MACROECONOMICS II

SECOND HALF

EXERCISE QUESTIONS

The Basic New Keynesian Business Cycle model

(1) Consider the Classical Model described by the following specification (notation as in Gali's textbook). The representative households maximizes

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right)$$

subject to budget constraint

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + D_t$$

for $t = 0, 1, 2, ...$ Goods are produced by a representative firm with technology

$$Y_t = A_t N_t^{1-\alpha}$$

where $a_t \equiv \log A_t$ follows an exogenous process

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

(a) What are the decisions taken by the household? What are the optimality conditions determining these decisions?
(b) Solve for equilibrium output and labor as functions of technology a_t. How does the labor response to a_t depend on σ?

(2) Consider the same model as above, except that there is now a continuum of differentiated goods indexed by $i \in (0, 1)$ so that

$$C_t \equiv \left(\int_0^1 C_{t,i}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} : \varepsilon > 1$$

and

$$C_{t,i} = Y_{t,i} = A_t N_{t,i}^{1-\alpha}$$

(a) Derive the optimal demand for good i as a function of aggregate output and the relative price of good i.
(b) Find the labor inputs used by firm i as a function of the relative price of good i and the aggregate price index P_t.
(c) Derive the employment subsidy τ such that the level of output is efficient.
(d) What is the optimal price set by firm i?

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(e) If prices are flexible and labor markets competitive, what is the dispersion in prices across different goods?

(3) Assume that goods prices are sticky a la Calvo (1983). Show how equilibrium allocations may be inefficient even if there is a production subsidy that makes the steady state level of output efficient.

(4) What parameters of the model determine the extent of the inefficiencies that arise from sticky prices? For what parameter values are the inefficiencies worse?

(5) Derive the New Keynesian Phillips curve from the optimal price of a firm that can reset its price in period t given by

$$p_t^* = \mu + (1 - \theta\beta) \sum_{k=0} (\beta\theta)^k E_t \left(\psi_{t+k|t}\right).$$

(6) Describe either with words or formulas how the price setting decision of firms is determined. How is it affected by Calvo-type price stickiness? What determines the degree to which a firm's price decision is forward looking?

(7) Show that in the absence of cost-push shocks (and in the presence of a employment subsidy) the divine coincidence holds.

(8) Consider the New Keynesian model with cost-push shocks u_t as the only exogenous source of shocks, i.e. consider the standard model but with the modified Phillips Curve given by

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa x_t + u_t$$

where $x_t \equiv y_t - y_t^e$ and

$$\kappa(y_t^e - y_t^n) \equiv u_t = \rho_u u_{t-1} + \varepsilon_t^u$$

(a) Solve for equilibrium using the method of undetermined coefficients under the assumption that monetary policy follows

$$i_t = \rho + \phi_\pi \pi_t$$

(b) What is the effect of a cost push shock on inflation and the output gap x_t? How does the response depend on the coefficient ϕ_π?

(c) Find the policy that maximizes the utility of the representative household and solve for the equilibrium values of inflation and the output gap. Can you find a Taylor-type rule that replicates this equilibrium?

Sticky Wages in the New Keynesian Model

(1) Describe some of the main facts about wage stickiness.

(2) What are the key difference in assumptions between the model with and without wage stickiness in the New Keynesian framework?

(3) Derive demand for labor of type j with wage $W_t(j)$. For what parameters does this function tend to a model with competitive labor markets?

(4) To make the steady state level of output efficient, do you need a larger or smaller production subsidy than in the model with competitive labor markets? Why?

(5) How should monetary policy change when wages are sticky relative to the model with flexible wages?

Unemployment in the New Keynesian Model

(1) What changes to the baseline model does Gali make to introduce unemployment in the New Keynesian model?
(2) What is the difference between the participation constraint/condition of a worker and actual labor supply?
(3) How is unemployment defined in Gali's framework? How does it differ from other common definitions, (e.g. the definition used to collect unemployment data)?
(4) How does the assumptions made by Gali limit the kind of questions the model can be used to address?

The Kalman Filter

(1) For the scalar process

$$\begin{aligned} x_t &= \rho x_{t-1} + u_t \\ z_t &= x_t + v_t \\ \begin{bmatrix} u_t \\ v_t \end{bmatrix} &\sim N\left(0, \begin{bmatrix} \sigma_u^2 & 0 \\ 0 & \sigma_v^2 \end{bmatrix}\right) \\ x_{0|0} &= \overline{x}_0 \\ E\left(\overline{x}_0 - x_0\right)^2 &= p_{0|0} \end{aligned}$$

   (a) find the Kalman gain k_t such that $x_{t|t}$ given by

$$x_{t|t} = \rho x_{t-1|t-1} + k_t\left[z_t - \rho x_{t-1|t-1}\right]$$

   is the expected value of x_t conditional on $\overline{x}_0$ and the history of z_t.
   (b) What is k_t if $\sigma_v^2 = 0$? Interpret.
   (c) What is k_t if $\sigma_v^2 = \infty$? Interpret.
   (d) What is k_t if $\sigma_u^2 = \infty$? Interpret.

(2) Consider the state space system of the form

$$\begin{aligned} X_t &= AX_{t-1} + C\mathbf{u}_t : \mathbf{u}_t \sim N(0, I) \\ Z_t &= DX_t + \mathbf{v}_t \end{aligned}$$

and define

$$P_{t|t-s} \equiv E\left[X_t - E\left(X_t \mid Z^{t-s}\right)\right]\left[X_t - E\left(X_t \mid Z^{t-s}\right)\right]'$$

What restrictions on A, C, D and Σ_v would imply that:
   (a) $P_{t|t} = 0$?
   (b) What are the upper and lower bounds of $P_{t|t}$ and $P_{t|t-1}$? For what values of A, C, D and Σ_v would these bounds be attained?

Calibration, Estimation, Maximization

(1) Describe the empirical strategy that Kydland and Prescott (1996) calls a *computational experiment*. What is the *calibration stage* and the *validation stage*? What are the weaknesses of calibration as a strategy to choose parameters for a model? Under what circumstances should calibration be the preferred strategy?

(2) What is the principal difference between *Calibration* and *Matching Moments*?
(3) Consider the model

$$y_t = \rho_1 y_{t-1} + u_t : u_t \sim N(0, \sigma_u^2)$$

   (a) What is the associated vector Θ of parameters?
   (b) Describe a grid search procedure to find the MLE $\widehat{\Theta}$.
   (c) What other method could you use to find the MLE of Θ?
   (d) What are the advantages and disadvantages of grid search?
(4) Describe the main components needed to estimate the structural parameters of the New Keynesian model by maximum likelihood.