1. Building a Ricardian trade model between England and Spain from scratch.

   (a) To construct the global production possibilities frontier, we will assume that each country has a comparative advantage in a different good. Following tradition, assume that England has a comparative advantage in mutton and Spain has a comparative advantage in wine. We have the two individual production possibility sets, for a certain level of production, in Figure 1.
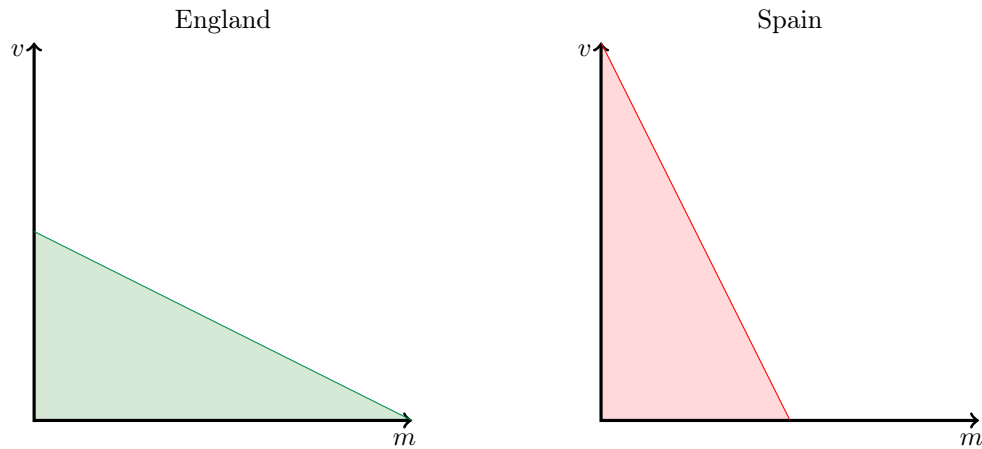

Figure 1: Production Posibility Sets for England and Spain

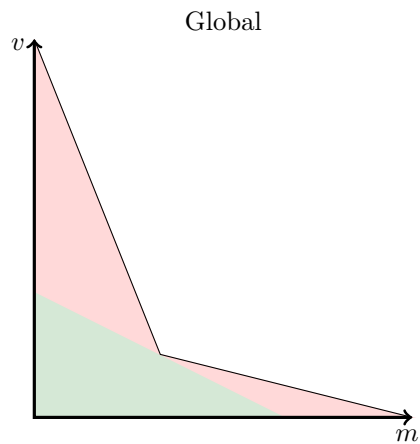   Summing them up, we can get the global production possibilities frontier, in Figure 2.

Figure 2: Global Production Possibility Set

We define England's labor stock as L_E and Spain's labor stock as L_S, and each country has a (different) labor requirement vector a^0_E, a^0_S respectively. England and Spain can also trade goods with each other, at a certain ratio determined by p_v and p_m, the prices. At a certain point x on the global production possibility frontier, the countries are both using the entirety of their labor stock. Each country will be producing the good that they're better at more than the good that they're worse at.

(b) This model says that a market equilibrium is a trade equilibrium between the two countries, where they each consume a certain amount in ratio to the price vector. Moreover, if the two labor requirement vectors are different, then there exist prices at which each country produces only the good that they have a comparative advantage in and they trade.

(c) Observe that if trade is positive (*i.e.* if each country produces a good cheaper than the other), then that means that each country has a comparative advantage in the good that they produce. Moreover, for any reasonable utility functions, if trade is positive then each country will produce only the good that they are better at producing.

(d) An efficient producer will produce a point on their own production possibility frontier. In equilibrium, both producers will of course be efficient here (as long as the goods are actually good).

(e) England has a production matrix $A_E \in \mathbb{R}^{2\times2}$, where a^0_E is the labor requirement vector. Spain has a production matrix $A_S \in \mathbb{R}^{2\times2}$, where a^0_S is the labor requirement vector. Now, England can produce $(I - A_E)^{-1}y$ of its own goods, and Spain can produce $(I - A_S)^{-1}y$ of its own goods. We no longer can use an easy story of comparative advantage, but the countries may still be incentivized to trade with each other. The pattern of trade could be the reverse, if wine is an input for mutton and vice versa. In that case, Spain would be very good at producing inputs for mutton and thus mutton, and vice versa for England, so they would trade in the other direction.

(f) If we add shipping costs which reduce the amount of inputs that go from one direction to the other, it is no longer so clear that the countries will trade everything and produce only one good. Instead, trade will need to be worth more than both the comparative price of production and the loss from shipping. If that does not hold, then the countries would rather use an autarkic regime.

(g) Equilibrium is dual to productive efficiency in the sense that production is efficient if and only if there is full trade in equilibrium. We could formulate the maximization problem as 'maximize (utility from) consumption subject to trade and labor being fulfilled'. Then the dual problem is 'minimize total costs to attain a certain utility from consumption'. As in producer theory, the two problems are dual and strong duality holds, meaning that the equilibrium will be the same no matter the formulation.