Assignment 1
Macro PhD Core
Spring 2025

The due date for this assignment is Thursday January 30th.

Feel free to make additional assumptions that you think are critical to solve a problem. These assumptions need to be clearly stated in your solution.

Skill-biased technical change

... Historical evidence suggests that complementarity between skilled labor and capital has characterized technological developments throughout the entire 20th century. (Goldin and Katz, 1998)

Consider an economy with two types of workers, skilled, s, and unskilled, u. All workers have identical preferences given by

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \qquad \beta \in (0,1)$$

where u is twice differentiable and concave, and satisfies the Inada conditions in leisure, l and consumption, c.
The technology for production is

$$f(z, k_t, n_t) = z(\lambda(\mu(k_t)^\rho + (1-\mu)(n_s)^\rho)^{\frac{\sigma}{\rho}} + (1-\lambda)n_u^\sigma)^{\frac{1}{\sigma}}$$

where n_s is the amount of skilled labor and n_u the amount of unskilled labor.
Feasibility dictates

$$c_t + x_{kt} \leq f(z, k_t, n_t) \qquad k_0 > 0$$

$$k_{t+1} \leq (1-\delta_k)k_t + x_{kt}$$

$$n_t + l_t \leq 1$$

$$n_s + n_u = n_t$$

where z is the overall productivity of the economy.

1. Show that this production technology displays a constant elasticity of substitution between capital and skilled labor. How does the elasticity depend on the parameter $\rho \in (-\infty, 1)$ and μ?

2. Go as far as you can providing conditions that guarantee existence of a steady state.

3. Go as far as you can characterizing the skill premium $(\frac{w_s}{w_u})$.

   Assume that $\sigma > \rho$ as estimated in Krusell et.al., Econometrica, 2000.

4. Assume that countries differ in the value of productivity z. If every country is in steady state, what does the model imply for the cross country differences in

   (a) the capital skills ratio k/n_s

   (b) output per worker

   (c) the skill-premium

5. Assume that countries differ in the relative supply of skill to unskill workers, $\frac{n_u}{n_s}$. If every country is in steady state, what does the model imply for the cross country differences in

   (a) the capital skills ratio k/n_s

   (b) output per worker

   (c) the skill-premium

6. Can you describe what's the incidence of skill-biased technical change on the skill-premium as function of the characteristics of f? HINT: think about different parameterizations.

Transition paths in the one sector growth model

Consider the one sector growth model as described in the class notes (lecture 1). Suppose that the economy is initiated with 85% of the steady state level of capital, $k_0 = 0.85k^*$ and that there is no government expenditure.

1. Implement the shooting algorithm described in lecture 2 and compute the saddle path of the economy. Assume the following values as benchmark:

   - $\Delta t = 1$.
   - $N = 600$.
   - A production function of the form, zk^α where $A = 1$ and $\alpha = 0.33$.
   - $\beta = 0.98$.
   - $\delta = 0.10$.
   - CES preferences, $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ with $\sigma = 0.95$.

2. Plot the phase diagram and the saddle path of the benchmark economy in the (k, c) space.

3. What happens with the transition dynamic when agents become more impatient? How does the speed of convergence to the steady state change?

4. Write the one sector model in recursive form. Solve for the value function of the representative household via value function iteration. Compute and plot the optimal accumulation policy as well as the residuals of the Euler equation.

5. Using the policy function from the previous question describe the transition path of capital when you start with $k_0 = 0.85k^*$. Compare this path to the one obtained in question (1).

You can write your code in your favorite language, maltab, python, fortran. Useful resources https://python.quantecon.org/cass_koopmans_1.html
The code (and documentation) should be uploaded to Canvas by the beginning of class on Thursday January 30th.