

Material / Practice Questions

We'll cover three problems from Homework 1 today:

- Let X be a finite set and let (\succsim, \succ) be a pair where \succsim is a (rational) preference relation and \succ is a transitive subrelation of \succsim . Assume that the pair satisfies extended transitivity – if $x \succsim y$ and $y \succsim z$, or if $x \succsim y$ and $y \succ z$, then $x \succsim z$. We say that a pair (\succsim, \succ) is represented by a function u if:

$$\begin{aligned} u(x) = u(y) &\iff x \sim y \\ u(x) - u(y) > 0 &\iff x \succ y \\ u(x) - u(y) > 1 &\iff x \succsim y \end{aligned}$$

Show that every extended preference relation (\succsim, \succ) can be represented by a utility function u .

- A consumer's preferences are defined on bundles $x = (x_1, x_2)$ in the consumption set $X = \mathbb{R}_+^2$. Their choices are based on the 'value' of a bundle, given by the function $v(x) = x_1 + x_2$. Their preferences, \succsim , are structured by two psychological rules that give rise to the following formal definitions, for any $x, y \in X$:

$$\begin{aligned} x \succ y &\iff (v(x) \geq 10, v(y) < 10) \text{ or } (v(x), v(y) < 10 \text{ and } v(x) > v(y) + 1) \\ x \sim y &\iff (v(x) \geq 10, v(y) \geq 10) \text{ or } (v(x), v(y) < 10 \text{ and } |v(x) - v(y)| \leq 1) \end{aligned}$$

Additionally, \succsim holds if either \succ or \sim hold, as always.

- Are these preferences rational?
 - Sketch the indifference map. What are the indifference sets for $A = (3, 3)$ and $B = (6, 6)$?
 - Can these preferences be represented by a real-valued utility function? Explain why or why not.
- A consumer's preferences are defined by a *bliss point* $\bar{x} \in \mathbb{R}_{++}^2$. Their preferences over bundles $x \in \mathbb{R}_+^2$ are defined by \succsim , where they prefer bundles closer to the bliss point. Formally, for $x, y \in X$:

$$x \succsim y \iff \|x - \bar{x}\| \leq \|y - \bar{x}\|$$

- a. Show that this preference relation is rational. Is it also continuous?
- b. Propose a simple, continuous utility function that represents these preferences. Is the preference relation convex? Is it strictly convex?
- c. Find the Marshallian demand correspondence for a consumer with these preferences. Your answer will depend on whether the bliss point is affordable.