Assignment 5

ECON 6140

Spring 2025

The due date for this assignment is Thursday March 6th.

Credit Markets and Economic Growth, continued

Consider an economy where individuals can either supply their labor to firms or operate an individual-specific technology. Individuals in this economy are heterogeneous in terms of their wealth and entrepreneurial ability. However, everyone has the same labor productivity, and behaves competitively.
Output is produced using labor (l), capital (k), and entrepreneurial input (e_i):

$$f(e_i, k, l) = e_i^v l^{(1-v)(1-\alpha)} k^{\alpha(1-v)}$$

Individuals' preference is given by

$$\int_0^\infty e^{-\rho t} \frac{c_i(t)^{1-\sigma}}{1-\sigma} dt$$

To simplify assume that the entrepreneurial ability can either be $e_L = 0$ or $e_H > 0$. Note that an individual's ability does not change over time. The measure of the high-ability individuals is normalized to one, and that of the low-ability ones is π. Assume that π is large enough that high-ability types always choose to operate their technology. Further assume that all the initial capital stock in the economy is owned by the individuals with high ability and is equally distributed among them.

No credit market

Assume that the capital rental market is shut down, i.e. to accumulate capital you depend on your own savings.

1. Can you compute the competitive equilibrium by solving the planner problem as before? Why or why not?

2. Consider the entrepreneur's static profit maximization. Taking the individual capital stock and the market wage as ggiven, derive the demand for labor of an individual entrepreneur.

3. Now compute its profits given k_t and w_t.
4. Express the wage that clears the labor market as a function of K_t. Note that $k_t = K_t$ in equilibrium, because all high ability entrepreneurs are identical.
5. Show that the profit of an entrepreneur can be expressed as $AK_t k_t^{\phi}$ where

$$AK_t = (\alpha + v - \alpha v)e^v \pi^{(1-v)(1-\alpha)} K_t^{\frac{-\alpha(1-\alpha)(1-v)^2}{\alpha+v-\alpha v}} \qquad \text{and } \phi = \frac{\alpha(1-v)}{\alpha + v - \alpha v}$$

6. The budget constraint of an individual entrepreneur is

$$\dot{k}(t) = AK_t k_t^{\phi} - \delta k_t - c_t$$

   where $c(t)$ is the consumption of a high-ability individual. Write down the Hamiltonian and derive a pair of ODEs in c(t) and k(t).
7. In equilibrium, $k_t = K_t$. Derive a pair of ODEs in $c(t)$ and $K(t)$.
8. If we assume that initial capital stock belongs to the low-ability agents, how would the dynamics of aggregate variables change?

Extra points. Go back to the economy with perfect credit markets (previous HW)

1. Let $y(t) = [C(t), K(t)]$. Linearize the system that describes the dynamics.
2. Describe the system in matrix form. Go as far as you can characterizing local stability of the system (i.e. what is the sign of the eigenvalues for alternative parameters?)

Optimal R&D in an AK world

Consider a closed economy populated by a continuum of households of measure 1 with homogeneous CES preferences,

$$U \equiv \int_0^\infty e^{-\rho t} \frac{c(t)^{1-\sigma}}{1-\sigma} dt$$

defined over a final good, Y. Final goods are produced using labor $L(t)$ and intermediate inputs, $x_i(t)$ with a constant returns to scale technology

$$Y(t) = L(t)^{1-\alpha} \left(\int_0^{A(t)} x_i(t)^{1-\mu} di \right)^{\frac{\alpha}{1-\mu}}$$

where $1/\mu$ is the elasticity of substitution across intermediate goods. We are interested in $\mu \in (0,1)$, i.e. some substitution; and non-trivial factor shares, $\alpha \in (0,1)$.
At each point in time, there exists a continuum of intermediate goods, x_i with endogenous support $i \in [0, A(t)]$. Intermediate goods are produced with a linear technology in labor

$$x_i(t) = a l_i(t)$$

for $a > 0$. Finally, the support of available intermediate goods for production evolves according to

$$\dot{A}(t) = bX(t)$$

where $X(t)$ are final goods devoted to R&D and $b > 0$.

1. Describe the planner's problem in this economy and characterize the optimal input and labor allocation.

2. Replace the optimal factor allocations into the problem. Does this problem have a recursive representation? If so describe it using the Hamilton-Jacobi-Belman equation, if not, explain what feature prevents recursivity. Finally, show under which conditions the economy reduces to an AK framework.

3. Characterize the growth rate of available intermediate inputs along a BGP, g_A. What is the role of R&D investment for long-run growth?

To decentralize the economy, assume that intermediate input producers compete monopolistically and charge $p_i(t)$ for each variety. In addition, assume that the final good sector as well as the production of new varieties of intermediate goods are competitive. The price of the final good is $p(t)$ while the cost of labor is $w(t)$. Whenever a new variety is available in the market, R&D firms charge $k(t)$ units for the innovation and an intermediate good producer can start producing such a variety. Because of free-entry into the R&D sector equilibrium profits are zero, so that in equilibrium, the return to innovation for the R&D firm, $k(t)$, equals the present discounted value of the profits of the intermediate good producer that manufactures such a variety.

4. What is the optimal pricing schedule for intermediate good producers? What features of the problem warrant this result?
5. How does R&D investment along the BGP compares to the efficient level under the assumptions that warrant an AK-specification?
6. Would a subsidy to R&D investment restore efficiency in the market? If yes, explain how. If not, describe an alternative policy that would restore efficiency.