Macroeconomics ECON 6140 (Second Half)

Lecture 2
A Classical Monetary Model

Cornell University
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The plan

- Present a simple “real” economy
- Derive equilibrium
- (Introduce monetary policy)

Based on Chapter 2 in Gali

A Classical Monetary Model

A Classical Monetary Model

Assumptions

- Two types of agents:
  1. A representative household
  2. Firms
- Perfect competition in goods and labor markets
- Flexible prices and wages
- No capital accumulation
- No fiscal sector
- Closed economy

The Representative Household

The representative household

The representative household takes two decisions:

- How much labor to supply
- How much of its income to consume and how much to save

The representative household also owns the firms.

The representative household

Representative household solves

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right) \tag{1}$$

subject to

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + D_t \tag{2}$$

for $t = 0, 1, 2, ...$and the solvency constraint

$$\lim_{T \to \infty} E_t \left\{ \Lambda_{t,T} (B_T / P_T) \right\} \geq 0 \tag{3}$$

where $\Lambda_{t,T} \equiv \beta^{T-t} U_{c,T} / U_{c,t}$ is the stochastic discount factor.

Labor supply decision

Optimality condition: Real wage $\times$ marginal utility of consumption $=$ marginal disutility of labor

$$\frac{W_t}{P_t} U_{c,t} = -U_{n,t} \tag{4}$$

Intuition:

- The additional disutility of working just a little bit more must be exactly offset by the utility of the consumption that the additional wage income can buy
- The labor supply decision responds to the **relative price of leisure vs consumption**

Consumption/savings decision

Optimality condition: Marginal utility of consumption today = expected marginal utility of consumption tomorrow × expected real return

$$U_{c,t} = \frac{\beta}{Q_t} \; E_t \left\{ U_{c,t+1} \; \frac{P_t}{P_{t+1}} \right\} \tag{5}$$

Intuition:

- The additional utility of consuming a little bit more today must be equal to the expected utility of consuming a little bit more tomorrow, while controlling for impatience, the expected real return and expected differences in the level of consumption
- Consumption and savings decision responds to the **relative price of consumption today vs consumption tomorrow**, i.e. the expected real interest rate

Explicit utility functions

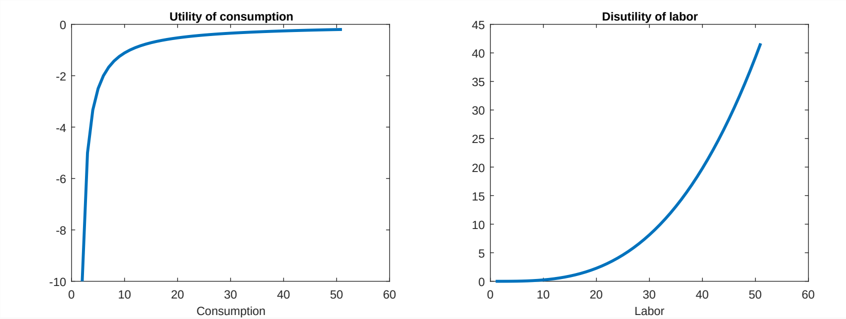
Households have CRRA utility functions that are separable in consumption and labor

$$U(C_t, N_t) = \begin{cases} \frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} & \text{for } \sigma \neq 1 \\ \log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} & \text{for } \sigma = 1 \end{cases}$$

- Decreasing marginal utility of consumption
- Increasing marginal disutility of working

"Separable" utility function $\Rightarrow$ marginal utility of consumption does not depend (directly) on labor and vice versa.

Utility of consumption and labor ($\sigma = \varphi = 2$)



Explicit utility functions: Optimality conditions

$$C_t^{-\sigma} \frac{W_t}{P_t} = N_t^{\varphi} \tag{6}$$

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\} \tag{7}$$

Log-linear optimality conditions

Labor supply decision

$$w_t - p_t = \sigma c_t + \varphi n_t \tag{8}$$

Consumption Euler equation

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) \tag{9}$$

where $\pi_t \equiv p_t - p_{t-1}$, $i_t \equiv -\log Q_t$ and $\rho \equiv -\log \beta$.

Convention: Lower case letters denote logs of corresponding upper case letters

Steady state real interest rate

In steady state and with zero growth rate we have that

$$i = \pi + \rho \tag{10}$$

so that the implied real rate r

$$r \equiv i - \pi = \rho \tag{11}$$

equals the (log of the) inverse of the discount rate.

Firms

Firms

Firms hire labor from households to produce a uniform good using the technology

$$Y_t = A_t N_t^{1-\alpha} \tag{12}$$

where $a_t \equiv \log A_t$ follows an exogenous process

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \tag{13}$$

Note:

- Labor is the only variable input
- $0 < \alpha < 1 \Rightarrow$ decreasing marginal productivity of labor

Profit maximization

A firm's profit is the difference between revenue and cost

$$\text{Firm profit} = P_t Y_t - W_t N_t \tag{14}$$

Maximizing profits subject to (12) while taking the price and wage as given (perfect competition) results in the optimality condition

$$\frac{W_t}{P_t} = (1-\alpha) A_t N_t^{-\alpha} \tag{15}$$

that equates the marginal product of labor with the real marginal cost.

In log-linear terms

$$w_t - p_t = a_t - \alpha n_t + \log(1-\alpha) \tag{16}$$

Equilibrium

Equilibrium

Goods market clearing (with no capital accumulation)

$$y_t = c_t$$

Household labor supply

$$w_t - p_t = \sigma c_t + \varphi n_t$$

Firm labor demand

$$w_t - p_t = a_t - \alpha n_t + \log(1 - \alpha)$$

Production function(aggregate output)

$$y_t = a_t + (1 - \alpha) n_t$$

Asset market clearing (and $B_t = 0$)

$$\begin{aligned} r_t &\equiv i_t - E_t\{\pi_{t+1}\} \\ &= \rho + \sigma E_t\{\Delta c_{t+1}\} \end{aligned}$$

Equilibrium

Implied equilibrium values for real variables as functions of productivity

$$n_t = \psi_{na} a_t + \psi_n$$

$$y_t = \psi_{ya} a_t + \psi_y$$

$$r_t = \rho - \sigma \psi_{ya} (1 - \rho_a) a_t$$

$$\begin{aligned} \omega_t &\equiv w_t - p_t \\ &= a_t - \alpha n_t + \log(1 - \alpha) \\ &= \psi_{\omega a} a_t + \psi_\omega \end{aligned}$$

where

$\psi_{na} \equiv \frac{1-\sigma}{\sigma(1-\alpha)+\varphi+\alpha}$; $\psi_n \equiv \frac{\log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha}$; $\psi_{ya} \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$;

$\psi_y \equiv (1-\alpha)\psi_n$; $\psi_{\omega a} \equiv \frac{\sigma+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$; $\psi_\omega \equiv \frac{(\sigma(1-\alpha)+\varphi)\log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha}$

Equilibrium properties

Nominal neutrality

- Real variables determined independently of monetary policy

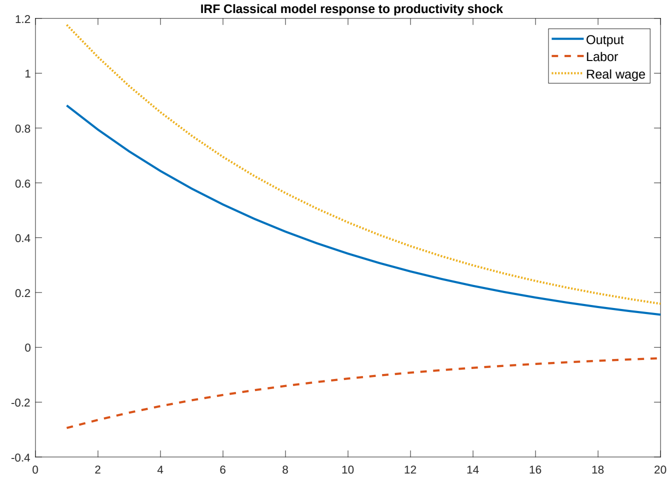
Optimal monetary policy is undetermined

- Inflation does not affect welfare

Price level is undetermined

- Rule for money supply or nominal interest rate is needed

Response of real variables to productivity shock



Monetary Policy and Price Level Determination

A Simple Interest Rate Rule

Nominal interest rate increases with inflation

$$i_t = \rho + \pi + \phi_\pi(\pi_t - \pi) + v_t$$

where $\phi_\pi \geq 0$. Combine with Fischer equation

$$r_t = i_t - E_t\{\pi_{t+1}\}$$

to get

$$\phi_\pi \widehat{\pi}_t = E_t\{\widehat{\pi}_{t+1}\} + \widehat{r}_t - v_t$$

where $\widehat{r}_t \equiv r_t - \rho$ and $\widehat{\pi}_t \equiv \pi_t - \pi$.

If $\phi_\pi > 1$,

$$\begin{aligned} \widehat{\pi}_t &= \sum_{k=0}^{\infty} \phi_\pi^{-(k+1)} E_t\{\widehat{r}_{t+k} - v_{t+k}\} \\ &= -\frac{\sigma(1-\rho_a)\psi_{ya}}{\phi_\pi - \rho_a}\ a_t - \frac{1}{\phi_\pi}\ v_t \end{aligned}$$

Summing up

Main economic mechanisms:

- The representative household trades off
  - Leisure against consumption
  - Consumption today against consumption tomorrow
- Firms hire labor until marginal cost = marginal revenue
- Monetary policy does not affect real variables

You should know:

- How to set up and solve model
- How parameters affect how the economy responds to exogenous variables