

Matrix exponential

- $\exp: M_n(\mathbb{C}) \rightarrow GL(n, \mathbb{C})$

$$X \mapsto \sum_{k=0}^{\infty} \frac{X^k}{k!} \quad \leftarrow \text{converges (absolutely) for all } X$$

- \exp is surjective

- $e^{A X A^{-1}} = A e^X A^{-1} \quad (A \in GL(n, \mathbb{C}))$

- $\det(e^X) = e^{\text{tr} X}$

- $e^{\overset{\text{zero matrix}}{0}} = \mathbb{1}$

- $(e^X)^* = e^{X^*}$

- $(e^X)^T = e^{X^T}$

- $(e^X)^{-1} = e^{-X}$

- $e^{(\alpha+\beta)X} = e^{\alpha X} e^{\beta X} \quad (\alpha, \beta \in \mathbb{C})$

- if $XY = YX$ then $e^{X+Y} = e^X e^Y = e^Y e^X$

- in general $e^{X+Y} = \lim_{k \rightarrow \infty} (e^{X/k} e^{Y/k})^k$ (Lie product formula)

- $\frac{d}{d\varepsilon} e^{\varepsilon X} = X e^{\varepsilon X} = e^{\varepsilon X} X \quad (\varepsilon \in \mathbb{R})$

- $\frac{d}{d\varepsilon} \Big|_{\varepsilon=0} e^{\varepsilon X} = X$

- $e^{\varepsilon X} = e^{\varepsilon Y} \quad \forall \varepsilon \in \mathbb{R} \quad \Leftrightarrow \quad X = Y$