Macroeconomics II ECON 6140 (Second Half)

Lecture 8
Sticky wages in the New Keynesian Model

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The plan

Introduce sticky wages into NK model

- “Love of diversity” in the production function
- Calvo-type stickiness in nominal wages

Sticky wages: The evidence

Wages are quite sticky:

- Probability of wage change is between 5% and 18% per quarter
- Probability of wage change is lower for salaried workers than for hourly workers
- Little heterogeneity across sectors
- Probability of a wage change increases with unemployment
- Wages are less likely to change downwards than upwards, even when controlling for inflation

See for instance Baratterieri, Basu and Gottschalk (2010) for more details.

A Model with Sticky Wages and Prices

Firms

Production function

$$Y_t(i) = A_t N_t(i)^{1-\alpha}$$

where $N_t(i) \equiv \left(\int_0^1 N_t(i,j)^{1-\frac{1}{\epsilon_w}} dj \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}$ and $a_t \equiv \log A_t \sim AR(1)$

Cost minimizing labor demand

$$N_t(i,j) = \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} N_t(i)$$

where $W_t \equiv \left(\int_0^1 W_t(j)^{1-\epsilon_w} dj \right)^{\frac{1}{1-\epsilon_w}}$

so that W_t is the ideal wage index implying

$$\int_0^1 W_t(j) N_t(i,j) dj = W_t N_t(i)$$

Price setting

Firm's problem

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta_p^k E_t \left\{ \Lambda_{t,t+k} (1/P_{t+k}) \left(P_t^* Y_{t+k|t} - \mathcal{C}_{t+k}(Y_{t+k|t}) \right) \right\}$$

subject to

$$\mathcal{C}_{t+k}(Y_{t+k|t}) = W_{t+k} \left(Y_{t+k|t}/A_{t+k} \right)^{\frac{1}{1-\alpha}}$$

$$Y_{t+k|t} = \left(P_t^*/P_{t+k} \right)^{-\epsilon_p} C_{t+k}$$

Implied price setting rule (log-linearized):

$$p_t^* = \mu^p + (1 - \beta\theta_p) \sum_{k=0}^{\infty} (\beta\theta_p)^k E_t\{\psi_{t+k|t}\}$$

where $\psi_{t+k|t} \equiv \log \Psi_{t+k|t}$ and $\mu^p \equiv \log \frac{\epsilon_p}{\epsilon_p - 1}$

So, nothing changed apart from notation!

Goods price Phillips Curve

Price inflation equation

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} - \lambda_p(\mu_t^p - \mu^p)$$

where $\lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}$

and $\mu_t^p - \mu^p = -(\sigma+\varphi)\widetilde{y}_t - (\mu_t^w - \mu^w)$.

Households

Households maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, \{\mathcal{N}_t(j)\}; Z_t)$$

subject to

$$\int_0^1 P_t(i) C_t(i) di + Q_t B_t \leq B_{t-1} + \int_0^1 W_t(j) \mathcal{N}_t(j) dj + D_t$$

where

$$U(C_t, \{\mathcal{N}_t(j)\}; Z_t) = \begin{cases} \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \int_0^1 \frac{\mathcal{N}_t(j)^{1+\varphi}}{1+\varphi} dj \right) Z_t & \text{for } \sigma \neq 1 \\ \left(\log C_t - \int_0^1 \frac{N_t(j)^{1+\varphi}}{1+\varphi} dj \right) Z_t & \text{for } \sigma = 1 \end{cases}$$

and $C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon_p}} di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$ and $z_t \equiv \log Z_t \sim AR(1)$.

Households, cont'd

Optimality conditions

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_p} C_t$$

$$Q_t = \beta E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{Z_{t+1}}{Z_t}\right)\left(\frac{P_t}{P_{t+1}}\right)\right\}$$

or, in log-linearized form:

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi^p_{t+1}\} - \rho) + \frac{1}{\sigma}(1-\rho_z)z_t$$

But no optimal labor supply condition....

Optimal wage setting

$$\max_{W_t^*} E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(C_{t+k}^{-\sigma} \frac{W_t^*}{P_{t+k}} N_{t+k|t} - \frac{N_{t+k|t}^{1+\varphi}}{1+\varphi} \right) Z_{t+k}$$

subject to

$$N_{t+k|t} = \left(\frac{W_t^*}{W_{t+k}} \right)^{-\epsilon_w} \left(\int_0^1 N_{t+k}(i) di \right)$$

First order condition

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ N_{t+k|t} Z_{t+k} C_{t+k}^{-\sigma} \left(\frac{W_t^*}{P_{t+k}} - \mathcal{M}_w MRS_{t+k|t} \right) \right\} = 0$$

where $MRS_{t+k|t} \equiv C_{t+k}^{\sigma} N_{t+k|t}^{\varphi}$ and $\mathcal{M}_w \equiv \frac{\epsilon_w}{\epsilon_w - 1}$.

Wage setting, cont'd

Log-linearized version:

$$w_t^* = \mu^w + (1-\beta\theta_w)\sum_{k=0}^{\infty}(\beta\theta_w)^k E_t\{mrs_{t+k|t} + p_{t+k}\}$$

or equivalently

$$w_t^* = \frac{1-\beta\theta_w}{1+\epsilon_w\varphi}\sum_{k=0}^{\infty}(\beta\theta_w)^k \; E_t\left\{(1+\epsilon_w\varphi)w_{t+k} - \widehat{\mu}_{t+k}^w\right\}$$

where $\mu_t^w \equiv (w_t - p_t) - mrs_t$ and $mrs_t = \sigma c_t + \varphi n_t$.

Wage inflation equation

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} - \lambda_w(\mu_t^w - \mu^w)$$

where $\lambda_w \equiv \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w\ (1+\varphi\epsilon_w)}$

$\Rightarrow$ Wage stickiness is introduced in a way that is completely analogous to price stickiness.

Equilibrium

Equilibrium

Goods market clearing

$$Y_t(i) = C_t(i) \text{ all } i \in [0,1] \Rightarrow Y_t = C_t$$

where $Y_t \equiv \left(\int_0^1 Y_t(i)^{1-\frac{1}{\epsilon_p}} di\right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$.

Aggregate employment

$$\begin{aligned} N_t &\equiv \int_0^1 \int_0^1 N_t(i,j) dj di = \int_0^1 N_t(i) \int_0^1 \frac{N_t(i,j)}{N_t(i)} dj di = \Delta_{w,t} \int_0^1 N_t(i) di \\ &= \Delta_{w,t} \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{Y_t(i)}{Y_t}\right)^{\frac{1}{1-\alpha}} di = \Delta_{w,t} \Delta_{p,t} \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \end{aligned}$$

where $\Delta_{w,t} \equiv \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} dj$ and $\Delta_{p,t} \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon_p}{1-\alpha}} di$.

$\Rightarrow$ Up to a first order approximation

$$(1-\alpha) n_t = y_t - a_t.$$

Wage gap

Define the new and useful variable the **wage gap** $\widetilde{\omega}_t$

$$\widetilde{\omega}_t \equiv \omega_t - \omega_t^n$$

where $\omega_t \equiv w_t - p_t$ and where ω_t^n is the *natural real wage*:

$$\begin{aligned} \omega_t^n &= \log(1-\alpha) + (a_t - \alpha n_t^n) - \mu^p \\ &= \log(1-\alpha) + \psi_{wa} a_t - \mu^p \end{aligned}$$

where $\psi_{wa} \equiv \frac{1-\alpha\psi_{ya}}{1-\alpha} > 0$ and $\psi_{ya} \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$.

Price markup gap

Define the price mark-up gap $\widehat{\mu}_t^p$ as

$$\begin{aligned} \widehat{\mu}_t^p &= (mpn_t - \omega_t) - \mu^p \\ &= -\frac{\alpha}{1-\alpha}\widetilde{y}_t - \widetilde{\omega}_t \end{aligned}$$

Hence

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} + \varkappa_p \widetilde{y}_t + \lambda_p \widetilde{\omega}_t$$

where $\varkappa_p \equiv \frac{\alpha\lambda_p}{1-\alpha}$.

Wage markup gap

Define the wage mark-up gap $\widehat{\mu}_t^w$ as

$$\begin{aligned}\widehat{\mu}_t^w &= \omega_t - mrs_t - \mu^w \\ &= \widetilde{\omega}_t - (\sigma \widetilde{y}_t + \varphi \widetilde{n}_t) \\ &= \widetilde{\omega}_t - \left(\sigma + \frac{\varphi}{1-\alpha}\right)\widetilde{y}_t\end{aligned}$$

so that

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} + \varkappa_w \widetilde{y}_t - \lambda_w \widetilde{\omega}_t$$

where $\varkappa_w \equiv \lambda_w \left(\sigma + \frac{\varphi}{1-\alpha}\right)$.

In addition we also have that

$$\widetilde{\omega}_t \equiv \widetilde{\omega}_{t-1} + \pi_t^w - \pi_t^p - \Delta\omega_t^n$$

Closing the model

We should not forget about the **IS equation**.....

$$\widetilde{y}_t = -\frac{1}{\sigma}(i_t - E_t\{\pi^p_{t+1}\} - r^n_t) + E_t\{\widetilde{y}_{t+1}\}$$

where $r^n_t \equiv \rho - \sigma(1-\rho_a)\psi_{ya}a_t + (1-\rho_z)z_t$

and we need to specify an **interest rate rule**

$$i_t = \rho + \phi_p \pi^p_t + \phi_w \pi^w_t + \phi_y \widehat{y}_t + v_t$$

Dynamical system

$$\mathbf{A}_0^w \mathbf{x}_t = \mathbf{A}_\mathbf{1}^w E_t\{\mathbf{x}_{t+1}\} + \mathbf{B}_0^w \mathbf{u}_t$$

where $\mathbf{x}_t \equiv [\widetilde{y}_t, \pi_t^p, \pi_t^w, \widetilde{\omega}_{t-1}]'$, $\mathbf{u}_t \equiv [\widehat{r}_t^n - v_t - \phi_y \widehat{y}_t^n, \Delta\omega_t^n]'$,

$$\mathbf{A}_0^w \equiv \begin{bmatrix} \sigma + \phi_y & \phi_p & \phi_w & 0 \\ -\varkappa_p & 1 & 0 & 0 \\ -\varkappa_w & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix}$$

$$\mathbf{A}_\mathbf{1}^w \equiv \begin{bmatrix} \sigma & 1 & 0 & 0 \\ 0 & \beta & 0 & \lambda_p \\ 0 & 0 & \beta & -\lambda_w \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ; \quad \mathbf{B}_0^w \equiv \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Conditions for uniqueness of the equilibrium

$$\phi_p + \phi_w + \phi_y \left(\frac{1-\beta}{\sigma + \frac{\alpha+\varphi}{1-\alpha}} \right) \left(\frac{1}{\lambda_p} + \frac{1}{\lambda_w} \right) > 1$$

Dynamic responses to shocks

Example: Dynamic Responses to a Monetary Policy Shock

Calibration **Interest rate rule**: $\phi_p = 1.5$, $\phi_y = \phi_w = 0$, $\rho_v = 0.5$

Labor demand elasticity: $\epsilon_w = 4.5$

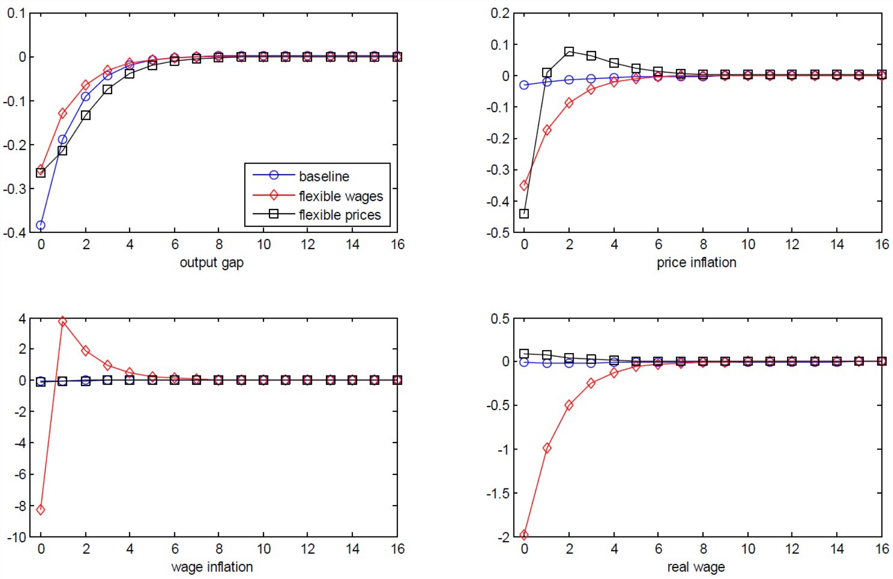
Three variations on stickiness:

Baseline: $\theta_p = 3/4$, $\theta_w = 3/4$

Flexible wages: $\theta_p = 3/4$, $\theta_w = 0$

Flexible price: $\theta_p = 0$, $\theta_w = 3/4$

Dynamic Responses to a Monetary Policy Shock



Monetary Policy Design: The Social Planner's Problem

$$\max U(C_t, \{\mathcal{N}_t(j)\}; Z_t)$$

subject to

$$C_t(i) = A_t N_t(i)^{1-\alpha}$$

$$\mathcal{N}_t(j) = \int_0^1 N_t(i,j) di$$

where $C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon_p}} di\right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$ and $N_t(i) \equiv \left(\int_0^1 N_t(i,j)^{1-\frac{1}{\epsilon_w}} dj\right)^{\frac{\epsilon_w}{\epsilon_w - 1}}$.

Optimality conditions

$$C_t(i) = C_t, \text{ all } i \in [0,1]$$

$$N_t(i,j) = \mathcal{N}_t(j) = N_t(i) = N_t, \text{ all } i, j \in [0,1]$$

$$-\frac{U_{n,t}}{U_{c,t}} = MPN_t$$

where $MPN_t = (1-\alpha) A_t N_t^{-\alpha}$.

Efficiency of the Natural Equilibrium

In the decentralized economy with flexible prices and wages:

$$P_t = \mathcal{M}_p \frac{(1-\tau)W_t}{MPN_t}$$

$$\frac{W_t}{P_t} = -\frac{U_{n,t}}{U_{c,t}} \mathcal{M}_w$$

for all goods and occupations, where $\mathcal{M}_p \equiv \frac{\epsilon_p}{\epsilon_p - 1}$ and $\mathcal{M}_w \equiv \frac{\epsilon_w}{\epsilon_w - 1}$.
Thus,

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{1}{\mathcal{M}(1-\tau)} MPN_t$$

where $\mathcal{M} \equiv \mathcal{M}_p \mathcal{M}_w$.

Condition for efficiency of the natural equilibrium: $\mathcal{M}(1-\tau) = 1$

Note: natural equilibrium generally not attainable with sticky prices and wages

Optimal Monetary Policy Problem

$$\min \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(\left(\sigma + \frac{\varphi + \alpha}{1-\alpha} \right) \widetilde{y}_t^2 + \frac{\epsilon_p}{\lambda_p} (\pi_t^p)^2 + \frac{\epsilon_w (1-\alpha)}{\lambda_w} (\pi_t^w)^2 \right)$$

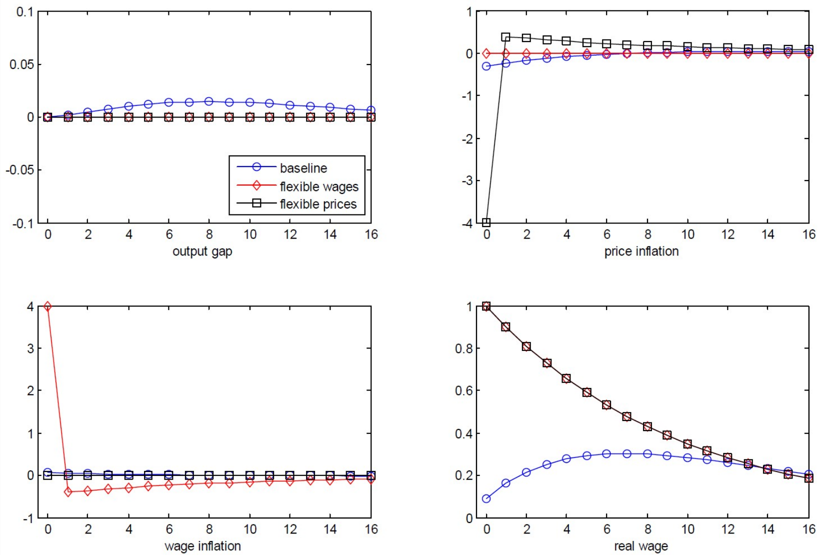
subject to:

$$\pi_t^p = \beta E_t \{\pi_{t+1}^p\} + \varkappa_p \widetilde{y}_t + \lambda_p \widetilde{\omega}_t$$

$$\pi_t^w = \beta E_t \{\pi_{t+1}^w\} + \varkappa_w \widetilde{y}_t - \lambda_w \widetilde{\omega}_t$$

$$\widetilde{\omega}_{t-1} \equiv \widetilde{\omega}_t - \pi_t^w + \pi_t^p + \Delta \omega_t^n$$

Responses to technology shock with optimal policy



Evaluation of Simple Interest Rate Rules

Consider two types of rules:

- Strict Targeting Rules: $\pi_t^i = 0$
- Flexible Targeting Rules: $i_t = 0.01 + 1.5\pi_t^i$

With a flexible rule, interest rates can respond to either price or wage inflation, or a composite measure.

Losses under different rules

Table 6.1 Evaluation of Simple Rules

	Optimal	*Strict Targeting*			*Flexible Targeting*		
		Price	Wage	Composite	Price	Wage	Composite
Technology shocks							
$\sigma(\pi^p)$	0.11	0	0.13	0.12	0.29	0.24	0.24
$\sigma(\pi^w)$	0.03	0.26	0	0.02	0.23	0.16	0.16
$\sigma(\widetilde{y})$	0.04	3.38	0.20	0	0.84	1.18	1.11
$\mathbb{L}$	0.0330	0.78	0.039	0.0337	0.47	0.305	0.307
Demand shocks							
$\sigma(\pi^p)$	0	0	0	0	0.02	0.04	0.03
$\sigma(\pi^w)$	0	0	0	0	0.05	0.06	0.06
$\sigma(\widetilde{y})$	0	0	0	0	1.08	1.05	1.06
$\mathbb{L}$	0	0	0	0	0.061	0.067	0.066

Summing up

- Wage stickiness can be introduced analogously to price stickiness
- First-best, i.e. efficient level of output not attainable with both price and wage stickiness even when the only disturbance are productivity shocks
- Composite inflation targeting is doing a pretty good job