

About TA sections:

TA: Ekaterina Zubova (ez268@cornell.edu)

Section time and location: 8:40am - 9:55am Rockefeller Hall 132

Office hours: Tuesday 4:30-5:30 pm in Uris Hall 451; other times available by appointment (just send me an email).

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1 Sources of Inefficiency in New Keynesian Model

There are three key **efficiency conditions**:

1. $-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = MPN_t$,
2. $C_t(i) = C(t) \quad \forall i$,
3. $N_t(i) = N(t) \quad \forall i$.

1. **Monopolistic competition** breaks

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = MPN_t.$$

Since

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}} < MPN_t.$$

\implies **level inefficiencies**: prices are too high, and as a result, consumption, output, and employment are inefficiently low.

These inefficiencies can be offset by implementing an optimal employment subsidy such that $\mathcal{M}(1 - \tau) = 1 \implies \tau = \frac{1}{\varepsilon}$.

2. **Sticky prices** break

$$C_t(i) = C(t) \quad \forall i,$$

$$N_t(i) = N(t) \quad \forall i,$$

because prices of individual goods adjust at different frequencies \implies **composition inefficiencies**: households prefer to consume relatively cheaper goods, and firms producing less tend to hire fewer workers.

Sticky prices also generate **level inefficiencies**, even with the optimal subsidy in place, because we have:

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = MPN_t \frac{\mathcal{M}}{\mathcal{M}_t} \neq MPN_t$$

unless $\frac{\mathcal{M}}{\mathcal{M}_t} = 1$ where \mathcal{M}_t denotes the average markup at time t .

Thus, the government can restore economic efficiency only if it succeeds in stabilizing the average markup at its frictionless level.

2 Divine Coincidence

In the baseline New Keynesian framework, the **divine coincidence** refers to the situation in which stabilizing inflation automatically stabilizes the output gap. This implies that a policy which stabilizes marginal costs at a level consistent with firms' desired markup can simultaneously ensure both price and output stability, i.e., there is no tradeoff.

With

- (a) optimal employment subsidy $\tau = \frac{1}{\varepsilon}$,
- (b) no inherited relative price distortions,
- (c) **only** demand and productivity shocks (no cost-push shock u_t !),

we have:

- The efficient output (y_t^e) equals the natural output (y_t^n), i.e., $y_t^e = y_t^n$.
Remark: Recall that y_t^n is the natural level of output of output under flexible prices, while y_t^e is the level of output that would be chosen by a social planner who can eliminate all distortions in the economy.
- Stabilizing inflation ($\pi_t = 0$) leads to a zero output gap ($\tilde{y}_t = 0$).
- Therefore, strict inflation targeting is both feasible and welfare-maximizing.

Why do we need the last assumption? With a cost-push shock, $y_t^e \neq y_t^n$ and the New Keynesian Phillips curve becomes

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \kappa(y_t - y_t^e) + u_t,$$

where u_t represents an exogenous cost-push shock, such as variations in firms' desired markups.

Remark: It is now helpful to introduce new notation: let $x_t \equiv y_t - y_t^e$ denote the welfare-relevant output gap, and define the cost-push shock as $u_t = \kappa(y_t^e - y_t^n)$. Substituting this into the NKPC yields the same expression seen earlier.

Note the key difference: previously, the output gap was defined as $\tilde{y}_t \equiv y_t - y_t^n$. This is no longer appropriate because, in the presence of a cost-push shock, $y_t^e \neq y_t^n$. That is, the allocation that would arise under flexible prices is no longer efficient, and thus there is no reason for the central bank to replicate it. As a result, stabilizing inflation may now require a non-zero output gap, leading to suboptimal output levels. This tradeoff motivates monetary policy decisions in the New Keynesian model.

3 Optional: New Keynesian Phillips Curve Derivations

Recall from the previous section

$$p_t^* = \mu + (1 - \theta\beta) \sum_{k=0}^{\infty} (\beta\theta)^k \mathbb{E}_t(\psi_{t+k|t}). \quad (1)$$

We will need to use some formulas and definitions

$$\widehat{mc}_t \equiv \psi_t - p_t + \mu, \quad (2)$$

$$\psi_{t+k|t} = \psi_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}(p_t^* - p_{t+k}), \quad (3)$$

$$\pi_t \equiv p_t - p_{t-1} \approx (1 - \theta)(p_t^* - p_{t-1}). \quad (4)$$

Substituting (2) and (3) into (1), we obtain

$$p_t^* = \mu + (1 - \theta\beta) \sum_{k=0}^{\infty} (\beta\theta)^k \mathbb{E}_t[\widehat{mc}_{t+k} + p_{t+k} - \mu - \frac{\alpha\varepsilon}{1-\alpha}(p_t^* - p_{t+k})].$$

Note that μ cancels out. Grouping terms with p_t^* and p_{t+k} , we can rewrite:

$$p_t^*(1 + \frac{\alpha\varepsilon}{1-\alpha}) = (1 - \theta\beta) \sum_{k=0}^{\infty} (\beta\theta)^k \mathbb{E}_t[\widehat{mc}_{t+k} + (1 + \frac{\alpha\varepsilon}{1-\alpha})p_{t+k}].$$

Now denote $\Theta \equiv 1/(1 + \frac{\alpha\varepsilon}{1-\alpha})$ and multiply both sides by Θ to obtain:

$$p_t^* = (1 - \theta\beta) \sum_{k=0}^{\infty} (\beta\theta)^k \mathbb{E}_t[\Theta \widehat{mc}_{t+k} + p_{t+k}],$$

which is equivalent to

$$p_t^* = \beta\theta \mathbb{E}_t[p_{t+1}^*] + (1 - \theta\beta)[\Theta \widehat{mc}_t + p_t].$$

Now subtract p_{t-1} from both sides

$$p_t^* - p_{t-1} = \beta\theta \mathbb{E}_t[p_{t+1}^*] + (1 - \theta\beta)[\Theta \widehat{mc}_t + p_t] - p_{t-1},$$

and add and subtract $\beta\theta p_t$

$$p_t^* - p_{t-1} = \beta\theta \mathbb{E}_t[p_{t+1}^* - \beta\theta p_t] + (1 - \theta\beta)[\Theta \widehat{mc}_t + p_t] - p_{t-1} + \beta\theta p_t.$$

Now, using (4), we obtain

$$\frac{\pi_t}{1-\theta} = \beta\theta\mathbb{E}_t\left[\frac{\pi_{t+1}}{1-\theta}\right] + (1-\theta\beta)\Theta\widehat{mc}_t + \pi_t,$$

which is equivalent to

$$\frac{\theta}{1-\theta}\pi_t = \beta\theta\mathbb{E}_t\left[\frac{\pi_{t+1}}{1-\theta}\right] + (1-\theta\beta)\Theta\widehat{mc}_t.$$

Now multiply both sides by $\frac{1-\theta}{\theta}$ to obtain

$$\pi_t = \beta\mathbb{E}_t[\pi_{t+1}] + (1-\theta\beta)\frac{1-\theta}{\theta}\Theta\widehat{mc}_t.$$

Define $\lambda \equiv (1-\theta\beta)\frac{1-\theta}{\theta}\Theta$, so we have:

$$\pi_t = \beta\mathbb{E}_t[\pi_{t+1}] + \lambda\widehat{mc}_t.$$

Now using

$$\widehat{mc}_t \equiv mc_t - mc = \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right)(y_t - y_t^n) = \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right)\tilde{y}_t,$$

we can write

$$\begin{aligned}\pi_t &= \beta\mathbb{E}_t[\pi_{t+1}] + \lambda\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right)\tilde{y}_t \\ \implies \pi_t &= \beta\mathbb{E}_t[\pi_{t+1}] + \kappa\tilde{y}_t, \quad \kappa \equiv \lambda\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right),\end{aligned}$$

which is the standard form of the New Keynesian Phillips Curve (NKPC) (in the absence of cost-push shocks).

4 Optional: Exam Practice

Try to answer the following questions:

- Assume that goods prices are sticky a la Calvo (1983). Show how equilibrium allocations may be inefficient even if there is a production subsidy that makes the steady state level of output efficient.
- Explain the divine coincidence and what it implies for optimal monetary policy. If the conditions for the divine coincidence hold, what does it imply for the design of a simple policy rule if we want to maximize the welfare of households?
- Show that in the absence of cost-push shocks (and in the presence of a employment subsidy) the divine coincidence holds.
- Consider the New Keynesian model with cost-push shocks u_t as the only exogenous source of shocks, i.e. consider the standard model but with the modified Phillips Curve given by

$$\pi_t = \beta \mathbb{E}_t \{\pi_{t+1}\} + \kappa x_t + u_t$$

where $x_t \equiv y_t - y_t^e$ and

$$\kappa(y_t^e - y_t^n) \equiv u_t = \rho_u u_{t-1} + \varepsilon_t^u$$

- (a) Solve for equilibrium using the method of undetermined coefficients under the assumption that monetary policy follows

$$i_t = \rho + \phi_\pi \pi_t$$

- (b) What is the effect of a cost-push shock on inflation and the output gap x_t ? How does the response depend on the coefficient ϕ_π ?
- (c) Find the policy that maximizes the utility of the representative household and solve for the equilibrium values of inflation and the output gap. Can you find a Taylor-type rule that replicates this equilibrium?