# Assignment 3
ECON 6140
Spring 2025

**The due date for this assignment is Thursday February 13th.**

## Health inequality and mitigation tools

Consider an economy populated with a continuum of households of measure 1. The population faces a mortality rate indexed by the expected lifetime of the household $1/(1-\pi)$. Half of the population lives in areas exposed to pesticides and faces a finite expected life $\pi < 1$, and the other half solves and infinite horizon problem, $\pi = 1$. We model the mortality rate as a higher discount factor on future consumption and normalize the utility of death to zero.

$$U(\mathbf{c}, \pi) = \sum_{t=0}^{\infty} (\beta\pi)^t u(c(t))$$

for a discount factor $\beta \in (0, 1)$.

There are three technologies available in the economy, one for capital accumulation, one to change the mortality rate in the economy (e.g. the health sector), and one for production of consumption goods and health investment, $h$ as follows

$$k_{t+1} = x_t + (1-\delta)k_t \quad \text{with } k_0 > 0$$
$$y_t = Ak_t^{\alpha} \quad \text{with } A > 0, \quad \text{and } \alpha \in (0, 1)$$

Health investments map into lower mortality through a technology $\pi_t = f(\frac{h_t}{y_t})$ with $f'(\frac{h_t}{y_t}) < 0$ and $f_0 = 0$.

The feasibility constraint of the economy is

$$c_t + h_t + x_t = y_t$$

1. Write the problem of utilitatian planner that cares equally of these two types of consumers. Characterize his optimal investment in capital and health in the steady state of the economy.

2. Write the problem of each type of household in this economy and characterize their investments in capital and health in the steady state of the competitive equilibrium.

3. Compare the health investment to output ratio of the planner to that of the competitive equilibrium. Explain how and if they differ and what is the economic reason for this result.

4. One can interpret the investment in health as investment in preventive care, or drug innovations that mitigate the effect of pesticides. Assume that as an alternative to these investment the government proposes banning the use of pesticides with an output cost modeled through $A' < A$. Characterize steady state investment in capital and health in the planner's problem. How do they compare to the your answers in the first question.

5. Define the value of life as $u(c)/u'(c)$ (the value of the utils yield by $c$ in consumption units). Go as far as you can characterizing it as a function of parameters in the economy. How do the incentives for health incentives vary with the value of life in your economy?

# Saving policies: risk and non-homotheticities

Consider an economy populated by a continuum of infinitely lived farmers with standard log preferences $u(c) = \log(c)$ over consumption $c$ and discount factor $\beta \in (0, 1)$. a household is born with an endowment for consumption $x_o$ that does not depreciate nor can it be accumulated.

1. Describe the Bellman equation associated to the problem of a farmer.

2. Solve for the value of the farmer through value function iteration assuming a discount factor $\beta = 0.98$. Plot the value to the farmer across different initial endowments.

3. Now assume that the farmer looses the entire cattle with probability $\delta = 0.5$ in each period. Rewrite the Bellman equation associated to the farmer and solve for its value. Plot the value of the farmer in this problem relative to the benchmark case. Explain why and how the optimal consumption decision's shift.

4. Now assume that farmers have non-homothetic preferences with some minimum desired consumption $\bar{c} = x_0/100$ (and eliminate the risk of loosing cattle). Rewrite the Bellman equation associated to the farmer and solve for its value. Plot the value of the farmer in this problem relative to the benchmark case. Explain why and how the optimal consumption decision's shift.

*Remark:* these are all versions of the cake eating problem.
https://python.quantecon.org/cake_eating_numerical.html