Taxation and Human Capital

Consider an economy populated by a continuum of identical households that live infinitely many periods with preferences

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \quad \beta \in (0,1)$$

Suppose that human capital is produced with existing human capital and additional investment

$$h_{t+1} = (1-\delta_h)h_t + B n_{ht}^{\gamma} h_t^{1-\gamma}$$

Additional investment in human capital is a function of hours, n_{ht} and the stock h_t. At the same time, the household uses n_m hours to work, that yield effective labor units $z_t = h_t n_{m,t}$. The total endowment of time per period is 1. At each point in time, the household can accumulate bonds, invest in capital, or consume. They hold initial asset holdings b_0, initial capital k_0, and initial human capital h_0. Suppose that the household pays consumption taxes, taxes on capital rents (net of depreciation), labor taxes, and taxes on capital gains (similarly to the notes). Finally, suppose that the household pays a lump sum tax on income. Firms maximize profits by choosing the amount of capital and effective labor units they demand each period and produce output using the technology $F(k_t, z_t)$. The feasibility constraint of the economy is as usual,

$$c_t + x_t + g_t \leq F(k_t, z_t)$$

as well as the law of motion for capital

$$k_{t+1} = (1-\delta)k_t + x_t$$

1. Describe the problem of the household and the problem of the firm.

Household's Problem:

$$\max_{c_t,l_t,x_t,k_{t+1},h_{t+1},n_{m,t},n_{h,t},b_{t+1}} \sum_{t=0}^{\infty} \beta^t u(c_t,l_t) \tag{1}$$

s.t.

$$\begin{aligned}
&(1+\tau_t^c)c_t + p_t^k x_t + b_{t+1} + T_t \leq (1-\tau_t^l)w_t h_t n_{m,t} + r_t k_t - \tau_k(r_t-\delta)k_t + (1+(1-\tau_t^b)r^b)b_t,\\
&k_{t+1} = (1-\delta)k_t + x_t,\\
&h_{t+1} = (1-\delta_h)h_t + Bn_{ht}^{\gamma}h_t^{1-\gamma},\\
&n_{m,t} + n_{h,t} + l_t = 1,\\
&(c_t,l_t,x_t,k_{t+1},h_{t+1},n_{m,t},n_{h,t}) \geq (0,0,0,0,0,0,0),\\
&k_0, b_0, h_0 \text{ - given.}
\end{aligned}$$

Remark: As we usually are only interested in relative prices, it is common to set one price, typically price of the output or consumption good, to 1, i.e., treat that good as a numeraire and measure all other prices relative to that. Since we assume a one-sector economy where consumption and investment goods are produced using the same technology and are perfect substitutes, prices for consumption and investment goods must be equal in equilibrium, i.e., in interior solution $p_t^k = 1,\ \forall t$. Hence, from now on I will omit p_t^k in the household's budget constraint to save on notation.

Firm's Problem

$$\max_{k_t,z_t} F(k_t,z_t) - r_t k_t - w_t z_t \tag{2}$$

where $z_t \equiv h_t n_{m,t}$.

2. Define a recursive competitive equilibrium

A **Recursive competitive equilibrium** (RCE) consists of price sequences $\{w_t^*, r_t^*, p_t^{k*}, r_t^{b*}\}_{t=0}^{\infty}$, allocation sequences $\{c_t^*, x_t^*, k_{t+1}^*, h_{t+1}^*, n_{m,t}^*, n_{h,t}^*, l_t^*\}_{t=0}^{\infty}$, government policy $\{(g_t, \tau_t^c, \tau_t^l, \tau_t^k, \tau_t^b)\}_{t=0}^{\infty}$ and bond holdings $\{b_{t+1}^*\}_{t=0}^{\infty}$ such that:

1. Given prices, the allocation and bond holdings sequence $\{b_{t+1}^*\}$ solve the household's problem (1).

2. Given prices, firms solve their profit maximization problem (2).

3. The government budget balances (policy is feasible):

$$g_t + (1 + r_t^b)b_t = \tau_t^c c_t + \tau_t^l w_t h_t n_{m,t} + \tau_t^k (r_t - \delta)k_t + \tau_t^b r_t^b b_t + T_t + b_{t+1}.$$

4. Markets clear:

$$\begin{aligned}
&c_t + p_t^k x_t + g_t \leq F(k_t, z_t), \\
&z_t = h_t^s n_{m,t}^s \\
&k_t^d = k_t^s = k_t, \\
&b_t^d = b_t^s = b_t.
\end{aligned}$$

Remark: The last three conditions are market clearing for labor, capital and bond markets. Even though I did not use those superscripts in 1.1 to save on notations, I implicitly assumed that households choose labor supply, while firms choose labor demand, and wages are determined to match them in equilibrium. Same is true for other markets.

5. Initial Conditions:

$$b_0^* = b_0 = 0, \quad k_0^* = k_0 > 0, \quad h_0^* = h_0 > 0.$$

3. Characterize the steady state allocation of this economy. How does the s.s. level of human capital depend on tax rates? Explain.

First, make our usual assumptions (Inada conditions, concavity, etc.) to guarantee that all functions are well-behaved and we can focus on the interior solution.

Let us start with the firm's problem. FOCs:

$$w_t = F_{z_t}(k_t, z_t)$$

$$r_t = F_{k_t}(k_t, z_t)$$

Now consider the household's problem. The Lagrangian is

$$\begin{aligned}\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Big\{ & u(c_t, l_t) \\ & + \lambda_t \Big[(1-\tau_t^l) w_t h_t n_{m,t} + r_t k_t - \tau_t^k (r_t - \delta) k_t + \big(1 + (1-\tau_t^b) r^b\big)\, b_t \\ & \qquad - (1+\tau_t^c) c_t - x_t - b_{t+1} - T_t \Big] \\ & + \mu_t \Big[(1-\delta) k_t + x_t - k_{t+1} \Big] \\ & + \nu_t \Big[(1-\delta_h) h_t + B\, n_{h,t}^{\gamma}\, h_t^{1-\gamma} - h_{t+1} \Big] \\ & + \xi_t \Big[1 - n_{m,t} - n_{h,t} - l_t \Big] \Big\}.\end{aligned}$$

FOC with respect to consumption c_t:

$$u_c(c_t, l_t) - (1+\tau^c)\lambda_t = 0 \quad \Longrightarrow \quad \lambda_t = \frac{u_c(c_t, l_t)}{1+\tau_t^c}.$$

FOC with respect to leisure l_t:

$$u_l(c_t, l_t) - \xi_t = 0 \quad \Longrightarrow \quad \xi_t = u_l(c_t, l_t).$$

FOC with respect to leisure x_t:

$$\lambda_t = \mu_t$$

FOC with respect to market labor $n_{m,t}$:

$$\lambda_t (1-\tau_t^l) w_t\, h_t - \xi_t = 0,$$

so that

$$u_c(c_t, l_t) \frac{1-\tau_t^l}{1+\tau_t^c} w_t\, h_t = u_l(c_t, l_t).$$

FOC with respect to human capital accumulation time $n_{h,t}$:

$$\nu_t\, B\, \gamma\, n_{h,t}^{\gamma-1}\, h_t^{1-\gamma} - \xi_t = 0,$$

so that

$$\nu_t\, B\, \gamma\, n_{h,t}^{\gamma-1}\, h_t^{1-\gamma} = u_l(c_t, l_t).$$

FOC with respect to k_{t+1}

$$\mu_t = \beta\left[\lambda_{t+1}\left(r_{t+1} - \tau_t^k\big(r_{t+1} - \delta\big)\right) + (1-\delta)\mu_{t+1}\right].$$

FOC with respect to h_{t+1}

$$\nu_t = \beta\left[\lambda_{t+1}(1-\tau_t^l)w_{t+1}n_{m,t+1} + \nu_{t+1}\left((1-\delta_h) + B\, n_{h,t+1}^{\gamma}(1-\gamma)h_{t+1}^{-\gamma}\right)\right].$$

FOC with respect to b_{t+1}

$$\lambda_t = \beta\,\lambda_{t+1}\left(1 + (1-\tau^b)r^b\right).$$

Steady state:

From the household's FOCs:

$$wh = \frac{1+\tau^c}{1-\tau^l}\frac{u_l(c,l)}{u_c(c,l)} \tag{3}$$

$$1 = \beta\left[r - \tau_k\big(r-\delta\big) + 1 - \delta\right]. \tag{4}$$

$$1 = \beta\,\left(1 + (1-\tau_t^b)r^b\right) \tag{5}$$

$$\nu\, B\,\gamma\, n_h^{\gamma-1}\, h^{1-\gamma} = u_l(c,l). \tag{6}$$

$$1 = \beta\left[u_c(c,l)\frac{1-\tau^l}{(1+\tau^c)\nu}wn_m + 1 - \delta_h + B\, n_h^{\gamma}(1-\gamma)h^{-\gamma}\right]. \tag{7}$$

Thus, (6) and (7) combined imply:

$$1 = \beta\left[\frac{B\,\gamma\,(1-\tau^l)w\,n_m}{1+\tau^c}\,\frac{u_c(c,l)}{u_l(c,l)}\,n_h^{\gamma-1}\,h^{1-\gamma} + 1 - \delta_h + B(1-\gamma)n_h^{\gamma}h^{-\gamma}\right]. \tag{8}$$

Human capital law of motion:

$$h = (1-\delta_h)h + B\,n_h^{\gamma}\,h^{1-\gamma} \implies \delta_h\,h = B\,n_h^{\gamma}\,h^{1-\gamma} \implies h = \left(\frac{B}{\delta_h}\right)^{\frac{1}{\gamma}} n_h. \tag{9}$$

Then, (8) and (9) combined imply:

$$1 = \beta\left[\frac{B\,\gamma\,(1-\tau^l)w\,n_m}{1+\tau^c}\,\frac{u_c(c,l)}{u_l(c,l)}\left(\frac{B}{\delta_h}\right)^{\frac{1-\gamma}{\gamma}} + 1 - \delta_h\gamma\right]. \tag{10}$$

Physical capital law of motion:

$$k = (1-\delta)k + x \implies x = \delta k \tag{11}$$

Budget constraint:

$$(1+\tau^c)c + x + T = (1-\tau^l)whn_m + rk - \tau_k(r-\delta)k + (1-\tau^b)r^b b \tag{12}$$

Time constraint:

$$n_h + n_m + l = 1 \tag{13}$$

Prices:

$$w = F_z(k, z) \tag{14}$$

$$r = F_k(k, z) \tag{15}$$

Feasibility constraint:

$$c + x + g = F(k, z) \tag{16}$$

Budget balance:

$$g + r^b b = \tau^c c + \tau^l whn_m + \tau^k(r-\delta)k + \tau^b r^b b + T \tag{17}$$

How does the s.s. level of human capital depend on tax rates?

Remark: First, note that consumption and labor taxes are equivalent for intratemporal decisions, while taxes on capital gains and capital returns are equivalent for intertemporal decisions.

In equilibrium:

$$h = \frac{1+\tau^c}{1-\tau^l}\frac{u_l(c,l)}{u_c(c,l)}\frac{1}{F_2(k, hn_m)},$$

$$h = \left(\frac{B}{\delta_h}\right)^{\frac{1}{\gamma}} n_h,$$

$$n_h = 1 - n_m - l.$$

Consider effects of taxes:

- An increase in the labor tax (or equivalently, an increase in the consumption tax) reduces the net return to market work. This incentivizes households to allocate more time to human–capital accumulation. Consequently, if the substitution effect dominates, the household may reduce its labor supply while human capital rises until wages adjust to clear the labor market.

- An increase in the tax on capital gains (or, equivalently, a decrease in the tax on capital rents) encourages the accumulation of physical capital. This, in turn, raises firms' demand for efficiency units of labor - if the production function exhibits complementarity between capital and labor - which can boost the accumulation of human capital. Again, this effect persists until wages and interest rates adjustments clear the market.

4. Go as far as you can describing the effect of changes in taxes on (a) capital gains and (b) labor income for the stock of capital, human capital, capital output ratios and labor. Explain

- A **tax on capital gains** (or on the appreciation of assets) reduces the net return received from holding capital or bonds. Because households choose freely between investing in physical capital and bonds, a higher capital gains tax leads to increased investment in physical capital until interest rates adjust to clear the market. Moreover, if physical capital and effective labor are complements in production, the resulting increase in the capital stock raises the demand for effective labor (which combines market labor and human capital).

- An increase in the **tax on labor income** reduces the net wage received by households, which discourages market labor and encourages a reallocation of time toward human capital accumulation. If the corresponding increase in human capital is sufficient to offset the negative effect on market labor and if the production function exhibits complementarities among inputs, then both output and investment in physical capital may increase.

In both cases, overall, the capital-output ratio, the capital stock, and labor allocation adjust depending on the interplay between substitution and income effects and on the degree of substitutability between inputs in production.

5. Suppose that the labor income tax changes by 10%. Can the effect on steady state allocations be replicated by a movements in the consumption tax? If yes, explain how. If no, explain what feature of the model prevents such an outcome.

Under general assumptions, these taxes are equivalent (see the optimality conditions in Part 3).

A primer to OLG

We will be working with the canonical OLG with log-utility and Cobb-Douglas production technologies. Each generation lives and consumes for two periods (young and adult). Each generation inelastically supplies 1 unit of labor when young at the market rate w_t and saves resources for consumption when old. The initial generation holds assets $(1+r_0)A_0$ and there is no population growth. This economy also has a government that finances a stream of expenditure g_t with debt d_{t+1} and lump-sum taxes on the young and the old, δ_{yt}, δ_{0t}. Notice that if "lump-sum taxes" are negative, these are effectively transfers from the government to the households in the economy. We will call the government policy a tuple $\delta_{yt}, \delta_{0t}, g_t$.The government finances itself at the risk-free rate r_t and we assume no initial debt $d_0 = 0$. In addition, impose a no-ponzi scheme restriction on the debt of the infinitely lived government.

1. Set up the problem of each generation and describe the optimal saving and consumption policy as a function of the government policy. Do not forget to characterize the optimal consumption policy for the initial old generation of the economy. Impose restrictions such that consumption is positive.

Household's problem for generation born at time t:

$$\max_{c_t^t, c_{t+1}^t, s_t} \quad \ln\left(c_t^t\right) \; + \; \beta \ln\left(c_{t+1}^t\right)$$

$$\begin{aligned} \text{subject to:} \quad & c_t^t + s_t \; = \; w_t - \delta_{yt}, \\ & c_{t+1}^t \; = \; (1+r_{t+1})\, s_t \; - \; \delta_{o,t+1}, \\ & c_t^t, \; c_{t+1}^t \; \geq 0, \quad s_t \; \geq 0. \end{aligned}$$

Euler equation (standard):

$$\frac{1}{c_t^t} \; = \; \beta \, \frac{1+r_{t+1}}{c_{t+1}^t} \quad \Longrightarrow \quad c_{t+1}^t \; = \; \beta \,(1+r_{t+1})\, c_t^t.$$

Budget constraints:

$$c_t^t \; = \; w_t - \delta_{yt} - s_t, \quad c_{t+1}^t \; = \; (1+r_{t+1}) s_t - \delta_{o,t+1}.$$

Substitute into the Euler equation to get:

$$(1+r_{t+1})\, s_t - \delta_{o,t+1} \; = \; \beta \,(1+r_{t+1}) \left[w_t - \delta_{yt} - s_t\right].$$

Rearranging yields:

$$s_t \;=\; \frac{\beta\,(w_t - \delta_{yt}) + \frac{\delta_{o,t+1}}{1+r_{t+1}}}{1+\beta}.$$

Optimal consumption:

$$c_t^t \;=\; w_t - \delta_{yt} - s_t \;=\; \frac{1}{1+\beta}\Big[w_t - \delta_{yt} \;-\; \tfrac{\delta_{o,t+1}}{1+r_{t+1}}\Big],$$

$$c_{t+1}^t \;=\; \beta\,(1+r_{t+1})\,c_t^t \;=\; \frac{\beta}{1+\beta}\Big[(1+r_{t+1})\,(w_t - \delta_{yt}) \;-\; \delta_{o,t+1}\Big].$$

To ensure positive consumption in each period, we impose:

$$c_t^t \;=\; w_t - \delta_{yt} - s_t \;>\; 0 \qquad \text{and} \qquad c_{t+1}^t \;>\; 0.$$

Depending on the sign and magnitude of the taxes/transfers δ_{yt} and δ_{ot+1}, these impose restrictions on s_t and on the government policy so that consumption is positive.

Initial old generation (at $t = 0$) simply consumes all the assets they have:

$$c_0^{\text{old}} \;=\; (1+r_0)\,A_0 \;-\; \delta_{o0},$$

with $c_0^{\text{old}} > 0$ implying $(1+r_0)\,A_0 \;-\; \delta_{o0} > 0$.

2. Assume that the government expenses are constant at g. Characterize the steady state of this economy.

Again, we will start with the **firm's problem**. In a steady state, $k_t = k_{t+1} = k$, and the firm's FOCs are as usual:

$$R \;=\; A\alpha\, k^{\alpha-1}, \qquad w \;=\; A(1-\alpha)\, k^{\alpha},$$

where $R = 1 + r$.

Remark: Recall from class that we assume full depreciation.

Now, consider the **household's problem**:

$$s \;=\; \frac{\beta}{1+\beta}\Big[w - \delta_y - \tfrac{\delta_o}{1+r}\Big],$$

$$c_{t,ss}^t \;=\; \frac{1}{1+\beta}\Big[w - \delta_y \;-\; \tfrac{\delta_{o,t+1}}{1+r_{t+1}}\Big],$$

$$c^t_{t+1,ss} \;=\; \frac{\beta}{1+\beta}\Big[(1+r)\,(w-\delta_y)\;-\;\delta_o\Big].$$

Because the next period's capital is just the (per-capita) saving of the young minus bond holdings ($b = d$ in equilibrium), the steady-state capital-labor ratio k satisfies

$$k \;=\; s - d.$$

Budget balance: With constant g and constant debt d, the steady-state government flow constraint is

$$\delta_y \;+\; \delta_o \;=\; g \;+\; rd.$$

A no-Ponzi condition requires that d remain finite and non-explosive.

Combining all the expressions above, we obtain the system of equations that characterizes the steady state.

3. What is the level of debt that government optimally holds? Explain why.

The optimal level of debt is $d^* = 0$. The government does not benefit from keeping a positive d^* forever in steady state.

In steady state, $k^* = s^* - d^*$. Holding $d^* > 0$ means k^* is smaller (since s^* is pinned down by preferences and after-tax wage). A smaller k^* implies a lower wage w^* and higher r^*. When k^* is lower, future young workers earn less w^*, so they consume less when young and save less for old age.

Since taxes are lump-sum, the government can cover g each period directly by taxes ($\delta_y^* + \delta_o^* = g$). Any interest obligations r^*d^* (with $r^* > 0$ if $k^* > 0$) must be raised via additional taxes. This purely redistributes from future cohorts to past cohorts without any efficiency gain, while permanently lowering k^*. Sustaining $d^* > 0$ forever simply imposes interest charges on all future generations, with no distortionary-tax benefit to offset it.

4. Bring in the steady state of the economy to the computer assuming a discount factor $\beta = 0.9$ a government to output ratio of 10% and a production technology with a capital share of $\alpha = 0.3$ and a TFP level normalized to 1. The steady state should include quantities, prices and policy.

Remark 1: In this section, we assume zero government debt because we have already explained why it is optimal.

Remark 2: In Table 2, I provide two examples with "extreme" taxation policies under which the government taxes only one of the two generations. If you choose a different policy, your results may differ.

Variable	Tax only young	Tax only old
k^*	0.1658	0.2721
y^*	0.5832	0.6768
w^*	0.4083	0.4737
R^*	1.0556	0.7460
g^*	0.0583	0.0677
δ_τ	0.0583	0.0677
s^*	0.1658	0.2721
c_{young}	0.1842	0.2016
c_{old}	0.1750	0.1354

Table 1: Comparison of steady-state results under different taxation policy

5. Show and describe what happens if the government to output ratio increases to 15% unexpectedly.

Variable	Tax only young	Tax only old
k^*	0.1464	0.3196
y^*	0.5619	0.7102
w^*	0.3933	0.4971
R^*	1.1515	0.6667
g^*	0.0843	0.1065
δ_τ	0.0843	0.1065
s^*	0.1464	0.2691
c_{young}	0.1627	0.2280
c_{old}	0.1686	0.3420

Table 2: Steady state results with $g^*/y^* = 0.15$

- **If the government taxes only the young generation:** They save less since they face a higher tax burden upfront, reducing capital accumulation. With less capital, output and consumption decline. A lower capital stock raises the marginal product of capital, increasing the interest rate, while the marginal product of labor falls, decreasing wages.

- **If the government taxes only the old generation:** The young are unaffected when deciding how much to save, leading to higher capital accumulation. More capital boosts output and consumption but also lowers the marginal product of capital, reducing the interest rate. The higher capital stock increases labor productivity, raising wages further.

6. Suppose now that the government finances a 10% government to output ratio with labor income taxes (instead of lump-sum) or debt. Characterize the steady state.

Household's problem for generation born at time t:

$$\max_{c_t^t, c_{t+1}^t, s_t} \ln\big(c_t^t\big) + \beta \ln\big(c_{t+1}^t\big)$$

$$\begin{aligned} \text{subject to:} \quad c_t^t + s_t &= (1-\tau_t^w) w_t - \delta_{yt}, \\ c_{t+1}^t &= (1+r_{t+1}) s_t - \delta_{o,t+1}, \\ c_t^t,\ c_{t+1}^t &\geq 0, \quad s_t \geq 0. \end{aligned}$$

Euler equation (unaffected):

$$\frac{1}{c_t^t} = \beta \frac{1+r_{t+1}}{c_{t+1}^t} \quad \Longrightarrow \quad c_{t+1}^t = \beta (1+r_{t+1}) c_t^t,$$

assuming $c_t^t > 0$ and $c_{t+1}^t > 0$.

At steady state:

$$s = \frac{\beta}{1+\beta} (1-\tau^w) w,$$

$$c_{t,ss}^t = \frac{1}{1+\beta} (1-\tau_t^w) w,$$

$$c_{t+1,ss}^t = \frac{\beta}{1+\beta} (1+r)(1-\tau_t^w) w.$$

The **firm's problem** does not change.

Budget balance:

$$d_{t+1} = (1+r_t) d_t + g - \tau_t w_t,$$

subject to a no–Ponzi condition. In steady state, we set $d_{t+1} = d$ and $d_t = d$, so

$$d = (1+r) d + g - \tau w \implies \tau w = g + rd.$$

Numerical solution is presented in Table 3.

Variable	No debt	With debt = 0.2
$k_\star$	0.1658	0.1220
$y_\star$	0.5832	0.5320
$w_\star$	0.4083	0.3724
$R_\star$	0.0556	0.3084
$g_\star$	0.0583	0.0532
$d_\star$	0.0000	0.2000
$\tau_{w,\star}$	0.1429	0.3085
$s_\star$	0.1658	0.1220
c_{young}	0.1842	0.1355
c_{old}	0.1750	0.1596

Table 3: Steady-state results comparison (with and without debt)

7. Compare the impact of an increase in the government to output ratio to 15% in the economy with labor taxes, relative to the one with lump-sum taxes.

Remark: Here I assume that the government only taxes the young generation. Your version can be different.

Case 1: Lump-Sum Taxes. Government sets a lump-sum tax $\delta_y^* = g$ on the young each period, no debt.

$$y^* = (k^*)^\alpha, \quad w^* = (1-\alpha)\,(k^*)^\alpha, \quad g = \gamma\, k^{*\alpha}.$$

$$s^* = \frac{\beta}{1+\beta}\big(w^* - \delta_y^*\big), \quad s^* = k^*, \quad \delta_y^* = g.$$

Hence

$$k^* \;=\; \frac{\beta}{1+\beta}\big((1-\alpha)(k^*)^\alpha - \gamma\,(k^*)^\alpha\big) \implies k^* \;=\; \left[\frac{\beta}{1+\beta}\,(1-\alpha-\gamma)\right]^{\frac{1}{1-\alpha}}.$$

An increase in $\gamma : 0.10 \to 0.15$ lowers k^*, lowering y^*, w^* and raising r^*.

Case 2: Labor Income Taxes. Government sets a lump-sum tax $\tau^*\, w^* = g = \gamma\, k^{*\alpha}$, no debt.

$$w^* = (1-\alpha)(k^*)^\alpha, \quad \tau^*(1-\alpha)k^{*\alpha} \;=\; \gamma\, k^{*\alpha} \implies \tau^* = \frac{\gamma}{(1-\alpha)}.$$

$$s^* = \frac{\beta}{1+\beta}\,(1-\tau^*)\,w^*, \quad s^* = k^*.$$

Thus

$$k^* \;=\; \frac{\beta}{1+\beta}\,\big(1-\tfrac{\gamma}{1-\alpha}\big)\,(1-\alpha)(k^*)^\alpha \implies k^* \;=\; \left[\frac{\beta}{1+\beta}\,(1-\alpha-\gamma)\right]^{\frac{1}{1-\alpha}}.$$

Again, raising γ from 0.10 to 0.15 lowers $(1-\tau^*)$, reducing k^*, y^* and w^*, raising r^*.