

## Material / Practice Questions

We'll cover three problems from Homework 1 today:

1. Let  $X$  be a finite set and let  $(\succsim, \succ)$  be a pair where  $\succsim$  is a (rational) preference relation and  $\succ$  is a transitive subrelation of  $\succsim$ . Assume that the pair satisfies extended transitivity – if  $x \succ y$  and  $y \succsim z$ , or if  $x \succsim y$  and  $y \succ z$ , then  $x \succ z$ . We say that a pair  $(\succsim, \succ)$  is represented by a function  $u$  if:

$$\begin{aligned} u(x) = u(y) &\iff x \sim y \\ u(x) - u(y) > 0 &\iff x \succ y \\ u(x) - u(y) > 1 &\iff x \succ \succ y \end{aligned}$$

Show that every extended preference relation  $(\succsim, \succ)$  can be represented by a utility function  $u$ .

3. A consumer's preferences are defined on bundles  $x = (x_1, x_2)$  in the consumption set  $X = \mathbb{R}_+^2$ . Their choices are based on the 'value' of a bundle, given by the function  $v(x) = x_1 + x_2$ . Their preferences,  $\succsim$ , are structured by two psychological rules that give rise to the following formal definitions, for any  $x, y \in X$ :

$$\begin{aligned} x \succ y &\iff (v(x) \geq 10, v(y) < 10) \text{ or } (v(x), v(y) < 10 \text{ and } v(x) > v(y) + 1) \\ x \sim y &\iff (v(x) \geq 10, v(y) \geq 10) \text{ or } (v(x), v(y) < 10 \text{ and } |v(x) - v(y)| \leq 1) \end{aligned}$$

Additionally,  $\succsim$  holds if either  $\succ$  or  $\sim$  hold, as always.

- a. Are these preferences rational?
  - b. Sketch the indifference map. What are the indifference sets for  $A = (3, 3)$  and  $B = (6, 6)$ ?
  - c. Can these preferences be represented by a real-valued utility function? Explain why or why not.
6. A consumer's preferences are defined by a *bliss point*  $\bar{x} \in \mathbb{R}_{++}^2$ . Their preferences over bundles  $x \in \mathbb{R}_+^2$  are defined by  $\succsim$ , where they prefer bundles closer to the bliss point. Formally, for  $x, y \in X$ :

$$x \succsim y \iff \|x - \bar{x}\| \leq \|y - \bar{x}\|$$

- a. Show that this preference relation is rational. Is it also continuous?
- b. Propose a simple, continuous utility function that represents these preferences. Is the preference relation convex? Is it strictly convex?
- c. Find the Marshallian demand correspondence for a consumer with these preferences. Your answer will depend on whether the bliss point is affordable.