Macroeconomics ECON 6140 (Second Half)

Lecture 7
Optimal Monetary Policy under Discretion and Commitment

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Optimal Monetary Policy under Discretion and Commitment

Today's plan

- Policy trade-offs: Cost-push shocks
- Optimal policy under discretion and under commitment

Readings: Gali Ch 5.1-5.2

The New Keynesian Model

New Keynesian Phillips Curve

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t$$

Dynamic IS Equation

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n)$$

where $r_t^n = \rho - \sigma\,(1 - \rho_a)\,\psi_{ya} a_t + (1 - \rho_z) z_t$.

Efficiency in the New Keynesian model

The Efficient Allocation

$$\max U(C_t, N_t; Z_t)$$

where $C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$ subject to:

$$C_t(i) = A_t N_t(i)^{1-\alpha}, \ \ all \ \ i \in [0,1]$$

$$N_t = \int_0^1 N_t(i)di$$

Efficiency conditions:

$$C_t(i) = C_t, \ \ all \ \ i \in [0,1]$$

$$N_t(i) = N_t, \ \ all \ \ i \in [0,1]$$

$$-\frac{U_{n,t}}{U_{c,t}} = MPN_t$$

where $MPN_t \equiv (1-\alpha)A_t N_t^{-\alpha}$

Sources of inefficiency: Market power

Under flexible prices, the optimal price is nominal marginal cost times a mark-up $P_t = \mathcal{M}\frac{W_t}{MPN_t}$, where $\mathcal{M} \equiv \frac{\varepsilon}{\varepsilon-1} > 1$

$$\Longrightarrow \quad -\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}} < MPN_t$$

Efficiency can be restored by employment subsidy τ so that $P_t = \mathcal{M}\frac{(1-\tau)W_t}{MPN_t}$.

$$\Longrightarrow \quad -\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}(1-\tau)}$$

The optimal subsidy is set so that $\mathcal{M}(1-\tau) = 1$ or, equivalently, $\tau = \frac{1}{\varepsilon}$.

Sources of inefficiency: Nominal rigidities

Level effects

With a constant employment subsidy that implies an efficient level of output under flexible prices, variation in mark-ups resulting from sticky prices are inefficient

$$\mathcal{M}_t \equiv \frac{P_t}{(1-\tau)(W_t/MPN_t)} = \frac{P_t \mathcal{M}}{W_t/MPN_t}$$

$$\implies \quad -\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = MPN_t \frac{\mathcal{M}}{\mathcal{M}_t} \neq MPN_t$$

Efficiency requires that average markup = desired markup $\forall t$

Welfare-based policy evaluation

We can approximate the welfare of the representative household as

$$\mathbb{W} \equiv -E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{U_t - U_t^n}{U_c C} \right) = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \widetilde{y}_t^2 + \frac{\epsilon}{\lambda} \pi_t^2 \right]$$

so that the expected average welfare loss per period $\mathbb{L}$ is given by

$$\mathbb{L} = \frac{1}{2} \left[\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) var(\widetilde{y}_t) + \frac{\epsilon}{\lambda} \; var(\pi_t) \right]$$

Optimal Monetary Policy in the Basic Model

Optimal Monetary Policy in the Basic Model

Key assumptions:

- Optimal (constant) employment subsidy as above
- No inherited relative price distortions, i.e. $P_{-1}(i) = P_{-1}$ for all $i \in [0, 1]$
- Only demand and productivity shocks (no shocks that make flex-price equilibrium inefficient).

Optimal policy and the Divine Coincidence

The optimal policy replicates the flexible price equilibrium allocation.

- Commit to stabilizing marginal costs at a level consistent with firms' desired markup at *given existing prices*
- No firm has an incentive to adjust its price, i.e. $P_t^* = P_{t-1}$ and, hence, $P_t = P_{t-1}$ for $t = 0, 1, 2, ...$(aggregate price stability)
- Equilibrium output and employment match their *natural* counterparts.

Equilibrium under the optimal policy then implies $y_t = y_t^n$, $\widetilde{y}_t = 0, \pi_t = 0, i_t = r_t^n$ for all t.

Policy trade-offs and the New Keynesian Phillips Curve

Cost-push shocks and Policy Trade-offs

In the simple New Keynesian model with only demand and productivity shocks there are no policy trade-offs

- Strict inflation targeting is then optimal even if we do not care about inflation per se

Implicit assumption in the simple model

- Natural and efficient level of output coincide, i.e. $y_t^e - y_t^n = 0$

Cost-push shocks and Policy Trade-offs

What if the **efficient and natural level of output do not coincide**?

- When actual output y_t coincide with the natural level of output y_t^n
  $\Rightarrow$ No inflation
- When actual output y_t coincide with the efficient level of output y_t^e
  $\Rightarrow$ There may be inflation, but the condition

$$-\frac{U_{n,t}}{U_{c,t}} = MPN_t$$

holds.

Cost-push shocks and Policy Trade-offs

If $y_t^e \neq y_t^n$ we need to modify the Phillips curve.

- Time-varying $y_t^e - y_t^n$ implies that

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa x_t + u_t$$

where $x_t \equiv y_t - y_t^e$ and $u_t \equiv \kappa(y_t^e - y_t^n)$

Optimal Policy under Discretion

The Monetary Policy Problem

$$\min E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \vartheta x_t^2\right)$$

subject to

$$\begin{aligned} x_t &= -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^e) + E_t\{x_{t+1}\} \\ \pi_t &= \beta E_t\{\pi_{t+1}\} + \kappa x_t + u_t \end{aligned}$$

for $t = 0, 1, 2, ...$where $\{u_t\}$ evolves exogenously according to

$$u_t = \rho_u u_{t-1} + \varepsilon_t$$

and where $r_t^e \equiv \rho - \sigma E_t \Delta y_{t+1}^e + (1 - \rho_z) z_t$.

Note: utility based criterion requires $\vartheta = \frac{\kappa}{\epsilon}$

Optimal policy under discretion

Each period the monetary authority chooses (x_t, π_t) to minimize

$$\pi_t^2 + \vartheta x_t^2$$

subject to

$$\pi_t = \kappa x_t + \upsilon_t$$

with $\upsilon_t \equiv \beta E_t\{\pi_{t+1}\} + u_t$ taken as given.

Why does the policy maker take inflation expectations as given?

Optimal policy under discretion

Optimality condition

$$x_t = -\frac{\kappa}{\vartheta}\pi_t$$

Equilibrium

$$\pi_t = \frac{\vartheta}{\kappa^2 + \vartheta(1-\beta\rho_u)}\ u_t$$

$$x_t = -\frac{\kappa}{\kappa^2 + \vartheta(1-\beta\rho_u)}\ u_t$$

$$i_t = r_t^e + \frac{\vartheta\rho_u + \sigma\kappa(1-\rho_u)}{\kappa^2 + \vartheta(1-\beta\rho_u)} u_t$$

Implementation

$$\begin{aligned} i_t &= r_t^e + \frac{\vartheta\rho_u + \sigma\kappa(1-\rho_u)}{\kappa^2 + \vartheta(1-\beta\rho_u)} u_t + \phi_\pi \left(\pi_t - \frac{\vartheta}{\kappa^2 + \vartheta(1-\beta\rho_u)}\ u_t\right) \\ &= r_t^e + \Theta_i u_t + \phi_\pi \pi_t \end{aligned}$$

where $\Theta_i \equiv \frac{\sigma\kappa(1-\rho_u) - \vartheta(\phi_\pi - \rho_u)}{\kappa^2 + \vartheta(1-\beta\rho_u)}$ and $\phi_\pi > 1$.

Optimal Policy with Commitment

Gains from Commitment

Solving the Phillips Curve forward gives

$$\pi_t = \kappa x_t + \kappa \sum_{k=1}^{\infty} \beta^k E_t\{x_{t+k}\} + \frac{1}{1-\beta\rho_u}\ u_t$$

By committing to future negative output gaps, the policy maker can reduce response of inflation today.

Given the convex loss function, smoothing out the response is optimal.

Optimal policy under commitment

State-contingent policy $\{x_t, \pi_t\}_{t=0}^{\infty}$ that minimizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \vartheta x_t^2\right)$$

subject to the sequence of constraints

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa x_t + u_t$$

Lagrangean

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2}\left(\pi_t^2 + \vartheta x_t^2\right) + \xi_t(\pi_t - \kappa x_t - \beta \pi_{t+1})\right] + t.i.p.$$

Optimality conditions

$$\vartheta x_t - \kappa \xi_t = 0$$

$$\pi_t + \xi_t - \xi_{t-1} = 0$$

for $t = 0, 1, 2, \ldots$with $\xi_{-1} = 0$.

Optimal policy under commitment

Eliminating multipliers

$$x_0 = -\frac{\kappa}{\vartheta}\pi_0$$

$$x_t = x_{t-1} - \frac{\kappa}{\vartheta}\pi_t$$

for $t = 1, 2, 3, ...$

Optimal policy under commitment

Alternative representation

$$x_t = -\frac{\kappa}{\vartheta}\widehat{p}_t$$

for $t = 0, 1, 2, ...$where $\widehat{p}_t \equiv p_t - p_{-1}$

Equilibrium

$$\widehat{p}_t = \gamma\widehat{p}_{t-1} + \gamma\beta E_t\{\widehat{p}_{t+1}\} + \gamma u_t$$

for $t = 0, 1, 2, ...$where $\gamma \equiv \frac{\vartheta}{\vartheta(1+\beta)+\kappa^2}$

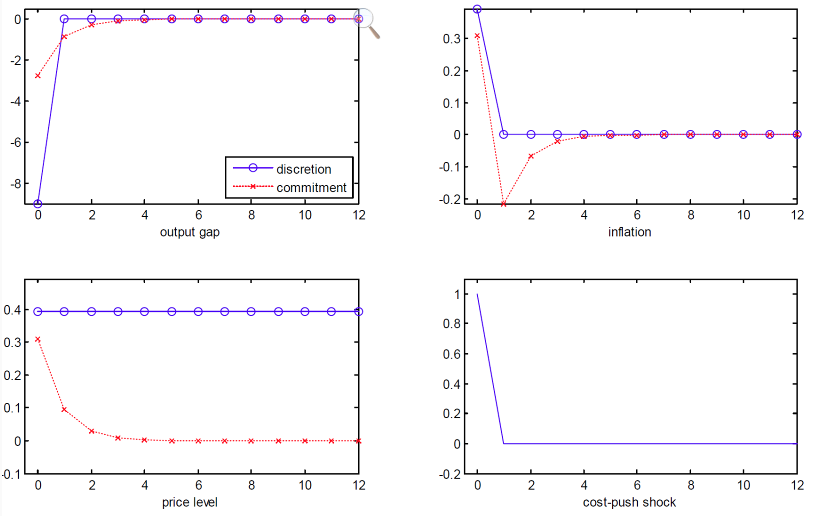
Stationary solution:

$$\widehat{p}_t = \delta\widehat{p}_{t-1} + \frac{\delta}{1-\delta\beta\rho_u}\ u_t$$

for $t = 0, 1, 2, ...$where $\delta \equiv \frac{1-\sqrt{1-4\beta\gamma^2}}{2\gamma\beta} \in (0, 1)$.

$\rightarrow$ *price level targeting !*

Optimal policy: Discretion vs Commitment



Optimal monetary policy in the New Keynesian model

What you need to know:

- It is possible to **derive optimal policy criteria from utility function** of representative household
- With CES utility and decreasing marginal productivity of labor production functions it is **optimal to produce the same amount of each good**
- In the presence of **only productivity and demand shocks**, optimal policy implies complete **price stability**
- In the presence of shocks that imply a **trade-off between stabilizing output and inflation**, the possibility of **committing to future policy actions** can lead to better outcomes

That's it for today.