Macroeconomics, PhD core

Lecture #10

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Intro

- Main challenges to endogenous growth theory
- AK model generates long run growth: The typical interpretation is that the marginal product of capital is non-decreasing because
  - spillovers of knowledge
  - learning by doing, etc.

Endogenous Growth Model

Ingredients

- What worked in the AK model?
  Constant returns in a reproducible factor (capital in a broad sense)
  We left out the non-reproducible factor (labor... or for what matters, physical capital)
- Dilemma:
  CRS to scale but non-reproducible factors claim a non-negligible share of income.
- Solutions in the literature:
  1. IRS, but not "perceived/priced in" by the firm: Externalities
  2. Monopolistic Competition

Romer (1986) simplified

- The economy is as in the AK model set up **but ...**
- There is a continuum of firms that produce output with technology

$$y_i(t) = F(k_i(t), l_i(t)K(t))$$

firms are indexed by $i \in [0, 1]$
$k_i(t)$ and $l_i(t)$ are capital and labor at firm i
$K(t) = \int k_i(t)di$ aggregate capital.
$F()$ is CRS with respect to k_i and l_i

Romer (1986) simplified

- Firms Problem

$$\max_{k_i, l_i} F(k_i(t), l_i(t)K(t)) - w(t)l_i(t) - r(t)k_i(t)$$

$K(t)$ is "exogenous to the firm"

- Note that there are *IRS* overall

$$\begin{aligned} F(\theta k_i(t), \theta l_i(t)\theta K(t)) &= \\ F(\theta k_i(t), \theta^2 l_i(t)K(t)) &> \theta F(k_i(t), l_i(t)K(t)) \text{ for all } \theta > 1 \end{aligned}$$

- The competitive equilibrium exists in this economy, because from the firm's perspective there are CRS.
- Will it be Pareto Optimal? Not in general.
- Households? As before, size L of identical people.
- No population growth.

Romer (1986)

Equilibrium

Definition

A competitive equilibrium are allocations $(\widehat{c}(t), \widehat{a}(t))_{t\in[0,\infty)}$ for the representative household, allocations $\left(\widehat{k}_i(t), \widehat{l}_i(t)\right)_{t\in[0,\infty)}$ for the firms, an aggregate capital stock $\widehat{K}(t)_{t\in[0,\infty)}$ and prices $(\widehat{r}(t), \widehat{w}(t))_{t\in[0,\infty)}$ such that ...

Romer (1986)

Equilibrium

Definition

1. Given prices, $(\widehat{c}(t), \widehat{a}(t))_{t\in[0,\infty)}$ solves

$$\max_{c(t),a(t)} \int \exp(-\rho t)\frac{c(t)^{1-\sigma}}{1-\sigma}dt$$

$$\text{s.t. } c(t) + \dot{a}(t) + na(t) = w(t) + (r(t) - \delta)a(t)$$

$$a(0) = k(0) \text{ given} \quad c(t) \geq 0$$

$$\lim_{t\to 0} a(t)\exp(-\int_0^t (r(\tau) - \delta)d\tau) \geq 0$$

2. Given $(\widehat{r}(t), \widehat{w}(t))_{t\in[0,\infty)}$ and $\widehat{K}(t)_{t\in[0,\infty)}$,the path $\left(\widehat{k}_i(t), \widehat{l}_i(t)\right)_{t\in[0,\infty)}$ maximizes firms profits ...

Romer (1986)

Equilibrium

Definition

1

3. (feasibility) For all t

$$L\widehat{c}(t) + \widehat{\dot{K}}(t) + \widehat{K}(t)\delta = \int_0^1 F(\widehat{k}_i(t), \widehat{l}_i(t)\widehat{K}(t))di$$

$$\int_0^1 \widehat{l}_i(t)di = L \qquad \int_0^1 \widehat{k}_i(t)di = L\widehat{a}(t)$$

4. (rational expectations)

$$\int_0^1 \widehat{k}_i(t)di = \widehat{K}(t)$$

Romer (1986)

Planner's problem

- The planner's problem is

$$\max_{c(t),K(t)} \int \exp(-\rho t)\frac{c(t)^{1-\sigma}}{1-\sigma}dt$$

$$Lc(t) + \dot{K}(t) + K(t)\delta = F(K(t), LK(t)) \quad \text{with } K(0) = Lk(0)$$

- Why is this the problem? Note that production decisions are static.

Romer (1986)

- The planner can choose first the aggregates and second, how to split it across firms and households. (HW)
- Optimality

$$\begin{aligned}\gamma_C^{SP}(t) &= \frac{\dot{c}(t)}{c(t)} \\ &= \frac{1}{\sigma}\left[F_1(K(t), LK(t)) + F_2(K(t), LK(t))L - (\delta + \rho)\right]\end{aligned}$$

Romer (1986)

Planner's problem

- F is HOD1, then F' is HOD0

$$F_1(K(t), LK(t)) + F_2(K(t), LK(t))L = F_1(1, L) + F_2(1, L)L$$

- Consumption growth

$$\gamma_C^{SP}(t) = \frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma}\left[F_1(1, L) + F_2(1, L)L - (\delta + \rho)\right]$$

Constant in time!

- From the aggregate resource constraint

$$L\frac{c(t)}{K(t)} + \frac{\dot{K}(t)}{K(t)} + \delta = F(1, L)$$

- In the BGP, $\gamma_C^{SP}(t) = \gamma_k^{SP}(t) = \gamma_K^{SP}(t)$

Romer (1986)

Competitive Equilibrium

- F is HOD1, then F' is HOD0

$$F_1(K(t), LK(t)) + F_2(K(t), LK(t))L = F_1(1, L) + F_2(1, L)L$$

- FOC household

$$\gamma_C^{CE}(t) = \frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma}\left[r(t) - (\delta + \rho)\right]$$

- Firm's problem

$$r(t) = F_1(k_i(t), l_i(t)K(t))$$

- All firms are identical and choose the same allocations

$$k_i(t) = k(t) = \int_0^1 k(t)di = K(t) \qquad \text{and } l_i = L$$

Romer (1986)

Competitive Equilibrium

- So $r(t) = F_1(1, L)$

$$\gamma_C^{CE}(t) = \frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma}\left[F_1(1, L) - (\delta + \rho)\right] < \gamma_C^{SP}(t)$$

  as $F_2(1, L)L > 0$
- In the BGP, $\gamma_C^{CE}(t) = \gamma_k^{CE}(t) = \gamma_K^{CE}(t)$
- How to bring $\gamma_K^{CE}(t)$ to $\gamma_K^{P}(t)$?
- Subsidy per unit of capital invested equal to $F_2(1, L)L$.
- Cost of capital for the firms

$$r(t) - F_2(1, L)L$$

Romer (1986)

Scale Effects

- A prediction of this model is that larger economies grow more

$$\gamma_C^{CE}(t) = \frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma}\left[F_1(1, L) - (\delta + \rho)\right] < \gamma_C^{SP}(t)$$

  as $\frac{\partial F_1(1,L)}{\partial L} > 0$
- If L is identified with the labor force of a country and or population, the data post-World War II across country shows no evidence.
- Technical reason, CRS in K and IRS in K and L.
- How to avoid it? Assume "productivity" depends on the average K/L rather than aggregate.
- Example, Lucas (1988)

Lucas (1988)

Externalities in Human Capital

- Household with measure 1.
- Human capital h_i

$$i \in (0,1) \quad \text{with } h_i(0) = h_0 \text{ and } k_i(0) = k_0$$

- Household chooses time to work, $(1 - s_i(t))$ and to accumulate human capital, $s_i(t)$
- Budget constraint

$$c_i(t) + \dot{a}_i(t) = (r(t) - \delta)\, a_i(t) + (1 - s_i(t))h_i(t)w(t)$$

$$\dot{h}_i(t) = \theta h_i(t)s_i(t) - \delta h_i(t)$$

Lucas (1988)

Externalities in Human Capital

- Production Technology

$$Y(t) = AK(t)^{\alpha} L(t)^{1-\alpha} H(t)^{\beta}$$

  Firms choose capital and labor, and they take $H(t)$ as given.

- Total supply of labor, $(1 - s_i(t))H(t)$. Hence,

$$AK(t)^{\alpha} H(t)^{1-\alpha} H(t)^{\beta} (1 - s(t))^{1-\alpha}$$

Lucas (1988)

Externalities in Human Capital

- Planner's problem

$$\max \int_0^\infty \exp(-\rho t)\frac{c(t)^{1-\sigma}}{1-\sigma}$$

subject to

$$C(t)+\dot{K}(t)+\delta K(t) = AK(t)^\alpha H(t)^{1-\alpha+\beta}(1-s(t))^{1-\alpha}$$

$$\dot{H}(t) = \theta H(t)s(t) - \delta H(t)$$

with $H(0)$ and $K(0)$ given, $s(t)\in[0,1]$ and I have used

$$\begin{aligned} \int (1-s_i(t))h_i(t) &= L(t) \\ \int a_i(t) &= K(t) \qquad \text{and} \quad \int c_i(t) = C(t) \end{aligned}$$

Lucas (1988)

Decentralized problems

- Firms

$$\max Y(t) - w(t)L(t) - r(t)K(t)$$

$$r(t) = \alpha \frac{Y(t)}{K(t)} \qquad \text{and } w(t) = (1-\alpha)\frac{Y(t)}{(1-s)H(t)}$$

- Households

$$\begin{aligned} H(.) \quad = \quad & \exp(-\rho t)\frac{c(t)^{1-\sigma}}{1-\sigma} + \lambda(t)[(r(t)-\delta)\, a_i(t) \\ & +(1-s_i(t))h_i(t)w(t) - c_t(t)] \\ & +\mu(t)\left[\theta h_i(t)s_i(t) - \delta h_i(t)\right] \end{aligned}$$

- Again, $\gamma_C^{SP} > \gamma_C^{CE}$