ECON 6110, Microeconomic Theory III	Section 11
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1 Bayesian Extensive Games and the Perfect Bayesian Equilibrium (PBE)

Definition 1.1. A **Bayesian extensive game with observed actions** is a tuple $\langle N, H, P, (\Theta_i), (p_i), (u_i)\rangle$ where:

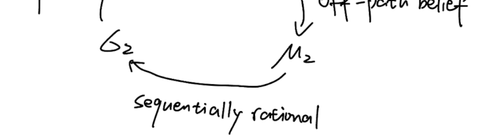
1. Set of N players, set of histories H, and player function P.
2. For each i:
   (a) A finite set of types Θ_i.
   (b) A probability measure p_i over Θ_i. (Assume independent types and common prior)
   (c) A preference relation $\succsim_i$ over $Z \times \Theta$.

Remark. In solving the game, we often recast the game as an extensive game with imperfect information, which is a tuple $\langle N, H, P, f_c, (\mathcal{I}_i), (u_i)\rangle$. We introduce Nature as another player, selecting types at time 0. (It will become clearer in the signaling game)

Definition 1.2 (Informal)**.** An assessment (σ, μ) is a **perfect Bayesian equilibrium** if

1. Sequentially rational: For each type θ_i, σ_i is the best response given μ_i and σ_{-i} at every information set I_i.
2. Bayesian updating whenever possible. (But no restriction on off-path beliefs)
3. Action determine beliefs: beliefs on i's type can only be changed by i's action. (True when independent types)

Remark. Solving for PBE often proceeds in a "loop":



2 Signaling Game

Consider the Spence's job-market signaling model with a discrete set of effort choices. The sender is a student, the receiver an employer. There are two types of students, defined by the value of their innate talent, $\theta \in \{2, 3\}$. Nature chooses θ with probability p that $\theta = 2$. The student chooses an effort level in college, $a_1 \in \{0, 1\}$. After observing a_1, the employer chooses a wage $a_2 \in [0, \infty)$. The student maximizes wage less cost of effort, the latter inversely related to talent:

$$v_1(a_1, a_2, \theta) = a_2 - \frac{ca_1}{\theta} \tag{1}$$

for some $c > 0$. The employer minimizes the expected squared difference between the wage and the student's innate talent.[1]

$$v_2(a_1, a_2, \theta) = -\mathbb{E}(a_2 - \theta)^2 \tag{2}$$

(a) Define a Bayesian extensive game with the information above. Specify the players, set of types, prior on types, player's actions, and utility functions. What are player's strategies and beliefs? Represent it with a graph.

(b) Does the above signaling game have a **separating PBE** where the low type chooses the low action and the high type chooses the high action?

(c) Does the above signaling game have a **separating PBE** where the low type chooses the high action and the high type chooses the low action?

(d) Does the above signaling game have a **pooling PBE** where both types chooses the low action?

(e) Does the above signaling game have a **pooling PBE** where both types chooses the high action?

(f) Does the above signaling game have a **semi-seperating PBE** where one type mixes?

[1]Note that the employer doesn't want to *underpay* the student either, perhaps because the student would then choose an alternative employer.