About TA sections:

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Section time and location: 8:40am - 9:55am Rockefeller Hall 132

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In these notes:

1 Notations

Let us first clarify the notation used throughout:

Output (all variables are in **logs**):

- y_t – output in period t,
- y – steady-state output,
- y^n – potential output (under flexible prices),
- $\tilde{y}_t \equiv y_t - y_t^n$ – output gap,
- $\hat{y}_t \equiv y_t - y$ – deviation of output from steady state.

Costs:

- $\psi_t \equiv \Psi'(Y_t)$ – nominal marginal cost,
- $mc_t \equiv \psi_t - p_t$ – real marginal cost,
- mc – steady-state real marginal cost,
- $\widehat{mc}_t \equiv mc_t - mc$ – deviation of marginal cost from its steady state.

2 Method of Undetermined Coefficients

We have two key equations in our model:

New Keynesian Phillips Curve (NKPC):

$$\pi_t = \beta \mathbb{E}_t\{\pi_{t+1}\} + \kappa \tilde{y}_t. \tag{1}$$

Dynamic IS equation:

$$\tilde{y}_t = \mathbb{E}_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma}(i_t - \mathbb{E}_t\{\pi_{t+1}\} - r_t^n) \tag{2}$$

where the natural rate of interest is given by

$$r_t^n = \rho - \sigma(1-\rho_a)\psi_{ya} a_t + (1-\rho_z) z_t \tag{3}$$

and the nominal interest rate is given by

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t + v_t \tag{4}$$

which corresponds to the ‘‘*simple interest rate rule*”.

Step 1. Rewrite the dynamic IS equation in terms of π_t, $\tilde{y}_t$ (the two variables we need to solve for), and shocks.

Plugging (3) and (4) into (2), we obtain:

$$\tilde{y}_t = \mathbb{E}_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma}\Big(\rho + \phi_\pi \pi_t + \phi_y \hat{y}_t + v_t - \mathbb{E}_t\{\pi_{t+1}\} - \rho + \sigma(1-\rho_a)\psi_{ya} a_t - (1-\rho_z) z_t\Big)$$

Note: Recall from the previous section that $y_t^n = \psi_{ya} a_t + \psi_y$. In the steady state, $y = \psi_y$. Hence, we can write $\hat{y}_t^n \equiv y_t^n - y = \psi_{ya} a_t$.

Using the identity $\hat{y}_t = \tilde{y}_t + \hat{y}_t^n = \tilde{y}_t + \psi_{ya} a_t$, the expression above becomes

$$\tilde{y}_t = \mathbb{E}_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma}\Big(\rho + \phi_\pi \pi_t + \phi_y(\tilde{y}_t + \psi_{ya} a_t) + v_t - \mathbb{E}_t\{\pi_{t+1}\} - \rho + \sigma(1-\rho_a)\psi_{ya} a_t - (1-\rho_z) z_t\Big).$$

Simplifying this expression, we obtain:

$$\tilde{y}_t = \mathbb{E}_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma}\Big(\phi_\pi \pi_t - \mathbb{E}_t\{\pi_{t+1}\} + \phi_y \tilde{y}_t + \psi_{ya}(\phi_y + \sigma(1-\rho_a)) a_t + v_t - (1-\rho_z) z_t\Big).$$

Note that the last three terms all contain shocks. We combine them together into one **composite shock**:

$$u_t \equiv \psi_{ya}(\phi_y + \sigma(1-\rho_a))a_t + v_t - (1-\rho_z)z_t.$$

Finally, we can rewrite the IS equation as:

$$\tilde{y}_t = \mathbb{E}_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma}\Big(\phi_\pi \pi_t - \mathbb{E}_t\{\pi_{t+1}\} + \phi_y \tilde{y}_t + u_t\Big).$$

We will assume that u_t follows an AR(1) process:

$$u_{t+1} = \rho_u u_t + \varepsilon_u$$

Step 2. Make a conjecture.

We conjecture that our two key variables take the form:

$$\pi_t = \psi_\pi u_t,$$

$$\tilde{y}_t = \psi_y u_t.$$

Hence, under rational expectations:

$$E_t \pi_{t+1} = \psi_\pi \rho_u u_t,$$

$$E_t \tilde{y}_{t+1} = \psi_y \rho_u u_t.$$

Step 3. Substitute our conjecture back into the original equations.

The NKPC becomes:

$$\pi_t = \beta \mathbb{E}_t\{\pi_{t+1}\} + \kappa \tilde{y}_t \iff \psi_\pi u_t = \beta \psi_\pi \rho_u u_t + \kappa \psi_y u_t$$

and the IS equation becomes:

$$\tilde{y}_t = \mathbb{E}_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma}\Big(\phi_\pi \pi_t - \mathbb{E}_t\{\pi_{t+1}\} + \phi_y \tilde{y}_t + u_t\Big) \iff \psi_y u_t = \psi_y \rho_u u_t - \frac{1}{\sigma}\Big(\phi_\pi \psi_\pi u_t - \psi_\pi \rho_u u_t + \phi_y \psi_y u_t + u_t\Big)$$

In both resulting equations, we can cancel u_t and simplify:

$$\psi_\pi - \beta\psi_\pi\rho_u - \kappa\psi_y = 0$$

$$\psi_y - \psi_y\rho_u + \frac{1}{\sigma}\Big(\phi_\pi\psi_\pi - \psi_\pi\rho_u + \phi_y\psi_y\Big) = -\frac{1}{\sigma}$$

Step 4. Solve the system.

We can rewrite the system of equations above as:

$$\begin{bmatrix} 1-\beta\rho_u & -\kappa \\ \frac{1}{\sigma}(\phi_\pi - \rho_u) & 1-\rho_u+\frac{\phi_y}{\sigma} \end{bmatrix} \begin{bmatrix} \psi_\pi \\ \psi_y \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{1}{\sigma} \end{bmatrix}.$$

Multiplying both sides by the inverse of the matrix on the left-hand side, we get:

$$\begin{bmatrix} \psi_\pi \\ \psi_y \end{bmatrix} = \begin{bmatrix} 1-\beta\rho_u & -\kappa \\ \frac{1}{\sigma}(\phi_\pi - \rho_u) & 1-\rho_u+\frac{\phi_y}{\sigma} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -\frac{1}{\sigma} \end{bmatrix},$$

that is

$$\begin{bmatrix} \psi_\pi \\ \psi_y \end{bmatrix} = \Lambda \begin{bmatrix} 1-\rho_u+\frac{\phi_y}{\sigma} & \kappa \\ -\frac{1}{\sigma}(\phi_\pi - \rho_u) & 1-\beta\rho_u \end{bmatrix} \begin{bmatrix} 0 \\ -\frac{1}{\sigma} \end{bmatrix}$$

where

$$\Lambda \equiv \frac{1}{(1-\rho_u+\frac{\phi_y}{\sigma})(1-\beta\rho_u) + \kappa\frac{1}{\sigma}(\phi_\pi - \rho_u)}.$$

This gives us

$$\begin{bmatrix} \psi_\pi \\ \psi_y \end{bmatrix} = \Lambda \begin{bmatrix} -\frac{\kappa}{\sigma} \\ \frac{\beta\rho_u - 1}{\sigma}. \end{bmatrix}.$$

So, the final solution is

$$\psi_\pi = -\Lambda\frac{\kappa}{\sigma},$$

$$\psi_y = \Lambda\frac{\beta\rho_u - 1}{\sigma}.$$

Awesome, we have solved the problem! Now we can plug this into the computer and compute the values.

3 Optimal Prices with Price Rigidities

With sticky prices, firms know they may not be able to adjust their prices in the next period. Hence, they take this into account when choosing current prices and set them to maximize expected discounted profits:

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k \, \mathbb{E}_t \left\{ \beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+k}} \right) \left(P_t^* Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t}) \right) \right\}$$

subject to the sequence of demand constraints

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} C_{t+k}, \quad \forall t.$$

Remark: The notation $t+k|t$ means that something is happening k periods after the firm last reset its price in period t.

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$$\sum_{k=0}^{\infty} \theta^k \, \mathbb{E}_t \left[\beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+k}} \right) Y_{t+k|t} \left(P_t^* - \mathcal{M} \, \psi_{t+k|t} \right) \right] = 0.$$

Remark: Recall from the previous section that $\mathcal{M} \equiv \frac{\varepsilon}{\varepsilon - 1}$. This is the optimal, or as we refer to it here, the **"desired" markup**. When there are no price rigidities (i.e., $\theta = 0$), the optimal price is:

$$P_t^* = \mathcal{M} \, \psi_{t|t}.$$

We can rewrite this expression, dividing it by P_{t-1} and denoting $\Pi_{t,t+k} \equiv \frac{P_{t+k}}{P_t}$:

$$\sum_{k=0}^{\infty} \theta^k \, \mathbb{E}_t \left[\beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+k}} \right) Y_{t+k|t} \left(\frac{P_t^*}{P_{t-1}} - \mathcal{M} \, MC_{t+k|t} \, \Pi_{t-1,t+k} \right) \right] = 0.$$

Note that we now use the **real marginal cost**, $MC_{t+k|t} \equiv \frac{\psi_{t+k|t}}{P_{t+k}}$, instead of the nominal one.

We can approximate this equation around the **zero-inflation steady state**. However, before doing so, note that this steady state implies: $P_t^*/P_{t-1} = 1$, $\Pi_{t-1,t+k} = 1$ - because it is "zero-inflation", $Y_{t+k|t} = Y$, and $MC_{t+k|t} = MC = \frac{1}{\mathcal{M}}$, and $\beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+k}} \right) = \beta^k$ - because it is a steady state. By a first-order Taylor expansion, we obtain:

$$p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k\,\mathbb{E}_t[mc_{t+k|t} + \mu + p_{t+k} - p_{t-1}],$$

or equivalently

$$p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k\,\mathbb{E}_t[\widehat{mc}_{t+k|t} + p_{t+k} - p_{t-1}].$$

If we cancel p_{t-1} from both sides, we can also rewrite this as:

$$p_t^* = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k\,\mathbb{E}_t[mc_{t+k|t} + p_{t+k}],$$

which is equivalent to

$$p_t^* = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k\,\mathbb{E}_t\psi_{t+k|t},$$

because $\psi_{t+k|t} \equiv mc_{t+k|t} + p_{t+k}$.

This expression provides an important intuition on **how firms choose their optimal prices**: they set a price equal to their desired markup plus a weighted average of current and expected future marginal costs, where the weights reflect the likelihood that the chosen price will remain in effect over time.

The two **key parameters** in this setting are:

- β - the discount factor, which tells us how much firms value future profits relative to current profits. A higher β means firms place more weight on expected future outcomes when setting prices.

- θ - the probability that a firm will **not** be able to reset its price in a given period. A higher θ implies more price rigidity, so firms must consider that today's price might stay in place for many periods.

These two parameters determine the degree to which a firm's price decision is **forward looking**.

4 Optional: Exam Practice

Try to answer the following questions:

- What additional structure do you need to add to the baseline model to introduce price stickiness a la Calvo?
- Describe either with words or formulas how the price setting decision of firms is determined. How is it affected by Calvo-type price stickiness? What determines the degree to which a firm's price decision is forward looking?
- What parameters of the model determine the extent of the inefficiencies that arise from sticky prices? For what parameter values are the inefficiencies worse?
- Derive the New Keynesian Phillips curve from the optimal price of a firm that can reset its price in period t.
- Solve for equilibrium using the method of undetermined coefficients under the assumption that monetary policy follows $i_t = \rho + \phi_\pi \pi_t$.