Numerical methods and condensed matter physics

Perimeter Institute Summer School 2020

Instructor information

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Course description

This course has two main goals: (1) to introduce some key models from condensed matter physics; and (2) to introduce some numerical approaches to studying these (and other) models. As a precursor to these objectives, we will carefully understand many-body states and operators from the perspective of condensed matter theory. (However, I will cover only spin models. We will not discuss or use second quantization.)

Once this background is established, we will study the method of exact diagonalization and write simple python programs to find ground states, correlation functions, energy gaps, and other properties of the transverse-field Ising model. We will also discuss the computational limitations of exact diagonalization. Finally, I will introduce the concept of matrix product states, and we will see that these can be used to study ground state properties for much larger systems than can be studied with exact diagonalization.

Each 90-minute session will include substantial programming exercises in addition to lecture. Prior programming experience is not expected or required, but I would like everyone to have python (version 3) installed on their computer prior to the first class, including Jupyter notebooks; see "Resources" below.

Learning outcomes

By the end of this course students should:

- Understand many-body Hilbert space and the operators that act on it, as well as the assumptions typically made in the setting of condensed matter physics
- Be familiar with one of the basic models of condensed matter, namely the transverse-field Ising model
- Be able to write simple python programs to study these models by exact diagonalization, and understand the limitations of this method
- Be aware that there are other numerical methods (eg based on matrix product states) that are more efficient than exact diagonalization

Resources

We will use Python 3 with Jupyter notebooks. If you have not used these before, I recommend installing them using Anaconda. I will distribute partially completed programs in the form of Jupyter notebooks, and your exercises will consist primarily of filling in the missing pieces.

Course schedule

Lecture 1	Introduction to Ising model; review spin-1/2 and matrix representation of states and operators; programming basics
Lecture 2	Tensor product spaces & many-body quantum states and operators; Limiting cases of two-site Ising model
Lecture 3	Exact diagonalization part 1: solve two-site Ising model, find energies, expectation values, etc.; Find matrices for N-site Ising model
Lecture 4	Exact diagonalization part 2: Continue with finding matrices for N-site Ising model, solve for eigenvalues and eigenstates of model
Lecture 5	Exact diagonalization part 3: Expectation values and correlation functions in the Ising model GS, using these and energy gaps to identify a quantum phase transition; Demonstration of using matrix product states for the Ising model