Macroeconomics ECON 6140 (Second Half)

Lecture 4
The Basic New Keynesian Business Cycle Model: Part II

Cornell University
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Today's plan

- The Basic New Keynesian Business Cycle Model
- Sticky prices and the New Keynesian Phillips Curve

Gali Ch 3 + Lecture Notes on Phillips Curve

Outline of basic New Keynesian model

Goods market:

- Demand side: Households consume a basket of goods
- Supply side: Firms produce different consumption goods (maximize profit under monopolistic competition)
- Price setting: Fixed probability of a firm resetting its price as in Calvo (1983)

Labor market

- Demand side: Firms hire labor (maximize profit in competitive markets)
- Supply side: Households supply labor

Financial markets

- Households optimally invest in a one-period nominally risk-less bond

The Basic New Keynesian Business Cycle Model

New Keynesian Phillips Curve

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t$$

Dynamic IS Equation

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n)$$

Monetary Policy Rule

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widehat{y}_t + v_t$$

The Basic New Keynesian Model

Households

Representative household solves

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t; Z_t\right)$$

where

$$U(C_t, N_t; Z_t) = \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) Z_t$$

and

$$C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

subject to

$$\int_0^1 P_t(i) C_t(i) di + Q_t B_t \leq B_{t-1} + W_t N_t + D_t$$

for $t = 0, 1, 2, ...$

Linearized equilibrium conditions

Allocation of expenditures across different goods

$$c_t(i) = -\epsilon\,(p_t(i) - p_t) + c_t$$

Labor supply

$$w_t - p_t = \sigma c_t + \varphi n_t$$

Intertemporal consumption

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) + \frac{1}{\sigma}(1-\rho_z)z_t$$

where $i_t \equiv -\log Q_t$ and $\rho \equiv -\log\beta$

Exogenous demand shocks

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z$$

Firms

Continuum of firms, indexed by $i \in (0, 1)$

- Each firm produces a differentiated good
- Identical technology

$$Y_t(i) = A_t N_t(i)^{1-\alpha}$$

where

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

- Probability of resetting price in any given period is $1 - \theta$ and independent across firms and time as in Calvo (1983).
- Implied average price duration $\frac{1}{1-\theta}$

The New Keynesian Phillips Curve

The mechanics of Calvo aggregate price dynamics

If a fraction $1-\theta$ of firms set price P_t^* the aggregate price level will follow

$$P_t = \left[\theta(P_{t-1})^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

which can be rearranged to

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon}$$

Log-linearization around zero inflation steady state give inflation as

$$\pi_t = (1-\theta)(p_t^* - p_{t-1})$$

or, equivalently

$$p_t = \theta p_{t-1} + (1-\theta)p_t^*$$

The optimal reset price P_t^*

Firms maximize expected discounted profits (using SDF of households)

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} (1/P_{t+k}) \left(P_t^* Y_{t+k|t} - \mathcal{C}_{t+k}(Y_{t+k|t}) \right) \right\}$$

subject to

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} Y_{t+k}$$

for $k = 0, 1, 2, \ldots$where $\Lambda_{t,t+k} \equiv \beta^k U_{c,t+k}/U_{c,t}$

Note that firm can only affect $\left(P_t^* Y_{t+k|t} - \mathcal{C}_{t+k}(Y_{t+k|t}) \right)$.

Optimal Price Setting

The first order condition for the price setting problem is given by

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} Y_{t+k|t} (1/P_{t+k}) \left(P_t^* - \mathcal{M} \Psi_{t+k|t} \right) \right\} = 0$$

where $\Psi_{t+k|t} \equiv \mathcal{C}'_{t+k}(Y_{t+k|t})$ is the nominal marginal cost in period $t+k$ of a firm producing a good with price P_t^* and $\mathcal{M} \equiv \frac{\epsilon}{\epsilon - 1}$ is the desired mark-up.

In the special case of flexible prices ($\theta = 0$) expression simplifies to a constant mark-up over marginal cost

$$P_t^* = \mathcal{M} \Psi_{t|t}$$

Linearized optimal price-setting

The linearized optimal price setting condition around a zero-inflation steady state is given by

$$p_t^* = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t\{\psi_{t+k|t}\}$$

where $\psi_{t+k|t} \equiv \log \Psi_{t+k|t}$ and $\mu \equiv \log \mathcal{M}$

With Calvo-pricing, firms aim to set a price p_t^* that results in an optimal expected average mark-up, weighted by the discount factor β and the probability θ^k of p_t^* still being in place in period $t + k$.

Finding marginal cost $\psi_{t+k|t}$ as a function of p_t^*

Nominal marginal cost is the wage divided by marginal productivity of labor

$$\begin{aligned} \psi_{t+k|t} &= w_{t+k} - mpn_{t+k|t} \\ &= w_{t+k} - (a_{t+k} - \alpha n_{t+k|t} + \log(1-\alpha)) \end{aligned}$$

Define the component of nominal marginal cost that is common across firms as

$$\psi_{t+k} \equiv w_{t+k} - (a_{t+k} - \alpha n_{t+k} + \log(1-\alpha))$$

We then have

$$\begin{aligned} \psi_{t+k|t} &= \psi_{t+k} + \alpha(n_{t+k|t} - n_{t+k}) \\ &= \psi_{t+k} + \frac{\alpha}{1-\alpha}(y_{t+k|t} - y_{t+k}) \\ &= \psi_{t+k} - \frac{\alpha\epsilon}{1-\alpha}(p_t^* - p_{t+k}) \end{aligned}$$

Deriving the Phillips curve

Define the following quantities

$$\begin{aligned} \psi_{t+k|t} &\equiv \psi_t - \frac{\alpha\varepsilon}{1-\alpha}\left(p_t^* - p_{t+k}\right) \\ \widehat{mc}_t &\equiv \psi_t - p_t + \mu \end{aligned}$$

and substitute into optimal price equation

$$p_t^* = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left(\psi_{t+k|t} + \mu\right)$$

to get

$$p_t^* = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left(\widehat{mc}_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}\left(p_t^* - p_{t+k}\right) + p_{t+k}\right)$$

Deriving the Phillips curve, cont'd

Collect all terms involving p_t^* on the left hand side

$$\frac{1-\alpha+\alpha\varepsilon}{1-\alpha} p_t^* = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left(\widehat{mc}_{t+k} + \frac{1-\alpha+\alpha\varepsilon}{1-\alpha} p_{t+k}\right)$$

and multiply both sides with $\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}$

$$p_t^* = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left(\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\widehat{mc}_{t+k} + p_{t+k}\right)$$

Deriving the Phillips curve, cont'd

Rewrite in recursive form

$$p_t^* = \beta\theta E_t p_{t+1}^* + (1-\beta\theta)\left(\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\widehat{mc}_t + p_t\right)$$

Subtract p_{t-1} from both sides and add and subtract $\beta\theta p_t$ from the right hand side

$$p_t^* - p_{t-1} = \beta\theta E_t\left(p_{t+1}^* - p_t\right) + (1-\beta\theta)\left(\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\widehat{mc}_t + p_t\right) - p_{t-1} + \beta\theta p_t$$

Use that $p_t^* - p_{t-1} = (1-\theta)^{-1}\pi_t$ so that

$$(1-\theta)^{-1}\pi_t = (1-\theta)^{-1}\beta\theta E_t\left(\pi_{t+1}\right) + (1-\beta\theta)\left(\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\widehat{mc}_t\right) + \pi_t$$

and solve for π_t

$$\pi_t = \beta E_t\left(\pi_{t+1}\right) + \frac{(1-\theta)(1-\beta\theta)}{\theta}\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\widehat{mc}_t.$$

Marginal cost, output and productivity

The real marginal cost is the real wage divided by the marginal productivity of labor

$$
\begin{aligned}
mc_t &\equiv \psi_t - p_t \\
&= (w_t - p_t) - (a_t - \alpha n_t + \log(1-\alpha)) \\
&= (\sigma y_t + \varphi n_t) - (a_t - \alpha n_t + \log(1-\alpha)) \\
&= \left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) y_t - \left(\frac{1+\varphi}{1-\alpha}\right) a_t - \log(1-\alpha)
\end{aligned}
$$

Real marginal cost thus depends on both output and productivity.

Marginal cost and the output gap

Under flexible prices the mark-up $\mu \equiv p - \psi$ is constant and equal to $-mc$

$$\begin{aligned} mc &= -\mu \\ &= \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n + \left(\frac{1+\varphi}{1-\alpha}\right) a_t - \log(1-\alpha) \end{aligned}$$

which we can use to solve for natural output

$$y_t^n = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha} a_t - \frac{(1-\alpha)(\mu - \log(1-\alpha))}{\sigma(1-\alpha)+\varphi+\alpha}$$

The deviation of marginal cost from steady state $\widehat{mc}_t \equiv mc_t - mc$ is proportional to the output gap

$$\widehat{mc}_t = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)(y_t - y_t^n)$$

Note: We **define** the natural level of output y_t^n as the level of output such that $mc = -\mu$.

Equilibrium/loose ends

Goods markets clearing

$$Y_t(i) = C_t(i)$$

for all $i \in [0,1]$ and all t.

Defining $Y_t \equiv \left(\int_0^1 Y_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$: so that

$$Y_t = C_t$$

and combine with Euler equation

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) + \frac{1}{\sigma}(1-\rho_z)z_t$$

Equilibrium/loose ends

Labor market clearing

$$
\begin{aligned}
N_t &= \int_0^1 N_t(i)di \\
&= \int_0^1 \left(\frac{Y_t(i)}{A_t}\right)^{\frac{1}{1-\alpha}} di \\
&= \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di
\end{aligned}
$$

Up to a first order approximation:

$$
n_t = \frac{1}{1-\alpha}(y_t - a_t)
$$

The Non-Policy Block of the Basic New Keynesian Model

New Keynesian Phillips Curve

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t$$

where
$\widetilde{y}_t \equiv y_t - y_t^n$, $\kappa \equiv \lambda \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$ and $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\alpha\varepsilon}$

Dynamic IS equation

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n)$$

where r_t^n is the *natural rate of interest*, given by

$$r_t^n = \rho - \sigma(1-\rho_a)\psi_{ya} a_t + (1-\rho_z) z_t$$

Monetary policy rule

We can close the model with a Taylor-type rule for nominal interest rate

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widehat{y}_t + v_t$$

where $\widehat{y}_t \equiv y_t - y$ and

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v$$

or equivalently:

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widetilde{y}_t + \phi_y \widehat{y}_t^n + v_t$$

where $\widehat{y}_t^n \equiv y_t^n - y$.

The solved model

We can use the method of undetermined coefficients to solve for inflation and the output gap

$$\begin{aligned} \widetilde{y}_t &= (1-\beta\rho_\upsilon)\,\Lambda_u u_t \\ \pi_t &= \kappa\Lambda_u u_t \end{aligned}$$

where

$$\Lambda_u \equiv \frac{1}{(1-\beta\rho_\upsilon)\left[\sigma\left(1-\rho_u\right)+\phi_y\right]+\kappa\left(\phi_\pi-\rho_u\right)} > 0$$

and u_t is the composite shock

$$u_t \equiv -\psi_{ya}\left(\phi_y+\sigma\left(1-\rho_a\right)\right)a_t+\left(1-\rho_z\right)z_t-v_t$$

Next time we will talk more about the dynamics of the solved model

Summary

Under Calvo pricing, firms set the price p_t^* so that the expected average mark-up equals the optimal mark-up, discounted by β and the probability θ that the price p_t^* stays in effect.

You should

- Understand why the optimal price is forward looking with sticky prices
- Understand how to find the optimal reset price p_t^*