Assignment 2
Macro PhD Core
Spring 2025

The due date for this assignment is Thursday February 6th.

Feel free to make additional assumptions that you think are critical to solve a problem. These assumptions need to be clearly stated in your solution.

Taxation and human capital

Consider an economy populated by a continuum of identical households that live infinitely many periods with preferences

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \qquad \beta \in (0,1)$$

Suppose that human capital is produced with existing human capital and additional investment

$$h_{t+1} = (1 - \delta_h)h_t + B n_{ht}^{\gamma} h_t^{1-\gamma}$$

Additional investment in human capital is a function of hours, n_{ht} and the stock h_t. At the same time, the household uses n_m hours to work, that yield effective labor units $z_t = h_t n_{m,t}$. The total endowment of time period is 1. At each point in time, the household can accumulate bonds, invest in capital or consume. They hold initial asset holdings b_0, initial capital k_0, and initial human capital h_0.

Suppose that the household pays consumption taxes, taxes on capital rents (net of depreciation), labor taxes and taxes on capital gains (similarly to the notes). Finally, suppose that the household pays a lump sum tax on income.

Firms maximize profits by choosing the amount of capital and effective labor units they demand each period, and produce out technology $F(k_t, z_t)$.

The feasibility constraint of the economy is as usual,

$$c_t + x_t + g_t \leq F(k_t, z_t)$$

as well as the law of motion for capital

$$k_{t+1} = (1 - \delta)k_t + x_t$$

1. Describe the problem of the household and the problem of the firm

2. Define a recursive competitive equilibrium

3. Characterize the steady state allocation of this economy. How does the s.s. level of human capital depend on tax rates? Explain

4. Go as far as you can describing the effect of changes in taxes on (a) capital gains and (b) labor income for the stock of capital, human capital, capital output ratios and labor. Explain

5. Suppose that the labor income tax changes by 10%. Can the effect on steady state allocations be replicated by a movements in the consumption tax? If yes, explain how. If no, explain what feature of the model prevents such an outcome.

A primer to OLG

We will be working with the canonical OLG with log-utility and Cobb-Douglas production technologies. Each generation lives and consumes for two periods (young and adult). Each generation inelastically supplies 1 unit of labor when young at the market rate w_t and saves resources for consumption when old. The initial generation holds assets $(1+r_0)A_0$ and there is no population growth.

This economy also has a government that finances a stream of expenditure g_t with debt d_{t+1} and lump-sum taxes on the young and the old, δ_{yt}, δ_{ot}. Notice that if "lump-sum taxes" are negative, these are effectively transfers from the government to the households in the economy. We will call the **government policy** a tuple $\delta_{yt}, \delta_{ot}, g_t$. The government finances itself at the risk-free rate r_t and we assume no initial debt $d_0 = 0$. In addition, impose a no-ponzi scheme restriction on the debt of the infinitely lived government.

1. Set up the problem of each generation and describe the optimal saving and consumption policy as a function of the government policy.

   Do not forget to characterize the optimal consumption policy for the initial old generation of the economy. Impose restrictions such that consumption is positive.

2. Assume that the government expenses are constant at g. Characterize the steady state of this economy.

3. What is the level of debt that government optimally holds? Explain why.

4. Bring in the steady state of the economy to the computer assuming a discount factor $\beta = 0.9$ a government to output ratio of 10% and a production technology with a capital share of $\alpha = 0.3$ and a TFP level normalized to 1. The steady state should include quantities, prices and policy.

5. Show and describe what happens if the government to output ratio increases to 15% unexpectedly.

6. Suppose now that the government finances a 10% government to output ratio with labor income taxes (instead of lump-sum) or debt. Characterize the steady state.

7. Compare the impact of an increase in the government to output ratio to 15% in the economy with labor taxes, relative to the one with lump-sum taxes.