ECON6110: Problem Set 4

Spring 2025

This problem set is due on at 23:59 on April 11, 2025. Every student must write their own solution and submit it individually. Problem set submissions are submitted electronically and may be typed or handwritten. If handwritten, please ensure your work and scan are legible. **Illegible submissions will not be graded.**

Problem 1: Repeated Games with Imperfect Public Monitoring

Consider an infinitely repeated game with imperfect public monitoring. There are two players $i = 1, 2$ and $A_i = \{C, D\}$. The public signals are $Y = \{G, B\}$ (good and bad) where:

$$\Pr(G \mid a) = \begin{cases} p & a = (C, C) \\ q & a = (C, D), (D, C) \\ r & a = (D, D) \end{cases}$$

with $p > q > r$ and $p + q + r = 1$. Assume $p - q > q - r$. The payoffs are:

$$r_i(a_i, y) = \begin{cases} 1 + \frac{2(1-p)}{p-q} & (a_i, y) = (C, G) \\ 1 - \frac{2p}{p-q} & (a_i, y) = (C, B) \\ \frac{2(1-r)}{q-r} & (a_i, y) = (D, G) \\ \frac{-2r}{q-r} & (a_i, y) = (D, B) \end{cases}$$

(a) What is the expected stage-game payoff of an action profile (C, C), i.e., $g_i(C, C)$, for each player? What is the expected payoff of the other action profiles? Discuss and provide an interpretation.

(b) Let

$$v = \frac{\delta r}{1 - \delta(p - r)} \quad \text{and} \quad v' = \frac{1 - \delta + \delta r}{1 - \delta(p - r)}$$

Find a condition on δ under which the payoff v is generated by δ and the set of continuation payoffs $W = \{(v, v), (v', v')\}$.

Hint: to generate v use the stage game action profile $a = (D, D)$ and the map $w(y) = \frac{\delta r}{1-\delta(p-r)}$ if $y = B$ and $w(y) = \frac{1-\delta+\delta r}{1-\delta(p-r)}$ if $y = G$.

(c) Let v, v' be the same as in (b). Find a condition on δ under which the payoff v' is generated by δ and the set of continuation payoffs $W = \{(v, v), (v', v')\}$.

Hint: use the same $w(y)$ as above and $a = (C, C)$.

Problem 2: Bayesian Games

Consider the following model of Bertrand duopoly with differentiated products and asymmetric information. Demand for firm $i = 1, 2$ is equal to:

$$q_i(p_i, p_j) = \begin{cases} a - p_i - b_i p_j & p_i \leq a - b_i p_j \\ 0 & \text{else} \end{cases}$$

Costs are zero for both firms. The sensitivity of firm i's demand to firm j's price is either high or low. That is, b_i is either b_H or b_L; where $1 > b_H > b_L > 0$. We also assume

$$2 - \theta (b_H - b_L) > 0.$$

For each firm i: $b_i = b_L$ with probability θ and $b_i = b_H$ with probability $1 - \theta$; independent of the realization of b_j. Each firm knows its own b_i but not its competitor's. All of this is common knowledge.

(a) What are the action spaces, state spaces, type spaces, prior beliefs, and utility functions in this game? What do pure strategies in this game look like?

(b) Find the symmetric pure-strategy Bayesian Nash equilibrium of this game, i.e. when both firms have the same strategy given their type.

   Hint. Assume an interior solution (i.e. that equilibrium prices and quantities are positive), and verify this is indeed the case.

Problem 3: Bayesian Games

Consider a Bayesian game in which player 1 may be either type a or type b; where type a has probability 0.9 and type b has probability 0.1. Player 2 has no private information. Depending on player 1's types, the payoffs to the two players depend on their actions in $A_1 = \{U, D\}$ and $A_1 = \{R, L\}$ as shown in the following tables:

$t_1 = a$

	L	**R**
U	**2, 2**	**−2, 0**
D	**0, −2**	**0, 0**

$t_1 = b$

	L	**R**
U	**0, 2**	**1, 0**
D	**1, −2**	**2, 0**

Find **all** (pure and mixed) Bayesian Nash equilibria of this game.