Macroeconomics ECON 6140 (Second Half)

Lecture 6
Monetary Policy in the New Keynesian Business Cycle Model

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Today's plan

- Sources of inefficiencies in the NK model
- Optimal policy without trade-offs
- Monetary policy and simple rules

Readings: Gali Ch 4

The Simple New Keynesian Model

New Keynesian Phillips Curve

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t$$

Dynamic IS Equation

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n)$$

where $r_t^n = \rho - \sigma\left(1 - \rho_a\right)\psi_{ya} a_t + (1 - \rho_z) z_t$.

Interest rate rule

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widetilde{y}_t + v_t$$

where $v_t = \rho_v v_{t-1} + \varepsilon_t^v$.

Solving Simple New Keynesian Model

Conjecture a solution of the form

$$\begin{aligned} \pi_t &= \psi_\pi u_t \\ \widetilde{y}_t &= \psi_y u_t \end{aligned}$$

Use structural equations to solve for ψ_π and ψ_y

$$\begin{aligned} \begin{bmatrix} \psi_\pi \\ \psi_y \end{bmatrix} &= \begin{bmatrix} 1-\beta\rho_u & \kappa \\ -\frac{1}{\sigma}\left[\phi_\pi - \rho_u\right] & 1+\frac{1}{\sigma}\phi_y \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} \kappa\Lambda^{-1} \\ \left(1-\beta\rho_u\right)\Lambda^{-1} \end{bmatrix} \end{aligned}$$

where

$$\Lambda = \left(1-\beta\rho_u\right)\left(1+\frac{1}{\sigma}\phi_y\right) + \kappa\frac{1}{\sigma}\left[\phi_\pi - \rho_u\right]$$

Efficiency in the New Keynesian model

Efficiency in the New Keynesian model

Two sources of inefficiencies:

1. Market power of individual firms in goods market
2. Nominal rigidities

The Efficient Allocation

$$\max U\left(C_t, N_t; Z_t\right)$$

where $C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$ subject to:

$$C_t(i) = A_t N_t(i)^{1-\alpha},\ \ all\ \ i \in [0,1]$$

$$N_t = \int_0^1 N_t(i) di$$

Optimality conditions:

$$C_t(i) = C_t,\ \ all\ \ i \in [0,1]$$

$$N_t(i) = N_t,\ \ all\ \ i \in [0,1]$$

$$-\frac{U_{n,t}}{U_{c,t}} = MPN_t$$

where $MPN_t \equiv (1-\alpha) A_t N_t^{-\alpha}$

Sources of inefficiency: Market power

Under flexible prices, the optimal price is nominal marginal cost times a mark-up $P_t = \mathcal{M}\frac{W_t}{MPN_t}$, where $\mathcal{M} \equiv \frac{\varepsilon}{\varepsilon-1} > 1$

$$\implies \quad -\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}} < MPN_t$$

Efficiency can be restored by employment subsidy τ so that $P_t = \mathcal{M}\frac{(1-\tau)W_t}{MPN_t}$.

$$\implies \quad -\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}(1-\tau)}$$

The optimal subsidy is set so that $\mathcal{M}(1-\tau) = 1$ or, equivalently, $\tau = \frac{1}{\varepsilon}$.

Sources of inefficiency: Nominal rigidities

Level effects

With a constant employment subsidy that implies an efficient level of output under flexible prices, variation in mark-ups resulting from sticky prices are inefficient

$$\mathcal{M}_t \equiv \frac{P_t}{(1-\tau)(W_t/MPN_t)} = \frac{P_t\mathcal{M}}{W_t/MPN_t}$$

$$\implies \quad -\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = MPN_t\frac{\mathcal{M}}{\mathcal{M}_t} \neq MPN_t$$

Efficiency requires that average markup = desired markup $\forall t$

Sources of inefficiency: Nominal rigidities

Composition effects

- Relative price distortions resulting from staggered price setting: $C_t(i) \neq C_t(j)$ if $P_t(i) \neq P_t(j)$.
- Decreasing marginal utility of individual goods
- Decreasing marginal productivity of labor in a given firm

Since utility of, and production technology for making, all goods are symmetric across all firms, optimal policy requires that prices and quantities (and hence marginal costs) are equalized

Optimal Monetary Policy in the Basic Model

Optimal Monetary Policy in the Basic Model

Key assumptions:

- Optimal (constant) employment subsidy as above
- No inherited relative price distortions, i.e. $P_{-1}(i) = P_{-1}$ for all $i \in [0, 1]$
- Only demand and productivity shocks (no shocks that make flex-price equilibrium inefficient).

Optimal policy and the Divine Coincidence

The optimal policy replicates the flexible price equilibrium allocation.

- Central bank should stabilize marginal costs at a level consistent with firms' desired markup at *given existing prices*

$\Rightarrow$ No firm has an incentive to adjust its price, i.e. $P_t^* = P_{t-1}$ and, hence, $P_t = P_{t-1}$ for $t = 0, 1, 2, ...$(aggregate price stability)

$\Rightarrow$ Equilibrium output and employment match their *natural* counterparts.

Equilibrium under the optimal policy then implies $y_t = y_t^n$, $\widetilde{y}_t = 0, \pi_t = 0, i_t = r_t^n$ for all t.

Implementing optimal policy: An exogenous interest rate rule

Inserting the exogenous interest rate rule

$$i_t = r_t^n$$

into the non-policy block gives

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t$$

$$\widetilde{y}_t = -\frac{1}{\sigma}(E_t\{\pi_{t+1}\}) + E_t\{\widetilde{y}_{t+1}\}$$

where $r_t^n = \rho - \sigma(1-\rho_a)\psi_{ya} a_t + (1-\rho_z) z_t$.

Equilibrium dynamics with exogenous rule

Equilibrium dynamics with an exogenous optimal rule can be represented as

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A_O} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix}$$

where

$$\mathbf{A_O} \equiv \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \beta + \frac{\kappa}{\sigma} \end{bmatrix}$$

Shortcoming: the solution $\widetilde{y}_t = \pi_t = 0$ for all t is *not* unique: one eigenvalue of $\mathbf{A_O}$ is strictly greater than one. $\rightarrow$ indeterminacy (real and nominal).

Interest rate reacts to target variables

We can resolve indeterminacy issues by specifying a rule that responds to inflation and output of the form

$$i_t = r_t^n + \phi_\pi \pi_t + \phi_y \widetilde{y}_t$$

Equilibrium dynamics are then given by

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix}$$

where

$$\mathbf{A}_T \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi} \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix}$$

Why does this work?

Finer points

Existence and uniqueness condition: (Bullard and Mitra (2002)):

$$\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0$$

Taylor-principle interpretation (Woodford (2000)):

$$\begin{aligned}\lim_{k\to\infty} \frac{\partial i_{t+k}}{\partial \pi_t} &= \phi_\pi + \phi_y \lim_{k\to\infty} \frac{\partial \widetilde{y}_{t+k}}{\partial \pi_t} \\ &= \phi_\pi + \frac{\phi_y(1-\beta)}{\kappa}\end{aligned}$$

Shortcomings of Optimal Rules

Optimal rules assumes implicitly assumes that the natural rate of interest is observable (in real time).

This requires policy maker to know

(i) the true model

(ii) true parameter values

(iii) realized shocks

Operational alternative: *Simple rules*

- the policy instrument depends only on observable variables
- do not require knowledge of the true parameter values if they approximate optimal rule across many different models

Welfare-based policy evaluation

We can approximate the welfare of the representative household as

$$\mathbb{W} \equiv - E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{U_t - U_t^n}{U_c C} \right) = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\left(\sigma + \frac{\varphi + \alpha}{1-\alpha} \right) \widetilde{y}_t^2 + \frac{\epsilon}{\lambda} \pi_t^2 \right]$$

so that the expected average welfare loss per period $\mathbb{L}$ is given by

$$\mathbb{L} = \frac{1}{2} \left[\left(\sigma + \frac{\varphi + \alpha}{1-\alpha} \right) var(\widetilde{y}_t) + \frac{\epsilon}{\lambda} \ var(\pi_t) \right]$$

A Taylor-type Simple Rule

The Taylor type rule

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widehat{y}_t$$

or equivalently

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widetilde{y}_t + v_t$$

where $v_t \equiv \phi_y \widehat{y}_t^n$ is one simple rule.
Equilibrium dynamics

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + \mathbf{B}_T(\widehat{r}_t^n - v_t)$$

where

$$\mathbf{A}_T \equiv \Omega \begin{bmatrix} \sigma & 1-\beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix} \quad ; \quad \mathbf{B}_T \equiv \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

and $\Omega \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi}$.
Note that $\widehat{r}_t^n - v_t = -\psi_{ya}(\sigma(1-\rho_a) + \phi_y)a_t + (1-\rho_z)z_t$

Taylor rules and the variance of inflation and the output gap

Taylor Rules								
ϕ_π	1.5	1.5	5	1.5	1.5	1.5	5	1.5
ϕ_y	0.125	0	0	1	0.125	0	0	1
Standard deviations								
	Technology shocks				Demand shocks			
$\sigma(y)$	1.85	2.07	2.25	1.06	0.59	0.68	0.28	0.31
$\sigma(\widetilde{y})$	0.44	0.21	0.03	1.23	0.59	0.68	0.28	0.31
$\sigma(\pi)$	0.69	0.34	0.05	1.94	0.20	0.23	0.09	0.10
$\mathbb{L}$	1.02	0.25	0.006	7.98	0.10	0.13	0.02	0.02

Taylor rules and the variance of inflation and the output gap

Conclusions from numerical exercise:

- Not optimal to stabilize output if fluctuations are mainly due to technology shocks
- Strongly stabilizing inflation leads to good outcomes regardless of whether fluctuations are due to technology or demand shocks

Summary

- Sticky prices introduce inefficiencies both through level and composition effects
- It is possible to derive welfare-based criteria for policy from the utility function of the representative agent
- With only productivity and demand shocks, it is possible to achieve first best by using a production subsidy and by stabilizing the price level completely

You should

- Understand the difference between level and compositional inefficiencies
- Understand how welfare is affected by policy
- Know difference between optimal and simple interest rate rules

The articles posted on Canvas by Clarida, Gali and Gertler (1999) and Woodford (2001, 2002) are useful references for the material covered today.