Macroeconomics II ECON 6140 (Second Half)

Lecture 9
Unemployment in the New Keynesian Model

Cornell University
Spring 2025

April 24, 2025

Unemployment in New Keynesian Models

One criticism of the baseline New Keynesian model is that is has no role for unemployment.

Today

- Reinterpretation of the standard NK model $\Rightarrow$ unemployment
- Alternative to search friction based framework: labor market frictions $+$ nominal rigidities

Note: Final exam will be at 2-3.30pm on May 8.

A Model of Unemployment and Inflation Fluctuations

Households

- Representative household with a continuum of members, indexed by $(j, s) \in [0, 1] \times [0, 1]$
- Continuum of occupations, indexed by $j \in [0, 1]$
- Disutility from (indivisible) labor: χs^{φ}, for $s \in [0, 1]$, where $\varphi \geq 0$
- Full consumption risk sharing within the household

A Model of Unemployment and Inflation Fluctuations

Households

$$
\begin{aligned}
U(C_t, \{\mathcal{N}_t(j)\}; Z_t) &\equiv \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \chi \int_0^1 \int_0^{\mathcal{N}_t(j)} s^{\varphi} ds dj \right) Z_t \\
&= \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \chi \int_0^1 \frac{\mathcal{N}_t(j)^{1+\varphi}}{1+\varphi} dj \right) Z_t
\end{aligned}
$$

where $C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon_p}} di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$

Households, cont'd

Budget constraint

$$\int_0^1 P_t(i)C_t(i)di + Q_t B_t \leq B_{t-1} + \int_0^1 W_t(j)\mathcal{N}_t(j)dj + D_t$$

Two optimality conditions

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_p} C_t$$

where $P_t \equiv \left(\int_0^1 P_t(i)^{1-\epsilon_p} di\right)^{\frac{1}{1-\epsilon_p}}$, implying $\int_0^1 P_t(i)C_t(i)di = P_t C_t$.

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{Z_{t+1}}{Z_t}\right) \left(\frac{P_t}{P_{t+1}}\right) \right\}$$

Wage Setting

Average wage dynamics

$$w_t = \theta_w w_{t-1} + (1-\theta_w) w_t^*$$

Optimal wage setting rule

$$w_t^* = \mu^w + (1-\beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ mrs_{t+k|t} + p_{t+k} \right\}$$

where $\mu^w \equiv \log \frac{\epsilon_w}{\epsilon_w - 1}$ and $mrs_{t+k|t} \equiv \sigma c_{t+k} + \varphi n_{t+k|t} + \xi$

Wage inflation equation

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} - \lambda_w(\mu_t^w - \mu^w)$$

where $\pi_t^w \equiv w_t - w_{t-1}$, $\mu_t^w \equiv w_t - p_t - mrs_t$, and $\lambda_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\varphi)}$.

Introducing Unemployment

Participation condition for an individual (j, s):

$$\frac{W_t(j)}{P_t} \geq \chi C_t^\sigma s^\varphi$$

Marginal participant, $L_t(j)$, given by:

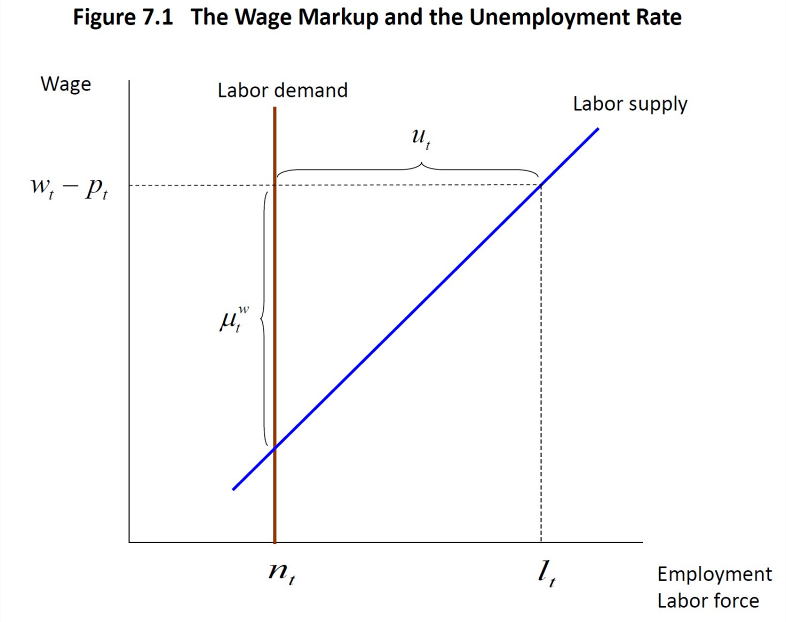
$$\frac{W_t(j)}{P_t} = \chi C_t^\sigma L_t(j)^\varphi$$

Aggregate labor force (in logs):

$$w_t - p_t = \sigma c_t + \varphi l_t + \xi$$

where $w_t \simeq \int_0^1 w_t(j)dj$ and $l_t \equiv \int_0^1 l_t(j)dj$

Labor supply and demand



Introducing Unemployment, cont'd

Unemployment rate

$$u_t \equiv l_t - n_t$$

Average wage markup and unemployment

$$\begin{aligned} \mu_t^w &= (w_t - p_t) - (\sigma c_t + \varphi n_t + \xi) \\ &= \varphi u_t \end{aligned}$$

Under flexible wages

$$\mu^w = \varphi u^n$$

$\Rightarrow u^n$: *natural* rate of unemployment

A New Keynesian Wage Phillips Curve

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} - \lambda_w \varphi (u_t - u^n)$$

Firms and Price Setting

Technology

$$Y_t(i) = A_t N_t(i)^{1-\alpha}$$

where $N_t(i) \equiv \left(\int_0^1 N_t(i,j)^{1-\frac{1}{\epsilon_w}} di \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}$

Average price dynamics

$$p_t = \theta_p p_{t-1} + (1 - \theta_p) p_t^*$$

Optimal price setting rule

$$p_t^* = \mu^p + (1 - \beta\theta_p) \sum_{k=0}^{\infty} (\beta\theta_p)^k E_t\{\psi_{t+k|t}\}$$

Firms and Price Setting

Implied price inflation equation

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} - \lambda_p(\mu_t^p - \mu^p)$$

where

$$\mu_t^p \equiv p_t - \psi_t$$

$$\psi_t \equiv w_t - (a_t - \alpha n_t + \log(1-\alpha))$$

and

$$\lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p} \; \frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}.$$

Equilibrium: Non-Policy block

$$\widetilde{y}_t = -\frac{1}{\sigma}(i_t - E_t\{\pi^p_{t+1}\} - r^n_t) + E_t\{\widetilde{y}_{t+1}\}$$

$$\pi^p_t = \beta E_t\{\pi^p_{t+1}\} + \varkappa_p \widetilde{y}_t + \lambda_p \widetilde{\omega}_t$$

$$\pi^w_t = \beta E_t\{\pi^w_{t+1}\} - \lambda_w \varphi \widehat{u}_t$$

$$\widetilde{\omega}_t \equiv \widetilde{\omega}_{t-1} + \pi^w_t - \pi^p_t - \Delta\omega^n_t$$

$$\begin{aligned}
\varphi \widehat{u}_t &= \widehat{\mu}^w_t \\
&= \widetilde{\omega}_t - (\sigma \widetilde{c}_t + \varphi \widetilde{n}_t) \\
&= \widetilde{\omega}_t - \left(\sigma + \frac{\varphi}{1-\alpha}\right) \widetilde{y}_t
\end{aligned}$$

Policy block: Example

Taylor-type rule:

$$i_t = \rho + \phi_p \pi_t^p + \phi_y \widehat{y}_t + v_t$$

Natural equilibrium

$$\widehat{y}_t^n = \psi_{ya} a_t$$

$$r_t^n = \rho - \sigma(1 - \rho_a)\psi_{ya} a_t + (1 - \rho_z) z_t$$

$$\widehat{\omega}_t^n = \psi_{wa} a_t$$

with $\psi_{ya} \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$ and $\psi_{wa} \equiv \frac{1-\alpha\psi_{ya}}{1-\alpha} > 0$.

Exogenous $AR(1)$ processes for $\{a_t\}$, $\{z_t\}$, and $\{v_t\}$

Calibration

Baseline calibration

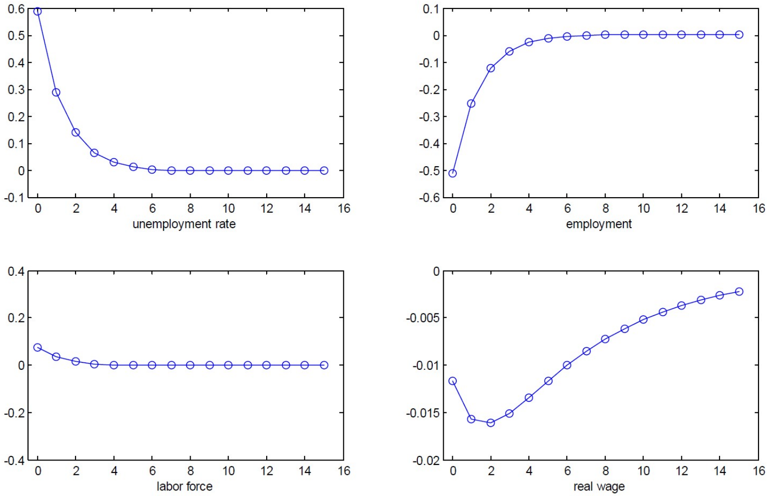
	Description	*Value*	*Target*
φ	Curvature of labor disutility	5	Frisch elasticity 0.2
α	Index of decrasing returns to labor	$1/4$	
ϵ_w	Elasticity of substitution (labor)	4.5	$u^n = 0.05$
ϵ_p	Elasticity of substitution (goods)	9	$S = \frac{1-\alpha}{\epsilon_p/(\epsilon_p-1)} = 2/3$
θ_p	Calvo index of price rigidities	$3/4$	avg. duration $= 4$
θ_w	Calvo index of wage rigidities	$3/4$	avg. duration $= 4$
ϕ_p	Inflation coefficient in policy rule	1.5	Taylor (1993)
ϕ_y	Output coefficient in policy rule	0.125	Taylor (1993)
β	Discount factor	0.99	
ρ_a	Persistence: technology shocks	0.9	
ρ_z	Persistence: demand shocks	0.5	
ρ_v	Persistence: monetary shocks	0.5	

Dynamic Effects of Monetary Policy Shocks on Labor Markets

- Impulse responses
- Wage rigidities and the volatility and persistence of unemployment

Response to a policy shock

Figure 7.2 Response of Labor Market Variables to a Monetary Policy Shock



Optimal Monetary Policy Problem

$$\min \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(\left(\sigma + \frac{\varphi + \alpha}{1-\alpha} \right) \widetilde{y}_t^2 + \frac{\epsilon_p}{\lambda_p} (\pi_t^p)^2 + \frac{\epsilon_w (1-\alpha)}{\lambda_w} (\pi_t^w)^2 \right)$$

subject to:

$$\pi_t^p = \beta E_t \{\pi_{t+1}^p\} + \varkappa_p \widetilde{y}_t + \lambda_p \widetilde{\omega}_t$$

$$\pi_t^w = \beta E_t \{\pi_{t+1}^w\} + \varkappa_w \widetilde{y}_t - \lambda_w \widetilde{\omega}_t$$

$$\widetilde{\omega}_t \equiv \widetilde{\omega}_{t-1} + \pi_t^w - \pi_t^p - \Delta \omega_t^n$$

Optimality conditions

$$\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) \widetilde{y}_t + \varkappa_p \zeta_{1,t} + \varkappa_w \zeta_{2,t} = 0 \tag{1}$$

$$\frac{\epsilon_p}{\lambda_p}\ \pi_t^p - \Delta \zeta_{1,t} + \zeta_{3,t} = 0 \tag{2}$$

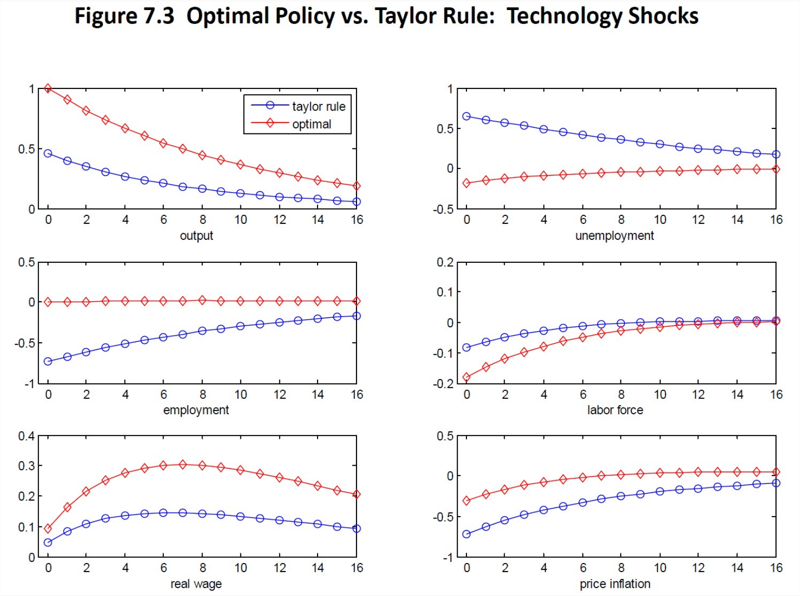
$$\frac{\epsilon_w (1 - \alpha)}{\lambda_w}\ \pi_t^w - \Delta \zeta_{2,t} - \zeta_{3,t} = 0 \tag{3}$$

$$\lambda_p \zeta_{1,t} - \lambda_w \zeta_{2,t} + \zeta_{3,t} - \beta E_t\{\zeta_{3,t+1}\} = 0 \tag{4}$$

A simple rule with unemployment (vs. optimal policy)

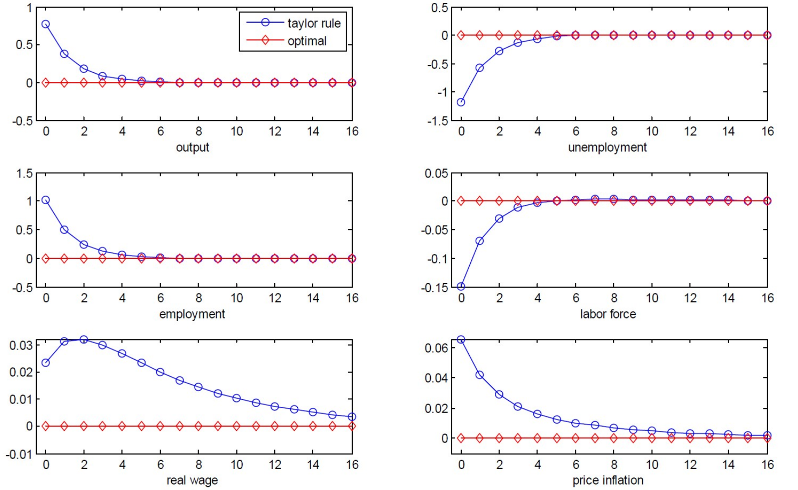
$$i_t = 0.01 + 1.5\pi_t^p - 0.5\widehat{u}_t \tag{5}$$

Response to a technology shock

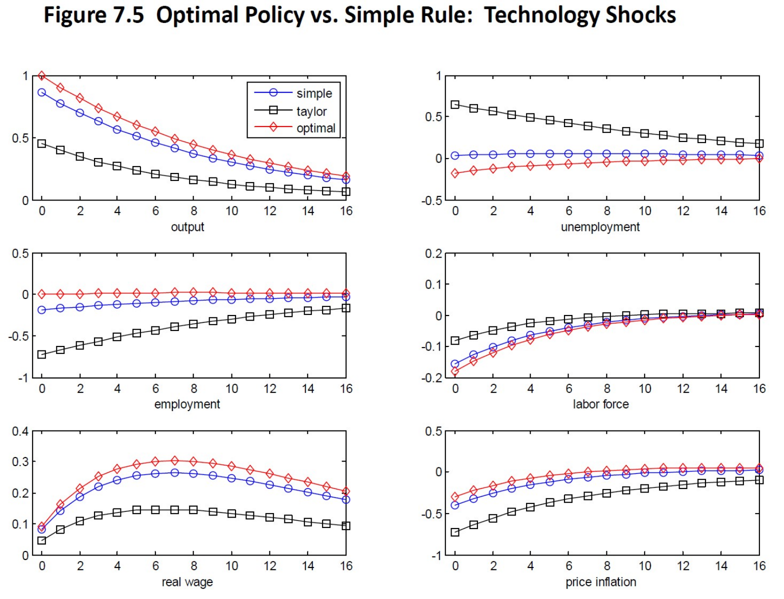


Response to a demand shock

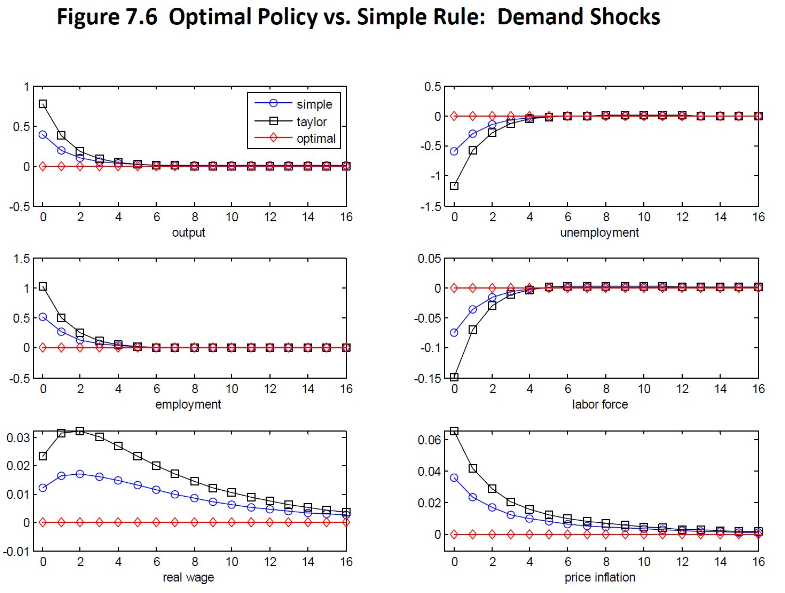
Figure 7.4 Optimal Policy vs. Taylor Rule: Demand Shocks



Response to technology shock: Simple rule



Response to a demand shock: Simple rule



Summing up unemployment a la Gali

- Wage stickiness and differentiated labor types allow for reinterpreting the New Keynesian model to include unemployment
- Unemployment = Labor force - employment
- Labor force includes all agents who would work for the aggregate real wage

3 Criticisms of the basic NK Model

Unemployment in the NK model and in reality

There is involuntary unemployment in the NK model

- At prevailing wage, some workers would prefer to work

What is missing?

- Search unemployment
- Adverse consequences of unemployment

Trigari (JMCB 2009) introduces search unemployment in NK model.

Household heterogeneity

Basic model uses a representative household. But:

- Not all households earn the same wage
- There is large heterogeneity on wealth holdings

Costs:

- Model makes inaccurate predictions if well-paid and wealthy households behave differently from low-paid and poor households.
- Model gives inaccurate normative recommendations if we care about inequality

Some Heterogenous Agent New Keynesian Models (or HANK models)

- McKay, A. and Reis, R., 2016. The role of automatic stabilizers in the US business cycle. Econometrica, 84(1), pp.141-194.
- Kaplan, G., Moll, B. and Violante, G.L., 2018. Monetary policy according to HANK. American Economic Review, 108(3), pp.697-743.

The Expected Real Interest Rate Channel

Consumption (and output) decision is completely determined by the expected real interest rate

- Little evidence that individual spending and investment decisions respond to expected real interest rate.

Costs

- Main model mechanism is not a good description of what drives consumption and investment in reality

Heterogenous agent models allow for wealth effects and precautionary motives to also affect spending/saving decision.

That's it for the NK model...