ECON6110: Problem Set 5

Spring 2024

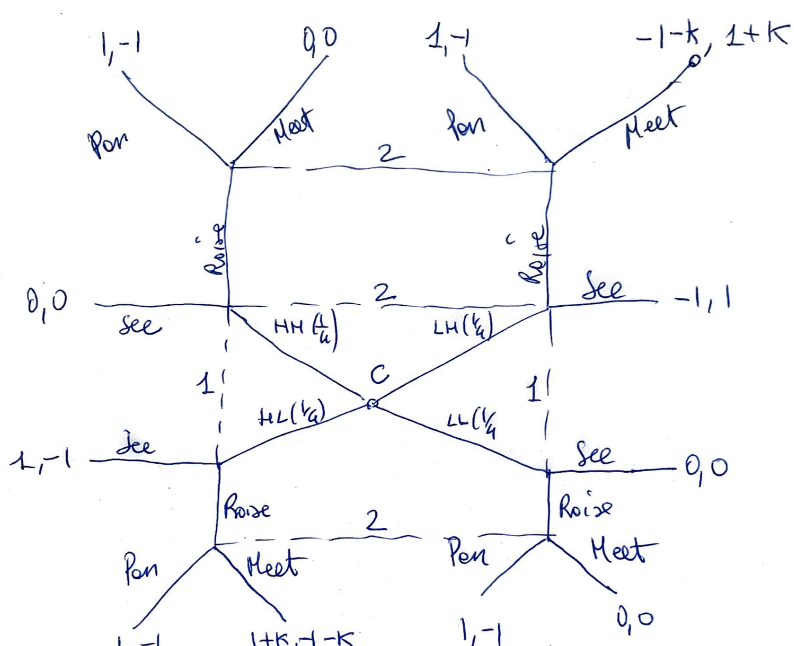
This problem set is due May 5, 2024 at 23:59. Every student must write their own solution and submit it individually. Problem set submissions are submitted electronically and may be typed or handwritten. If handwritten, please ensure your work and scan are legible. **Illegible submissions will not be graded.**

Problem 1

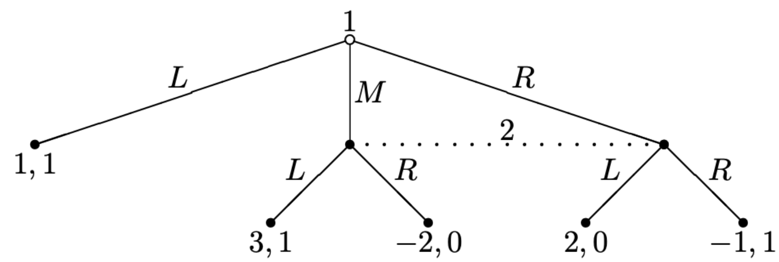
Consider the following game. Initially each player puts a dollar in the pot. Then each player is dealt a card; each player's card is equally likely to be High or Low, independent of the other player's card. Each player sees only her own card. Player 1 may see or raise. If she sees, then the players compare their cards. The one with the higher card wins the pot; if the cards are the same, then each player takes back the dollar she had put in the pot. If player 1 raises, then she adds \$ k to the pot (where k is a fixed positive number), and player 2 may pass or meet. If player 2 passes, then player 1 takes the money in the pot. If player 2 meets, then she adds \$ k to the pot and the players compare cards, the one with the higher card winning the pot; if the cards are the same, then each player takes back the \$ $(1 + k)$ she had put in the pot.

Model this card game as an extensive game. Drawing a diagram is sufficient; you can avoid the need for information sets to cross histories by putting the initial move in the center of your diagram.

Solution:



Problem 2



Find the set of Perfect Bayesian equilibria of the game.

Solution:

Denote player 1's strategy by (α, β, ζ). In all Perfect Bayesian equilibria:

• If $\beta > \zeta$ then player 2 chooses L and hence $\beta = 1$; (M,L) is indeed a Perfect Bayesian equilibrium strategy profile.

• If $\beta < \zeta$ then player 2 chooses R, so that player 1 chooses L and $\beta = \zeta = 0$, a contradiction.

• If $\beta = \zeta > 0$ then player 2 must choose L with probability $1/2$, in which case player 1

is better off choosing L, a contradiction.

• If $\beta = \zeta = 0$ then player 2's strategy $(\delta, 1-\delta)$ has to be such that

$$1 \geq 3\delta - 2(1-\delta) = 5\delta - 2$$

or $3/5 \geq \delta$, and

$$1 \geq 2\delta - (1-\delta) = 3\delta - 1$$

or $2/3 \geq \delta$. For each $0 < \delta \leq 3/5$ the strategy is supported by the belief $(1/2, 1/2)$ of player 2. For $\delta = 0$ the strategy is supported by any belief $(p, 1-p)$ with $p \leq 1/2$.

In summary, there are two types of Perfect Bayesian equilibria: one in which the strategy profile is $((0,1,0),(1,0))$ and player 2's belief is $(1,0)$, and one in which the strategy profile is $((1,0,0),(\delta, 1-\delta))$ for some $\delta \in [0, 3/5]$ and player 2's belief is $(1/2, 1/2)$ for $\delta > 0$ and $(p, 1-p)$ for some $p \leq 1/2$ for $\delta = 0$.

Problem 3

Gilligan and Krehbiel (1988) depict the open rule in Congress as a cheap-talk game, that is, as a signaling game in which signals are costless. As a rough approximation, the committee proposes a policy, but the floor can introduce amendments and choose the policy it likes. The open rule is depicted as a two-player game, with a single member in the committee and a single representative on the floor (who stands for the median voter). The object of the decision is a policy a_2 in R. The outcome given policy a_2 is $x = a_2 + w$, where w is a random variable uniformly distributed between 0 and 1. The committee knows w; the floor does not. The committee moves first and suggests a policy a_1 to the floor. The preferences of both are quadratic with bliss points $x = 0$ for the floor and $x = x_c \in (0, 1)$ for the committee:

$$\begin{aligned} u_1(x) &= -(x - x_c)^2 \\ u_2(x) &= -(x)^2 \end{aligned}$$

(a) Show that there always exists a "babbling" Perfect Bayesian equilibrium in which a_1 is uninformative and $a_2 = -1/2$.

(b) Look for informative Perfect Bayesian equilibria. In particular, find an equilibrium in which the committee "reports low" when $w \in [0, w^*]$ and "reports high" when $w \in [w^*, 1]$.

Solution:

(a) Suppose that there exists a "babbling" equilibrium where $a_1(w) = 0 \; \forall w$ (i.e., the committee sends the same signal regardless of the state). Therefore, the floor doe not update its prior upon receiving a signal (assuming no updating also for off-path signals):

$$\mu(w \mid a_1 = 0) = \text{Unif}[0, 1]$$
$$\mu(w \mid a_1 \neq 0) = \text{Unif}[0, 1]$$

Thus, it chooses a_2 that maximizes the unconditional expected utility. Formally,

$$
\begin{aligned}
\mathbb{E}[u_2] =& \mathbb{E}[-(a_2 + w)^2 \mid a_1] \\
=& \mathbb{E}[-(a_2 + w)^2 \mid a_1] \\
=& \mathbb{E}[-a_2^2 - 2a_2 w - w^2 \mid a_1] \\
=& - a_2^2 - 2a_2 \, \mathbb{E}[w \mid a_1] - \mathbb{E}[w^2 \mid a_1] \\
\Longrightarrow a_2^* =& - \mathbb{E}[w \mid a_1] = - \mathbb{E}[w] = -\frac{1}{2}
\end{aligned}
$$

Given that $a_2^* = -\frac{1}{2} \; \forall a_1$, sending an uninformative a_1 is a best response.

(b) Suppose that the committee uses the following strategy:

$$
a_1(w) = \begin{cases} L & w \in [0, w^*] \\ H & w \in [w^*, 1] \end{cases}
$$

Specify the belief system of player 2. In particular, suppose that player 2 thinks the committee "reports low" when receiving any off-path signals.

$$
\begin{aligned}
\mu(w \mid a_1 = L) =& \text{Unif}[0, w^*] \\
\mu(w \mid a_1 = H) =& \text{Unif}[w^*, 1] \\
\mu(w \mid a_1 \notin \{L, H\}) =& \text{Unif}[0, w^*]
\end{aligned}
$$

Therefore, player 2's best response is:

$$
\begin{aligned}
a_2^*(a_1 = L) &= - \mathbb{E}[w \mid a_1 = L] = -\frac{w^*}{2} \\
a_2^*(a_1 = H) &= - \mathbb{E}[w \mid a_1 = H] = -\frac{1 + w^*}{2} \\
a_2^*(a_1 \notin \{L, H\}) &= - \mathbb{E}[w \mid a_1 \notin \{L, H\}] = -\frac{w^*}{2}
\end{aligned}
$$

Given a_2^*, we find the optimal threshold w^* for player 1. The optimal threshold must be such that when $w = w^*$, the committee is indifferent between L or H.

$$\begin{aligned} U_1(L, a_2^*(L), w^*) &= U_1(H, a_2^*(H), w^*) \\ -(-\frac{w^*}{2} + w^* - x_c)^2 &= -(-\frac{1+w^*}{2} + w^* - x_c)^2 \\ (\frac{w^*}{2} - x_c)^2 &= (\frac{w^*}{2} - \frac{1}{2} - x_c)^2 \\ \implies w^* &= 2x_c + \frac{1}{2} \text{ if } x_c < \frac{1}{4} \end{aligned}$$

We must also check that L is the best response when $w \leq w^*$ and H is the best response otherwise.

$$\begin{aligned} U_1(L, a_2^*(L), w) &= -(-\frac{w^*}{2} + w - x_c)^2 \\ U_1(H, a_2^*(H), w) &= -(-\frac{1+w^*}{2} + w - x_c)^2 \end{aligned}$$

When $w \leq w^*$, $-\frac{w^*}{2} + w - x_c \leq \frac{1}{4}$ and $-\frac{w^*}{2} + w - x_c \leq -\frac{1}{4}$. Therefore $L \succsim_1 H$. When $w \geq w^*$, the reverse inequalities are true and therefore $H \succsim_1 L$. Moreover, since player 2's off-path belief is the same as receiving the low signal, there is no strict incentive for player 1 to deviate off-path as well.

Taking together, for $x_c \in (0, \frac{1}{4})$, there exists an informative PBE with the following strategies of the committee,

$$a_1(w) = \begin{cases} L & w \in [0, 2x_c + \frac{1}{2}] \\ H & w \in [2x_c + \frac{1}{2}, 1] \end{cases}$$

the strategy of the floor,

$$\begin{aligned} a_2^*(a_1 = L) &= -\mathbb{E}[w \mid a_1 = L] = -x_c - \frac{1}{4} \\ a_2^*(a_1 = H) &= -\mathbb{E}[w \mid a_1 = H] = -x_c - \frac{3}{4} \\ a_2^*(a_1 \notin \{L, H\}) &= -\mathbb{E}[w \mid a_1 \notin \{L, H\}] = -x_c - \frac{1}{4} \end{aligned}$$

and the floor's beliefs,

$$\begin{aligned}
\mu(w \mid a_1 = L) =& \text{Unif}[0, 2x_c + \frac{1}{2}] \\
\mu(w \mid a_1 = H) =& \text{Unif}[2x_c + \frac{1}{2}, 1] \\
\mu(w \mid a_1 \notin \{L, H\}) =& \text{Unif}[0, 2x_c + \frac{1}{2}].
\end{aligned}$$