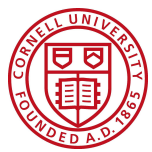
Macroeconomics, PhD core

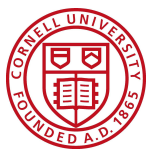
Lecture #12

Julieta Caunedo



Intro

- Variety models deal with horizontal innovation but typically innovations:
  - improve the quality of the good
  - lowers the cost of production
- Endogenous growth: schumpeterian models
  - price competition
  - replacement of old vintages and business stealing effects (entrants)
- Aghion-Howitt (1992-1998), Grossman and Helpman (1991).



Environment

- Representative household with CRRA preferences, constant population and inelastic labor supply L.
- Resource constraint:

$$C(t) + X(t) + Z(t) = Y(t)$$

- R&D, $Z(t)$, investment $X(t)$ and consumption $C(t)$
- Production of final good

$$Y(t) = \frac{1}{1-\beta} L(t)^{\beta} \left(\int_0^1 q(v,t) x(v,t|q)^{1-\beta} dv \right)$$

$x(v,t|q)$ quantity of machines of vintage v of quality $q(v,t)$.

- Source of growth: quality improvements

Environment

- $q(v,t)$ quality of machine of vintage v at time t
- "Quality ladder" for each machine type

$$q(v,t) = \lambda^{n(v,t)} q(v,0)$$

for $\lambda > 1$, and $n(v,t)$ the number of innovations up to t.

- At any point in time, only one quality of any machine v is used.
- Creative destruction: an invention of a higher-quality machine "destroys"/replaces and older machine.
- Once a machine of quality $q(v,t)$ is invented, any quantity can be produced at cost $\psi q(v,t)$
- Innovation is driven by entrants (arrow's replacement effect: incumbents have weaker incentives to innovate, i.e. destroy its own profits)

Environment

- Innovation requires investment
- $Z(v,t)$ units of the final good are used for research in line v with quality $q(v,t)$.
- Rate of innovation

$$z(v,t|q) = \eta \frac{Z(v,t)}{q(v,t)}$$

- Free entry into research
- Firm that innovate has a perpetual patent.

Allocation

Definition

An allocation in this economy is a time path for

- consumption levels, aggregate spending on machines and aggregate R&D spending $\{C(t), X(t), Z(t)\}_{t=0}^{\infty}$
- machine qualities $\{q(v,t)\}_{t=0}^{\infty}$ for $v \in [0,1]$
- prices and quantities of each machine and the NPV of profits from each machine $\{p^x(v,t|q), x(v,t|q), V(v,t|q)\}_{t=0}^{\infty}$ for $v \in [0,1]$
- and interest rates and wages for $\{r(t), w(t)\}_{t=0}^{\infty}$

Equilibrium characterization

- Final good producer

$$\max_{L,x(v)} \frac{1}{1-\beta} L(t)^{\beta} \left(\int_0^1 q(v,t) x(v,t|q)^{1-\beta} dv \right)$$
$$- w(t) L(t) - \int_0^1 p^x(v,t) x(v,t|q) dv$$

- Optimal machine demand

$$x(v,t|q) = \left(\frac{q(v,t)}{p^x(v,t|q)} \right)^{\frac{1}{\beta}} L$$

- Two regimes:
  1. Drastic innovation, firm charge monopoly prices
  2. Limit prices
- For now on we assume drastic innovations, i.e. λ large

$$\lambda \geq \left(\frac{1}{1-\beta} \right)^{\frac{1-\beta}{\beta}}$$

Innovator

- Normalize $\psi = 1 - \beta$

$$\pi(v,t) = \max_x p^x(v,t|q)x(v,t|q) - \psi q(v,t)x(v,t|q)$$

$$\max_x q(v,t)L^\beta x(v,t|q)^{1-\beta} - \psi q(v,t)x(v,t|q)$$

- Profit maximizing-monopoly

$$x(v,t|q) = L$$

$$p^x(v,t|q) = q(v,t)$$

$$\pi(v,t) = \beta q(v,t)L$$

Aggregates

- Total output

$$Y(t) = \frac{1}{1-\beta} Q(t) L$$

$$Q(t) = \int_0^1 q(v,t)dv$$

- Aggregate spending in machines

$$\int_0^1 p^x(v,t|q)x(v,t|q) = Q(t)L$$

- Equilibrium wage rate

$$w(t) = \frac{\beta}{1-\beta} Q(t)$$

Innovation ctn'd

- Value function for the monopolist of variety v and quality $q(v,t)$

$$r(t)V(v,t|q) - \dot{V}(v,t|q) = \pi(v,t|q) - z(v,t|q)V(v,t|q)$$

where $z(v,t|q)$ is the rate of arrival of innovations to variety v

- Last term "Schumpeterian growth"
  - when an innovation occurs, the monopolist loses its monopoly and is replaced by a higher quality producer
  - From then on, it has zero value
  - $z(v,t|q)$ is the rate of of replacement of incumbents in variety/sector v
- Entrants (free entry)

$$\eta V(v,t|q) \leq \frac{q(v,t)}{\lambda} \text{ with equality if } Z(v,t|q) > 0$$

Innovation ctn'd

- Consumer maximization problem, i.e. Euler equation

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta}(r(t) - \rho)$$

- Tranversality condition:

$$\lim_{t \longrightarrow \infty} \exp(-\int_0^t r(s)ds) \int_0^1 V(v, t|q)dv = 0$$

for all q.

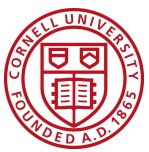
Equilibrium

- $V(v,t)$ is nonstochastic:

Definition

An equilibrium is an allocation that

1. satisfies the aggregate feasibility constraint for goods, machines and the TVC
2. the value of a firm and the average quality satisfy optimality of the monopolist and machine demands, and free entry
3. prices and quantities of machines are as described in the monopolist problem
4. interest rate and wages are consistent with the euler equation and feasibility in labor markets.



BGP

- Given that consumption grows at a constant rate in a BGP, feasibility implies that output grows at constant rate
- From the euler equation, the interest rate is constant
- Is there is positive growth, there must be reasearch in at least a sector.
- Homogeneity of the value of the firm and innovation costs on quality implies free-entry holds for all varieties
- If free entry holds in every period, $\dot{V}(v,t|q) = 0$
- R&D for each machine type has the same productivity, $z(v,t) = z(t) = z*$

BGP ctn'd

- Firm value

$$V(v,t|q) = \frac{\beta q(v,t)L}{r^* + z^*}$$

- Effective discount, $r^* + z^*$.
- By Free entry

$$r^* + z^* = \eta\lambda\beta L$$

- Through the euler equation

$$g^* = \frac{(r^* - \rho}{\theta})$$

$$r^* = g^*\theta + \rho$$

BGP ctn'd

- From the definition of output

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{Q}(t)}{Q(t)}$$

- Dynamics

$$Q(t+\Delta t) = \lambda Q(t) z(t) \Delta t + (1 - z(t)\Delta t) Q(t) + o(\Delta t)$$

- Note: measure of varieties experiencies more than one innovation is second order in Δt, $\frac{o(\Delta t)}{\Delta t} \longrightarrow 0$

$$\dot{Q}(t) = (\lambda - 1) z(t) Q(t)$$

$$g^* = (\lambda - 1) z^*$$

- Equilibrium growth rate

$$g^* = \frac{\eta \lambda \beta L - \rho}{\theta + (\lambda - 1)^{-1}}$$