RL Chapter 3 - Finite Markov Decision Process

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Introduction

1. Actions influence not only immediate rewards, but also subsequent situations.
2. Trade off between immediate and delayed reward.

Definition

1. The **agent** is the learner and decision maker.
2. The **environment** is everything the agent interacts with. The environment usually include anything that cannot be arbitrarily changed by the agent.

Setup

1. Time steps are discrete: $t = 0, 1, 2, \cdots$

2. At each step, the agent receives information on the current state $S_t \in S$ and selects their action $A_t \in A(S_t)$.

3. Depending on the action, the agent receives a reward $R_{t+1} \in R \subset \mathbb{R}$ and moves to the next state S_{t+1}.

4. The **dynamics** can be described using the probability p defined by

$$p(s', r|s, a) = P\{S_t = s', R_t = r|S_{t-1} = s, A_{t-1} = a\}$$

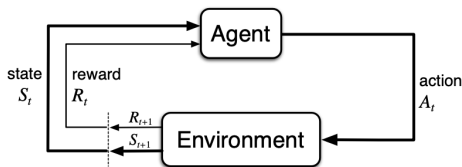A sequence follows like $S_0, A_0, R_1, S_1, A_1, R_2, S_2, \cdots$

Figure 3.1: The agent–environment interaction in a Markov decision process.

Returns

1. **Episodes** are cases where there is a natural notion of final time step.
2. **Continuing Tasks** are those going on continuously without limit.
3. The agent's goal is to maximize the expected discount return:

$$G_t \equiv R_{t+1} + R_{t+2} + \cdots = \sum_{k=0}^{\infty} \delta^k R_{t+k+1}$$

where $\delta \in [0, 1]$ is the discount rate. (I refuse to use γ to represent it.)

4. By introducing absorbing state after the terminal nodes for episodes, we can use the same notation to describe both situations.

Policies and Value Functions

1. A **policy** is a mapping from states to probabilities of selecting each possible actions. $\pi(a|s)$ describes the probability that $A_t = a$ given $S_t = s$ when the agent follows policy π.

2. The **value function** of a state s under policy π is denoted $v_\pi(s)$.

3. The value function v_π is the unique solution to its Bellman equation defined by

$$v_\pi(s) = \sum_a \pi(a|s) \sum_{s',r} p(s', r|s, a)[r + \delta v_\pi(s')], \quad \forall s \in S$$

Grid World (Example 3.5)

The world is defined as a 5×5 grid. At each cell on the grid, the actions are {north, south, east, west}.

If the agent takes an action that will bring them off grid, their location will remain unchanged and receive a reward of -1.

Any action at state A brings the agent to A' and gives a reward of 10. Any action at state B brings the agent to B' and gives a reward of 5. All other actions give a reward of 0.

Pseudo Code for Grid World

- ▶ π : each direction is played with same probability.
- ▶ Define the world size, possible actions, and rewards.

1. Solve v_π with linear system
    - ▶ Solve for

 $$v_\pi(s) = \sum_a \pi(a|s) \sum_{s',r} p(s',r|s,a)[r + \delta v_\pi(s')], \quad \forall s \in S$$

2. Solve v_π by value function iteration
    - ▶ Set an initial guess for v_k.
    - ▶ **Loop**
      Update v_k with Bellman equation using π given.
      If $|v_{k+1} - v_k|$ is small enough, the loop ends and we find the solution.

Pseudo Code for Grid World

- ▶ We want to find the optimal policy π^* and corresponding v_π^*.
- ▶ Set an initial guess for v_k.
- ▶ **Loop**
  Sweeping through all $a \in A$ to find the maximum value possible and update v_{k+1} with it.
  If $|v_{k+1} - v_k|$ is small enough, the loop ends and we find the optional v_π^*.
- ▶ Giving the optimal value function, sweep through possible actions to find the action that yields the optimal value. This gives the optimal policy π^*.
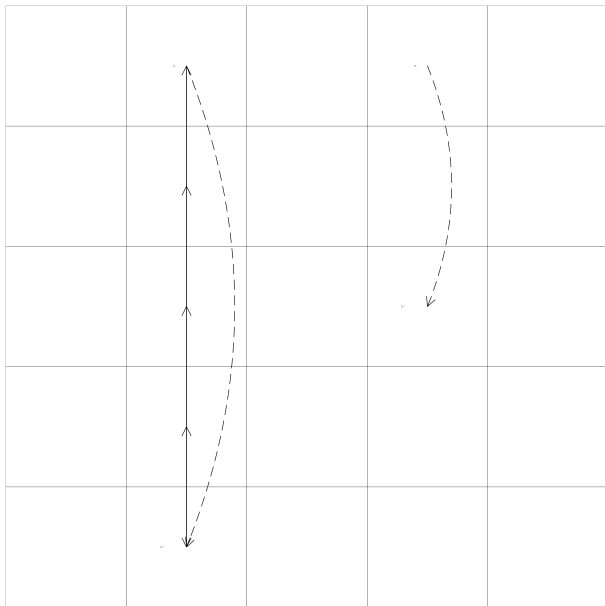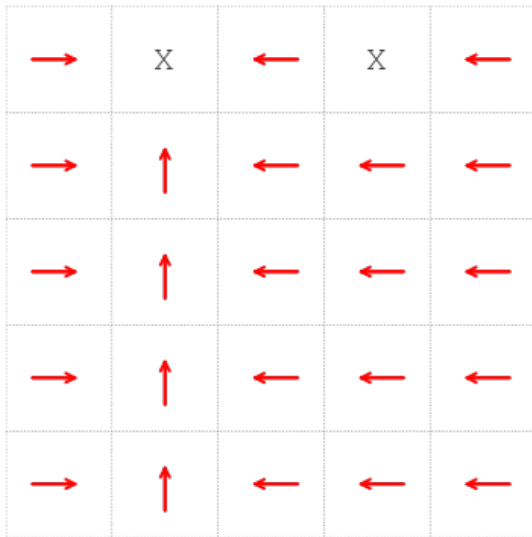
Grid World Map

Grid World Value ($\delta = 0.9$)

Value Function (gamma=0.9)

22.0	24.4	22.0	19.4	17.5
19.8	22.0	19.8	17.8	16.0
17.8	19.8	17.8	16.0	14.4
16.0	17.8	16.0	14.4	13.0
14.4	16.0	14.4	13.0	11.7

Grid World Policy ($\delta = 0.9$)

Best Policy (gamma=0.9)

Grid World Value ($\delta = 0.5$)

Value Function (gamma=0.5)

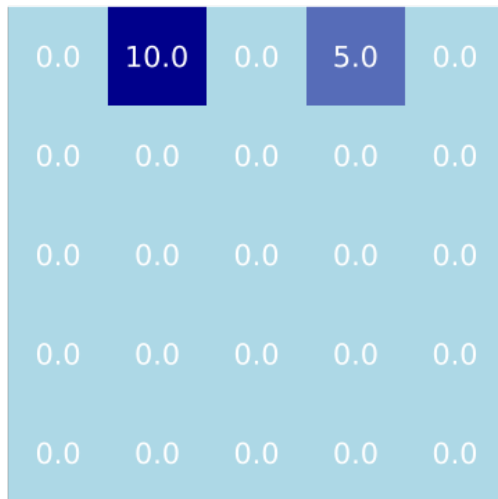| 5.2	10.3	5.2	5.7	2.9
2.6	5.2	2.6	2.9	1.4
1.3	2.6	1.3	1.4	0.7
0.6	1.3	0.6	0.7	0.4
0.3	0.6	0.3	0.4	0.2

Grid World Policy ($\delta = 0.5$)

Best Policy (gamma=0.5)
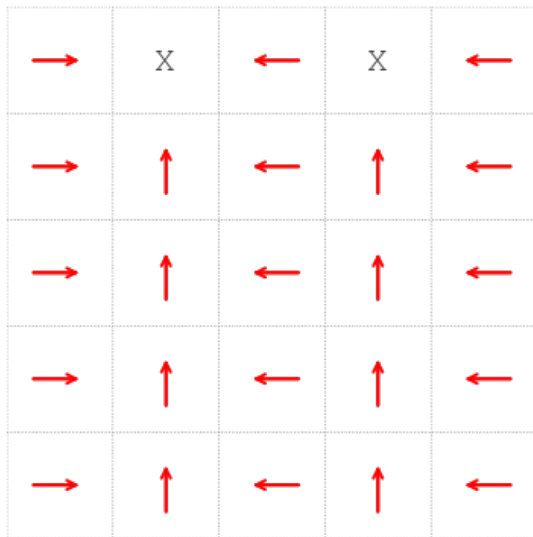
Grid World Value ($\delta = 1$)

Value Function (gamma=1)

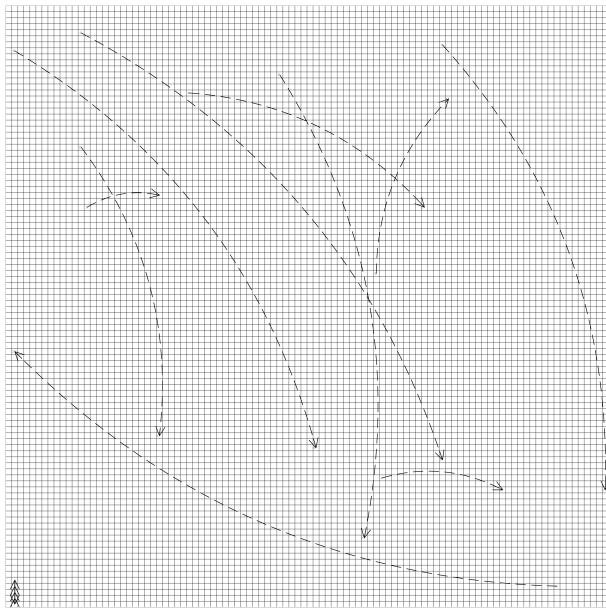|				
0.0	10.0	0.0	5.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0

Grid World Policy ($\delta = 1$)
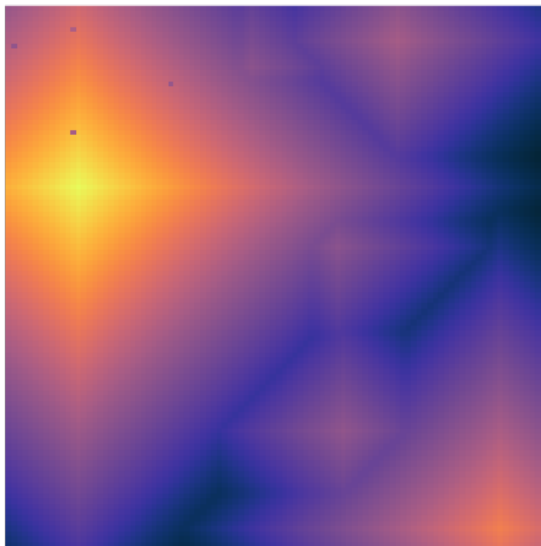
Best Policy (gamma=1)
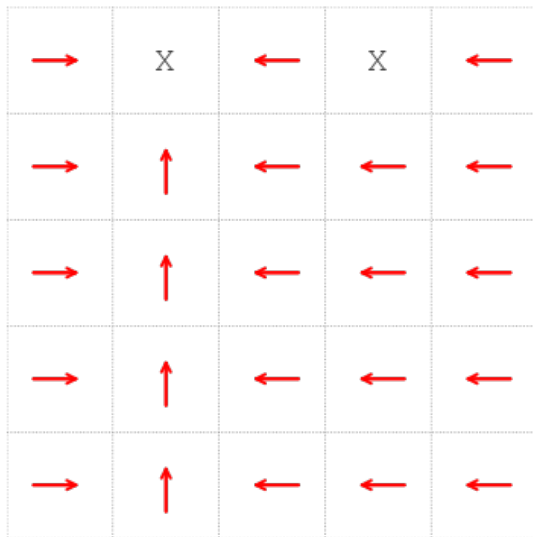
Large Grid World Map

Large Grid World Value (choice)

Value Function (gamma=0.995, 100x100 grid)

Large Grid World Value (random)

Value Function (random policy)