

# Artificial Intelligence, Algorithmic Pricing, and Collusion

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# Background

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# Motivation

- Algorithms becoming increasingly prevalent in practice
  - German gasoline markets (Assad et al. 2024)
  - Smartphone price discrimination (Kehoe et al. 2020)
- Regulatory questions:
  - How do algorithms get to collusive prices?
  - Can they do so in the absence of active principals?
  - Is algorithmic collusion visibly different than tacit collusion?
- Massive lack of theoretical guarantees for this (see Banchio and Mantegazza (2023); Possnig (2024); Lamba and Zhuk (2025))

# Q-Learning Algorithms

- We're familiar with Q-learners
- Specifically, they learn *slowly*. This example has a massive state space and is trying to learn the opponent's policy as part of the state
- Our biggest criticisms are related to this. Specifically:
  - What is the loss in the learning phase?
  - How sensitive are these results to the initialization of the Q-matrix?

# Model

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# Environment

Canonical oligopoly pricing game, with  $n$  firms / products and an outside good, where in each period  $t$  the demand for good  $i$  is

$$q_{i,t} = \frac{e^{\frac{a_i - p_{i,t}}{\mu}}}{\sum_{j=1}^n e^{\frac{a_j - p_{j,t}}{\mu}} + e^{\frac{a_0}{\mu}}}$$

where  $a_i$  is an index of quality,  $\mu$  is in index of differentiation, and  $a_0$  is an outside good. Firms choose  $p_{i,t}$ , and we have exogenous marginal costs  $c_i$ . The stage problem is:

$$\max_{p_{i,t}} q_{i,t}(p_{i,t}) \cdot p_{i,t} - q_{i,t}(p_{i,t}) \cdot c_{i,t}$$

This is quasiconcave but does not in general have a nice closed form solution.

## Simplified Stage Environment

Assume  $n = 2$ ,  $c_i = 1$ ,  $a_i = 2$ ,  $a_0 = 0$ , and  $\mu = \frac{1}{4}$ . Then the stage game reduces to

$$\max_{p_i} \frac{(p_i - 1)e^{8-4p_i}}{e^{8-4p_i} + e^{8-4p_j} + 1}$$

This is strictly concave, and we have that it admits Nash prices

$$p_i^N = p_j^N \approx 1.473$$

and monopoly prices are obtained from setting  $n = 1$ , where we attain

$$p^M = \frac{5}{4} - \frac{1}{4}W_n(2e^3) \approx 1.925$$

## Simplified Stage Environment

We basically have an extension of a Prisoner's Dilemma:

	$N$	$M$
$N$	(0.22, 0.22)	(0.37, 0.12)
$M$	(0.12, 0.37)	(0.34, 0.34)

Since all of the involved functions are continuous and concave, this extends fairly nicely.

So we're making our Q-learners play a repeated Prisoner's Dilemma, and the strategies they learn *should* be similar to canonical repeated PD strategies.