Path Integrals

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1 Composition

(a) In lecture we argued that the propagator K can be written as a sum (or integral) over paths γ of a functional $A[\gamma]$

$$K = \sum_{\gamma} A[\gamma]\,. \tag{1}$$

Show that the propagator obeys the composition property

$$K(q_f, t_f, q_i, t_i) = \int dy K(q_f, t_f, y, t_{int}) K(y, t_{int}, q_i, t_i) \tag{2}$$

if A has the factorization property

$$A[\gamma_{12}] = A[\gamma_1] A[\gamma_2]\,. \tag{3}$$

(b) Explain why the factorization property

$$A[\gamma_{12}] = A[\gamma_1] A[\gamma_2] \tag{4}$$

holds if $A[\gamma] = e^{iS[\gamma]/\hbar}$ where $S[\gamma]$ is the classical action of the trajectory γ.

(c) Suppose $A[\gamma] = C_1 e^{C_2 S[\gamma]}$ for some constants C_1 and C_2. Which values of C_1 and C_2 give rise to a composition property?

2 Classical limit

(a) Suppose $f(x)$ is a real function. Which values of x provide the most important contribution to

- $\int e^{-f(x)} dx$? Why? (Optional: what conditions should be placed on $f(x)$?)
- $\int e^{if(x)} dx$? Why? (Optional: what conditions should be placed on $f(x)$?)

(b) The classical equation of motion for a particle with classical action S is that the first-order variation vanishes

$$\delta S = 0\,. \tag{5}$$

A trajectory with $\delta S = 0$ is a classical trajectory. No extra conditions on the second-order variation of the action are required. Why does $A[\gamma] = e^{iS[\gamma]/\hbar}$ give the correct classical limit?

(c) What other choices $A[\gamma]$ give rise to the correct classical limit? Do they have a composition property?

(d) Do any of the other possibilities you identified in 1 (c) have the correct classical limit?