ECON6110: Problem Set 2

Spring 2025

This problem set is due on at 23:59 on February 28, 2025. Every student must write their own solution and submit it individually. Problem set submissions are submitted electronically and may be typed or handwritten. If handwritten, please ensure your work and scan are legible. **Illegible submissions will not be graded.**

Problem 1

Consider the following game between a union (player 1) and a monopolist (player 2). The union moves first and sets wage $w \geq 0$. The monopolistic firm, after observing the wage set by the union, chooses output $q \geq 0$. Assume that workers have opportunistic cost $l \in (0, 1)$ of their time, that one unit of labor is needed for each unit of output, and firms faces downward-sloping inverse demand $p(q) = 1 - q$. Overall:

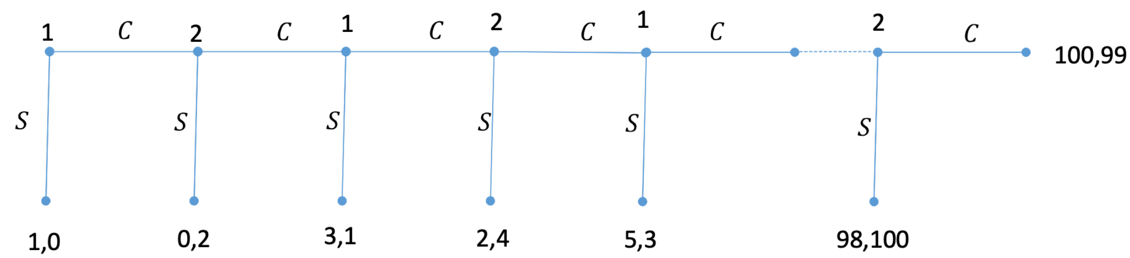
$$v_1(w, q) = (w - l)q \quad \text{and} \quad v_2(w, q) = q((1 - q) - w).$$

Question 1: Find a subgame perfect equilibrium.

Question 2: Find a Nash equilibrium that is not subgame perfect.

Problem 2

Consider the following Centipede game:



Question 1: Find a Nash equilibrium that is not a SPE, or prove that there are none.

Question 2: Is the subgame-perfect equilibrium outcome the unique Nash equilibrium outcome of the centipede game?

Problem 3

Suppose a parent and child play the following game, first analyzed by Gary Becker. First, the child takes an action, A; that produces income for the child, $I_C(A)$; and income for the parent, $I_P(A)$. (Think of $I_C(A)$ as the child's income net of any cost of the action A). Second, the parent observes the incomes $I_C(A)$ and $I_P(A)$ and then chooses a bequest, B; to leave to the child. The child's payoff is:

$$U(I_C(A) + B)$$

The parent's is:

$$V(I_P(A) - B) + kU(I_C(A) + B)$$

where $k > 0$ reflects the parent's concern for the child's well-being. Assume that: the action is a nonnegative number, $A > 0$; the income functions $I_C(A)$ and $I_P(A)$ are strictly concave and maximized at $A_C > 0$ and $A_P > 0$, respectively; the bequest B can be positive or negative; and the utility functions U and V are increasing and strictly concave. Prove the "Rotten Kid" Theorem (G. Becker): in the subgame-perfect equilibrium outcome, the child chooses the action that maximizes the family's aggregate income, $I_C(A) + I_P(A)$; even though only the parent's payoff exhibits altruism.

Problem 4

Two partners would like to complete a project. Each partner receives the payoff V when the project is completed but neither receives any payoff before completion. The cost remaining before the project can be completed is R. Neither partner can commit to making a future contribution towards completing the project, so they decide to play the following two-period game: In period one partner 1 chooses to contribute $c_1 \geq 0$ towards completion. If this contribution is sufficient to complete the project then the game ends and each partner receives V: If this contribution is not sufficient to complete the project (i.e., $c_1 < R$) then in period two partner 2 chooses to contribute $c_2 \geq 0$ towards completion. If the (undiscounted) sum of the two contributions is sufficient to complete the project then the game ends and each partner receives V: If this sum is not sufficient to complete the project then the game ends and both partners receive zero.

Each partner must generate the funds for a contribution by taking money away from other profitable activities. The optimal way to do this is to take money away from the least profitable alternative first. The resulting (opportunity) cost of a contribution is thus convex in the size of the contribution. Suppose that the cost of contribution c is c^2 for each partner. Assume that partner 1 discounts second-period benefits by the discount factor $\delta \in (0, 1)$: Compute the unique backwards-induction outcome of this two-period contribution game for each triple of parameters $\{V, R, \delta\}$.