

NE 401 Computational Project

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1 Solving the Discretized P-1 Equations

1.1 Cross Section Relations

From lecture 18 [1].

$$D_{i,g} = \frac{1}{3\Sigma_{tr,i,g}} \quad (1)$$

$$\Sigma_{r,i,g} = \Sigma_{t,i,g} - \Sigma_{s0,i,g \rightarrow g} \quad (2)$$

$$\Sigma_{tr,i,g} = \Sigma_{t,i,g} - \Sigma_{s1,i,g \rightarrow g} \quad (3)$$

1.1.1 General Form

From Lecture 13 [1]

$$\Sigma_{s0,g \rightarrow g}(\vec{r}, t) = \int_{-1}^1 \int_0^{2\pi} \Sigma_{s,g \rightarrow g}(\vec{r}, \mu_0, t) d\gamma d\mu_0 = \Sigma_{s,g \rightarrow g}(\vec{r}, t) \quad (4)$$

Equation 4-125, page 133, [3]

$$\Sigma_{s1,g \rightarrow g}(\vec{r}, t) = \int_{-1}^1 \int_0^{2\pi} \mu_0 \Sigma_{s,g \rightarrow g}(\vec{r}, \mu_0, t) d\gamma d\mu_0 = \bar{\mu}_0 \Sigma_{s,g \rightarrow g}(\vec{r}, t) \quad (5)$$

1.1.2 Discrete Form

$$\Sigma_{s0,i,g \rightarrow g} = \Sigma_{s,i,g \rightarrow g} \quad (6)$$

$$\Sigma_{s1,i,g \rightarrow g} = \bar{\mu}_0 \Sigma_{s,i,g \rightarrow g} \quad (7)$$

$$\Sigma_{r,i,g} = \Sigma_{t,i,g} - \Sigma_{s,i,g \rightarrow g} \quad (8)$$

$$\Sigma_{tr,i,g} = \Sigma_{t,i,g} - \bar{\mu}_0 \Sigma_{s,i,g \rightarrow g} \quad (9)$$

1.2 Equations

$$J_{g,i+1}^{(s)} - J_{g,i}^{(s)} + \Sigma_{r,g,i} \bar{\phi}_{g,i}^{(s)} h_i = Q_{g,i}^{(s)} h_i \quad (10)$$

$$Q_{g,i}^{(s)} = \sum_{g'=1}^{g-1} \Sigma_{g' \rightarrow g} \bar{\phi}_{g',i} + Q_{g,i}^{(s-1)} \quad (11)$$

$$Q_{g,i}^{(s-1)} = \frac{\chi_{g,i}}{k^{(s-1)}} \sum_{g'=1}^{N_g} \nu_{f,g',i} \Sigma_{f,g',i} \phi_{g',i}^{(s-1)} + \sum_{g'=g+1}^{N_g} \Sigma_{s,g' \rightarrow g,i} \phi_{g',i}^{(s-1)} \quad (12)$$

$$J_{g,i+1}^{(s)} - J_{g,i}^{(s)} + \Sigma_{r,g,i} \bar{\phi}_{g,i}^{(s)} h_i - \sum_{g'=1}^{g-1} \Sigma_{g' \rightarrow g} \bar{\phi}_{g',i} = Q_{g,i}^{(s-1)} h_i \quad (13)$$

$$J_{g,i+1}^{(s)} - J_{g,i}^{(s)} + \Sigma_{t,g,i} \bar{\phi}_{g,i}^{(s)} h_i - \sum_{g'=1}^g \Sigma_{g' \rightarrow g} \bar{\phi}_{g',i} = Q_{g,i}^{(s-1)} h_i \quad (14)$$

1.3 Final Form of Equations

In order to solve these equations with a matrix formulation of the problem the right hand side needs to be in terms of knowns and the lefthand unknowns. So I move all the parts of the equations that are unknown to the left and all the parts that are known to the right.

$$J_{g,i+1}^{(s)} - J_{g,i}^{(s)} + \Sigma_{t,g,i} \bar{\phi}_{g,i}^{(s)} h_i - \sum_{g'=1}^g \Sigma_{g' \rightarrow g} \bar{\phi}_{g',i} h_i = Q_{g,i}^{(s-1)} h_i \quad i = 1, \dots, N \quad (15)$$

$$J_{g,i}^{(s)} + \frac{\tilde{D}_{g,i}}{\tilde{h}_{g,i}} \left(\bar{\phi}_{g,i+1}^{(s)} - \bar{\phi}_{g,i}^{(s)} \right) = 0 \quad i = 1, \dots, N+1 \quad (16)$$

$$Q_{g,i}^{(s-1)} = \frac{\chi_{g,i}}{k^{(s-1)}} \sum_{g'=1}^{N_g} \nu_{f,g',i} \Sigma_{f,g',i} \phi_{g',i}^{(s-1)} + \sum_{g'=g+1}^{N_g} \Sigma_{s,g' \rightarrow g,i} \phi_{g',i}^{(s-1)} \quad (17)$$

1.3.1 Additional Relations

$$\tilde{D}_{g,i} = \frac{D_{g,i} D_{g,i-1} (h_i + h_{i-1})}{D_{g,i-1} h_i + D_{g,i} h_{i-1}} \quad i = 2, \dots, N \quad (18)$$

$$\tilde{D}_{g,1} = D_{g,1} \quad (19)$$

$$\tilde{D}_{g,N+1} = D_{g,N} \quad (20)$$

$$\tilde{h}_i = \frac{1}{2} (h_i + h_{i-1}) \quad i = 2, \dots, N \quad (21)$$

$$\tilde{h}_1 = \frac{1}{2}h_1 \tag{22}$$

$$\tilde{h}_{N+1} = \frac{1}{2}h_N \tag{23}$$

1.3.2 Boundary Conditions

$$\bar{\phi}_{g,1} - \bar{\phi}_{g,0} = 0 \tag{24}$$

$$\bar{\phi}_{g,N+1} - \bar{\phi}_{g,N} = 0 \tag{25}$$

1.4 Matrix Formulation

To start I formed the system of equations as a matrix equation in the form

$$Ax^{(s)} = b^{(s-1)} \quad (26)$$

$$J^{(s)} = \begin{bmatrix} J_{1,1}^{(s)} \\ J_{2,1}^{(s)} \\ \vdots \\ J_{N_g,N}^{(s)} \end{bmatrix} \quad (27)$$

$$\phi^{(s)} = \begin{bmatrix} \phi_{1,1}^{(s)} \\ \phi_{2,1}^{(s)} \\ \vdots \\ \phi_{N_g,N}^{(s)} \end{bmatrix} \quad (28)$$

$$x^{(s)} = \begin{bmatrix} J^{(s)} \\ \phi^{(s)} \end{bmatrix} \quad (29)$$

$$b^{(s-1)} = \begin{bmatrix} \frac{\chi_{1,1}}{k^{(s-1)}} \sum_{g'=1}^{N_g} \nu_{f,g',1} \Sigma_{f,g',1} \phi_{g',1}^{(s-1)} + \sum_{g'=2}^{N_g} \Sigma_{s,g' \rightarrow g,1} \phi_{g',1}^{(s-1)} \\ \frac{\chi_{2,1}}{k^{(s-1)}} \sum_{g'=1}^{N_g} \nu_{f,g',1} \Sigma_{f,g',1} \phi_{g',1}^{(s-1)} + \sum_{g'=3}^{N_g} \Sigma_{s,g' \rightarrow g,1} \phi_{g',1}^{(s-1)} \\ \vdots \\ \frac{\chi_{g,i}}{k^{(s-1)}} \sum_{g'=1}^{N_g} \nu_{f,g',i} \Sigma_{f,g',i} \phi_{g',i}^{(s-1)} + \sum_{g'=g+1}^{N_g} \Sigma_{s,g' \rightarrow g,i} \phi_{g',i}^{(s-1)} \end{bmatrix} \quad (30)$$

1.4.1 Current Terms in Equation 15

$$\tilde{J}_1 = \begin{bmatrix} & 1 & 2 & \dots & Ng+1 & Ng+2 & \dots & \dots & NgN+1 & \dots & \dots & Ng(N+1) \\ 1 & -1 & 0 & \dots & 1 & 0 & \dots & \dots & 0 & \dots & \dots & 0 \\ 2 & 0 & -1 & 0 & \dots & 1 & 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & 0 & \ddots & 0 & \dots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ Ng+1 & \vdots & \vdots & 0 & -1 & 0 & \dots & \ddots & \vdots & \vdots & \vdots & \vdots \\ Ng+2 & \vdots & \vdots & \vdots & 0 & -1 & 0 & \dots & 1 & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & 0 & \ddots & 0 & \dots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & \ddots & 0 & \dots & \ddots & \vdots \\ NgN & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & -1 & 0 & \dots & 1 \end{bmatrix}$$

1.4.2 Flux Terms in Equation 15

$$\bar{\Phi}_1 = \begin{bmatrix} & 1 & \dots & Ng & \dots & Ng(N-1)+1 & \dots & NgN \\ & \Sigma_{r,1,1}h_1 & 0 & \dots & \dots & \dots & \dots & \dots \\ \vdots & \vdots & \ddots & 0 & \dots & \dots & \dots & \dots \\ Ng & \Sigma_{s,1 \rightarrow Ng,1}h_1 & \dots & \Sigma_{r,Ng,1}h_1 & 0 & \dots & \dots & \dots \\ \vdots & 0 & \dots & \dots & \ddots & 0 & \dots & \dots \\ Ng(N-1)+1 & 0 & \dots & \dots & \dots & \Sigma_{r,1,N}h_N & 0 & \dots \\ \vdots & 0 & \dots & \dots & \dots & \vdots & \ddots & 0 \\ NgN & 0 & \dots & \dots & \dots & \Sigma_{s,1 \rightarrow Ng,N}h_N & \dots & \Sigma_{r,Ng,N}h_N \end{bmatrix}$$

1.4.3 Current Terms in equation 16

$$\tilde{J}_2 = \begin{bmatrix} & 1 & 2 & \dots & Ng(N+1) \\ & 1 & 1 & 0 & \dots & 0 \\ 2 & 0 & 1 & 0 & \vdots & \\ \vdots & \vdots & 0 & \ddots & \vdots & \\ Ng(N+1) & 0 & \dots & \dots & 1 & \end{bmatrix} \quad (31)$$

1.4.4 Flux terms in equation 16

$$\bar{\Phi}_2 = \begin{bmatrix} & 1 & 2 & \dots & Ng+1 & Ng+2 & \dots & \dots & NgN & \dots & \dots & NgN \\ 1 & 0 & 0 & \dots & 0 & 0 & \dots & \dots & 0 & \dots & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \dots & \dots & \vdots & \dots & \dots & \vdots \\ Ng+1 & -\frac{\bar{D}_{1,1}}{h_1} & 0 & \dots & \frac{\bar{D}_{1,2}}{h_2} & 0 & \dots & \dots & 0 & \dots & \dots & 0 \\ Ng+2 & 0 & -\frac{\bar{D}_{2,1}}{h_1} & 0 & \dots & \frac{\bar{D}_{2,2}}{h_2} & 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & 0 & \ddots & 0 & \dots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 2Ng+1 & \vdots & \vdots & 0 & -\frac{\bar{D}_{1,2}}{h_2} & 0 & \dots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 2Ng+2 & \vdots & \vdots & \vdots & 0 & -\frac{\bar{D}_{2,2}}{h_2} & 0 & \dots & \frac{\bar{D}_{2,3}}{h_3} & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & 0 & \ddots & 0 & \dots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & \ddots & 0 & \dots & \ddots & \vdots \\ NgN & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & -\frac{\bar{D}_{Ng,N}}{h_N} & 0 & \dots & \frac{\bar{D}_{Ng,N+1}}{h_{N+1}} \\ NgN+1 & 0 & 0 & \dots & 0 & 0 & \dots & \dots & 0 & \dots & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \dots & \dots & \vdots & \dots & \dots & \vdots \\ Ng(N+1) & 0 & 0 & \dots & 0 & 0 & \dots & \dots & 0 & \dots & \dots & 0 \end{bmatrix}$$

1.5 Full A Matrix

We can then get our System of equations by concatenating the four terms.

$$A = \begin{bmatrix} \tilde{J}_1, \bar{\Phi}_1 \\ \tilde{J}_2, \bar{\Phi}_2 \end{bmatrix} \quad (32)$$

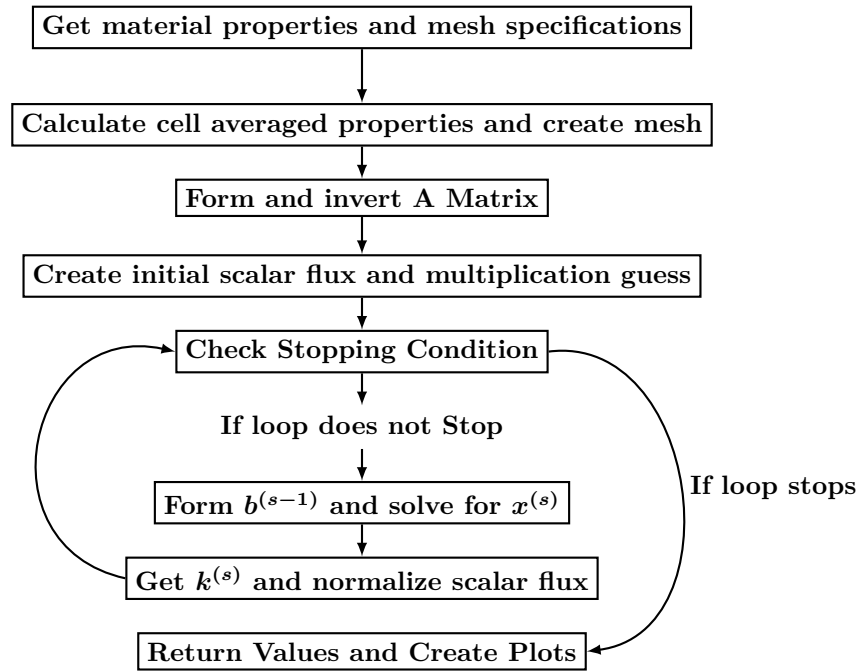
Doing so gives a $N_g(2N+1)$ square matrix which can be inverted to solve for the current and flux.

1.6 Solution to the system

In order to solve the system of equations we can simply take the inverse of the A matrix.

$$x^{(s)} = A^{-1}b^{(s-1)} \quad (33)$$

2 Program Flow Chart



3 Test A Results

3.1 Multiplication Factor and Number of Power Iterations

k	Iterations
1.147588e+00	3.000000e+00

Table 1: Multiplication Factor and Number of Power Iterations

When this problem was solved using the L and P matrices the same result was obtained for multiplication factor as when using the finite volume method algorithm.

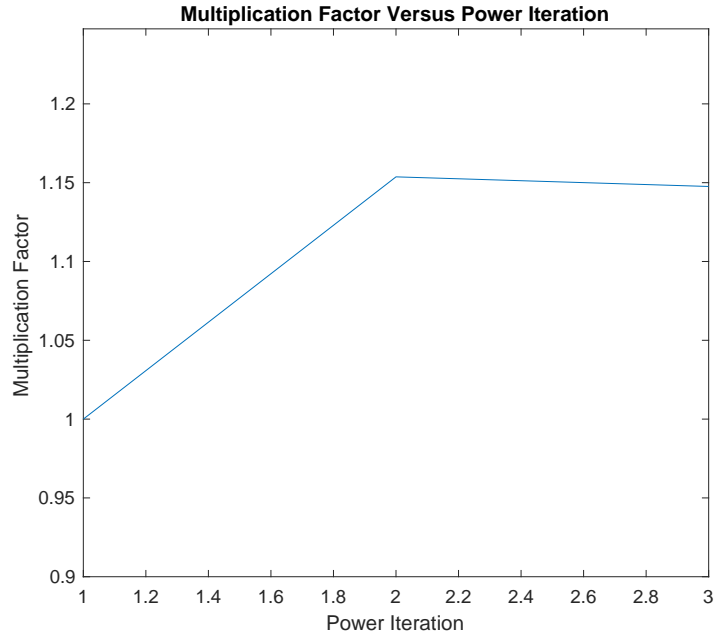


Figure 1: Multiplication Factor versus Iterations

3.2 Cell Average Scalar Flux at $x = 0$ and \bar{x}_5

Group	ϕ at $x = 0$	$\bar{\phi}$ at \bar{x}_5
1	1.434937e-02	1.434937e-02
2	1.779309e-01	1.779309e-01
3	7.580497e-03	7.580497e-03
4	1.358082e-04	1.358082e-04
5	3.400972e-06	3.400972e-06
6	4.241979e-08	4.241979e-08
7	7.476735e-10	7.476735e-10

Table 2: Normalized Cell Averaged Scalar Flux at \bar{x}_5

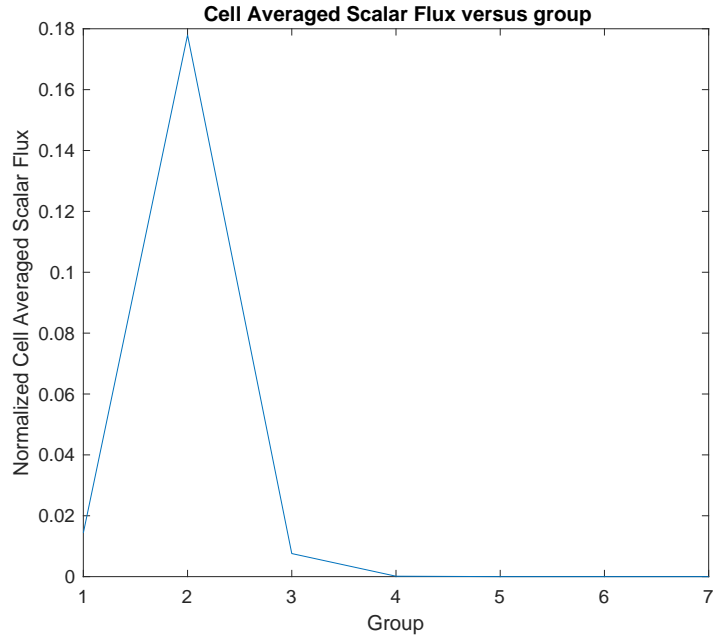


Figure 2: Cell Averaged Scalar Flux versus group at \bar{x}_5

4 Test B Results

4.1 Multiplication Factor and Number of Power Iterations

k	Iterations
1.500000e+00	1.100000e+01

Table 3: Multiplication Factor and Number of Power Iterations

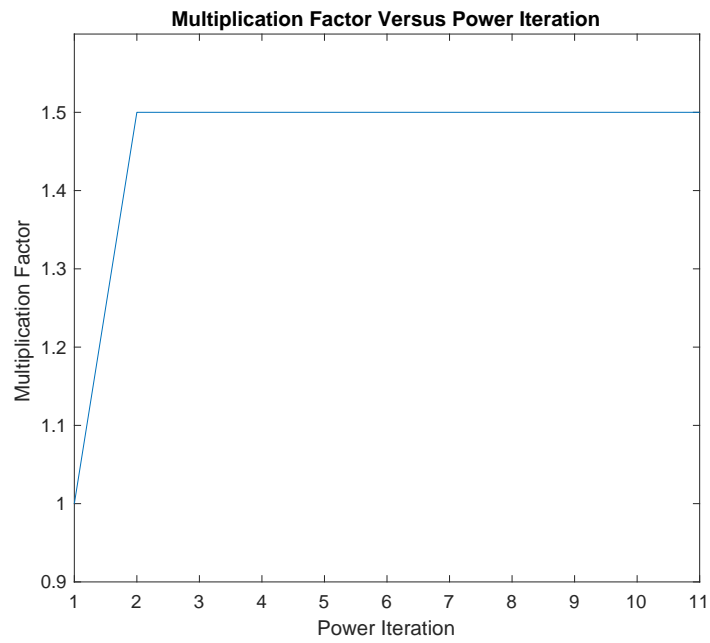


Figure 3: Multiplication Factor versus Iterations

4.2 Normalized Cell Average Scalar Flux Group Wise Plots

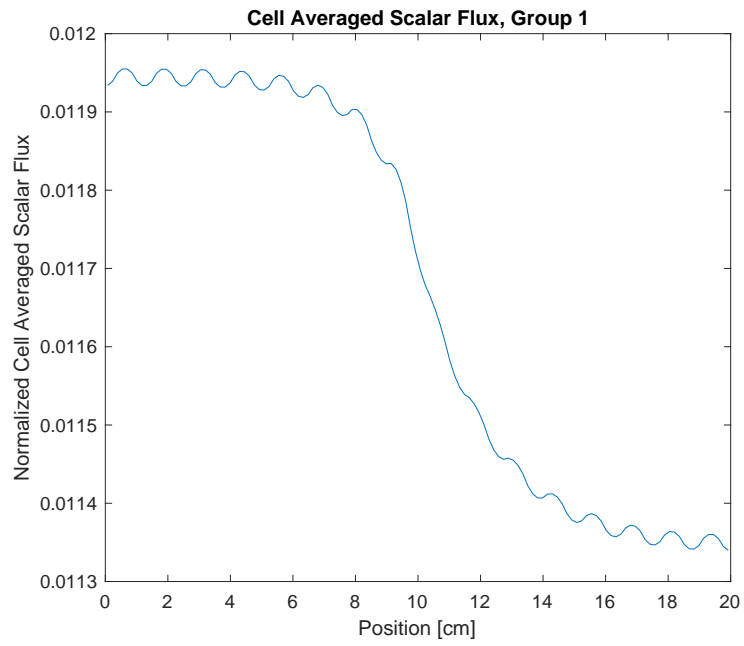


Figure 4: Cell Averaged Scalar Flux versus position in group 1

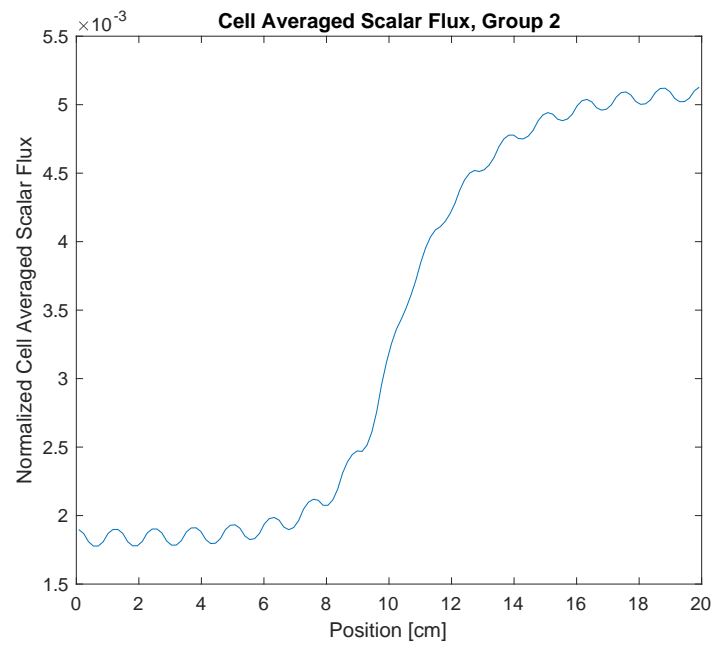


Figure 5: Cell Averaged Scalar Flux versus position in group 2

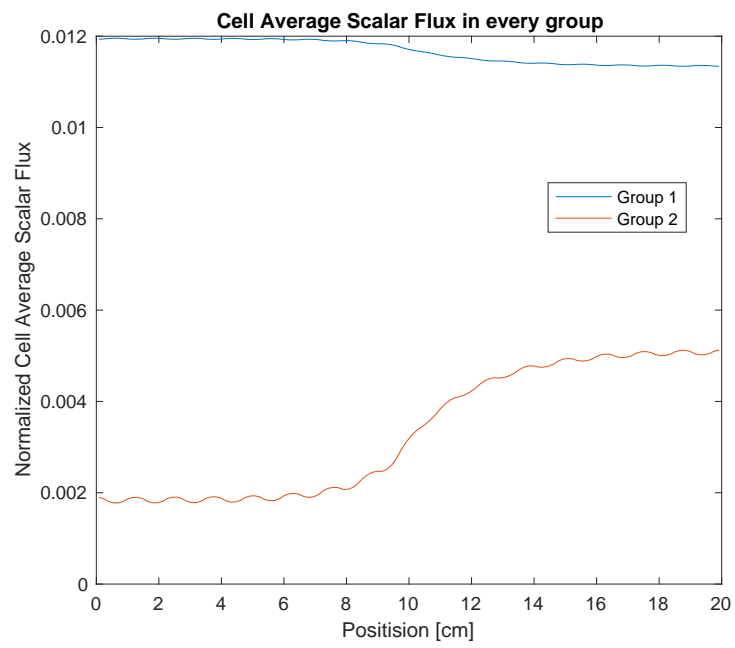


Figure 6: Cell Average Flux versus position for all groups

4.3 Normalized Cell Edge Current Group Wise Plots

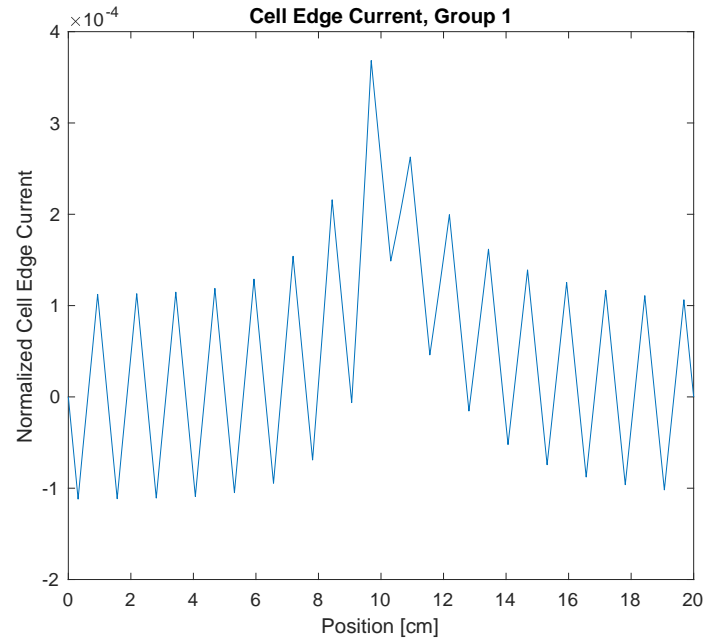


Figure 7: Cell Edge Current versus position in group 1

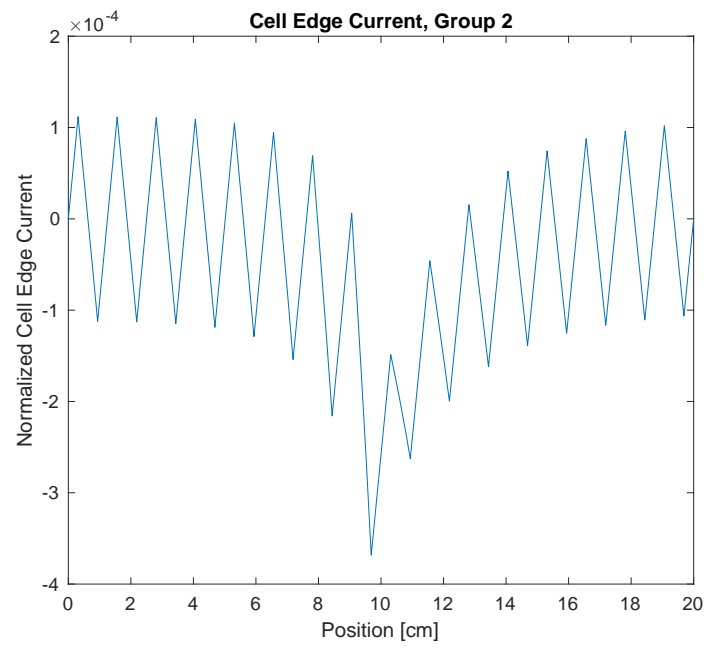


Figure 8: Cell Edge Current versus position in group 2

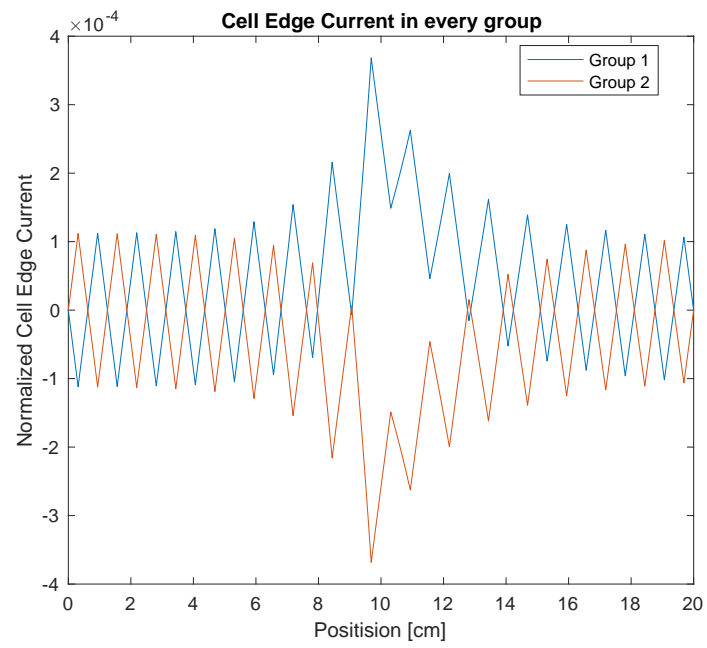


Figure 9: Cell Edge Current versus position for all groups

4.4 Total Normalized Cell Average Scalar Flux

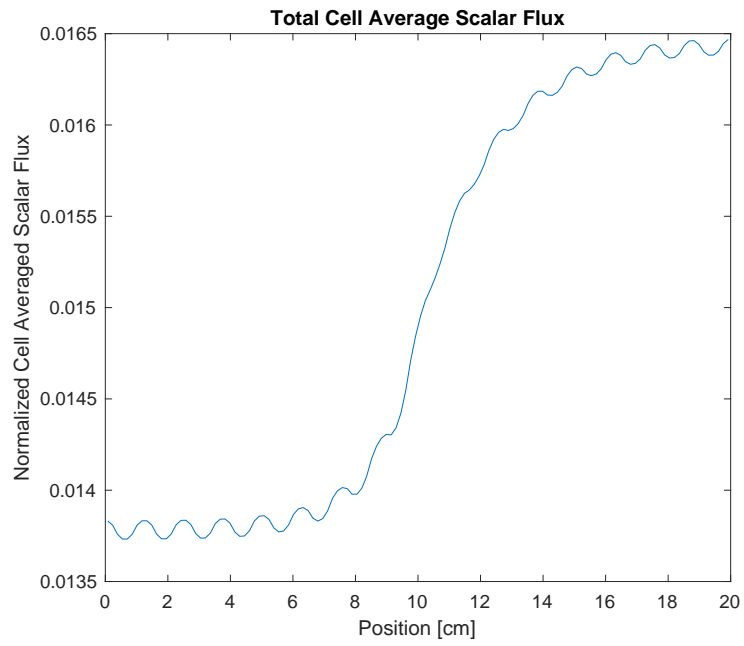


Figure 10: Total scalar flux versus position

4.5 Group Wise Cell Averaged Scalar Flux at \bar{x}_4 and \bar{x}_{124}

Group	$\bar{\phi}$ at \bar{x}_4	$\bar{\phi}$ at \bar{x}_{124}
1	1.195488e-02	1.136020e-02
2	1.777727e-03	5.021816e-03

Table 4: Cell Average Scalar Flux versus group \bar{x}_4 and \bar{x}_{124}

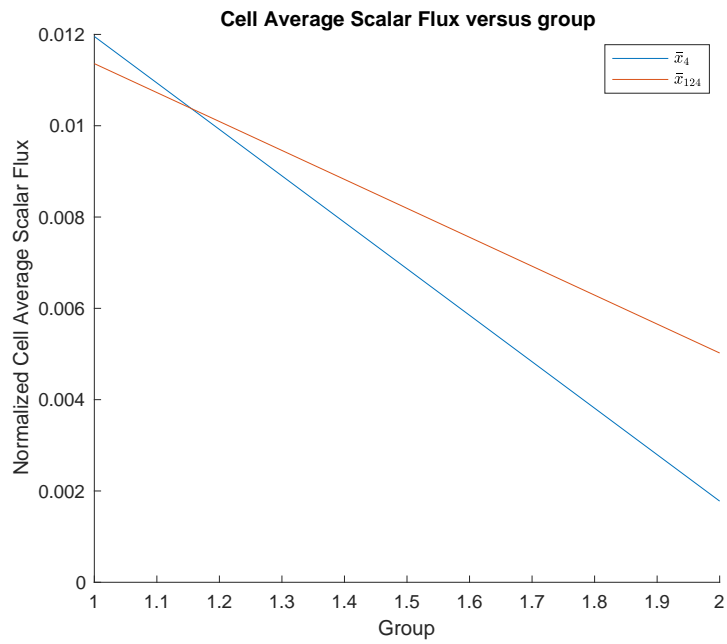


Figure 11: Cell Average Flux versus group at \bar{x}_4 and \bar{x}_{124}

4.6 Group Wise Cell Edge Current at $x = 10$

Group	J at $x = 10$
1	2.583440e-04
2	-2.583368e-04

Table 5: Cell Edge Current versus group at $x = 10$

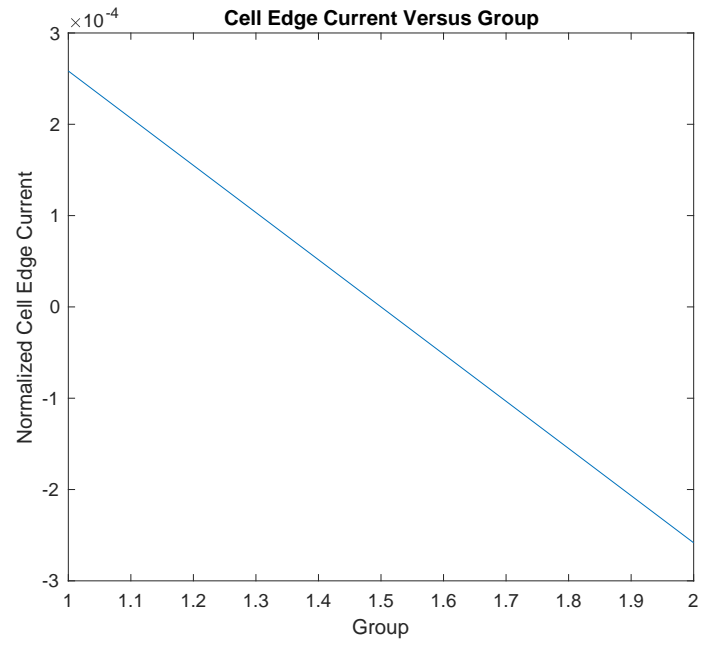


Figure 12: Cell Edge Current versus group at $x = 10$

5 Test C Results

5.1 Multiplication Factor and Number of Power Iterations

k	Iterations
1.064958e+00	1.200000e+01

Table 6: Multiplication Factor and Number of Power Iterations

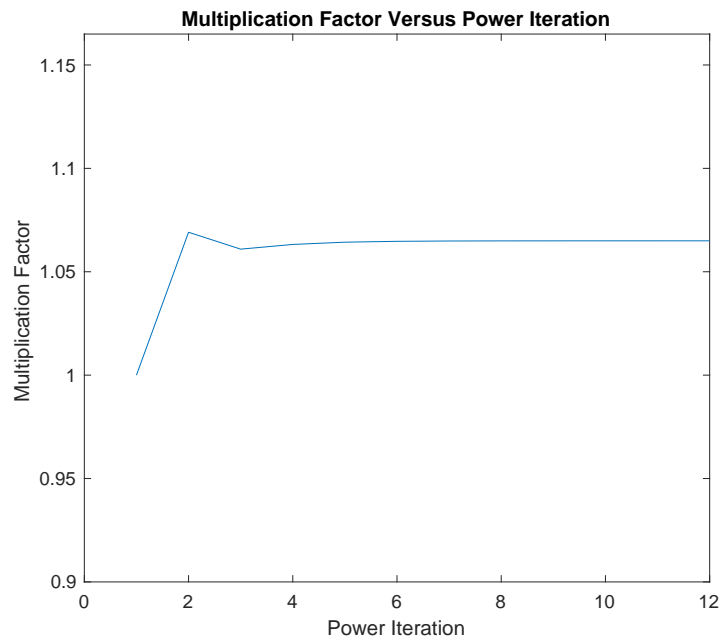


Figure 13: Multiplication Factor versus Iterations

5.2 Normalized Cell Average Scalar Flux Group Wise Plots

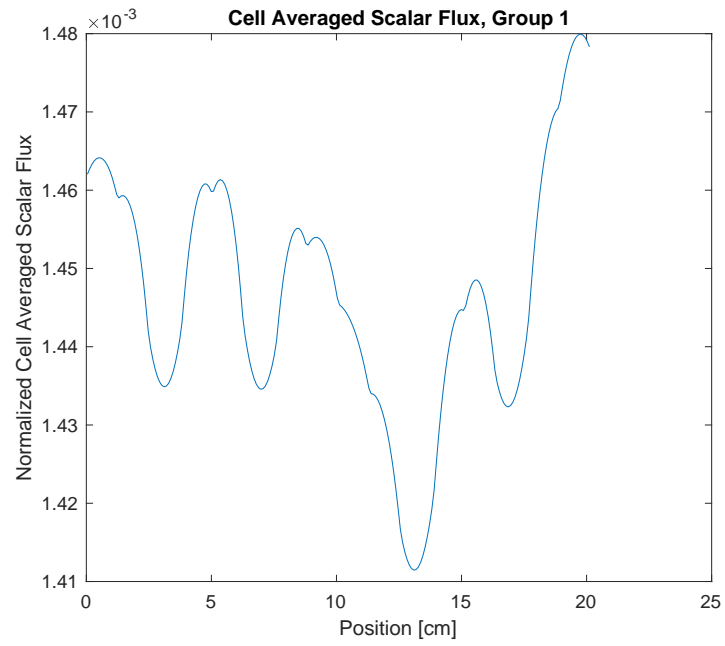


Figure 14: Cell Averaged Scalar Flux versus position in group 1

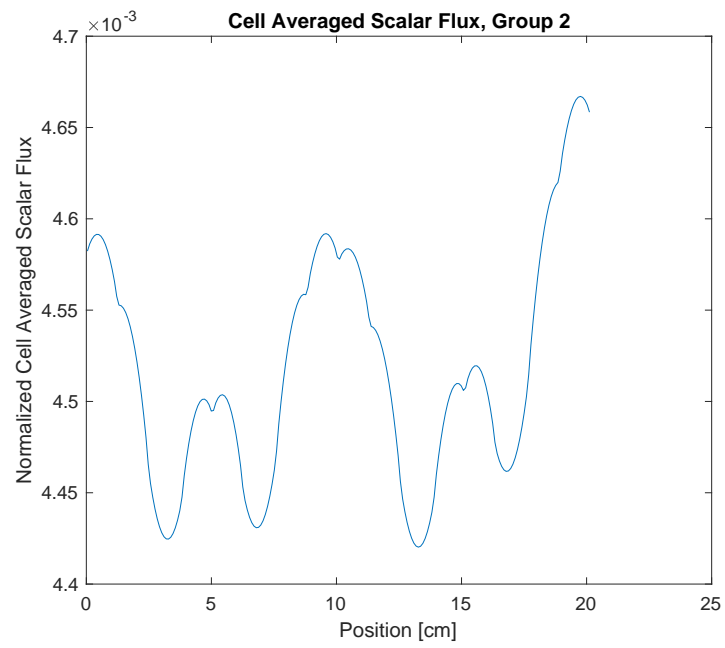


Figure 15: Cell Averaged Scalar Flux versus position in group 2

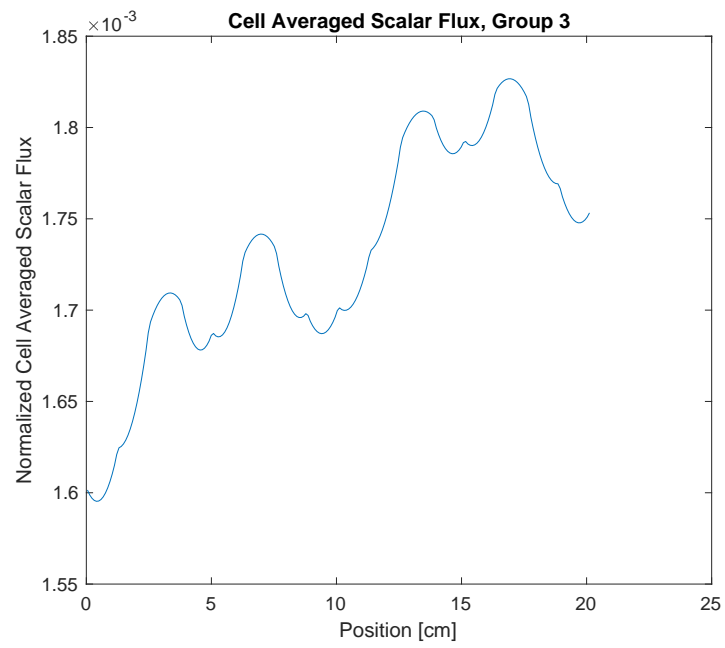


Figure 16: Cell Averaged Scalar Flux versus position in group 3

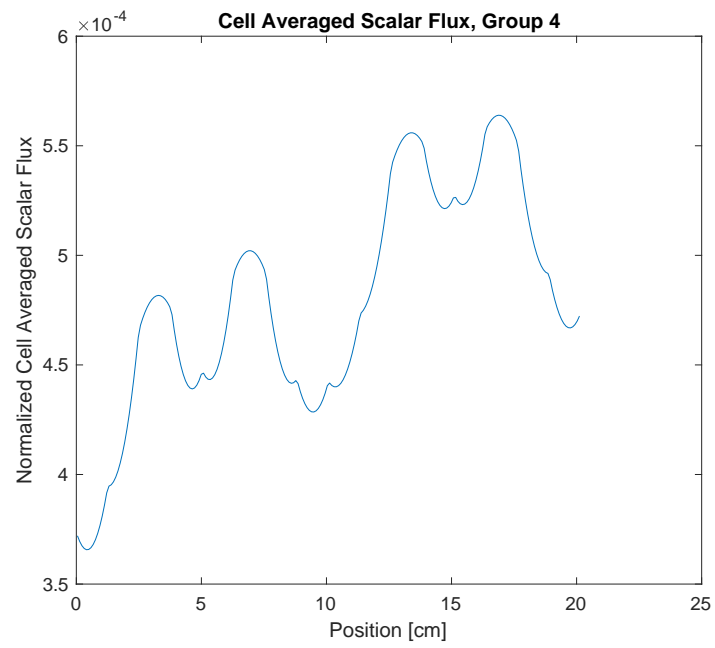


Figure 17: Cell Averaged Scalar Flux versus position in group 4

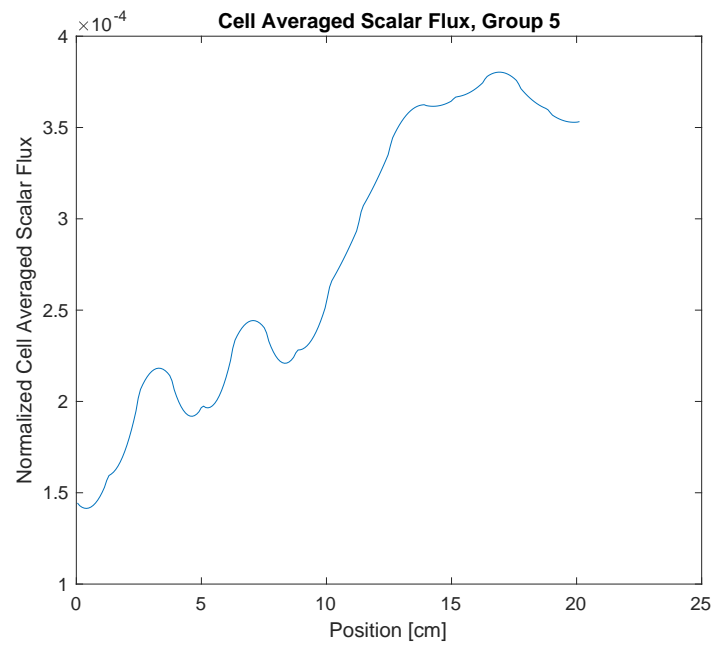


Figure 18: Cell Averaged Scalar Flux versus position in group 5

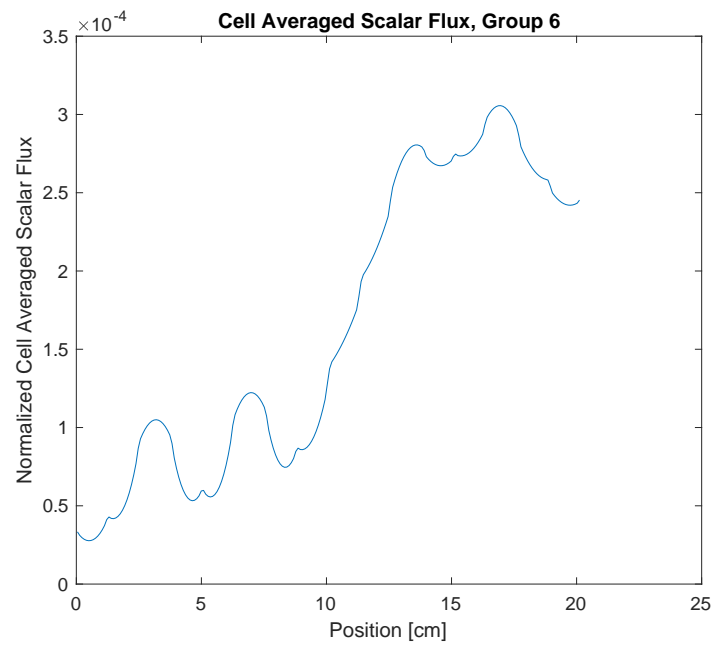


Figure 19: Cell Averaged Scalar Flux versus position in group 6

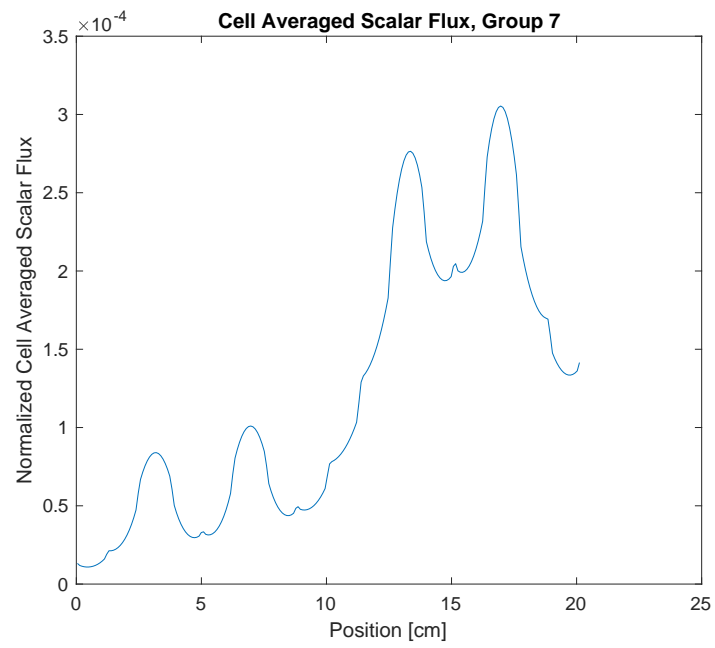


Figure 20: Cell Averaged Scalar Flux versus position in group 7

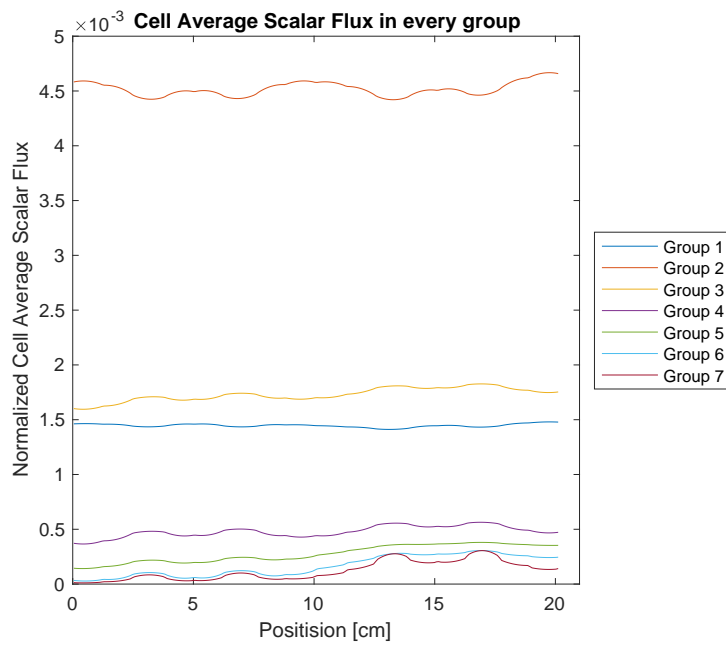


Figure 21: Cell Average Flux versus position for all groups

5.3 Normalized Cell Edge Current Group Wise Plots

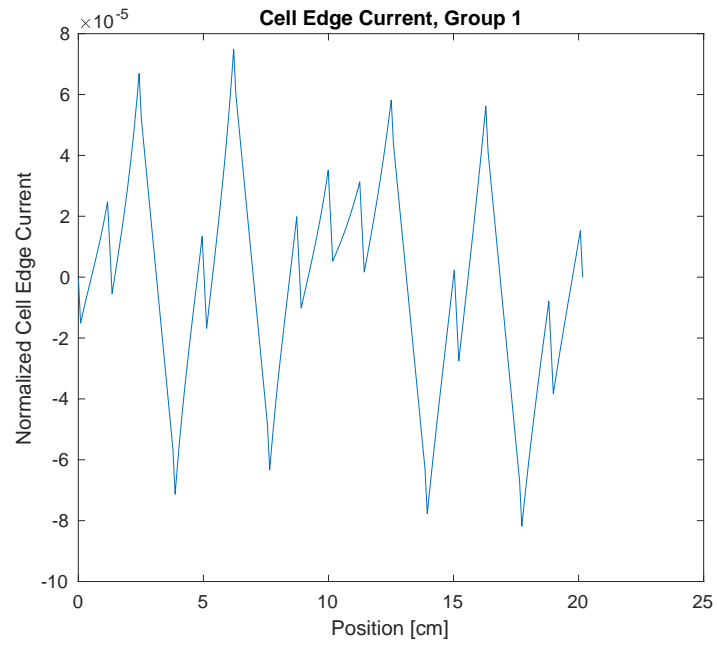


Figure 22: Cell Edge Current versus position in group 1

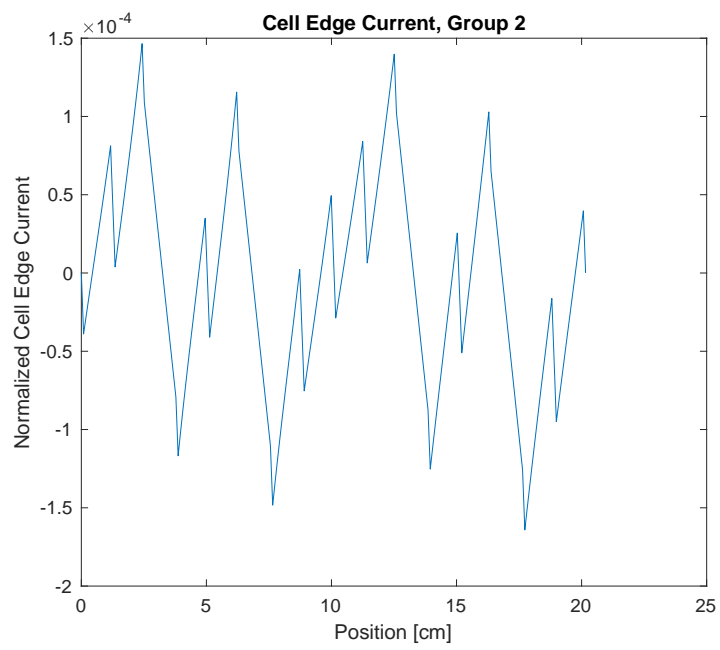


Figure 23: Cell Edge Current versus position in group 2

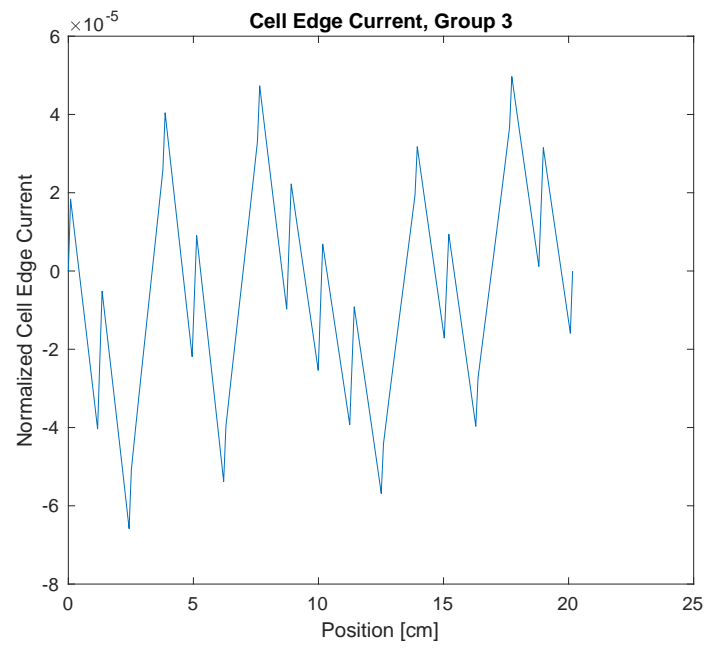


Figure 24: Cell Edge Current versus position in group 3

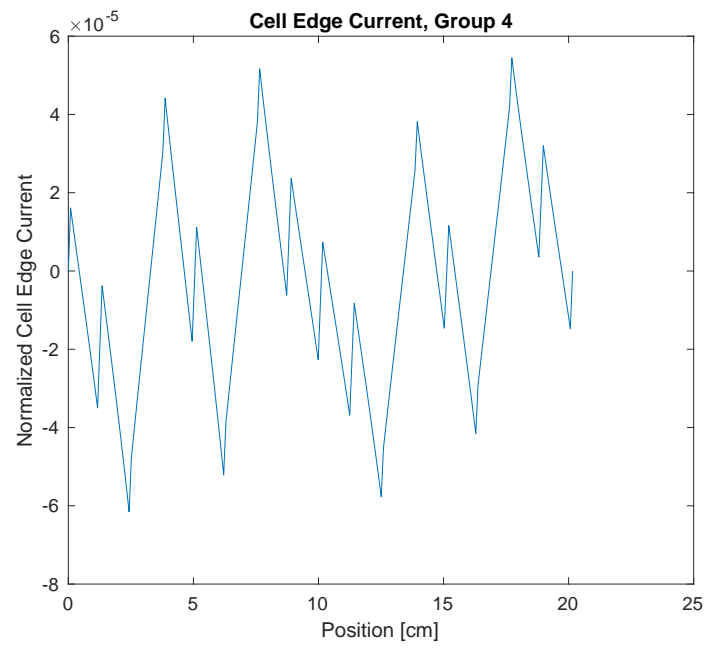


Figure 25: Cell Edge Current versus position in group 4

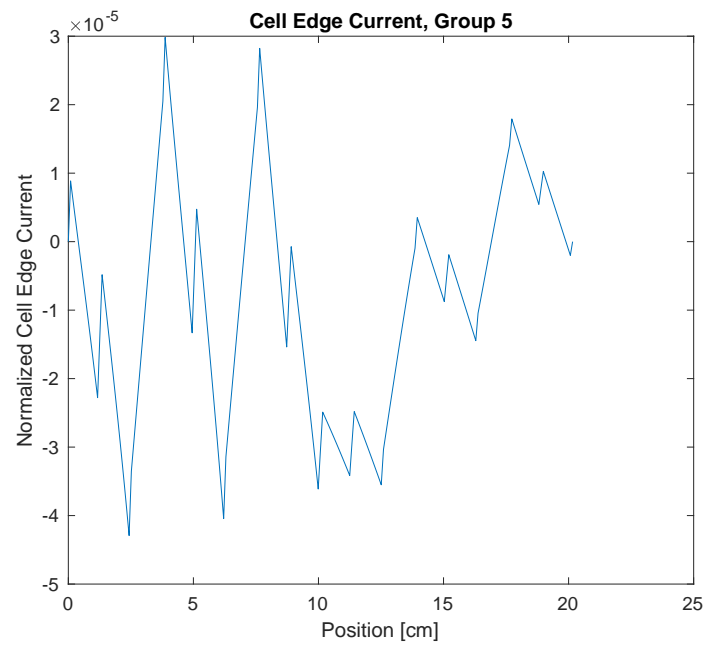


Figure 26: Cell Edge Current versus position in group 5

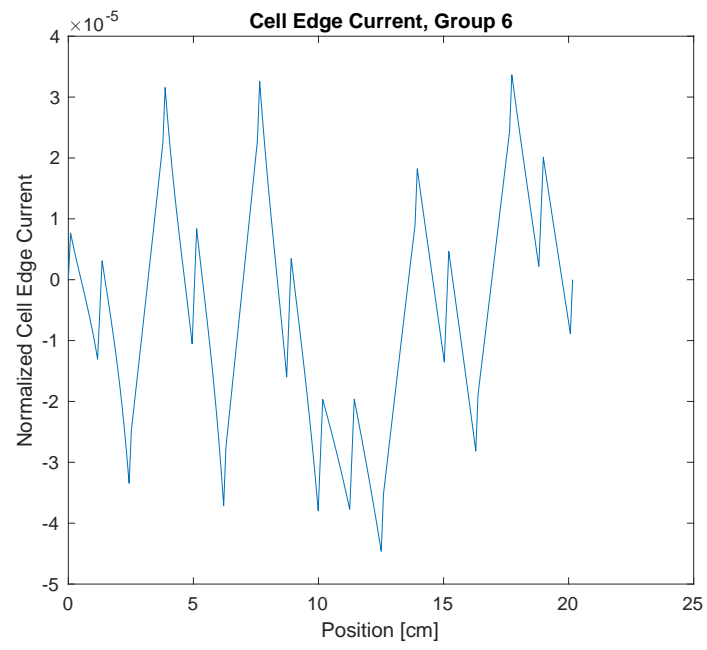


Figure 27: Cell Edge Current versus position in group 6

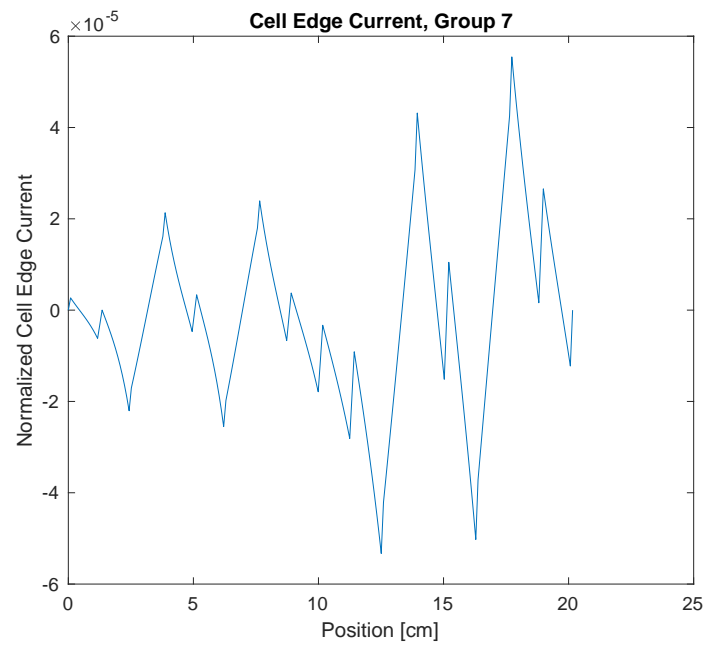


Figure 28: Cell Edge Current versus position in group 7

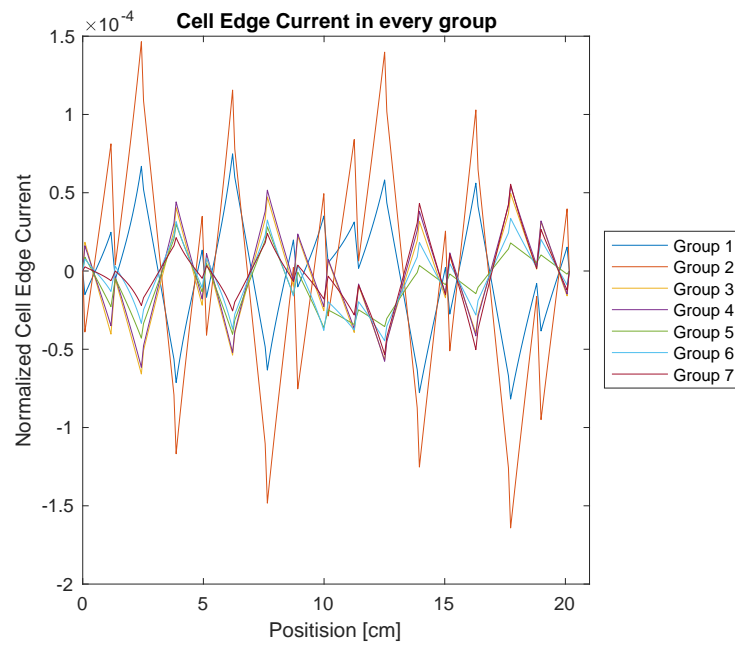


Figure 29: Cell Edge Current versus position for all groups

5.4 Total Normalized Cell Average Scalar Flux

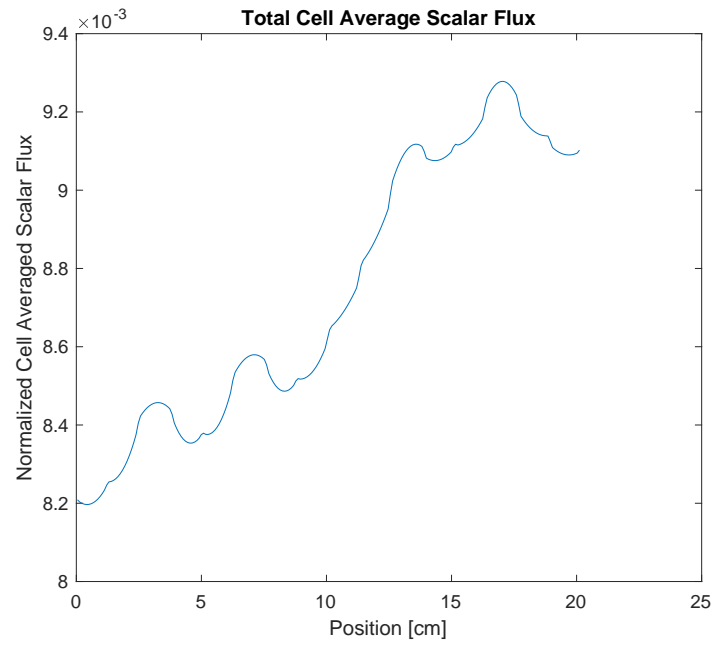


Figure 30: Total scalar flux versus position

5.5 Group Wise Cell Averaged Scalar Flux at \bar{x}_7 and \bar{x}_{214}

Group	$\bar{\phi}$ at \bar{x}_7	$\bar{\phi}$ at \bar{x}_{214}
1	1.464108e-03	1.476168e-03
2	4.590459e-03	4.649489e-03
3	1.596284e-03	1.755405e-03
4	3.665948e-04	4.762296e-04
5	1.421810e-04	3.552932e-04
6	2.783075e-05	2.463252e-04
7	1.107330e-05	1.411047e-04

Table 7: Cell Average Scalar Flux at \bar{x}_7 and \bar{x}_{214}

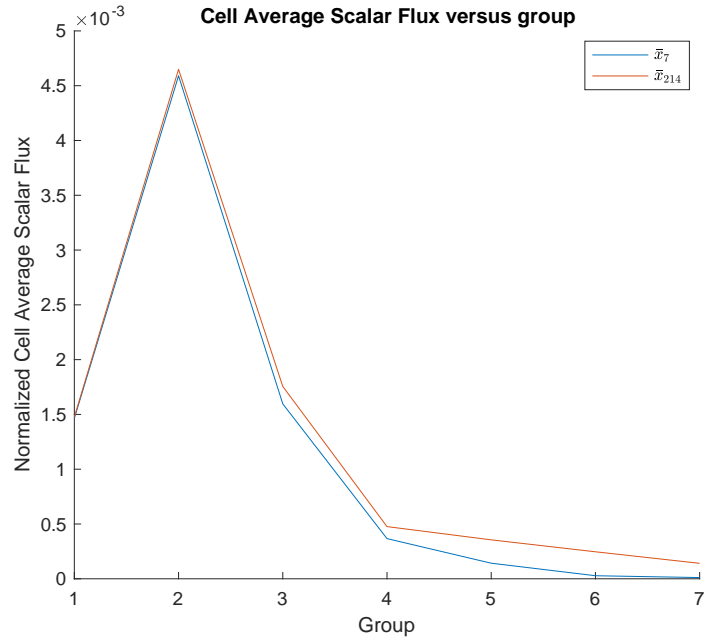


Figure 31: Cell Average Flux versus group at \bar{x}_7 and \bar{x}_{214}

5.6 Group Wise Cell Edge Current at $x = 10.8$

Group	J at $x = 10.8$
1	1.857595e-05
2	3.578377e-05
3	-1.992772e-05
4	-1.805769e-05
5	-3.016380e-05
6	-2.964959e-05
7	-1.676897e-05

Table 8: Cell Edge Current at $x = 10.8$

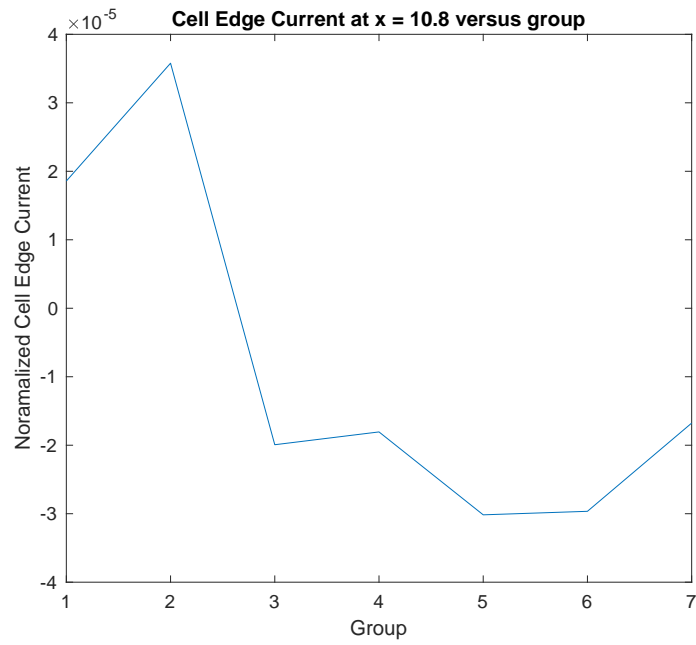


Figure 32: Cell Edge Current at $x = 10.8$ versus group

6 Program Usage

6.1 Usage

The use of the program is quite straight forward as long as you have a version of MATLAB that supports live scripts all of the code will be included in zip file with this submission. Simply unzip the file and run the live script in its entirety. Optionally each test is in its own section and can be run individually. Finally if you are interested in the proof of concept for my matrix formulation please see "Test/Test_Main.mlx". Please note that this program does require the MATLAB symbolic toolbox in order to run to completion. NC State has a full Licence for all toolboxes so this should be free to download if you do not already have it. For additional documentation please see the block comments on individual functions.

6.2 Output

When my program is run it will generate all of the plots and tables needed within the MATLAB live script. I also have several functions that automatically convert MATLAB figures and tables to \LaTeX . These will generate 3 additional folders and text files within your working directory.

7 References

- [1] Dr. Dmitriy Anistratov. *NE 401 Lecture Notes*. NC State University Department of Nuclear Engineering, 2022.
- [2] Dr. Dmitriy Anistratov. *Numerical Methods for Solving k -Eigenvalue Problems for the Multigroup Neutron Diffusion Equations*. NC State University Department of Nuclear Engineering, 2022.
- [3] James Johnson Duderstadt and Louis James Hamilton. Wiley, 1976.