Problem Set on Zero-Sum Games

1. A two person zero-sum game is described by a single matrix M; m_{ij} is the payoff to the row player of choosing strategy i when the column player chooses strategy j. Since the game is zero-sum, the payoff to the column player of the i, j pair is $-m_{ij}$.
   (a) Suppose row chooses mixed strategy p. What is the vector of payoffs against each pure strategy j that column could choose.
   (b) Suppose that column's goal is to give row as little utility as possible. Row's *security level* is the maximal amount of utility that row can guarantee himself by a suitable strategy choice no matter what column does after seeing row's strategy choice. Formulate the problem of finding row's security level as a linear program. The solution should give you both a value z^* and a strategy p^*.
   (c) Write down and interpret the dual of your linear program. The dual should give you both a value w^* and a strategy q^*.
   (d) The fundamental theorem of two-person zero sum games, due to von Neumann in 1927, is that finite games have a value, that is, a (p^*, q^*) pair such that p maximizes row's security value and q maximizes column's security value. Show that (p^*, q^*) is such a pair.
   (e) Suppose we think of the (p^*, q^*) pair as a solution concept for this class of games. How is this solution related to Nash equilibrium?
   (f) Describe some properties of the solution set to a given game. Compare to Nash equilibrium in other kinds of finite games.