Health inequality and mitigation tools

Consider an economy populated with a continuum of households of measure 1. The population faces a mortality rate indexed by the expected lifetime of the household $1/(1-\pi)$. Half of the population lives in areas exposed to pesticides and faces a finite expected life $\pi < 1$, and the other half solves an infinite horizon problem, $\pi = 1$. We model the mortality rate as a higher discount factor on future consumption and normalize the utility of death to zero.

$$U(c, \pi) = \sum_{t=0}^{\infty} (\beta\pi)^t u(c_t)$$

for a discount factor $\beta \in (0, 1)$.

There are three technologies available in the economy: one for capital accumulation, one to change the mortality rate in the economy (e.g., the health sector), and one for the production of consumption goods and health investment, h, as follows:

$$k_{t+1} = x_t + (1-\delta)k_t \quad \text{with } k_0 > 0$$

$$y_t = Ak_t^{\alpha} \quad \text{with } A > 0, \quad \text{and } \alpha \in (0, 1)$$

Health investments map into lower mortality through a technology:

$$\pi_t = f\left(\frac{h_t}{y_t}\right) \quad \text{with } f'\left(\frac{h_t}{y_t}\right) > 0, \quad \text{and } f(0) = 0.$$

The feasibility constraint of the economy is:

$$c_t + h_t + x_t = y_t$$

1. Write the problem of a utilitarian planner that cares equally about these two types of consumers. Characterize the optimal investment in capital and health in the steady state of the economy.

Social Planner's Problem: The planner maximizes social welfare as

$$
\begin{aligned}
\max_{\{c_t^1, c_t^2, h_t, k_{t+1}, x_t, \pi_t\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} \beta^t \Big[(\pi_t)^t u(c_t^1) + u(c_t^2) \Big] \\
\text{s.t.} \quad & c_t^1 + c_t^2 + h_t + x_t = A k_t^{\alpha}, \quad \forall t, \\
& k_{t+1} = x_t + (1-\delta) k_t, \quad \forall t, \\
y \equiv A k^{\alpha}, & \\
& \pi_t = f\left(\frac{h_t}{y_t}\right), \quad \forall t, \\
& \pi_t \in [0,1], \quad \forall t, \\
& (c_t^1, c_t^2, k_{t+1}, x_t, h_t) \geq (0,0,0,0,0), \quad \forall t, \\
& k_0 > 0, \text{ given},
\end{aligned}
$$

where superscripts $\{1,2\}$ denote household types.

We impose our usual assumptions (concavity and monotonicity of preferences, the Inada conditions, and no free lunch) and focus on the interior solution.

The Lagrangian is:

$$
\begin{aligned}
\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Big(& (\pi_t)^t u(c_t^1) + u(c_t^2) \\
& + \lambda_t \Big[A k_t^{\alpha} - c_t^1 - c_t^2 - h_t - x_t \Big] + \\
& + \mu_t \Big[x_t + (1-\delta) k_t - k_{t+1} \Big] + \\
& + \theta_t \Big[f\Big(\frac{h_t}{A k_t^{\alpha}}\Big) - \pi_t \Big] + \\
& + \gamma_t^1 \Big[1 - \pi_t \Big] + \\
& + \gamma_t^2 \pi_t \Big).
\end{aligned}
$$

FOCs:

$$
\frac{\partial \mathcal{L}}{\partial c_t^1}: \quad (\pi_t)^t u'(c_t^1) - \lambda_t = 0 \implies (\pi_t)^t u'(c_t^1) = \lambda_t.
$$

$$
\frac{\partial \mathcal{L}}{\partial c_t^2}: \quad u'(c_t^2) - \lambda_t = 0 \implies u'(c_t^2) = \lambda_t.
$$

$$\implies (\pi_t)^t u'(c_t^1) = u'(c_t^2) = \lambda_t$$

$$\frac{\partial \mathcal{L}}{\partial x_t}: \quad -\lambda_t + \mu_t = 0 \implies \mu_t = \lambda_t.$$

$$\frac{\partial \mathcal{L}}{\partial h_t}: \quad -\lambda_t + \theta_t f'\Big(\frac{h_t}{Ak_t^\alpha}\Big) \cdot \frac{1}{Ak_t^\alpha} = 0 \implies \lambda_t = \theta_t \frac{1}{Ak_t^\alpha} f'\Big(\frac{h_t}{Ak_t^\alpha}\Big).$$

$$\implies \theta_t = u'(c_t^2)\frac{Ak_t^\alpha}{f'\Big(\frac{h_t}{Ak_t^\alpha}\Big)} = \pi^t u'(c_t^1)\frac{Ak_t^\alpha}{f'\Big(\frac{h_t}{Ak_t^\alpha}\Big)}$$

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}}: \quad -\mu_t + \beta\mu_{t+1}(1-\delta) + \beta\lambda_{t+1}\, A\alpha k_{t+1}^{\alpha-1} + \beta\theta_{t+1}\, f'\Big(\frac{h_{t+1}}{Ak_{t+1}^\alpha}\Big)\frac{\alpha h_{t+1}}{Ak_{t+1}^{\alpha+1}} = 0.$$

$$\frac{\partial \mathcal{L}}{\partial \pi_t}: \quad t(\pi_t)^{t-1}u(c_t^1) - \theta_t - \gamma_t^1 + \gamma_t^2 = 0.$$

From the FOCs with respect to k_{t+1} and h_t, we obtain the following condition:

$$-u'(c_t^2) + \beta u'(c_{t+1}^2)(1-\delta) + \beta u'(c_{t+1}^2)\, A\alpha k_{t+1}^{\alpha-1} + \beta u'(c_{t+1}^2)\frac{Ak_{t+1}^\alpha}{f'\Big(\frac{h_{t+1}}{Ak_{t+1}^\alpha}\Big)} f'\Big(\frac{h_{t+1}}{Ak_{t+1}^\alpha}\Big)\frac{\alpha h_{t+1}}{Ak_{t+1}^{\alpha+1}} = 0$$

$$\implies -u'(c_t^2) + \beta u'(c_{t+1}^2)(1-\delta) + \beta u'(c_{t+1}^2)\, A\alpha k_{t+1}^{\alpha-1} + \beta u'(c_{t+1}^2)\frac{\alpha h_{t+1}}{k_{t+1}} = 0$$

Steady State: In a steady state we have $c_t^1 = c^1$, $c_t^2 = c^2$, $h_t = h$, $k_t = k$ for all t. The resource constraint becomes

$$c^1 + c^2 + h + x = Ak^\alpha,$$

and since

$$k = x + (1-\delta)k \implies x = \delta k,$$

it follows that

$$c^1 + c^2 = Ak^\alpha - \delta k - h.$$

From FOCs for consumption:

$$\pi^t u'(c^t) = u'(c^2),$$

which holds only if $\pi_t = \pi = 1$. This implies

$$c^1 = c^2 = c$$

We also know

$$\pi = f\Big(\frac{h}{Ak^\alpha}\Big) \implies f\Big(\frac{h}{Ak^\alpha}\Big) = 1 \implies h = f^{-1}(1)Ak^\alpha,$$

assuming that $f^{-1}(1)$ is well-defined.

The Euler equation:

$$-u'(c^2) + \beta u'(c^2)(1-\delta) + \beta u'(c^2)\, A\alpha k^{\alpha-1} + \beta u'(c^2)\frac{\alpha h}{k} = 0$$

$$\implies A\alpha k^{\alpha-1} + \frac{\alpha h}{k} = \frac{1}{\beta} - 1 + \delta$$

Substituting our result for h:

$$A\alpha k^{\alpha-1} + \alpha f^{-1}(1)Ak^{\alpha-1} = \frac{1}{\beta} - 1 + \delta$$

$$\implies k^{\alpha-1} = \frac{\frac{1}{\beta} - 1 + \delta}{A\alpha(1 + f^{-1}(1))}$$

$$k = \Big[\frac{A\alpha(1 + f^{-1}(1))}{\frac{1}{\beta} - 1 + \delta}\Big]^{\frac{1}{1-\alpha}}$$

An allocation $(c^{1*}, c^{2*}, x^*, h^*, k^*, \pi^*)$ that satisfies

$$\begin{cases} c^1 = c^2 = \frac{Ak^\alpha - \delta k - h}{2}, \\ x = \delta k, \\ k = \Big[\frac{A\alpha(1+f^{-1}(1))}{\frac{1}{\beta}-1+\delta}\Big]^{\frac{1}{1-\alpha}}, \\ h = f^{-1}(1)Ak^\alpha, \\ \pi = 1, \end{cases}$$

is a steady-state equilibrium allocation.

Remark. Recall our FOC with respect to π_t:

$$\frac{\partial \mathcal{L}}{\partial \pi_t}: \quad t(\pi_t)^{t-1}u(c_t^1) - \theta_t - \gamma_t^1 + \gamma_t^2 = 0.$$

Due to the concavity and monotonicity of preferences, setting $\pi_t = 0$ is never optimal for the social planner. By complementary slackness, we then have $\gamma_t^2 = 0$. We also know that θ and $u(c^1)$ are constant in the steady state and $\pi^* = 1$. Hence, the equation simplifies to

$$tu(c^1) - \theta = \gamma_t^1.$$

As $t \to \infty$, we observe that γ_t^1 grows unbounded, implying that this constraint becomes increasingly binding unless we normalize the Lagrange multiplier. One way to normalize is

$$\tilde{\gamma}_t^1 \equiv \frac{\gamma_t^1}{t}.$$

Then, in the steady state, $\tilde{\gamma}_t^1 = \tilde{\gamma}^1$ is constant.

2. Write the problem of each type of household in this economy and characterize their investments in capital and health in the steady state of the competitive equilibrium.

To simplify notation, normalize all prices so that the price of the consumption good is set to 1.

Households of the first type (who are exposed to health risk):

$$\begin{aligned}
\max_{\{c_t^1, h_t^1, k_{t+1}^1, x_t^1, h_t\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} \beta^t f(\frac{h_t}{y_t})^t u(c_t^1), \\
\text{s.t.} \quad & c_t^1 + p_t^h h_t^1 + p_t^x x_t^1 \leq r_t k_t^1 + 1/2 w_t, \\
& k_{t+1}^1 = x_t^1 + (1-\delta) k_t^1, \\
& (c_t, k_{t+1}, x_t, h_t) \geq (0, 0, 0, 0), \\
& k_0^1 > 0, \text{ given.}
\end{aligned}$$

Here I assumed that only this type invests in health as if they have complete information that another type is not exposed to risk.

The Lagrangian is given by:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Big(f(\frac{h_t}{y_t})^t u(c_t^1) + \lambda_t \Big[r_t k_t^1 + 1/2 w_t - c_t^1 - p_t^h h_t^1 - p_t^x x_t^1 \Big] + \mu_t \Big[x_t^1 + (1-\delta) k_t^1 - k_{t+1}^1 \Big] + \gamma_t^1 \Big[1 - f(\frac{h_t}{y_t}) \Big] + \gamma_t^2 f(\frac{h_t}{y_t}) \Big).$$

FOCs:

$$\frac{\partial \mathcal{L}}{\partial c_t^1} : \quad f(\frac{h_t}{y_t})^t u'(c_t^1) = \lambda_t,$$

$$\frac{\partial \mathcal{L}}{\partial x_t^1} : \quad p_t^x \lambda_t = \mu_t,$$

$$\frac{\partial \mathcal{L}}{\partial k^1_{t+1}}: \quad -\mu_t + \beta\mu_{t+1}(1-\delta) + \beta\lambda_{t+1}\, r_{t+1} = 0,$$

$$\frac{\partial \mathcal{L}}{\partial h_t}: \quad \lambda_t p^h_t = u(c^1_t) t f(\frac{h_t}{y_t})^{t-1} f'(\frac{h_t}{y_t})\frac{1}{y_t} - \gamma^1_t f'(\frac{h_t}{y_t})\frac{1}{y_t} + \gamma^2_t f'(\frac{h_t}{y_t})\frac{1}{y_t}$$

$$\Longrightarrow \; f(\frac{h_t}{y_t})^t u'(c^1_t) p^h_t = u(c^1_t) t f(\frac{h_t}{y_t})^{t-1} f'(\frac{h_t}{y_t})\frac{1}{y_t} - \gamma^1_t f'(\frac{h_t}{y_t})\frac{1}{y_t} + \gamma^2_t f'(\frac{h_t}{y_t})\frac{1}{y_t}.$$

Again, by complementary slackness and since it is never optimal for the household to choose zero probability of survival, we set $\gamma^2_t = 0$. We can also normalize $\tilde{\gamma}_1$ analogously to Part 1.

Households of the second type (who do not face health risk):

$$\begin{aligned}
\max_{\{c^2_t, k^2_{t+1}, x^2_t\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} \beta^t\, u(c^2_t) \\
\text{s.t.} \quad & c^2_t + p^x_t x^2_t \le r_t k^2_t + 1/2 w_t, \\
& k^2_{t+1} = x^2_t + (1-\delta)k^2_t, \\
& (c^2_t, k^2_{t+1}, x^2_t) \ge (0,0,0), \\
& k^2_0 > 0, \text{ given.}
\end{aligned}$$

The Lagrangian is:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Big(u(c^2_t) + \lambda_t \Big[r_t k^2_t + 1/2 w_t - c^2_t - p^x_t x^2_t \Big] + \mu_t \Big[x^2_t + (1-\delta)k^2_t - k^2_{t+1} \Big] \Big).$$

FOCs:

$$\frac{\partial \mathcal{L}}{\partial c^2_t}: \quad u'(c^2_t) = \lambda_t,$$

$$\frac{\partial \mathcal{L}}{\partial x^2_t}: \quad p^x_t \lambda_t = \mu_t,$$

$$\frac{\partial \mathcal{L}}{\partial k^2_{t+1}}: \quad \mu_t = \beta\lambda_{t+1} r_{t+1} + \beta\mu_{t+1}(1-\delta).$$

Firm's profit maximization problem gives us:

$$r_t = \frac{\partial y_t}{\partial k_t} = \alpha A (k^1_t + k^2_t)^{\alpha-1},$$

$$w_t = A(k_t^1 + k_t^2)^\alpha - r_t(k_t^1 + k_t^2) = (1-\alpha)(k_t^1 + k_t^2)^\alpha.$$

At equilibrium, since consumption and both investment goods are produced with the same technology and are perfectly substitutable from the firm's perspective, in the interior case, their prices must be equal: $p_t^x = p_t^h = 1$, where 1 is the normalized price of the consumption good.

Steady state: Euler equations for both types of households:

$$1 = \beta f(\frac{h}{y})(\alpha A(k^1 + k^2)^{\alpha-1} + 1 - \delta),$$

$$1 = \beta(\alpha A(k^1 + k^2)^{\alpha-1} + 1 - \delta),$$

which only holds if $f(\frac{h}{y}) = 1$.

Hence,

$$h = f^{-1}(1)A(k^1 + k^2)^\alpha.$$

That is, in complete markets risk-exposed households are fully insured against the pesticide-induced risk. Technically, this result follows from the common interest rate, as the group exposed to pesticide risk must invest sufficiently in health so that their effective survival probability equals 1. They increase health investments at the expense of capital accumulation, which increases the interest rate and incentivizes risk-free agents to invest more in capital until markets clear.

Then,

$$x^1 = \delta k^1$$

$$c_t^1 = \alpha A(k^1 + k^2)^{\alpha-1}k^1 + 1/2(1-\alpha)A(k^1 + k^2)^{\alpha-1} - \delta k^1 - f^{-1}(1)A(k^1 + k^2)^\alpha$$

$$x^2 = \delta k^2$$

$$c_t^2 = \alpha A(k^1 + k^2)^{\alpha-1}k^2 + 1/2(1-\alpha)A(k^1 + k^2)^{\alpha-1} - \delta k^2$$

$$k^1 + k^2 = \Big[\frac{\alpha A}{\frac{1}{\beta} - 1 + \delta}\Big]^{\frac{1}{1-\alpha}}$$

$$h = f^{-1}(1)A(k^1 + k^2)^\alpha$$

$$u'(c^1) = (u(c^1) - \tilde{\gamma}^1)f'(f^{-1}(1))\frac{1}{A(k^1 + k^2)^\alpha}$$

$$c^1 + c^2 + x^1 + x^2 + h = A(k^1 + k^2)^\alpha$$

3. Compare the health investment to output ratio of the planner to that of the competitive equilibrium. Explain how and if they differ and what is the economic reason for this result.

From Part 1:

$$\left(\frac{h}{y}\right)^{SPP} = \frac{f^{-1}(1)Ak^{\alpha}}{Ak^{\alpha}} = f^{-1}(1).$$

From Part 2:

$$\left(\frac{h}{y}\right)^{CE} = \frac{f^{-1}(1)A(k^1+k^2)^{\alpha}}{A(k^1+k^2)^{\alpha}} = f^{-1}(1).$$

Thus, in both cases, these ratios are equal, ensuring that the probability of survival for risk-exposed households remains constant and equal to one. In the first case, this result follows from the fact that the planner cares equally about both types of households. In the second case, it is a consequence of complete risk-sharing between households of different types in a complete market setting.

4. One can interpret the investment in health as investment in preventive care, or drug innovations that mitigate the effect of pesticides. Assume that as an alternative to these investment the government proposes banning the use of pesticides with an output cost modeled through $A' < A$. Characterize steady state investment in capital and health in the planner's problem. How do they compare to the your answers in the first question.

As the planner is now banning the use of pesticides, both types of households become equivalent since no one bears risk, and no health investments are needed. However, total factor productivity is now lower.

Social Planner's Problem: The planner maximizes social welfare as

$$\begin{aligned}
\max_{\{c_t,h_t,k_{t+1},x_t,\pi_t\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty}\beta^t u(c_t) \\
\text{s.t.} \quad & c_t + x_t = A'k_t^{\alpha}, \quad \forall t, \\
& k_{t+1} = x_t + (1-\delta)k_t, \quad \forall t, \\
& (c_t, k_{t+1}, x_t) \geq (0,0,0), \quad \forall t, \\
& k_0 > 0, \text{ given,}
\end{aligned}$$

where $c_t = c_t^1 + c_t^2$ with $c_t^1 = c^2 = \frac{1}{2}c_t$, since both types of households are now identical.

This is now a standard neoclassical growth model problem. The solution to the SPP yields

$$\frac{u'(c_t')}{u'(c_{t+1}')} = \beta(\alpha A' k_{t+1}'^{\alpha-1} + 1 - \delta)$$

At steady state:

$$k' = \left[\frac{\alpha A'}{\frac{1}{\beta} - 1 + \delta}\right]^{\frac{1}{1-\alpha}}$$

$$x' = \delta k'$$

$$c' = A' k'^{\alpha} - x'$$

and $h' = 0$, as there is no longer a need for preventive health investment.

From Part 1:

$$k = \left[\frac{\alpha A(1 + f^{-1}(1))}{\frac{1}{\beta} - 1 + \delta}\right]^{\frac{1}{1-\alpha}} \text{ with } A > A'$$

Since $A > A'$ and $f^{-1}(1) > 0$, $k > k'$ and $y > y'$. That is, in the baseline scenario, capital investments are higher due to higher return to capital investments and, hence, output is higher (because k and A are larger), but part of that output must be diverted to health investment in order to insure survival.

5. Define the value of life as $u(c)/u'(c)$ (the value of the utils yield by c in consumption units). Go as far as you can characterizing it as a function of parameters in the economy. How do the incentives for health incentives vary with the value of life in your economy?

From Part 2[1]:

$$u'(c^1) = (u(c^1) + \tilde{\gamma}^1) f'(f^{-1}(1)) \frac{1}{A(k^1 + k^2)^{\alpha}}.$$

Thus, we obtain

$$V_L = \frac{u(c^1)}{u'(c^1)} = \frac{A(k^1 + k^2)^{\alpha}}{f'(f^{-1}(1))} - \frac{\tilde{\gamma}^1}{u'(c^1)} \frac{A(k^1 + k^2)^{\alpha}}{f'(f^{-1}(1))},$$

where $k^1 + k^2 = \left[\frac{\alpha A}{\frac{1}{\beta} - 1 + \delta}\right]^{\frac{1}{1-\alpha}}$, so we can rewrite the expression above as:

$$V_L = \frac{u(c^1)}{u'(c^1)} = \frac{A\left[\frac{\alpha A}{\frac{1}{\beta}-1+\delta}\right]^{\frac{\alpha}{1-\alpha}}}{f'(f^{-1}(1))} - \frac{\tilde{\gamma}^1}{u'(c^1)} \frac{A\left[\frac{\alpha A}{\frac{1}{\beta}-1+\delta}\right]^{\frac{\alpha}{1-\alpha}}}{f'(f^{-1}(1))} = \left[1 - \frac{\tilde{\gamma}^1}{u'(c^1)}\right] \frac{A\left[\frac{\alpha A}{\frac{1}{\beta}-1+\delta}\right]^{\frac{\alpha}{1-\alpha}}}{f'(f^{-1}(1))},$$

where $\frac{\tilde{\gamma}^1}{u'(c^1)}$ is positive since $\tilde{\gamma}^1 > 0$ and $u'(c^1) > 0$. Since, given general assumptions, $u(\cdot)$ and $u'(\cdot)$ are both positive, $1 - \frac{\tilde{\gamma}^1}{u'(c^1)} > 0$.

Both terms are increasing in A and β and decreasing in δ. Therefore, we conclude that the value of life is increasing in productivity and discounting factor and decreasing in depreciation.

[1]For the planner (Part 1), derivations are analogous.

Saving policies: risk and non-homotheticities

Consider an economy populated by a continuum of infinitely lived farmers with standard log preferences $u(c) = log(c)$ over consumption c and discount factor $\beta \in (0, 1)$. A household is born with an endowment for consumption x_0 that does not depreciate nor can it be accumulated.

1. Describe the Bellman equation associated to the problem of a farmer.

The original problem:

$$\begin{aligned} \max_{\{c_t\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} \beta^t ln(c) \\ \text{s.t.} \quad & 0 \leq c_t \leq x_t, \\ & x_{t+1} = x_t - c_t, \\ & x_0 > 0, \text{ given.} \end{aligned}$$

That is, each period, the farmer chooses how much to consume (c_t) and how much to save $(x_t - c_t)$, given the amount of wealth from the previous period (x_t). The Bellman equation associated with this problem is

$$V(x) = \max_{0 \leq c \leq x} ln(c) + \beta V(x - c).$$

2. Solve for the value of the farmer through value function iteration assuming a discount factor $\beta = 0.98$. Plot the value to the farmer across different initial endowments.

Remark: To plot these graphs (see Figures 1, 3 and 4), I used the normalization $V(0) = 0$. It is not necessary and if you didn't do that, it is completely fine, just your graphs would look slightly differently (for example, see Figure 2).

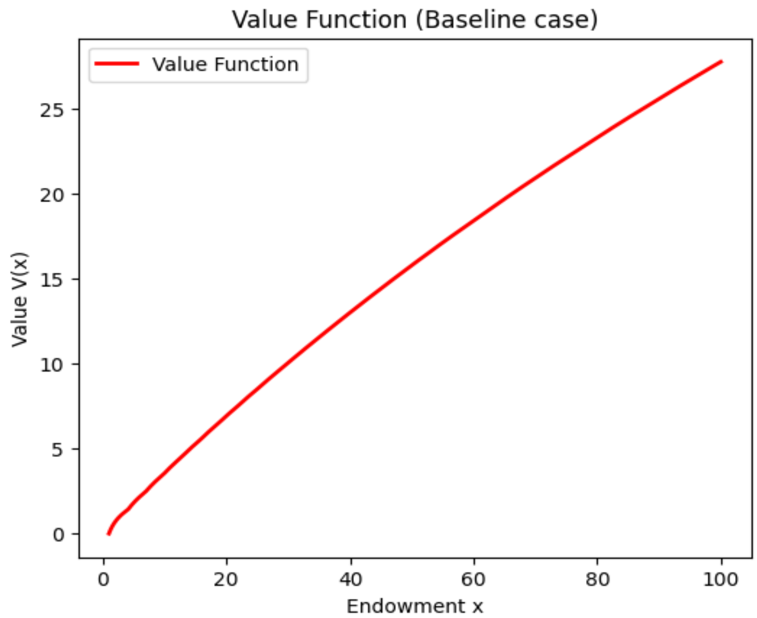

Figure 1: With normalization V(0)=0.

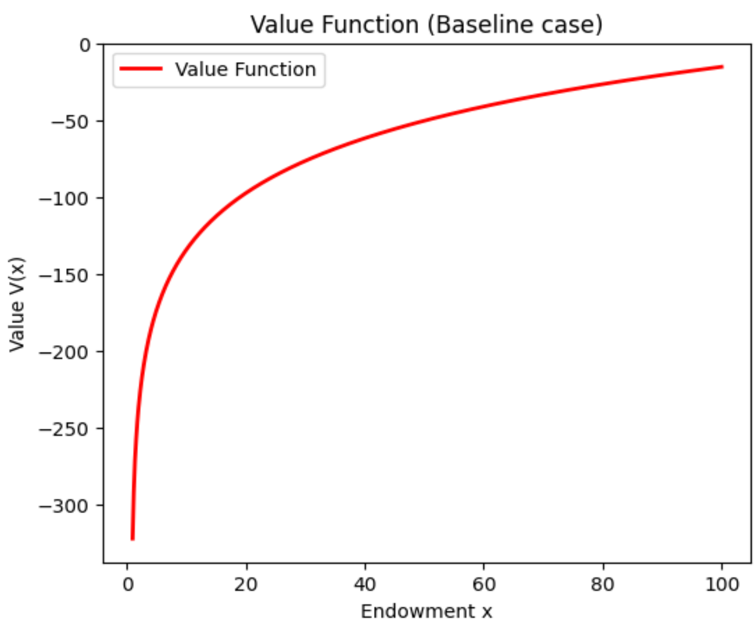

Figure 2: Without normalization.

3. Now assume that the farmer looses the entire cattle with probability $\delta = 0.5$ in each period. Rewrite the Bellman equation associated to the farmer and solve for its value. Plot the value of the farmer in this problem relative to the benchmark case. Explain why and how the optimal consumption decision's shift.

The farmer's problem in the recursive form now is:

$$V(x) = \max_{0\leq c\leq x} ln(c) + \beta\mathbb{E}[V(x-c)] = \max_{0\leq c\leq x} ln(c) + \beta[(1-\delta)V(x-c) + \delta V(0)]$$

Note that $x = 0$ is an absorbing state: if the farmer ever reaches this state, they stay there forever. In other words, if the farmer has zero cattle, there is nothing available to consume or save.

Intuition: In the baseline case, saving is "safe" so the farmer may postpone consumption to benefit from future utility and optimally allocate their consumption along infinite horizon. In this case, in each period the farmer only consumes a tiny portion of their initial endowment and saves the rest for the future. Under risk, however, the expected payoff from saving is reduced, so the farmer will tend to consume more today relative to the risk-free case. The lifetime value $V(x)$ will be lower because the risk of total loss drags it down.

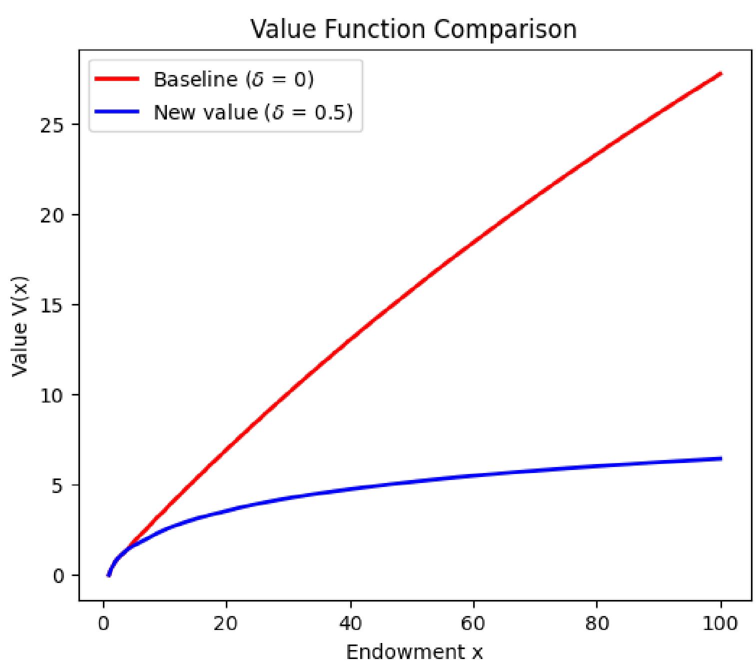

Figure 3: Comparison with and without risk.

4. Now assume that farmers have non-homothetic preferences with some minimum desired consumption $\bar{c} = x/100$ (and eliminate the risk of loosing cattle). Rewrite the Bellman equation associated to the farmer and solve for its value. Plot the value of the farmer in this problem relative to the benchmark case. Explain why and how the optimal consumption decision's shift.

The farmer's problem in the recursive form now is:

$$V(x) = \max_{\bar{c} \leq c \leq x} ln(c - \bar{c}) + \beta V(x - c) = \max_{0.01x \leq c \leq x} ln(c - \frac{x}{100}) + \beta V(x - c)$$

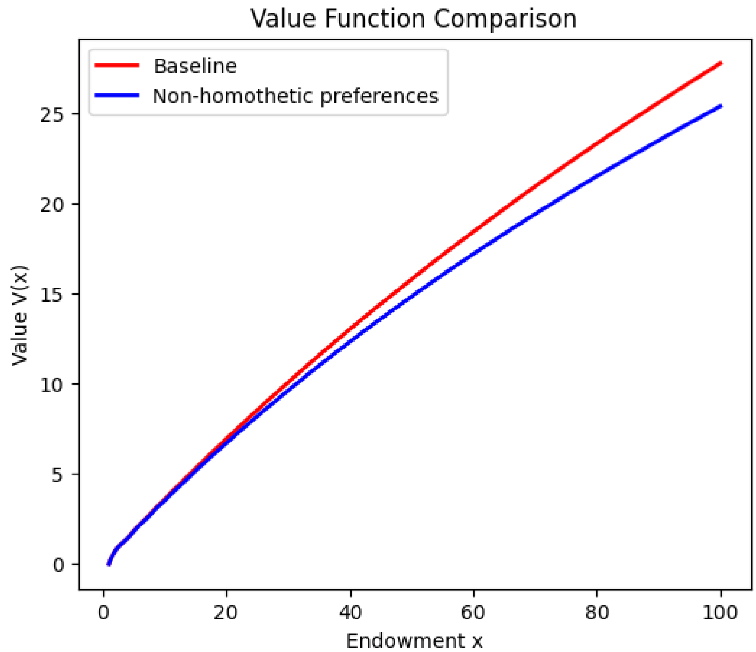

Figure 4: Comparison with and without risk.

Intuition: In the baseline case, the farmer can freely pick any $c \in [0, x]$. In contrast, in the new setup, the farmer cannot reduce consumption below the threshold. This loss of flexibility lowers total lifetime utility because the farmer cannot smooth consumption across time as efficiently.