

ECON6110: Problem Set 3

Spring 2025

This problem set is due on at 23:59 on March 14, 2024. Every student must write their own solution and submit it individually. Problem set submissions are submitted electronically and may be typed or handwritten. If handwritten, please ensure your work and scan are legible. **Illegible submissions will not be graded.**

Problem 1

Consider a repeated game with finite horizon $T < \infty$ and discount factor $\delta = 1$. The stage game is as follows:

| | X | Y | Z |
|-----|------|------|------|
| A | 5, 5 | 0, 7 | 2, 5 |
| B | 5, 2 | 4, 3 | 2, 1 |
| C | 7, 0 | 3, 3 | 3, 4 |

- (a) Suppose that $T = 1$, that is, the game is played two times. Is there a SPE in which (A, X) is played in the first period? Explain.
- (b) Find a value of $T < \infty$ and a SPE in which (A, X) is played in the first period.

Problem 2

Consider the infinitely repeated game where the stage game is Bertrand's duopoly game. In the stage game, each firm i chooses simultaneously a price $p_i \in [0, 1]$. If the market price is p_i , the demand is $1 - p_i$. The marginal cost of production is constant and equal to $c = 0.5$. The payoff in the stage game is per-period profit:

$$\pi_i(p_i, p_{-i}) = \begin{cases} (1 - p_i)(p_i - 0.5) & \text{if } p_i < p_{-i} \\ \frac{1}{2}(1 - p_i)(p_i - 0.5) & \text{if } p_i = p_{-i} \\ 0 & \text{otherwise.} \end{cases}$$

Denote by $p_i^* \in [0, 1]$ the price that firm i would choose if she was a monopolist, that is, p_i^* is the price that maximizes $(1 - p_i)(p_i - 0.5)$ over $p_i \in [0, 1]$.

- (a) Consider the following strategy profile: "Each firm i chooses $p_i = p_i^*$ in the first period and subsequently as long as the other firm continues to charge $p_{-i} = p_{-i}^*$ and punishes any deviation from $p_{-i} = p_{-i}^*$ by the other firm by choosing the price $p_i = c$ for a finite number $T \geq 1$ of periods, then reverting to $p_i = p_i^*$." Given any $\delta \in (1/2, 1)$, for what values of T this strategy profile is a subgame perfect equilibrium?
- (b) Consider the following strategy profile: "In the first period, each firm i chooses $p_i = p_i^*$. In every subsequent period, each firm i charges the lowest of all the prices charged by the other firm in all previous periods." Is there a value of $\delta \in (0, 1)$ for which this strategy profile is a SPE? Explain.

Problem 3

Consider an infinite repetition of the following stage game:

| | A | B | C |
|-----|------|------|------|
| A | 4, 4 | 3, 0 | 1, 0 |
| B | 0, 3 | 2, 2 | 1, 0 |
| C | 0, 1 | 0, 1 | 0, 0 |

Assume the discount factor is very close to one; to be concrete, assume $\delta = 0.99$.

- (a) Find a subgame perfect equilibrium where both players get a payoff of 4.
- (b) Is there a SPE where player 1 gets a payoff of 0.5?
- (c) Find a SPE where both players get a payoff of 2.

Problem 4

Consider the following infinite-horizon game between a single firm and a sequence of workers, each of whom lives for one period. In each period the worker chooses either to expend effort and so produce output y at effort cost c or to expend no effort, produce no output, and incur no cost. If output is produced, the firm owns it but can share it with the worker by paying a wage w , as described next. Assume that at the beginning of the period the worker has an alternative opportunity worth zero (net of effort cost) and that the worker cannot be forced to accept a wage less than zero. Assume also that $y > c$ so that expending effort is efficient. Within each period, the timing of events is as follows: first the worker chooses an effort level, then output is observed by both the firm and the worker, and finally the firm chooses a wage to pay the worker. Assume that no wage contracts can be enforced by law: the firm's choice of wage is completely unconstrained. In a one-period game, therefore, subgame-perfection implies that the firm will offer a wage zero independent of the worker's output, so the worker will not expend any effort.

Now consider the infinite-horizon problem. Recall that each worker lives for only one period. Assume, however, that at the beginning of period t ; the history of the game through period $t - 1$ is observed by the worker who will work in period t . Suppose the firm discounts the future according to the discount factor δ per period. Describe strategies for the firm and each worker in a subgame-perfect equilibrium in the infinite horizon game in which in equilibrium each worker expends effort and so produces output y ; provided the discount factor is high enough. Give a necessary and sufficient condition for your equilibrium to exist.