ECON6110: Problem Set 1

Spring 2025

This problem set is due at 23:59 on Friday February 14, 2025. You are encouraged to work in groups, but every student has to write their own solution and submit it individually. Problem set submissions are submitted electronically and may be typed or handwritten. If handwritten, please ensure your work and scan are legible. **Illegible submissions will not be graded.**

Problem 1

The answer of this problem may be useful to solve some of the following problems (but the proofs of these findings are not essential for the next problems).

Question 1: Prove that a profile is a Nash equilibrium of a game only if it is the Nash equilibrium of the game in which strategies have been removed by iterated **strict** dominance.

Question 2: Prove that a profile is a Nash equilibrium of a game if it is the Nash equilibrium of the game in which strategies have been removed by iterated **strict** dominance.

Question 3: Prove that a profile is a Nash equilibrium of a game if it is the Nash equilibrium of a game in which strategies have been removed by iterated **weak** dominance.

Question 4: Give an example of a Nash equilibrium of a game that is not a Nash equilibrium of the game where strategies have been removed by iterated **weak** dominance.

Problem 2

Find all the equilibria of these games:

Question 1:

	L	R
U	$4,5$	$3,1$
D	$4,0$	$0,6$

Question 2:

	L	R
U	$3,4$	$-2,6$
D	$0,3$	$-5,1$

Question 3:

	L	C	R
U	$6,6$	$1,2$	$3,3$
M	$2,1$	$4,7$	$4,3$
D	$3,4$	$2,5$	$3,9$

Problem 3

Consider this game:

	L	R
U	0, 0	2, 1
D	1, 2	0, 0

Consider a correlated equilibrium with this distribution over actions (the table represents the joint distribution):

	L	R
U	1/3	1/3
D	1/3	0

We want to precisely define the correlated equilibrium that generates this outcome.

Question 1: Define the probability space (Ω, π). Argue that the state space Ω can be restricted to coincide the action space A.

Question 2: Define the partitions for each player associated to the state space described above.

Question 3: Define the strategies associated to the probability space described above and the distribution described in the table.

Question 4: Are the strategies a correlated equilibrium? Prove your answer.

Problem 4

Consider an election with two candidates, Ann and Bob. There is a continuum of citizens, whose most preferred policies are distributed on $[0, 1]$ according to a cumulative distributive function $F : [0, 1] \to [0, 1]$. We assume that F is continuous and strictly increasing.[1] Candidates choose their policy simultaneously from $[0, 1]$. Each citizen votes for the candidate whose policy is closer to his or her most preferred policy. The candidate with majority votes wins.

This game can be formalized as follows. The player set is $N = \{Ann, Bob\}$. The set of actions is $[0, 1]$ for both players. Let's call $s_i(a_i, a_{-i}) \in [0, 1]$ the share of votes for candidate i:

$$s_i(a_i, a_{-i}) = \begin{cases} 1 - F\left(\frac{1}{2}a_i + \frac{1}{2}a_{-i}\right) & \text{if } a_i > a_{-i}, \\ 1/2 & \text{if } a_i = a_{-i}, \\ F\left(\frac{1}{2}a_i + \frac{1}{2}a_{-i}\right) & \text{if } a_i < a_{-i}. \end{cases}$$

The payoff function is

$$v_i(a_i, a_{-i}) = \begin{cases} 1 & \text{if } s_i(a_i, a_{-i}) > 1/2, \\ 1/2 & \text{if } s_i(a_i, a_{-i}) = 1/2, \\ 0 & \text{else.} \end{cases}$$

Hint. It could be useful to keep in mind a concrete example for F. For example, F could be the uniform distribution. In such case,

$$s_i(a_i, a_{-i}) = \begin{cases} 1 - \frac{1}{2}a_i + \frac{1}{2}a_{-i} & \text{if } a_i > a_{-i}, \\ 1/2 & \text{if } a_i = a_{-i}, \\ \frac{1}{2}a_i + \frac{1}{2}a_{-i} & \text{if } a_i < a_{-i}. \end{cases}$$

Question 1: What policies are rationalizable?

Question 2: Find a Nash equilibrium in "weakly dominant actions," that is, find a

[1] Recall that, in addition, $F(0) = 0$ and $F(1) = 1$.

pure Nash equilibrium a^* such that for all $i \in N$ and $a_i, a_{-i} \in [0,1]$

$$v_i(a_i^*, a_{-i}) \geq v_i(a_i, a_{-i}).$$

Question 3: Show that the equilibrium above is the unique (pure or mixed) Nash equilibrium.

Problem 5

Consider a voting game with $M + N$ voters. The first M voters $M = \{1, ..., M\}$ are in team T_1, the remaining N voters in team T_2. Each voter can either abstain or vote for candidate 1 or 2. The candidate with the highest number of votes is elected. In case of a tie candidate 2 wins. A member of team i receives a payoff of 1 if candidate i is elected and a payoff of 0 otherwise. If a voter votes, then s/he pays a participation cost $c \in (0, 1)$.

Question 1. Is it possible that a member of team 1 (resp. team 2) votes for candidate 2 (resp. candidate 1)?

Question 2. Lets try to construct an Nash equilibrium in which each member of team 1 votes with probability q for candidate 1, and the members of team 2 abstain with certainty. Under what conditions does such an equilibrium exists? What is the strategy used by the member of team 1? What is the probability that a voter of team 1 votes if $c = 1/2$ and $M = 25$?

Question 3. Lets study the existence of a Nash equilibrium in which the members of team 1 (i.e., T_1) vote with probability q for 1 and abstain otherwise; and exactly $k \in (0, \min(M - 1, N)]$ members of T_2 vote with probability 1 for candidate 2, and all the others abstain. Characterize q and establish a necessary and sufficient condition for such an equilibrium to exist.

Question 4. Use the answer of Q2-3 to show that an equilibrium in which the members of T_1 adopt a totally mixed strategy q and k members of T_2 always vote exists for all $M, N \geq 2$ and $c \in (0, 1)$ for some $k \geq 0$.