About TA sections:

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Section time and location: 8:40am - 9:55am Rockefeller Hall 132

Office hours: Tuesday 4:30-5:30 pm in Uris Hall 451; other times available by appointment (just send me an email).

Remark: Today, we will primarily discuss PS1. These notes below contain information you are likely already familiar with, but you can use them as a reference to refresh your memory.

In these notes:

1 CES production function

The constant elasticity of substitution (CES) production (as well as utility) function, such as the one you saw in PS1, is very common in macroeconomic models. One advantage of this function is its flexibility in allowing different degrees of substitutability between inputs.

General form:

$$Y = A\Big(\sum_{i=1}^{n} \alpha_i^{\frac{1}{\eta}} X_i^{\frac{\eta-1}{\eta}}\Big)^{\frac{\eta}{\eta-1}},$$

where $\eta > 0$ - elasticity of substitution, $\alpha_i > 0$ and $\sum_{i=1}^{n} \alpha_i = 1$.

Another common form is:

$$Y = A\Big(\sum_{i=1}^{n} X_i^{\rho}\Big)^{\frac{\nu}{\rho}},$$

where $\rho \in (-\infty, 1)$, $\rho \neq 0$, $\nu > 0$. In this case, the elasticity of substitution equals $\frac{1}{1-\rho}$.

ν determines return to scale:

- $\nu = 1$ - constant return to scale,
- $\nu > 1$ - increasing return to scale,
- $\nu < 1$ - decreasing return to scale,

Example: A two-factor CES production function is given by:

$$Y = A\left[\alpha K^{\rho} + (1-\alpha) L^{\rho}\right]^{\frac{1}{\rho}}, \tag{1}$$

where all the variables are defined as usual and ρ determines the elasticity of substitution between inputs.

The elasticity of substitution between capital and labor (σ), as the name suggests, is constant and given by:

$$\sigma = -\frac{dln(K/L)}{dln(MP_k/MP_l)} = \frac{1}{1-\rho}. \tag{2}$$

This determines how easily labor and capital can be substituted for one another:

- If $\rho < 0$, then $0 < \sigma < 1$, meaning inputs are complements.
- If $\rho > 0$, then $\sigma > 1$, meaning inputs are substitutes.

- If $\rho = 0$, then $\sigma = 1$, and it reduces to the Cobb-Douglas function:

$$Y = AK^{\alpha}L^{1-\alpha}$$

- As $\rho \to -\infty$, $\sigma \to 0$, which corresponds to a Leontief production function (perfect complements):

$$Y = \min\left(\frac{K}{\alpha}, \frac{L}{1-\alpha}\right),$$

implying that inputs must be used in fixed proportions.

- As $\rho \to 1$, $\sigma \to \infty$, and it reduces to a linear production function, indicating perfect substitutes:

$$Y = A\left(\alpha K + (1-\alpha)L\right)$$

2 Taxation

If we want to understand the role of taxation in shaping savings, labor supply, investment, and other agents' decisions, we usually classify taxes as:

- **Non-distortionary**: These do not alter ***relative prices or incentives***. Since they are independent of economic decisions, they do not affect marginal conditions. The usual example is lump-sum taxes (or transfers).

- **Distortionary**: These influence economic choices by altering relative prices and incentives. That is, such taxes create a wedge between pre-tax and post-tax returns on economic activities, affecting optimal decision-making. In the lecture slides, we saw examples such as labor income tax, consumption tax (i.e., VAT), and taxes on capital and bond returns.

For the government, taxes create revenue to finance government spending, while, from the household perspective, taxes are usually included in the budget constraint. Consider the model we studied in class:

The representative household who maximizes lifetime utility:

$$\max_{\{c_t, k_{t+1}, n_t\}} \sum_{t=0}^{\infty} \beta^t U(c_t, 1 - n_t) \tag{3}$$

subject to the budget constraint:

With non-distortionary taxes:

$$c_t + k_{t+1} - (1 - \delta)k_t = w_t n_t + r_t k_t - T_t, \tag{4}$$

where T_t represents lump-sum taxes or transfers.

With a lump-sum tax T_t, the household's first-order conditions remain:

$$U_c(c_t, 1 - n_t) = \beta U_c(c_{t+1}, 1 - n_{t+1})(1 + r_{t+1}), \tag{5}$$
$$U_n(c_t, 1 - n_t) = U_c(c_t, 1 - n_t) w_t. \tag{6}$$

So, these are identical to the no-tax case, implying no distortions.

With distortionary taxes:

For example, if labor income is taxed at rate τ_n, the household's budget constraint becomes:

$$c_t + k_{t+1} = (1 - \tau_n) w_t n_t + r_t k_t. \tag{7}$$

The labor-leisure first-order condition modifies to:

$$U_n(c_t, 1 - n_t) = (1 - \tau_n) U_c(c_t, n_t) w_t. \tag{8}$$

or equivalently,

$$MRS_{n_t, c_t} = \frac{(1 - \tau_n) w_t}{1}. \tag{9}$$

where 1 is the normalized price of consumption goods.

Since $\tau_n > 0$ lowers the after-tax wage, relative prices of consumption and leisure change. In response, households may decrease their supply of labor. However, to determine its effect on equilibrium, we must first solve the firm's problem and impose market-clearing conditions.

The key point here is that, in competitive equilibrium, in the absence of labor income tax households are paid wage equal to the marginal product of labor, while with the tax their payments decrease proportionally, creating a wedge between the marginal product of labor and the wage received by workers.