About TA sections:

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Section time and location: 8:40am - 9:55am Rockefeller Hall 132

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In these notes:

1 Sticky Wages

1.1 Assumptions

New assumptions:

- Workers have monopoly power which comes from a "love of variety" in firms' production function (introduced as CES aggregation of labor types similarly to the "love of variety" in consumption),
- Workers face constraints on the frequency of wage changes a la Calvo (similar to prices).

$\Longrightarrow$ Fluctuations in wages introduce an additional source of inefficiency. Focusing solely on stabilizing price inflation is no longer optimal for a central bank as it also has to care about wage inflation. As a result, the central bank now has **three** objectives: stabilizing prices, wages, and the output gap.

1.2 Firm Problem and Labor Demand

As before, we have a continuum of firms indexed by $i \in [0, 1]$. Given the new assumptions, we now also have a continuum of labor types indexed by $j \in [0, 1]$. The production function for firm i is given by:

$$Y_t(i) = A_t N_t(i)^{1-\alpha}$$

where $N_t(i)$ is an index of labor input used by firm i:

$$N_t(i) \equiv \left[\int_0^1 N_t(i,j)^{1-\frac{1}{\varepsilon_w}} dj\right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}}.$$

Given wages, the firm chooses $\{N_t(i,j)\}_{j\in[0,1]}$ to minimize the total cost of labor, subject to producing a given level of output (which is equivalent to using a specific effective labor input $N_t(i)$):

$$\begin{aligned} \min_{\{N_t(i,j)\}_{j\in[0,1]}} \quad & \int_0^1 W_t(j) N_t(i,j)\, dj \\ \text{subject to} \quad & N_t(i) = \left[\int_0^1 N_t(i,j)^{1-\frac{1}{\varepsilon_w}} dj\right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}} \end{aligned}$$

The Lagrangian for this problem is:

$$\mathcal{L} = \int_0^1 W_t(j) N_t(i,j)\, dj - \lambda \left(\left[\int_0^1 N_t(i,j)^{1-\frac{1}{\varepsilon_w}} dj\right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}} - N_t(i) \right)$$

Solving for $N_t(i,j)$ similarly as we did for consumption (Section 1) yields the demand for type-j labor:

$$N_t(i,j) = \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w} N_t(i),$$

where $W_t \equiv \left[\int_0^1 W_t(j)^{1-\varepsilon_w} dj\right]^{\frac{1}{1-\varepsilon_w}}$.

Remark: Note that we have added one additional layer to the firm's problem as firms now choose the composition of labor. Given the optimal allocation of labor types, the firm chooses an optimal price just as before:

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta_p^k \, \mathbb{E}_t \left\{\Lambda_{t,t+k} \left(P_t^* Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t})\right)\right\}$$

subject to

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon_p} C_{t+k}.$$

Compare this with the CMM, where firms were perfectly competitive and only chose their total demand for labor. Later on, in the presence of monopoly power, firms were choosing their prices, and the total amount of labor was then chosen implicitly. In our current setting with sticky prices and wages, firms choose both labor allocation and optimal prices. So, the firm's problem is now two-fold: choose the composition of different types of labor, and, given that composition, choose the optimal price for their differentiated goods.

1.3 Household Problem and Wage Setting

As before, the household solves a utility maximization problem and chooses an optimal consumption basket, driven by a love of variety. The difference is that each household member, worker j, where $j \in [0,1]$, specializes in supplying a specific type of labor j and, having monopoly power in the labor market, sets the wage at which they are willing to supply that labor. However, as wages are sticky, at each period some workers may not be able to reset their wages with probability θ_w.

While choosing what wages to demand, workers take into account that they may not be able to reset their wage in future periods (similar with firms and sticky prices). Optimal wage setting problem is identical for workers of all labor types and is given by

$$\max_{W_t^*} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(C_{t+k}^{-\sigma} \frac{W_t^*}{P_{t+k}} N_{t+k|t} - \frac{N_{t+k|t}^{1+\varphi}}{1+\varphi}\right) Z_{t+k}$$

subject to

$$N_{t+k|t} = \left(\frac{W_t^*}{W_{t+k}}\right)^{-\varepsilon_w} \left(\int_0^1 N_{t+k}(i)\, di\right)$$

Optimality condition:

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k \, \mathbb{E}_t \left\{ N_{t+k|t} Z_{t+k} C_{t+k}^{-\sigma} \left(\frac{W_t^*}{P_{t+k}} - \mathcal{M}_w MRS_{t+k|t} \right) \right\} = 0$$

where

$$MRS_{t+k|t} \equiv C_{t+k}^{\sigma} N_{t+k|t}^{\varphi}, \quad \mathcal{M}_w \equiv \frac{\varepsilon_w}{\varepsilon_w - 1}.$$

Log-linearize this optimality condition around steady state to obtain

$$w_t^* = \mu^w + (1 - \beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k \mathbb{E}_t \left\{ mrs_{t+k|t} + p_{t+k} \right\}$$

Important remark: That is, in this setup, households **choose wages optimally but then supply labor passively, determined by firms' demand**. This is an important assumption because, when wages are sticky, labor demand is well-defined only if **households are willing to supply whatever labor is demanded at the preset wage**. This is ensured in the New Keynesian model by full risk-sharing within households. Note the difference: in competitive markets, labor supply is determined by households' marginal rate of substitution between consumption and leisure, balancing supply and demand for labor. With sticky wages, labor supply is demand-determined, and the MRS may not equal the real wage.

By analogy with price inflation, **wage inflation** is given by

$$\pi_t^w = \beta \mathbb{E}_t \left\{ \pi_{t+1}^w \right\} - \lambda_w \hat{\mu_t}^w$$

where $\lambda_w \equiv \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w(1+\varphi\epsilon_w)}$. Again, by analogy with prices, $\hat{\mu_t}^w \equiv \mu_t^w - \mu^w = \omega_t - mrs_t - \mu^w$ is the deviation of the actual markup with sticky prices from the desired steady state markup.

Intuition: The markup term appears here because when wages in the economy are too low compared to what workers need to maintain their desired markup, those workers who get the chance to renegotiate their wages will ask for higher wages. As more workers do this over time, overall wage levels start to rise, leading to wage inflation. It can be interpreted as a catch-up effect: because wages are lagging behind their "desired" levels, workers who can adjust pull them upward.

1.4 Optimal Subsidy

Assuming flexible prices and wages, inefficiencies in levels can be eliminated by an optimal subsidy. In the presence of monopoly power in both goods and labor markets, this condition takes the form

$$-\mathcal{M}^w \frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}^p}.$$

We know that the steady state level of output is efficient if

$$-\frac{U_{n,t}}{U_{c,t}} = MPN_t,$$

and this is what we want to achieve. To find the optimal subsidy, we can rewrite

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{MPN_t}{(1-\tau^w)\mathcal{M}^p\mathcal{M}^w}.$$

Hence, for optimality, we need the denominator on the RHS to be equal 1, which can be achieved by setting

$$\tau^{w*} = 1 - \frac{1}{\mathcal{M}^p\mathcal{M}^w} = 1 - \frac{(\varepsilon^p - 1)(\varepsilon^w - 1)}{\varepsilon^p \varepsilon^w}.$$

Recall that when only firms had monopoly power, the optimal subsidy was determined by $(1-\tau^{p*})\mathcal{M}^p = 1$ and was given by $\tau^{p*} = \frac{1}{\varepsilon^p}$.

Since both $\mathcal{M}^p > 1$ and $\mathcal{M}^w > 1 \implies \mathcal{M}^p\mathcal{M}^w > \mathcal{M}^p$, it follows that $\tau^{w*} > \tau^{p*}$.

That is, when we have monopoly power in labor market in addition to goods market, we need even larger subsidy to restore efficiency in steady state.

2 Unemployment

To account for unemployment, Gali extends the baseline New Keynesian framework with the following modifications:

- A representative household is now assumed to consist of a continuum of members indexed by $(j, s) \in [0,1] \times [0,1]$, where:
  - j indexes a continuum of occupations.
  - s indexes heterogeneity in labor disutility.
- Labor is modeled as indivisible, i.e., each member either works full time or not at all. The disutility from working is given by χs^{φ}, where $s \in [0,1]$ reflects individual-specific taste for leisure.
- Employment becomes endogenous: only those with low enough s (low disutility from labor) will work at a given wage, leading naturally to unemployment.
- Despite heterogeneous labor participation, full consumption risk-sharing ensures all household members consume the same amount. This allows household-level optimization over both consumption and labor supply (via the participation margin).

Each household member (j, s) decides whether to work based on a comparison between their real wage and their perceived disutility of labor, which is scaled by the marginal utility of consumption (i.e., similarly as before - MRS):

$$\frac{W_t(j)}{P_t} \geq \chi C_t^{\sigma} s^{\varphi}$$

$\Longrightarrow$ Individuals with lower disutility s are more likely to participate. Participation is thus determined at the extensive margin.

The marginal individual $L_t(j)$ in occupation j who is indifferent between working and not working satisfies:

$$\frac{W_t(j)}{P_t} = \chi C_t^{\sigma} L_t(j)^{\varphi}$$

$\Longrightarrow$ $L_t(j)$ is the threshold value of s such that all individuals with $s \leq L_t(j)$ participate. Higher real wages or lower disutility shift more individuals into the labor force.

Log-linearizing the previous equation, we obtain:

$$w_t - p_t = \sigma c_t + \varphi l_t + \xi \implies \varphi l_t = w_t - p_t - \sigma c_t - \xi.$$

Unemployment in this model is simply the difference between those willing to work (l_t) at the given wage and those who actually have a job (n_t):

$$u_t \equiv l_t - n_t.$$

Recall that the average wage markup is defined by

$$\mu_t^w = w_t - p_t - (\sigma c_t + \varphi n_t + \xi).$$

Note that this looks a lot like the last equation on the previous page. Now we can combine this with $u_t \equiv l_t - n_t \implies n_t = l_t - u_t$ to obtain

$$\mu_t^w = w_t - p_t - (\sigma c_t + \varphi(l_t - u_t) + \xi) = w_t - p_t - \sigma c_t - \xi - \varphi l_t + \varphi u_t =$$

$$= w_t - p_t - \sigma c_t - \xi - (w_t - p_t - \sigma c_t - \xi) + \varphi u_t = \varphi u_t.$$

Recall the expression for the **wage inflation**:

$$\pi_t^w = \beta \mathbb{E}_t \left\{ \pi_{t+1}^w \right\} - \lambda_w \hat{\mu_t}^w.$$

Using the result above, we can now rewrite it as

$$\pi_t^w = \beta \mathbb{E}_t \left\{ \pi_{t+1}^w \right\} - \lambda_w \varphi \hat{u}_t.$$

Intuition: This final equation provides a wage Phillips Curve: wage inflation depends negatively on the deviation of unemployment from its natural rate. When unemployment is above the efficient level ($\hat{u}_t > 0$), fewer individuals are employed, reducing upward pressure on wages, and wage inflation falls. Conversely, when unemployment is low ($\hat{u}_t < 0$), wage inflation rises as labor becomes scarcer.

3 Optional: Exam Practice

Try to answer the following questions:

- Describe some of the main facts about wage stickiness. (*See slide 3, lecture 8*).
- What are the key difference in assumptions between the model with and without wage stickiness in the New Keynesian framework?
- If wages are sticky, what other assumptions do you need to make equilibrium labor demand well-defined? Explain how labor supply is determined in the New Keynesian model when wages are sticky. How does it differ from labor supply in the basic model with competitive labor markets?
- Derive demand for labor of type j with wage $W_t(j)$. For what parameters does this function tend to a model with competitive labor markets?
- To make the steady state level of output efficient, do you need a larger or smaller production subsidy than in the model with competitive labor markets? Why?
- How should monetary policy change when wages are sticky relative to the model with flexible wages?
- What changes to the baseline model does Gali make to introduce unemployment in the New Keynesian model?
- Explain how Gali introduce the notion of unemployment into the New Keynesian model. What aspects of actual unemployment does this version of the New Keynesian model capture well? What does it capture less well?
- How is unemployment defined in Gali's framework? How does it differ from other common definitions, (e.g. the definition used to collect unemployment data)?
- What is the difference between the participation constraint/condition of a worker and actual labor supply?
- How does the assumptions made by Gali limit the kind of questions the model can be used to address? (*See slides 24-26, lecture 9*).