RL Chapter 3 - Finite Markov Decision Process

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Introduction

1. Actions influence not only immediate rewards, but also subsequent situations.
2. Trade off between immediate and delayed reward.

Definition

1. The **agent** is the learner and decision maker.
2. The **environment** is everything the agent interacts with. The environment usually include anything that cannot be arbitrarily changed by the agent.

Setup

1. Time steps are discrete: $t = 0, 1, 2, \cdots$
2. At each step, the agent receives information on the current state $S_t \in S$ and selects their action $A_t \in A(S_t)$.
3. Depending on the action, the agent receives a reward $R_{t+1} \in R \subset \mathbb{R}$ and moves to the next state S_{t+1}.

A sequence follows like:

$$S_0, A_0, R_1, S_1, A_1, R_2, S_2, \cdots$$

Returns

1. **Episodes** are cases where there is a natural notion of final time step.
2. **Continuing Tasks** are those going on continuously without limit.
3. The agent's goal is to maximize the expected discount return:

$$G_t \equiv R_{t+1} + R_{t+2} + \cdots = \sum_{k=0}^{\infty} \delta^k R_{t+k+1}$$

   where $\delta \in [0, 1]$ is the discount rate. (I refuse to use γ to represent it.)

4. By introducing absorbing state after the terminal nodes for episodes, we can use the same notation to describe both situations.

Policies and Value Functions

1. A **policy** is a mapping from states to probabilities of selecting each possible actions. $\pi(a|s)$ describes the probability that $A_t = a$ given $S_t = s$ when the agent follows policy π.
2. The **value function** of a state s under policy π is denoted $v_\pi(s)$.
3. The value function v_π is the unique solution to its Bellman equation defined by

$$v_\pi(s) = \sum_a \pi(a|s) \sum_{s',r} p(s', r|s, a)[r + \delta v_\pi(s')], \quad \forall s \in S$$

Grid World (Example 3.5)

The world is defined as a 5×5 grid. At each cell on the grid, the actions are $\{$north, south, east, west$\}$.
If the agent takes an action that will bring them off grid, their location will remain unchanged and receive a reward of -1.
Any action at state A brings the agent to A' and gives a reward of 10. Any action at state B brings the agent to B' and gives a reward of 5. All other actions give a reward of 0.