Problem Set — The Roy Model

N people live in a village. They can either hunt or fish. Each villager is described by two numbers that, in calorie-equivalent units, describe their productivity: An individual of type $(\alpha, \beta) \gg (0, 0)$ would generate α calories hunting and β calories fishing. Each individual can choose one task or the other. Suppose that an efficient village assigns individuals to tasks so as to maximize total caloric intake.

This problem set will run through several versions of this problem, each more complex than the preceding case. The requirement is to characterize equilibrium allocations. What does it mean "to characterize"? This involves deriving a number of necessary conditions, and then showing that these conditions are sufficient.

1. Suppose that every individual can decide to be either a hunter or a fisher. This is a deterministic version of the Roy (1951) model, which is perhaps the earliest paper to be concerned with the implications of selection. The analysis is sharp, but entirely verbal. Remarkable! Everyone should read it. But start here by describing the integer program.

2. What is its linear relaxation?

3. Find the set of efficient matches and stable payoffs, and interpret it.

4. Suppose now that there is capacity for only $V_h < N$ hunters. We might suppose the village auctions off these slots. What is its linear relaxation of the assignment problem? What is the dual of the relaxation?

5. Does the best hunter always hunt?

6. Describe all the stable allocations and interpret your answer fully, both with respect to the meaning of the variables and with respect to how the allocation works.

7. Suppose now that there is capacity for only V_h hunters and V_f fishers. We might suppose the village auctions off these slots. Those not assigned to a slot produce nothing. What is its linear relaxation of the assignment problem? What is the dual of the relaxation?

8. If both V_h and V_f are at least N, we have the scenario of questions 2 and 3. If only one is at least N we have the scenario of questions 4 through 6. Now consider the case where both are less than N, but $V_h + V_f \geq N$. Describe the stable allocations.

9. Now suppose that $V_h + V_f < N$ Since all types are positive, all slots will be filled, but there is now a third group of villagers who will neither hunt nor fish. Describe the stable allocations as best you can.