Macroeconomics, PhD core

Lecture #7

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Bewley Economies

- Agents are ex-ante identical. Only source of (ex-post) heterogeneity is income risk.
- Agents cannot insure against these shocks due to incomplete markets.

Today:

1. Simplest set up.
2. Mechanisms and results.

Endowment economy

- Time discrete.
- No aggregate uncertainty. Aggregate endowment $\bar{e}$ constant.

Household:

- Measure 1 of households.
- Each one of them lives forever.
- Preferences $E_o \sum_{t=0}^{\infty} \beta^t u(c_t)$.

Technology:

- Random endowments $e_{it} \in E$ (finite)
- Transition matrix

$$\pi(e_{it+1}|e_t)$$

  - Let $\Pi(e)$ be the stationary distribution of e.
  - "Law of large numbers" $\Pi(e) =$ measure of households with endowment e.

Household problem

$$\max_{c_{it}, a_{it+1}} E_o \sum_{t=0}^{\infty} \beta^t u(c_{it})$$

subject to

- budget constraint

$$c_{it} + a_{it+1} = e_{it} + (1+r) a_{it}$$

- borrowing constraint

$$a_{it+1} \geq -b$$

where b could be 0, or some arbitrary number b.

Household problem

- state vector?: a, e.
- control? c

Recursive form

$$v(a,e) = \max_c u(c) + \beta \sum_{e'} \pi(e'|e) v(a', y')$$

subject to

- budget constraint

$$c + a' = e + (1+r)a$$

- borrowing constraint

$$a' \geq -b$$

Solution?
Careful! the borrowing constraint may bind $\rightarrow$ K-T

Bellman equation

$$v(a,e) = \max_{a'} u(e + (1+r)a - a') + \beta \sum_{e'} \pi(e'|e) v(a', y') + \mu(a' + b)$$

Sufficient conditions for an optimum (K-T)

$$\frac{\partial u(c)}{\partial c} = \beta \sum_{e'} \pi(e'|e) \frac{\partial v(a', y')}{\partial a'} + \mu \qquad (a')$$

$$\frac{\partial v(a,e)}{\partial a} = \frac{\partial u(c)}{\partial c}(1+r) \qquad (a)$$

$$\mu(a' + b) = 0 \qquad (comp.slackness)$$

Household problem

Recursive form

- Euler equation

$$\frac{\partial u(c)}{\partial c} \geq \beta \sum_{e'} \pi(e'|e) \frac{\partial u(c')}{\partial c'}(1+r) \qquad (a')$$

with equality if $a' > -b$ and therefore $\mu = 0$.

- Solution to the household problem
Functions $v(a, e)$, $a'(a, e)$ that satisfy the sufficient conditions, given r.

- Assume shocks are i.i.d. $e \approx i.i.d.$
- Consumption and savings decisions depend on the current value of income.

$$x \equiv e + (1 + r)a$$

- Savings are increasing in the current value of income, $\partial a' / \partial x > 0$
  - If x is sufficiently high: choose $a' > -b$ and satisfy the standard Euler equation.
  - If x is below some cutoff, choose $a' = -b$ and let the Euler equation be violated.
- Hence, current consumption is lower than "it should" when the constraint is binding.

Stationary Equilibrium

- Aggregate state: The joint distribution of assets and endowment $\Phi(a, e)$.
- In a stationary recursive competitive equilibrium aggregate quantities and prices are constant over time.
- A *stationary equilibrium* is an allocation and prices such that
  1. Households maximize utility $\rightarrow v(a, e)$, $a'(a, e)$,
  2. Markets clear,
  3. Φ is time invariant.

Market Clearing

- Goods

$$C = \int_a \int_e c(a, e)\Phi(da, de) = \int_0^1 e\Pi(de) = \bar{e}$$

- Bonds/assets

$$\int_a \int_e a'(a, e)\Phi(da, de) = 0$$

why? who are the households borrowing from?

Law of motion aggregate distribution Φ.

- Define a transition function

$$Q((a,e),(A,E))$$

the probability (or mass of households) in state (a,e) that transition to $(a',e') \in (A,E)$ tomorrow.

$$Q((a,e),(A,E)) = \sum_{e' \in E} \pi(e'|e) \quad \text{if} \quad a'(a,e) \in A$$

$$0 \text{ otherwise.}$$

notice that a' is determined today.

- Law of motion

$$\Phi'(A,Y) = \int_e \int_e Q((a,e),(A,E))\Phi(da,dy)$$

Algorithm.

1. Given the interest rate, solve the policy function of the household.
2. Given the policy function, iterate over the law of motion of the aggregate state until $\Phi' = \Phi$.
3. Using the stationary distribution, check market clearing.
4. If aggregate asset positions are positive, lower the interest rate and go back to 1.
5. If aggregate asset positions are negative, increase the interest rate and go back to 1.
6. iterate until the market clearing condition is satisfied.

Example, Huggett (1993)

- Six periods per year
- Time discount $\beta = 0.66^{1/6} = 0.993$ per period
- CRRA u(c)
- Two-state Markov chain with $y_H = 1$, $y_L = 0.1$
- Transition probabilities

$$\pi_{HH} = 0.925, \qquad \pi_{LL} = 0.5$$

- Solved on a grid of borrowing constraints, $\bar{a}$

Asset policy $a' = g(a, y)$.

assetpolicy.pdf

Consumption policy $c(a, y)$.

consumptionpolicy.pdf

Excess demand $F(q) = \int_a \int_e g(a, e; q)\Phi(da, de; q) = 0.$

...for bond price $q = \frac{1}{1+r}$.

excessdemand.pdf

Complete market benchmark

- Complete risk-sharing

$$c_{it} = C = Y.$$

- Bond price (interest rate)

$$q \equiv = \frac{1}{1+r} = \beta$$

- Dynamic of assets

$$a_{it+1} = (1+r)(a_{it} + e_{it} - Y)$$

Low-risk aversion results

- Risk-free rate r in annual percent

$\underline{a}$	r	q
-2	-7.1%	1.0124
-4	2.3%	0.9962
-6	3.4%	0.9944
-8	4.0%	0.9935

- Tighter borrowing constraint (higher $\underline{a}$), higher demand for savings ($q \uparrow$, $r \downarrow$)
- Slacker borrowing constraint (lower $\underline{a}$), lower demand for savings ($q \downarrow$, $r \uparrow$)
- Convergence to the complete market equilibrium

High-risk aversion

- Risk-free rate r in annual percent

$\underline{a}$	r	q
-2	-23%	1.0448
-4	-2.6%	1.0045
-6	1.8%	0.9970
-8	3.7%	0.9940

- Higher risk aversion α, higher demand for savings, lower $r \forall \underline{a}$.