Macroeconomics ECON 6140 (Second Half)

Lecture 3
The Basic New Keynesian Business Cycle Model

Cornell University
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Today's plan

- Dixit-Stiglitz (CES) Demand Systems
- The Basic New Keynesian Business Cycle Model
- Sources of inefficiency

Gali Ch 3 + Lecture Notes on CES Demand Systems

The Basic New Keynesian Model: Why is it useful?

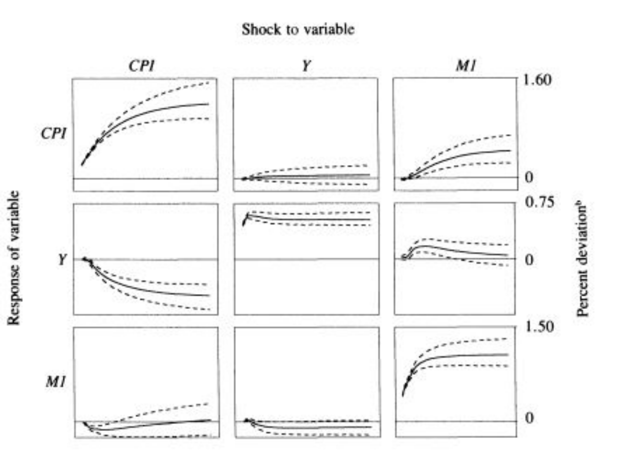
Macro evidence on the effects of monetary policy shocks

- persistent effects on real variables
- slow adjustment of aggregate price level
- liquidity effect

These stylized facts are in conflict with the predictions of classical monetary models

Micro evidence also suggest that there are significant price and wage rigidities

Responses to changes in M1 (Leeper, Sims and Zha, 1996)



Micro-evidence on price stickiness, Alvarez et al (2006)

TABLE 1. Measures of price stickiness in the euro area and the US (% per month unless otherwise stated).

	Statistics	Euro area	US
CPI*	Frequency	15.1	24.8
	Average duration (*months*)	13.0	6.7
	Median duration (*months*)	10.6	4.6
PPI†	Frequency	20.0	n.a
Surveys‡	Frequency	15.9	20.8
	Average duration (*months*)	10.8	8.3
NKPC§	Average durations (*months*)	13.5–19.2	7.2–8.4
Internet prices¶	Frequency	79.2	64.3

Notes: *Dhyne et al. (2006) for the euro area, Bils and Klenow (2004) for the US. Euro area refers to the aggregate of Austria, Belgium, Finland, France, Germany, Italy, Luxembourg, the Netherlands, Portugal, and Spain.

The New Keynesian business cycle model: Outline

Outline of basic New Keynesian model

Goods market:

- Demand side: Households consume a basket of goods
- Supply side: Firms produce different consumption goods (maximize profit under monopolistic competition)
- Price setting: Fixed probability of a firm resetting its price as in Calvo (1983)

Labor market

- Demand side: Firms hire labor (maximize profit in competitive markets)
- Supply side: Households supply labor

Financial markets

- Households optimally invest in a one-period risk-less bond

Our destination: The 3-equation New Keynesian model

New Keynesian Phillips Curve

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t$$

Dynamic IS Equation

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n)$$

Monetary Policy Rule

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widehat{y}_t + v_t$$

Dixit-Stiglitz Demand Systems

Utility over differentiated goods

Consider the utility function

$$U(C) = \frac{C^{1-\sigma}}{1-\sigma}$$

where C is CES aggregator over a continuum of goods

$$C \equiv \left(\int_0^1 C_i^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} : \varepsilon > 1$$

and $i \in (0, 1)$.

For a finite ε, goods are imperfect substitutes and firms therefore have some market power over the pricing of goods.

Choosing the optimal consumption basket

We want to maximize C subject to the budget constraint

$$\int_0^1 P_i C_i di \leq R$$

where R is the nominal budget constraint. Set up the Lagrangian

$$\max_{C_i} \left(\int_0^1 C_i^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} - \lambda \left(\int_0^1 P_i C_i di - R \right)$$

and take f.o.c.

$$\frac{\varepsilon}{\varepsilon-1} \frac{\varepsilon-1}{\varepsilon} \left(\int_0^1 C_i^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}-1} C_i^{\frac{\varepsilon-1}{\varepsilon}-1} = \lambda P_i \qquad (1)$$

Why is the derivative of $\partial C/\partial C_i$ not zero?

The integral is defined as the limit of the weighted sum over n goods

$$\left(\int_0^1 C_i^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}} \equiv \left(\lim_{n\to\infty}\frac{1}{n}\sum_{i=0}^{n} C_i^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

and

$$\int_0^1 P_i C_i di \equiv \lim_{n\to\infty}\frac{1}{n}\sum_{i=0}^{n} P_i C_i.$$

The first order condition then becomes

$$\frac{\varepsilon}{\varepsilon-1}\frac{\varepsilon-1}{\varepsilon}\left(\int_0^1 C_i^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}-1}\frac{1}{n}C_i^{\frac{\varepsilon-1}{\varepsilon}-1} = \frac{1}{n}\lambda P_i.$$

Since this expression holds for any n, we can multiply both sides with n to get the f.o.c. (1).

Simplifying demand for C_i

The f.o.c. can be simplified to

$$\left(\int_0^1 C_i^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{1}{\varepsilon-1}} C_i^{-\frac{1}{\varepsilon}} = \lambda P_i. \tag{2}$$

Now use that the definition of C implies that

$$C^{\frac{\varepsilon-1}{\varepsilon}} \equiv \int_0^1 C_i^{\frac{\varepsilon-1}{\varepsilon}} di$$

so that (2) can be rewritten as

$$C^{\frac{1}{\varepsilon}} C_i^{-\frac{1}{\varepsilon}} = \lambda P_i \tag{3}$$

implying

$$C_i = (\lambda P_i)^{-\varepsilon} C.$$

Simplifying demand for C_i, cont'd.

To express demand as a function of the aggregate consumption and the relative price multiply both sides of (3) with C_i

$$C^{\frac{1}{\varepsilon}} C_i^{1-\frac{1}{\varepsilon}} = \lambda P_i C_i$$

and integrate over i

$$C^{\frac{1}{\varepsilon}} \int_0^1 C_i^{1-\frac{1}{\varepsilon}} di = \lambda \int_0^1 P_i C_i di$$

to get

$$C^{\frac{1}{\varepsilon}+1-\frac{1}{\varepsilon}} = \lambda \int_0^1 P_i C_i di$$

which by the definition of $P \equiv C^{-1} \int_0^1 P_i C_i di$ implies that $P = \lambda^{-1}$ so that

$$C_i = \left(\frac{P_i}{P}\right)^{-\varepsilon} C.$$

The price index P

The following manipulations

$$\begin{aligned}
C_i &= \left(P^{-1}P_i\right)^{-\varepsilon} C \\
\left(P^{-1}P_i\right)^{\varepsilon} &= CC_i^{-1} \\
P^{-1}P_i &= C^{\frac{1}{\varepsilon}} C_i^{-\frac{1}{\varepsilon}} \\
\left(P^{-1}P_i\right)^{1-\varepsilon} &= C^{\frac{1-\varepsilon}{\varepsilon}} C_i^{-\frac{1-\varepsilon}{\varepsilon}} \\
P^{\varepsilon-1}\int_0^1 P_i^{1-\varepsilon} di &= C^{\frac{1-\varepsilon}{\varepsilon}} \int_0^1 C_i^{-\frac{1-\varepsilon}{\varepsilon}} \\
P^{\varepsilon-1}\int_0^1 P_i^{1-\varepsilon} di &= C^{\frac{1-\varepsilon}{\varepsilon}} C^{-\frac{1-\varepsilon}{\varepsilon}} \\
&= 1
\end{aligned}$$

delivers the expression

$$P = \left(\int_0^1 P_i^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}.$$

The optimal price under monopolistic competition

The optimal price in a static setting

Consider firm i facing the demand curve

$$Y_i = C_i = \left(\frac{P_i}{P}\right)^{-\varepsilon} C.$$

The maximization problem can then be written as

$$\max_{P_i} P_i Y_i - \mathcal{C}\left(Y_i\right).$$

where $\mathcal{C}\left(Y_i\right)$ is nominal cost as a function of output Y_i.

Optimal price in a static setting

Use the product rule for differentiation

$$(f \cdot g)^{'} = g \cdot f' + f \cdot g'$$

to get the f.o.c.

$$Y_i + P_i \frac{\delta Y_i}{\delta P_i} - \frac{\delta \mathcal{C}}{\delta Y_i} \frac{\delta Y_i}{\delta P_i} = 0$$

Optimal price in a static setting

Denote the nominal marginal cost as $\Psi_i \equiv \delta \mathcal{C} / \partial Y_i$. The following steps

$$
\begin{aligned}
Y_i + P_i \frac{\delta Y_i}{\delta P_i} - \Psi_i \frac{\delta Y_i}{\delta P_i} &= 0 \\
1 + \frac{P_i}{Y_i} \frac{\delta Y_i}{\delta P_i} - \Psi_i \frac{1}{Y_i} \frac{\delta Y_i}{\delta P_i} &= 0 \\
1 - \varepsilon + \Psi_i \frac{\varepsilon}{P_i} &= 0 \\
\Psi_i \frac{\varepsilon}{P_i} &= \varepsilon - 1 \\
P_i &= \frac{\varepsilon}{\varepsilon - 1} \Psi_i
\end{aligned}
$$

then gives the optimal price as the markup $\mathcal{M} = \frac{\varepsilon}{\varepsilon - 1}$ times nominal marginal cost.

The Basic New Keynesian Model

Households

Representative household solves

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t; Z_t\right)$$

where

$$U(C_t, N_t; Z_t) = \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) Z_t$$

and

$$C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

subject to

$$\int_0^1 P_t(i) C_t(i) di + Q_t B_t \leq B_{t-1} + W_t N_t + D_t$$

for $t = 0, 1, 2, ...$

Optimality conditions

Allocation of expenditures across different goods

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t$$

Labor supply

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$$

Intertemporal consumption

$$Q_t = \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \frac{Z_{t+1}}{Z_t} \right\}$$

Linearized equilibrium conditions

Allocation of expenditures across different goods

$$c_t(i) = -\epsilon\left(p_t(i) - p_t\right) + c_t$$

Labor supply

$$w_t - p_t = \sigma c_t + \varphi n_t$$

Intertemporal consumption

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) + \frac{1}{\sigma}(1-\rho_z)z_t$$

where $i_t \equiv -\log Q_t$ and $\rho \equiv -\log\beta$

Exogenous demand shocks

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z$$

Firms

Continuum of firms, indexed by $i \in (0, 1)$

- Each firm produces a differentiated good
- Identical technology

$$Y_t(i) = A_t N_t(i)^{1-\alpha}$$

where

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

- Probability of resetting price in any given period is $1 - \theta$ and independent across firms and time as in Calvo (1983).
- Implied average price duration $\frac{1}{1-\theta}$

Sources of inefficiency

Two sources of inefficiency

1. Decreasing marginal utility of individual goods ("love of variety")
2. Decreasing returns to scale in production

Love of variety preferences

Consider the CES aggregator with only two goods

$$C_t \equiv \left(\frac{1}{2} \sum_{i=1}^{2} C_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

Which bundle is better?

$$\{C_t(1), C_t(2)\} \textbf{ or } \{2 \times C_t(1)\}$$

Why?

Decreasing return to scale

Consider the production function with common productivity A_t

$$Y_t(i) = A_t N_t(i)^{1-\alpha}$$

Which output bundle takes more labor inputs to produce?

$$\{Y_t(1), Y_t(2)\} \textbf{ or } \{2 \times Y_t(1)\}$$

Why?

Summing up

- Monopolistic competition is introduced via household preferences
  - CES demand system
  - Optimal price is fixed mark-up over marginal cost
- Potential sources of inefficiency
  - Decreasing marginal utility of individual goods ("*love of variety*")
  - Decreasing marginal productivity of labor

You should

- Know how to derive demand for individual goods
- Know how to derive the CES price index
- Know how to find the optimal price P_i
- Understand the economic forces that determine efficiency in allocation of labor inputs