the associated choice correspondence to a rational preference relation is nonempty for nonempty inputs.

Remark. The Easley notes define power sets slightly differently. This is unnecessary and (I feel) less intuitive.

Proof. Proof by induction on $n = |B|$. Suppose $|B| = 1$, so $B = \{x\}$ for some $x \in X$. Then by completeness, $x \succsim x$, and $C^\star(B, \succsim) = \{x\} \in \mathcal{P}(X) \setminus \varnothing$. Suppose next that for any Y where $|Y| = n$, $C^\star(Y, \succsim)$ is nonempty. Take some arbitrary B, where $|B| = n + 1$. Define $B' \coloneqq B \setminus \{x\}$, and let x' be an element of $C^\star(B', \succsim)$, which is nonempty by the inductive hypothesis. By completeness, either $x \succ x'$, $x' \succ x$, or $x \sim x'$. Case by case, we would have that $C^\star(B, \succsim) \in \{\{x\}, C^\star(B', \succsim), C^\star(B', \succsim) \cup \{x\}\} \subseteq \mathcal{P}(X)$, by transitivity. $\qquad\square$

Definition. $C^\star$ satisfies *Sen's α*, also known as *independence of irrelevant alternatives*, if $x \in A \subseteq B$ and $x \in C^\star(B, \succsim)$ implies that $x \in C^\star(A, \succsim)$.

Remark. The classical example of a preference relation that violates Sen's α is 'choosing the second-cheapest wine.' It should be fairly clear to see why this violates Sen's α. Is it a rational preference relation?

Proposition 1.2. *If $\succsim$ is a rational preference relation, then $C^\star(\cdot, \succsim)$ satisfies Sen's α.*

Proof. The result is trivially true if $A = B$. Suppose that $A \subset B$. Let $x \in C^\star(B, \succsim)$. Then $x \succsim y$ for all $y \in B$. In particular, if $y \in A \subseteq B$, then $x \succsim y$. Thus, $x \in C^\star(A, \succsim)$. $\qquad\square$

Definition. $C^\star$ satisfies *Sen's β*, also known as *expansion consistency*, if $x, y \in C^\star(A, \succsim)$, $A \subseteq B$, and $y \in C^\star(B, \succsim)$ implies that $x \in C^\star(B, \succsim)$.

Remark. I couldn't find a classical example violating Sen's β, but a simple one is as follows: assume that the waiter offers you French or Italian wine. You are indifferent between them, but then they remember that they also have California wine. You say 'in that case, I'll have the French wine'. Again, this directly violate's Sen's β, but is it rational? Why or why not?

Proposition 1.3. *If $\succsim$ is a rational preference relation, then $C^\star(\cdot, \succsim)$ satisfies Sen's β.*

Proof. Let $x, y \in C^\star(A, \succsim)$, $A \subseteq B$, and $y \in C^\star(B, \succsim)$. Since $x \in C^\star(A, \succsim)$, we have $x \succsim y$ since $y \in A$. Since $y \in C^\star(B, \succsim)$, we have $y \succsim z$ for all $z \in B$. By transitivity, $x \succsim y$ and $y \succsim z$ implies that $x \succsim z$ for all $z \in B$, so $x \in C^\star(B, \succsim)$. $\qquad\square$

Definition. $C^\star$ satisfies *Houthaker's weak axiom of revealed preference* (often called either *HWARP* or *HARP*) if for all $A, B \in \mathcal{P}(X)$ if $x, y \in A \cap B$, $x \in C^\star(A, \succsim)$ and $y \in C^\star(B, \succsim)$, then $x \in C^\star(B, \succsim)$ and $y \in C^\star(A, \succsim)$.

Proposition 1.4. *$C^\star : \mathcal{P} \rightrightarrows X$ satisfies Sen's α and β if and only if it satisfies Houthaker's weak axiom of revealed preference.*

Proof.

(i) $(\alpha + \beta \implies \text{HWARP})$ Suppose $x, y \in A \cap B \subseteq \mathcal{P}(X)$, $x \in C^\star(A, \succsim)$, and $y \in C^\star(B, \succsim)$. By Sen's α, both x and y are in $C^\star(A \cap B, \succsim)$. Then by Sen's β, $x \in C^\star(B, \succsim)$ and $y \in C^\star(A, \succsim)$.

(ii) $(\text{HWARP} \implies \beta)$ Say $x, y \in C^\star(A, \succsim)$, $A \subseteq B$ and $y \in C^\star(B, \succsim)$. Because $A = A \cap B$, $x, y \in C^\star(A \cap B, \succsim)$. Applying HWARP, we have that $x \in C^\star(B, \succsim)$.

4

(iii) (HWARP $\implies \alpha$) Say $x \in A \subseteq B$ and $x \in C^\star(B, \succsim)$. Suppose $x \notin C^\star(A, \succsim)$. Then by Proposition 1.1, there exists $y \in C^\star(A, \succsim)$. Note that $x, y \in A = A \cap B$, $x \in C^\star(B, \succsim)$ and $y \in C^\star(A, \succsim)$. By HWARP, $x \in C^\star(A, \succsim)$, which is a contradiction.

$\square$

Proposition 1.5. *The following are equivalent for $C^\star(\cdot, \succsim)$, where $C^\star : \mathcal{P}(X) \to \mathcal{P}(X)$*

(i) $\succsim$ *is rational*

(ii) $C^\star$ satisfies Sen's α and β

(iii) $C^\star$ satisfies HWARP

Proof. (ii) and (iii) are equivalent by Proposition 1.4. (i) $\implies$ (ii) is given by Propositions 1.2 and 1.3. Finally, (iii) $\implies$ (i) is given below, in the proof of Proposition XXX $\square$

2 Consumer Theory (Kircher)

3 Producer Theory (Harris)

4 Uncertainty Theory (Blume)

5   Uncertainty Applications (Barseghyan)

6 Information Theory (Battaglini)

7 Exercises