1 Econometrics II: Homework 6

Due: Sunday, May 4^{rd}

1 Bootstrap CI for the Median.

1.1 Draw $n = 100$ i.i.d. realizations from a distribution of your choice. Do not hesitate to use a distribution whose density would be slightly harder to estimate, though you should keep in mind that 3.2 will require thousands of draws. Using that sample, compute two 90% bootstrap confidence intervals for the median, one using a bootstrap standard error and one percentile interval. Do they cover? (Note: The 90% size allows you to get away with relatively low B if computation is a concern. The B you need in practice increases as you move into the tails of distributions.)

1.2 Code a little Monte Carlo simulation to assess finite sample size of your CI.

2 Bootstrap Failures. The sampling distribution of the following test statistics cannot be bootstrapped. (In all examples, assume that a sequence of i.i.d. realizations X_i of a r.v. X is available.)

- X follows the Cauchy distribution.
- X follows a well-behaved distribution. The functional of interest is $T_n = \sqrt{n}(|\mathbb{E}_n(X)| - |\mathbb{E}(X)|)$ and the true d.g.p. has $\mathbb{E}(X) = 0$.
- $X \sim N(\mu, \sigma^2)$. We are interested in μ and we know that $\mu \geq 0$, and we want to do maximum likelihood estimation incorporating that constraint. Please verify that the estimator therefore is $\hat{\mu} = \max\{\overline{X}, 0\}$. We are interested in the distribution of $\sqrt{n}(\hat{\mu} - \mu_0)$. Suppose that the true value of μ equals 0.
- $X \sim \text{unif}[0, \alpha]$ and we use the order statistics $\hat{\alpha} = \max_i X_i$ to estimate α.

For each case, give an intuition of why there is a red flag regarding applicability of (simple nonparametric) bootstrap inference. For one case of your choice, verify that the bootstrap fails. (Numerical is ok, though for extra challenge, closed-form is available in most of these cases.)