11/6

Levon

Expected Utility Theory will not get us that far

State is the most important thing
↳ events

States have to be payoff relevant
↳ $X :=$ collection of possible realizations
$= \{X_1, \dots, X_N\}$
$P = \{p = \{p_1, \dots, p_N\}\}$
$\sum_{i=1}^{N} p_i = 1 \;,\; p_i > 0$

Can we expectation to derive stuff
$EV = \sum_{i=1}^{N} p_i X_i$

Suppose $EV = \$72$
How much you are willing to pay to play determines if you are risk averse

---

Say we have another $q := \{q_1, \dots, q_N\}$
probabilities
$p \succsim q$
→ complete and transitive
→ continuous
→ independence

If I prefer p to q then I prefer a mixture of p and r to a mix of q and r

X (states) are fixed

$\alpha p + (1-\alpha) r \succsim \alpha q + (1-\alpha) r$
EUs are linear in probabilities

$p \succsim q$
$\Rightarrow \sum_{i=1}^{M} p_i U(x_i) \geq \sum_{i=1}^{N} q_i U(x_i)$

Must know U!

Let's do stuff with U

① U is concave (risk averse)
, convex (risk preferred)
U is "shape restricted" because of patterns in the data

U depends on wealth

We define
$V = EU = \sum_{i=1}^{N} p_i U(x_i)$

Flip 2 coins
heads → 0
tails → I pay you $-T$

$\{0 \;;\; -T\}$
$\;\; p_1 \quad\; p_2$

Utility fn: $-\exp(-\gamma x)$

$\left\{ \rightarrow p_1(-\exp(0)) + p_2(-\exp(\gamma T)) \atop - (p_1 + (1-p_1)\exp(\gamma T)) \right\}$ How much you should pay to play

Certainty Equivalent

$= -\exp(-\gamma\, CE)$

Take logs,

$-\frac{1}{\gamma}\log(p_1 + (1-p_1)\exp(\gamma T)) = CE$

The assumption you make about utility and probability are what's important

"If you raise your hand and are right, I'll bring you a potato" - Levon

		0.1-E	E
→	0	+1m	-10
→	0	+2m	

Ch 6 of MWG

11/8
TA

SEU: A & A

S: states

X: outcomes

$\mathbb{P}$: all possible distribution over X

H: acts $= \{h \mid h : S \to \mathbb{P}\}$

π: prior / belief over states

SEU: one belief $\pi \in \Delta(S) = \mathbb{P}$

$$SEU(h) = \sum_{s \in S} \pi(s) \sum_{x \in X} h(x|s)\, u(x)$$

$$\max_h SEU(h)$$

Beyond SEU (Gilboa-Schmeidler type)
Only have a set of beliefs Q

<u>max-min utility</u>

$$\max \{ \min_{\pi \in Q} [\sum_{s \in S} \pi(s) \sum h(x|s) u(x)]$$

Ex 1)

$\bar{w}$ — guess right → $\beta \bar{w}$; guess wrong → $\alpha \bar{w}$

$u(w) = \ln(w)$

In state H: heads is $\frac{2}{3}$

$$\begin{aligned} SEU = & P\{\text{heads} \mid \text{guess heads}\} \ln(\beta \bar{w}) \\ & + P\{\text{heads} \mid \text{guess tails}\} \ln(\alpha \bar{w}) \\ & + P\{\text{tails} \mid \text{guess heads}\} \ln(\alpha \bar{w}) \\ & + P\{\text{tails} \mid \text{guess tails}\} \ln(\beta \bar{w}) \\ = & \tfrac{2}{3} \ln(\beta \bar{w}) + \tfrac{2}{3} \ln(\alpha \bar{w}) \\ & + \tfrac{1}{3} \ln(\alpha \bar{w}) + \tfrac{1}{3} \ln(\beta \bar{w}) \end{aligned}$$

In state T: heads is $\frac{1}{3}$

$$\begin{aligned} SEU = & P\{\text{heads} \mid \text{guess heads}\} \ln(\beta \bar{w}) \\ & + P\{\text{heads} \mid \text{guess tails}\} \ln(\alpha \bar{w}) \\ & + P\{\text{tails} \mid \text{guess heads}\} \ln(\alpha \bar{w}) \\ & + P\{\text{tails} \mid \text{guess tails}\} \ln(\beta \bar{w}) \\ = & \tfrac{1}{3} \ln(\beta \bar{w}) + \tfrac{1}{3} \ln(\alpha \bar{w}) \\ & + \tfrac{2}{3} \ln(\alpha \bar{w}) + \tfrac{2}{3} \ln(\beta \bar{w}) \end{aligned}$$

$$\tfrac{2}{3} \ln(\beta \bar{w}) + \tfrac{2}{3} \ln(\alpha \bar{w}) + \tfrac{1}{3} \ln(\alpha \bar{w}) + \tfrac{1}{3} \ln(\beta \bar{w})$$
$$= \tfrac{1}{3} \ln(\beta \bar{w}) + \tfrac{1}{3} \ln(\alpha \bar{w}) + \tfrac{2}{3} \ln(\alpha \bar{w}) + \tfrac{2}{3} \ln(\beta \bar{w})$$

a) $\pi(T) = \pi(H) = \frac{1}{2}$

If accept,

$$SEU(h) = \tfrac{1}{2} [\tfrac{2}{3} \ln(\beta \bar{w}) + \tfrac{1}{3} \ln(\alpha \bar{w})) + \tfrac{1}{2} (\tfrac{1}{3} \ln(\beta \bar{w}) + \tfrac{2}{3} \ln(\alpha \bar{w})]$$
$$= \tfrac{1}{2} \ln(\beta \alpha \bar{w}^2)$$

You choose which gives higher SEU

$$SEU(t) = \tfrac{1}{2} \ln(\beta \alpha \bar{w}^2)$$

If no accept,

$$SEU(n) = \ln(\bar{w})$$
$$\Rightarrow \ln(\bar{w}) = \tfrac{1}{2} \ln(\beta \alpha \bar{w}^2)$$
$$= \ln(\sqrt{\alpha \beta \bar{w}})$$
$$w = \sqrt{\alpha \beta}\, \bar{w}$$
$$\alpha \beta = 1$$

b)

$$SEU(h) = \min_{\pi(H) \in [0,1]} \pi(H) [\tfrac{2}{3} \ln(\beta \bar{w}) + \tfrac{1}{3} \ln(\alpha \bar{w})] + (1 - \pi(H)) [\tfrac{1}{3} \ln(\beta \bar{w}) + \tfrac{2}{3} \ln(\alpha \bar{w})]$$

"worst case" occurs when $\pi(H) = 0$

$$= \tfrac{1}{3} \ln(\beta \bar{w}) + \tfrac{2}{3} \ln(\alpha \bar{w})$$

$$SEU(t) = \tfrac{1}{3} \ln(\beta \bar{w}) + \tfrac{2}{3} \ln(\alpha \bar{w})$$

no bet

$$SEU(n) = \ln(\bar{w})$$

$$\Rightarrow \ln(\bar{w}) = \tfrac{1}{3} \ln(\beta \bar{w}) + \tfrac{2}{3} \ln(\alpha \bar{w})$$

$$\Rightarrow (\beta \bar{w})^{1/3} (\alpha \bar{w})^{2/3} = w$$

$$\alpha^2 \beta = 1$$

c) For part (a),
It holds

For part (b),

$$SEU(h) = \tfrac{9}{10} \ln(\alpha \bar{w}) + \tfrac{1}{10} \ln(\beta \bar{w})$$
$$SEU(n) = \ln(\bar{w})$$
$$\alpha^9 \beta = 1$$

because worst case vs. worst

2) a)

Correct → \$1

Wrong → −\$1

$u(w) = \log(w)$

$P\{H \mid s_A\} = \frac{1}{3}$

$P\{H \mid s_B\} = \frac{3}{4}$

$\ln(1)$

$u(1) = 1$

$$\begin{aligned} SEU(h) = & \tfrac{1}{2} [P\{H \mid s_A\} \cdot 1 + P\{T \mid s_A\} \cdot (-1)\} \\ & + \tfrac{1}{2} [P\{H \mid s_B\} \cdot 1 + P\{T \mid s_B\} \cdot (-1)\} \\ = & \tfrac{1}{2} (\tfrac{1}{3} - \tfrac{2}{3}) + \tfrac{1}{2} (\tfrac{3}{4} - \tfrac{1}{4}) \\ = & -\tfrac{1}{6} + \tfrac{1}{4} = \tfrac{1}{12} \end{aligned}$$

Check Online

$$\begin{aligned} SEU(t) = & \tfrac{1}{2} [P\{H \mid s_A\}(-1) + P\{T \mid s_A\} \cdot (1)\} \\ & + \tfrac{1}{2} [P\{H \mid s_B\}(-1) + P\{T \mid s_B\}(1)\} \\ = & \tfrac{1}{2} (-\tfrac{1}{3} + \tfrac{2}{3}) + \tfrac{1}{2} (-\tfrac{3}{4} + \tfrac{1}{4}) \\ = & \tfrac{1}{6} - \tfrac{1}{4} = -\tfrac{1}{12} \end{aligned}$$

$$SEU(n) = 0$$

Yes, bet on h

b) $$EU(H) = \min_{p \in [\frac{1}{3}, \frac{2}{3}]} p(\tfrac{1}{3}(w+1) + \tfrac{2}{3}(w-1)] + (1-p)[\tfrac{3}{4}(w+1) + \tfrac{1}{4}(w-1)]$$

Let $p = \frac{2}{3}$ to minimize RHS

$$= \tfrac{2}{3}(w - \tfrac{1}{3}) + \tfrac{1}{3}(w + \tfrac{1}{2})$$

$$= w - \tfrac{1}{18}$$

$$EU(T) = w - \tfrac{2}{9}$$

$$EU(N) = w$$

Choose not to bet

11/11
HW due Friday

States of the World

Potential outcomes
↳ $X_1, \dots, X_N$

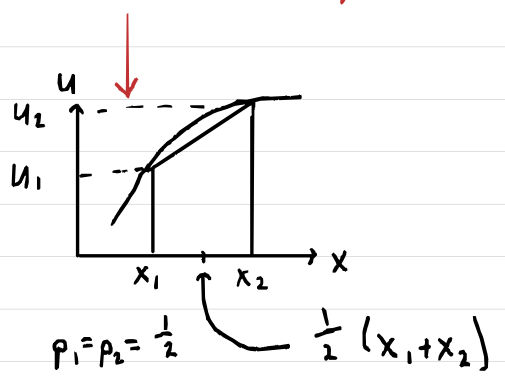
Probability
↳ $p_1, \dots, p_N$

Expected Value $= \sum_{i=1}^{N} p_i X_i$

Expected Utility $= \sum_{i=1}^{N} p_i \underbrace{U(x_i)}$

There is a transformation from X_i's to Utility

The utility curve is the "certain" lottery
The expected value is less
↳ people are risk averse



You should only gamble if your preferences are convex!

Certainty Equivalent
↳ allows you to evaluate any lottery In the world of outcomes by which the lottery is realized
↳ e.g. the dollar amount between certainty and lottery

$$U\left(\sum_{i=1}^{N} p_i x_i\right) \geq \sum_{i=1}^{N} p_i U(x_i)$$

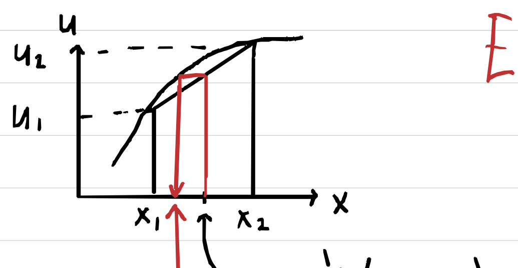
$$U(CE) := \sum_{i=1}^{N} p_i U(x_i)$$

↳ You are willing to accept a lower payoff in order to recei less risk

$\frac{dF(x)}{dx} = f(x)$

$dF(x) = f(x)dx$

$E = \int x f(x) dx$



Certainty Equivalence

Lower CE ⇒ More Risk Averse

How do convex preferences ⇒ quasi concave utility?

$\min X_i \leq CE \leq \max X_i$
$\leq \sum_{i=1}^{N} p_i X_i$

Continuous Case

Expected Value $= \int_{\mathbb{R}} x \, dF(x)$

Expected Utility $= \int_{\mathbb{R}} u(x) \, dF(x)$

Two losses (will come out of initial wealth w)

		prob.	price	how much you buy
Loss 1	$w - L_1$	p_1	q_1	π_1
Loss 2	$w - L_2$	p_2	q_1	π_1
No Loss	$W - 0$	p_0 ($= 1 - p_1 - p_2$)	q_1	π_1

?

Loss 1: $w - L_1 - q_1\pi_1 - q_2\pi_2 + \pi_1$

Loss 2: $w - L_2 - q_1\pi_1 - q_2\pi_2 + \pi_1$

No Loss: $w - 0 - q_1\pi_1 - q_2\pi_2$

Maximize this guys utility

$$\max_{\{\pi_1, \pi_2\}} p_1 U(w - L_1 - q_1\pi_1 - q_2\pi_2 + \pi_1) + p_2 U(w - L_2 - q_1\pi_1 - q_2\pi_2 + \pi_1) + (1 - p_1 - p_2) U(w - q_1\pi_1 - q_2\pi_2)$$

FOC

$[\pi_1]$ $p_1(1 - q_1)U'(\cdot_1) + p_2(-q_2)U'(\cdot_2) + (1 - p_1 - p_2)q_1 U'(\cdot_3) = 0$

$[\pi_2]$ $-p_1 q_2 U'(\cdot_1) + p_2(1 - q_2)U'(\cdot_2) - (1 - p_1 - p_2)q_2 U'(\cdot_3) = 0$

↳ $p_2 U'(\cdot_2)$ $-p_1 q_2 U'(\cdot_1) + -(p_2 q_2)U'(\cdot_2) - (1 - p_1 - p_2)q_2 U'(\cdot_3) = 0$

$$\frac{p_1 U'(\cdot_1)}{p_2 U'(\cdot_2)} = \frac{q_1}{q_2}$$

↳ the rate at which you can transfer resources between states

A perfectly competitive firm would price:

$q_1 = p_1$
$q_2 = p_2$

If $p_1, p_2 \uparrow$ by scalar α then you will be poorer because you are paying more not to take heavy losses in L1 or L2

This required assuming that
↳ we knew u
↳ we knew p
↳ we knew w

11/13

"Actuarilly fair" means that the expected value of the loss equals the price

Former example (where to put wine tree)

		Returns w/o flood	Prob. of flood
L (Left)	200	l	f_l
R (Right)	200	r	f_r
M (midde)	50	m	0

We should ask if the floods are mutually exclusive (ME)

If L and R are divisible and their combinations dominate M then M is useless

Reservation price to pay insurance is M (50)

Now with math,

states of the world

$$\max_{x_l + x_r + x_m \leq 1} \sum_{i \in \{\emptyset, l, r, lr\}} f_i \, u(w_i)$$

$$w_\emptyset = x_m m$$

Let's say the floods are indep.
Then,

$P\{\text{no flood}\} = (1-f_l)(1-f_r) \to x_l l + x_r r + x_m m$

$P\{l \text{ flood}\} = f_l(1-f_r) \to x_r r + x_m m$

$P\{r \text{ flood}\} = f_r(1-f_l) \to x_l l + x_m m$

$P\{lr \text{ flood}\} = f_l f_r \to x_m m$

Suppose preferences are CARA

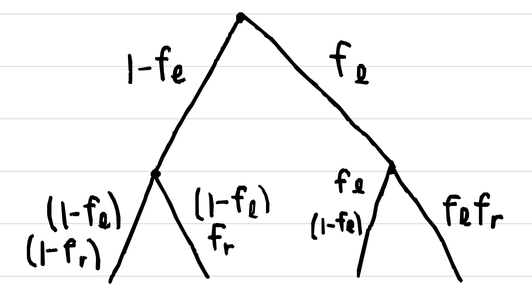
then we maximize $\underbrace{-\sum f_i \exp(-\gamma \cdot w_i)}_{\text{CARA}}$

$\to f_l(1-f_r)\exp(-\gamma(x_r r + x_m m))$

$\to (1-f_l)(1-f_r)\exp(-\gamma(x_l l + x_r r + x_m m))$

$\to (1-f_l)f_r \exp(-\gamma(x_l l + x_m m))$

$\to f_l f_r \exp(-\gamma(x_m m))$



Recall that $x_l + x_r + x_m \leq 1$
↳ she will never leave the constraints slack

We rewrite

$\to f_l(1-f_r)\exp(-\gamma(x_r r + (1-x_r-x_l)m))$

$\to (1-f_l)(1-f_r)\exp(-\gamma(x_l l + x_r r + (1-x_r-x_l)m))$

$\to (1-f_l)f_r \exp(-\gamma(x_l l + (1-x_r-x_l)m))$

$\to f_l f_r \exp(-\gamma((1-x_r-x_l)m))$

So we maximize

$$f_l\left[(1-f_r)\exp(-\gamma(x_r r + (1-x_r-x_l)m)) + f_r \exp(-\gamma((1-x_r-x_l)m))\right]$$
$$+ (1-f_l)\left[(1-f_r)\exp(-\gamma(x_l l + x_r r + (1-x_r-x_l)m)) + f_r \exp(-\gamma(x_l l + (1-x_r-x_l)m))\right]$$

$\downarrow$

$$f_l\left[(1-f_r)\exp(-\gamma(x_r r + (1-x_r-x_l)m)) + f_r \exp(-\gamma((1-x_r-x_l)m))\right]$$
$$+ (1-f_l)\exp(-\gamma x_l l)\left[(1-f_r)\exp(-\gamma(x_r r + (1-x_r-x_l)m)) + f_r \exp(-\gamma((1-x_r-x_l)m))\right]$$

Since the term with x_r is the same, the amount of x_r you choose doesn't depend on the state

Can see more clearly by

$$\exp(\gamma x_l m)\Big\{f_l\left[(1-f_r)\exp(-\gamma(x_r r + (1-x_r)m)) + f_r \exp(-\gamma((1-x_r)m))\right]$$
$$\exp(-\gamma x_l l)(1-f_l)\left[(1-f_r)\exp(-\gamma(x_r r + (1-x_r)m)) + f_r \exp(-\gamma((1-x_r)m))\right]\Big\}$$

Background risk simply means an event that can occur with simple prob., but the occurance (not the prob.) is independent from something?

11/15
TA

$$\max_{\{x_1,x_2\}} \overbrace{\pi U(x_1+ax_2)+(1-\pi)U(x_1+bx_2)}^{EU}$$
s.t. $x_1+x_2=1$
$x_1, x_2 \in [0,1]$

by substitution

$$\Leftrightarrow \max_{x_2\in[0,1]} \pi U(1-x_2+ax_2)+(1-\pi)U(1-x_2+bx_2)$$

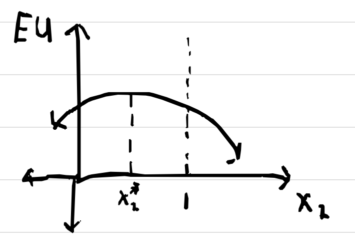
a) Assume $U'' \leq 0$ (by risk aversion)

$$\frac{\partial EU(x_2)}{\partial x_2} = \pi U'(1-x_2+ax_2)(a-1)+(1-\pi)U'(1-x_2+bx_2)(b-1)$$

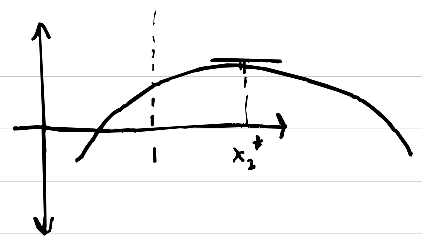
$$\frac{\partial^2 EU(x_2)}{\partial x_2^2} = \pi \underbrace{U''(1-x_2+ax_2)}_{\leq 0}\underbrace{(a-1)^2}_{\geq 0}+(1-\pi)\underbrace{U''(1-x_2+bx_2)}_{\leq 0}\underbrace{(b-1)^2}_{\geq 0}$$

$$\Rightarrow \frac{\partial^2 EU(x_2)}{\partial x_2^2} \leq 0$$

One necessary condition is $\left.\frac{\partial EU(x)}{\partial x_2}\right|_{x_2=1} < 0$



If $x_2^* > 1$



So,
$$\pi U'(a)(a-1)+(1-\pi)U'(b)(b-1) < 0$$

is a suff. and necessary condition due to concavity.

A simple necessary condition is $\min\{a,b\} < 1$

↳ Not sufficient

b) $\left.\frac{\partial EU(x_2)}{\partial x_2}\right|_{x_2=0} > 0$

So,
$$\pi U'(1)(a-1)+(1-\pi)U'(1)(b-1) > 0$$

cancel

c) Assuming (a) and (b) hold let's us ignore corner solutions

$$\frac{\partial EU(x_2)}{\partial x_2} = \pi U'(1-x_2+ax_2)(a-1)+(1-\pi)U'(1-x_2+bx_2)(b-1) \overset{set}{=} 0$$

d) $x_1 = 1-x_2$
$dx_1 = -dx_2$

$$\frac{dx_1}{da} \leq 0 \Leftrightarrow \frac{dx_2}{da} \geq 0$$

Recall our FOC

$$\frac{\partial EU(x_2)}{\partial x_2} = \pi U'(1-x_2+ax_2)(a-1)+(1-\pi)U'(1-x_2+bx_2)(b-1) = 0$$

Take deriv. of a on FOC

$$\pi U''(1-x_2+ax_2)(a-1)\left(-\frac{dx_2}{da}+x_2+a\frac{dx_1}{da}\right) + \pi U'(1-x_2+ax_2) + (1-\pi)U''(1-x_2+bx_2)(b-1)^2\frac{dx_2}{da} = 0$$

Collect terms of $\frac{\partial x_2}{\partial a}$

$$\Rightarrow \pi U''(1-x_2+ax_2)(a-1)x_2 + \pi U'(1-x_2+ax_2)$$
$$= -\left[\pi U''(1-x_2+ax_2)(a-1)^2\frac{dx_2}{da} + (1-\pi)U''(1-x_2+bx_2)(b-1)^2\frac{dx_2}{da}\right]$$
$$= -\left[\pi U''(1-x_2+ax_2)(a-1)^2 + (1-\pi)U''(1-x_2+bx_2)(b-1)^2\right]\frac{dx_2}{da}$$

$$\frac{dx_2}{da} = -\frac{\pi U''(1-x_2+ax_2)(a-1)x_2+\pi U'(1-x_2+ax_2)}{\left[\pi U''(1-x_2+ax_2)(a-1)^2+(1-\pi)U''(1-x_2+bx_2)(b-1)^2\right]} \quad \begin{matrix}\geq 0\\ \leq 0\end{matrix}$$

$$\Rightarrow \frac{dx_2}{da} \geq 0$$

e) π is probability of bad state / lose

So the more likely you are going to lose, the less you will invest in risky asset.

So, $\frac{\partial x_1}{\partial \pi} < 0$ and $\frac{\partial x_2}{\partial \pi} > 0$

e) Recall our FOC

$$\frac{\partial EU(x_2)}{\partial x_2} = \pi U'(1-x_2+ax_2)(a-1)+(1-\pi)U'(1-x_2+bx_2)(b-1) = 0$$

Take deriv w.r.t. π?

"Take derivative of π on FOC"

11/18

Short Quiz on Friday

$\int U(w)\,dF(w)$ is the bread and butter of Expected Utility

Equity Premium
↳ return of risky asset minus safe asset?

---

① Risk free asset $1 \to r$, $r > 1$

② Stochastic return $1 \to q$ $\quad E[q] < r$

Inconsistent with what we expect. So it better be that $E[q] > r$

Consider $E[q] > r$

$$\max_x \int U\left(w\{(1-x)\cdot r + xq\}\right) dF(q)$$

Deriv. w.r.t. x

The sum of 2 concave fns. is concave so the sum of n (aka the integral) is still concave

$$\int U'(w(1-x)r + xq)\cdot W(-r+q)\,dF(q) = 0$$

If $x=0$ and it is the optimum, then $U'(wr)\,W(q-r)\,dF = 0$

Since this is never true, $x=0$ is not optimal.

Asset 1	Rain $\frac{1}{2}$	No Rain $\frac{1}{2}$
Asset 2	Chiefs W $\frac{1}{2}$	Chiefs Lose $\frac{1}{2}$
Asset 3	People Drank in Kansas City $\frac{1}{2}$	People Not Drank in Kansas City $\frac{1}{2}$

Correlated

So one of these assets is irrelevant

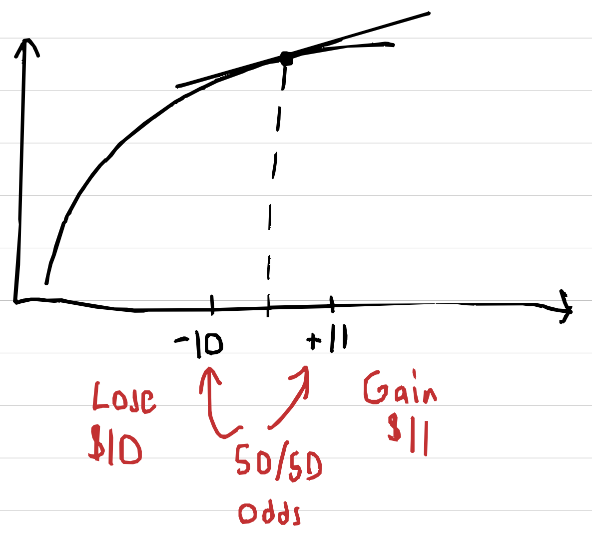
Now we choose between Asset 1 and 2

If two assets have the same profile then their price profile should be basically the same

If Taylor Swift benefits from KC winning, then she should bet on weather in Ithaca

↳ A Ford worker should invest in Tesla

Must know W $\quad F(w) \quad U(w)$
↳ states



Ex: $\frac{1}{2}U(w+L-p) + \frac{1}{2}U(w-p)$

$$\pi\left(\tfrac{1}{2}\right)U(w+L-p) + \pi\left(\tfrac{1}{2}\right)U(w-p)$$

Rank dependence

$$[1-\pi]\left(\tfrac{1}{2}\right)U(w+L-p) + \pi\left(\tfrac{1}{2}\right)U(w-p)$$

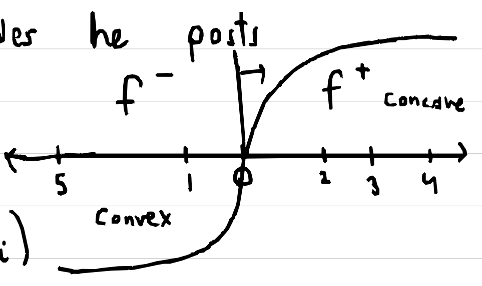
$(w-p_1-p_2)$	$w-p_1-p_2+L_1$	$w-p_1-p_2+L_2$	$W-p_1-p_2+L_1+L_2$
f_1	f_2	f_3	f_4
$\pi(f_1)$	$\pi(f_1+f_2)-\pi(f_1)$	$\pi(f_1+f_2+f_3)-\pi(f_1+f_2)$	↑ $1-\pi(f_1+f_2+f_3)$

Learn the sliders he posts

$\sum f_i U(w_i)$

$\Rightarrow \sum g_i U(w_i)$

$\Rightarrow \sum g_i U(w_i)$



11/20

Review of Decision Making under Risk

Probability is measure of likelihood

Know the paradoxes!
↳ Which assumptions they violate
↳ Ellsberg, St. Petersberg, etc.

MWG 6.C.17

$$W_t = [(1-\alpha_{t-1})R + \alpha_{t-1}X]\,W_{t-1}$$

$$\max E[U(w_2)]$$
$$\text{s.t. } w_2 = [(1-\alpha_1)R + \alpha_1 X]w_1$$

$$= \max \int U\big(((1-\alpha_1)R + \alpha_1 x)w_1\big)\, dF(x)$$

$$\Rightarrow \int w_1 U_1(\cdot)'(X-R)\, dF(x) = 0 \quad \text{by FOC}$$

Using our functional form:

$$U' = [(1-\alpha_1)R + \alpha_1 x]^{-\sigma} w_1^{-\sigma}$$

$$\int [(1-\alpha_1)R + \alpha_1 x]^{-\sigma} w_1^{1-\sigma}(x-R)\, dF(x) = 0$$
by FOC

$$= \int w_1 \{\exp\{-\gamma[[(1-\alpha_1)R+\alpha_1 x]w_1]\}\}(X-R)\, dF(x) = 0$$

$$= \int (X-R)\exp\{-\gamma[[(1-\alpha_1)x]w_1]\, dF(x) = 0$$

Tangent: Apply basic IFT,

$$A(\alpha_1, w_1) = 0$$
$$A(\alpha_1(w_1), w_1) = 0$$

$$A_\alpha \cdot \frac{d\alpha_1}{dw_1} + A_w = 0$$

A