Assignment 4
ECON 6140
Spring 2025

The due date for this assignment is Thursday February 27th.

Income Fluctuations Problem

Feel free to pair with a student that has working knowledge of python to complete this question, you can submit one answer per UP TO 3 students.
Consider an economy populated by a continuum of households that live forever and per period preferences

$$u(c_{it}) = \frac{c_{it}}{1-\gamma}^{1-\gamma}$$

with $\gamma = 1.5$ and discount factor $\beta = 0.98$. The household has a savings technology with return r, and initial asset holdings that for simplicity we set equal to zero for all households. Households are constrained in the amount of borrowing by b

$$a_{t+1} \geq -b$$

Labor income is a two-state Markov chain with state $y_H = 2$ and $y_L = 0$ with transition probabilities $\pi_{HH} = 0.95$ and $\pi_{LL} = 0.4$.

1. Assume an interest rate of $r = 0.1$. Solve for the savings and consumption function of the household assuming no-borrowing is allowed whatsoever ($b = 0$).
2. Plot the policies in the asset space.
3. Increase the curvature of the preferences to $\gamma = 3$, plot the new policies and explains why they move in such a direction.
4. Now go back to the baseline framework and set the borrowing constraint $b = 4$. Plot the new policies and explain why they move in such a direction.
5. Now compute the aggregate assets of the economy in the stationary distribution. Plot the level of aggregate assets for different levels of the interest rate.

As usual, you can code in whichever language you want, but these exercises can be fully solved following the codes in Quantecon section 49
https://python.quantecon.org/ifp.htmlid16
Alternatively, you can use the compecon plug-in into matlab.

Credit Markets and Economic Growth

Consider an economy where individuals can either supply their labor to firms or operate an individual-specific technology. Individuals in this economy are heterogeneous in terms of their wealth and entrepreneurial ability. However, everyone has the same labor productivity, and behaves competitively.
Output is produced using labor (l), capital (k), and entrepreneurial input (e_i):

$$f(e_i, k, l) = e_i^v l^{(1-v)(1-\alpha)} k^{\alpha(1-v)}$$

Individuals' preference is given by

$$\int_0^\infty e^{-\rho t} \frac{c_i(t)^{1-\sigma}}{1-\sigma} dt$$

To simplify assume that the entrepreneurial ability can either be $e_L = 0$ or $e_H > 0$. Note that an individual's ability does not change over time. The measure of the high-ability individuals is normalized to one, and that of the low-ability ones is π. Assume that π is large enough that high-ability types always choose to operate their technology. Further assume that all the initial capital stock in the economy is owned by the individuals with high ability and is equally distributed among them.

Perfect credit markets

1. Argue that you can compute the competitive equilibrium by solving the planner's problem.

2. Show that it is possible to write the aggregate production function as

   $$F(K_t) = e^v \pi^{(1-v)(1-\alpha)} K_t^{\alpha(1-v)}$$

   where K_t is the aggregate capital stock at t.

   Thus, the resource constraint of the planner will be

   $$\dot{K}(t) = F(K_t) - \delta K_t - C_t$$

   where δ is the depreciation rate of capital and C_t stands for aggregate consumption.

3. Why it is acceptable to write the resourse constraint in terms of aggregate consumption instead of c_H and c_L the consumption of each ability type household?

4. Write down the Hamiltonian of the problem adn derive the first-order conditions. Then substitute out the co-state variable to obtain a pair of first-order differential equations in $C(t)$ and $K(t)$.

5. If we assume that all the initial capital stock belongs to the low ability agents, how should the dynamics of aggregate variables change?