Competitive Equilibrium and Uncertainty

1. In a two-period economy with risk-averse von Neumann-Morgenstern preferences and a single good in each state, suppose that the aggregate endowment is independent of the state and that all individuals have the same beliefs. Show that the second-period equilibrium allocation is independent of state. Relate the equilibrium price ratios to primitives of the model.

2. Consider an exchange economy with 1 commodity. Each of N consumers flip a coin, that comes up H with probability p and T with probability $(1p)$. If H, the consumer's endowment is e_H, otherwise it is e_T. Suppose all flips are independent. All consumers have identical preferences. They are expected utility maximizers with strictly concave and increasing payoff function u. Let $\omega = (s_1, \ldots, s_N)$ denote a state of the economy, the outcome of each of the N flips. Let $r(\omega)$ denote the fraction of flips that come up H. Notice that the aggregate endowment e_ω depends only upon r, so let e_r denote the aggregate endowment for all states ω such that $r(\omega) = r$.

   (a) Show that in any Pareto optimal allocation, for each r, eachconsumer i's consumption is identical in all states ω such that $r(\omega) = r$.

   (b) Show that in any equilibrium allocation, that for all N the amount any consumer receives in the equilibrium allocation in state ω depends only upon r and i's flip, and not on N.

   (c) Suppose that the common payoff function is CRRA. Compute the contract curve and the set of competitive equilibria.

3. Show directly that if markets are complete, then Radner equilibrium consumption allocations are Pareto optimal. By directly I mean **not** this: Radner equilibrium consumption bundles are A-D equilibrium consumption bundles and the 1st welfare theorem applies. Go directly from Radner equilibrium conditions to first-order conditions for Pareto optimality.

4. Show that if ϕ is an Arrow-Debreu price vector for the two-period economy, and if the matrix A of asset returns has full row rank, then there are spot prices p and asset prices q such that the set of affordable consumption bundles in the sequential trading model is identical. Conversely, for any p and q there is a ρ such that the set of affordable consumption bundles is the same.