

# **CS 5003: Parameterized Algorithms**

**Lectures 16-17**

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**Reference Books: Parameterized Algorithms by Cygan et al. and Kernelization by Fomin et al.**

# Vertex Cover Above LP

## Vertex Cover Above LP

Instance: A graph  $G$  on  $n$  vertices  $m$  edges and integer  $k$

Question: Does  $G$  have a vertex cover of size at most  $k$ ?

Parameter:  $k - \text{lp}(G)$

$$\text{lp}(G) \geq |M|$$

## Vertex Cover Above Matching

Instance: A graph  $G$  on  $n$  vertices  $m$  edges, integer  $k$ , a matching  $M$

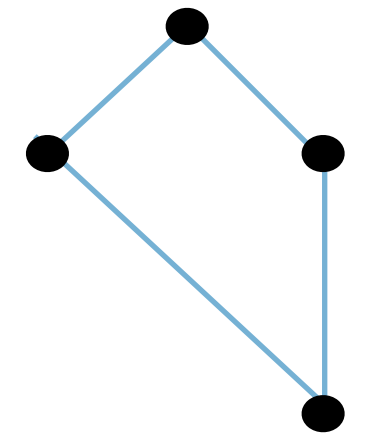
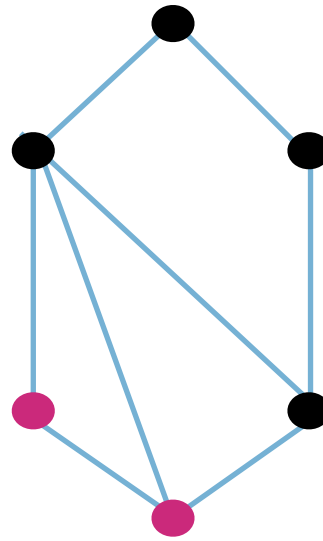
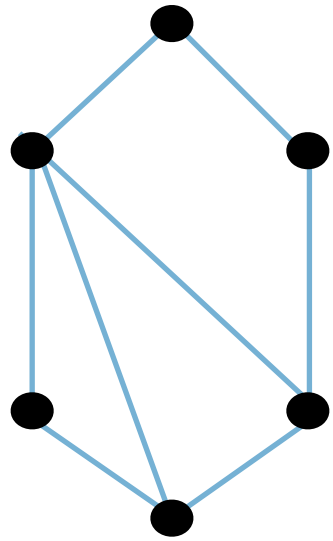
Question: Does  $G$  have a vertex cover of size at most  $k$ ?

Parameter:  $k - |M|$

$4^{(k - \text{lp}(G))} n^{O(1)}$  time algorithm is a  $4^{(k - |M|)} n^{O(1)}$  time algorithm

# Odd Cycle Transversal

**OCT** - set of vertices that has at least one vertex of every odd length cycle



bipartite

A graph is bipartite iff it has no odd cycle

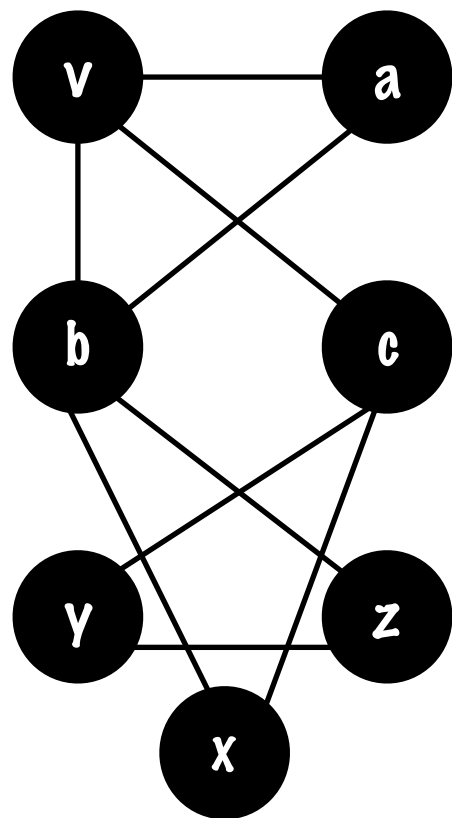
## Odd Cycle Transversal

Instance: A graph  $G$  on  $n$  vertices  $m$  edges and integer  $k$

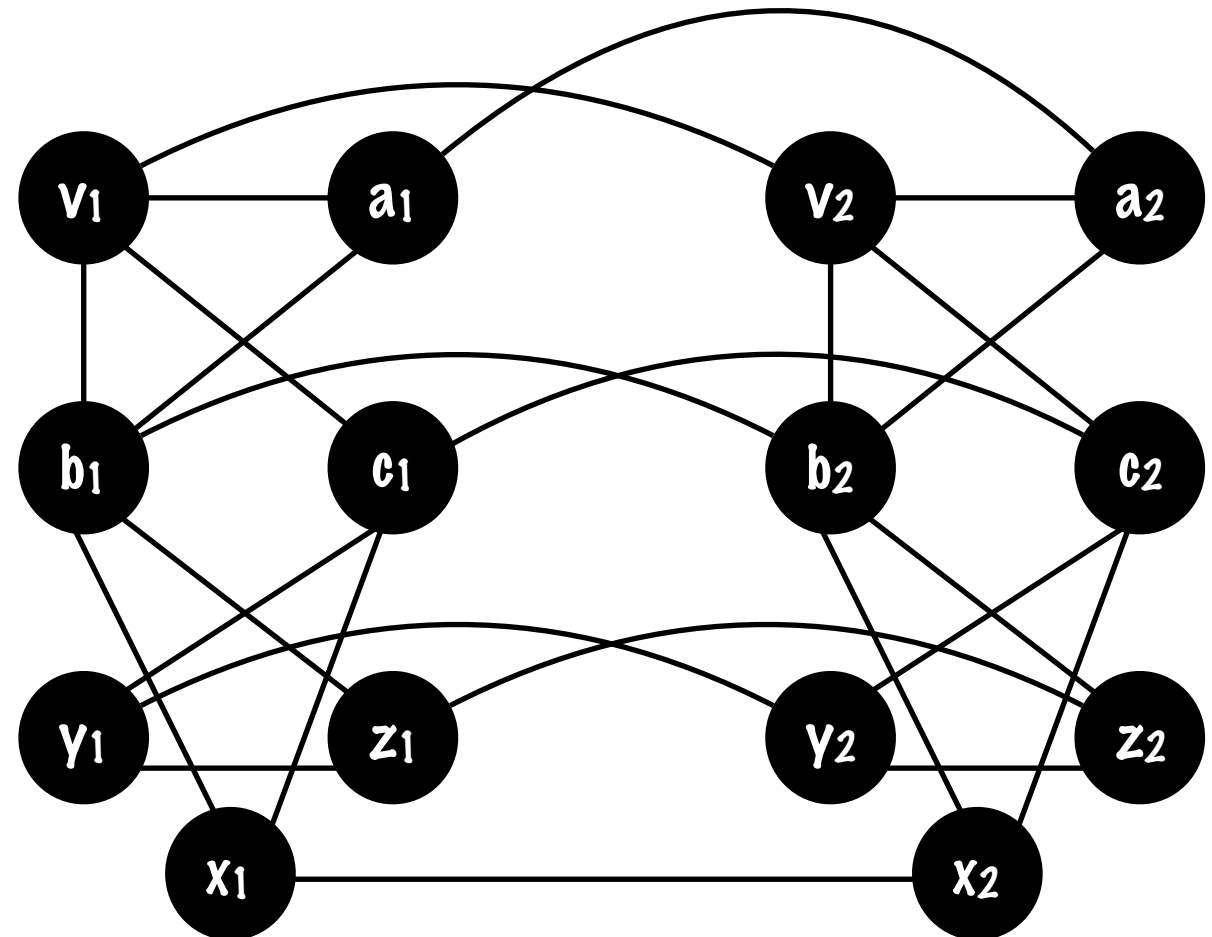
Question: Does  $G$  have an oct of size at most  $k$ ?

Parameter:  $k$

# OCT Reduces to VC Above LP



$(G, k)$  OCT

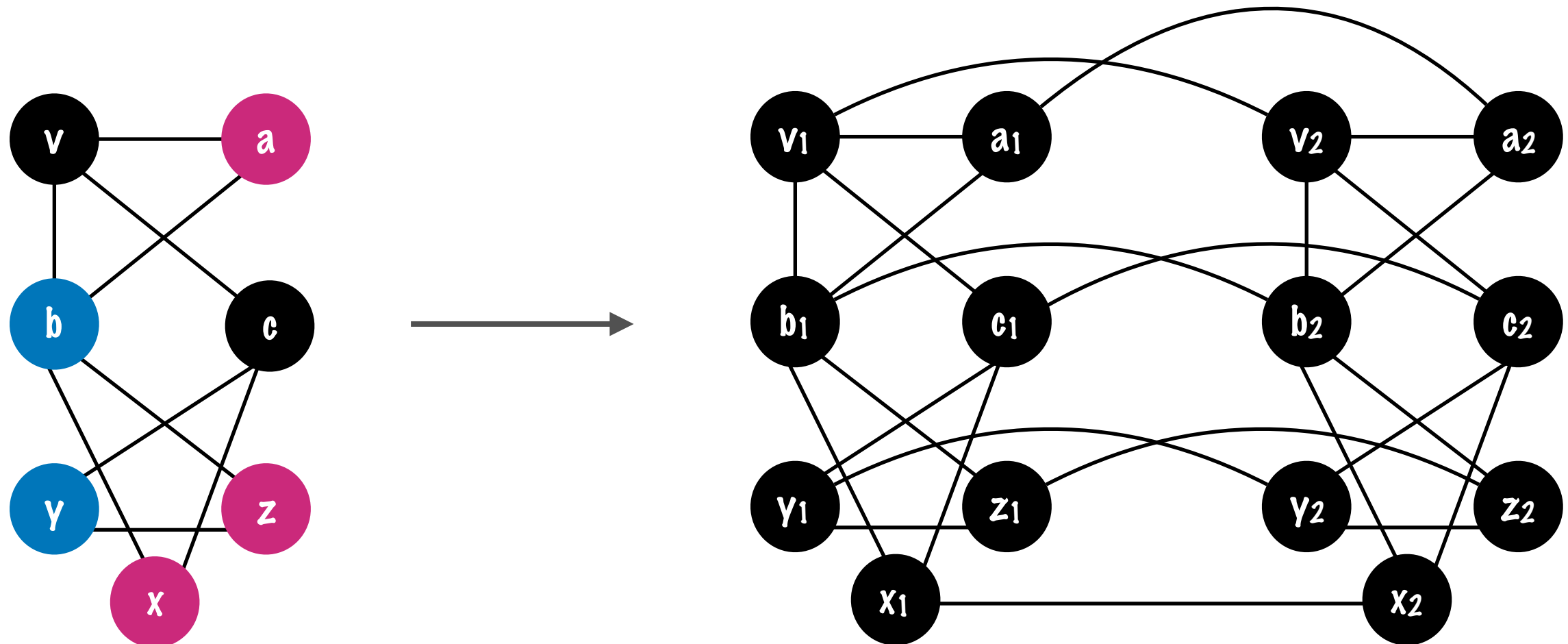


$(H, |V(G)|+k)$  Vertex Cover

$G$  has an OCT of size  $k$  iff  $H$  has a VC of size  $|V(G)|+k$

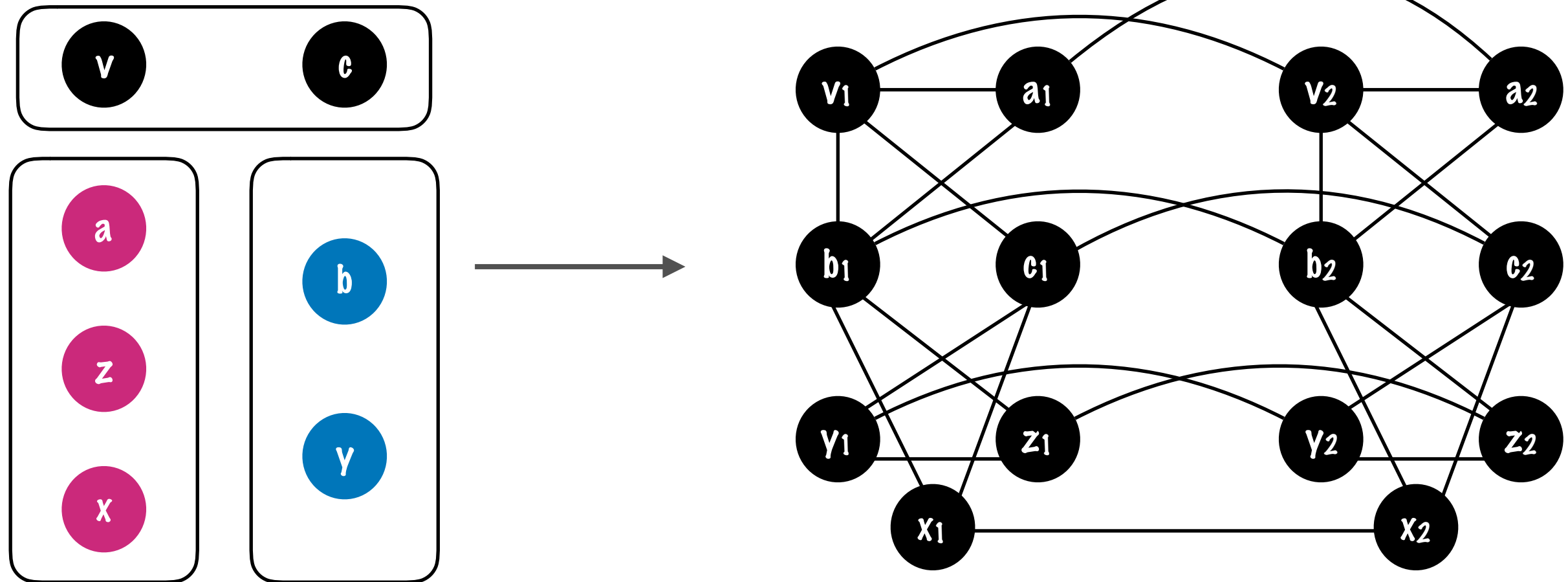
# OCT Reduces to VC Above LP

Suppose  $G$  has an OCT of size  $k$



# OCT Reduces to VC Above LP

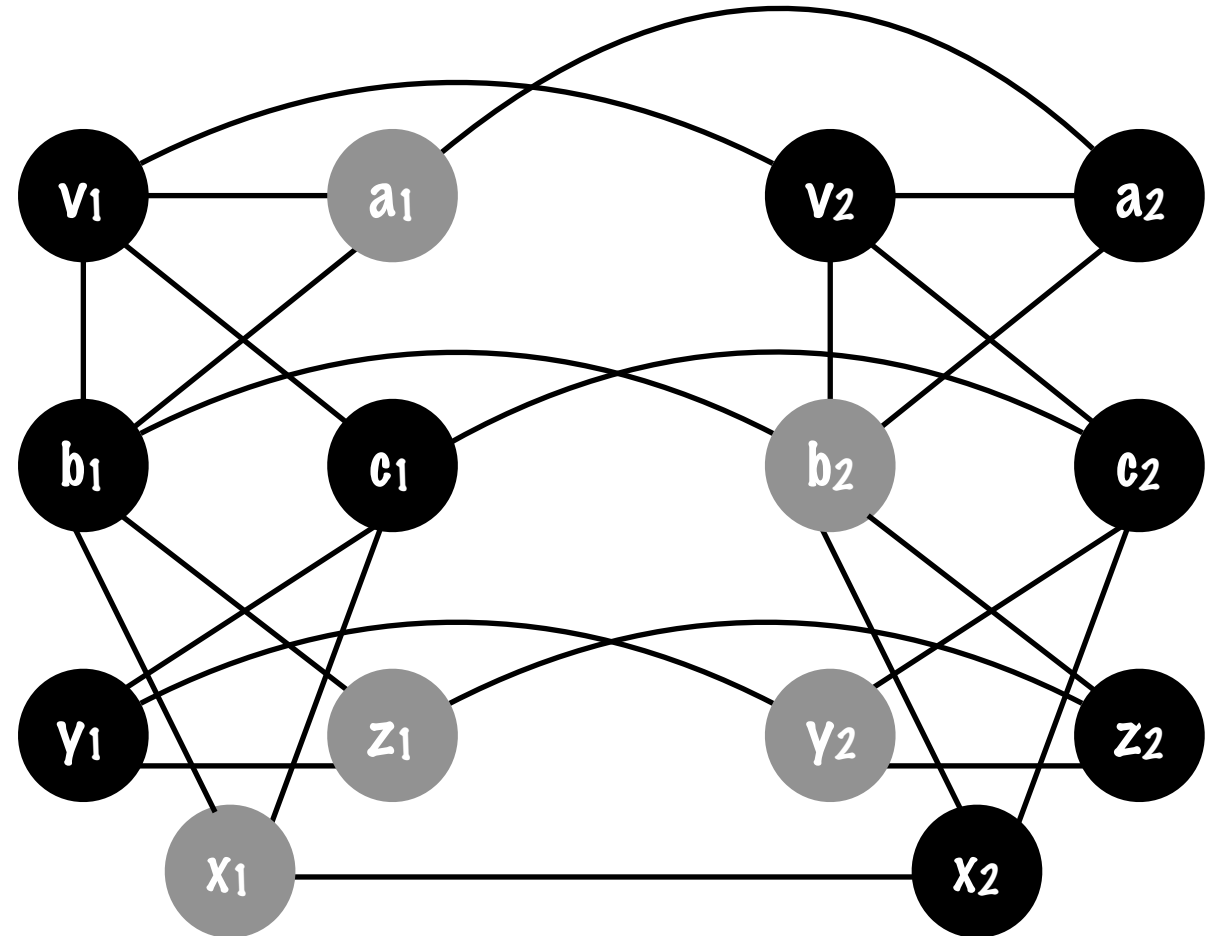
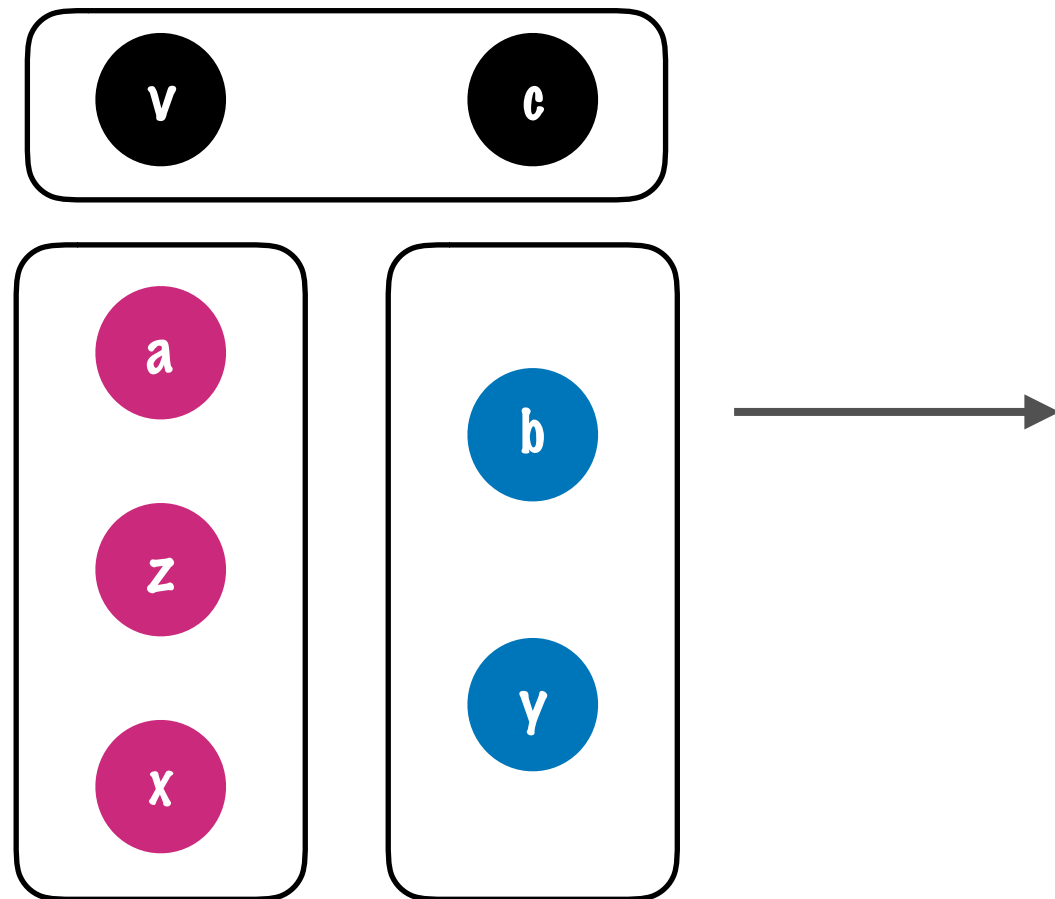
Suppose  $G$  has an OCT of size  $k$



$G$  has a bipartite graph of size  $|V(G)| - k$

# OCT Reduces to VC Above LP

Suppose  $G$  has an OCT of size  $k$

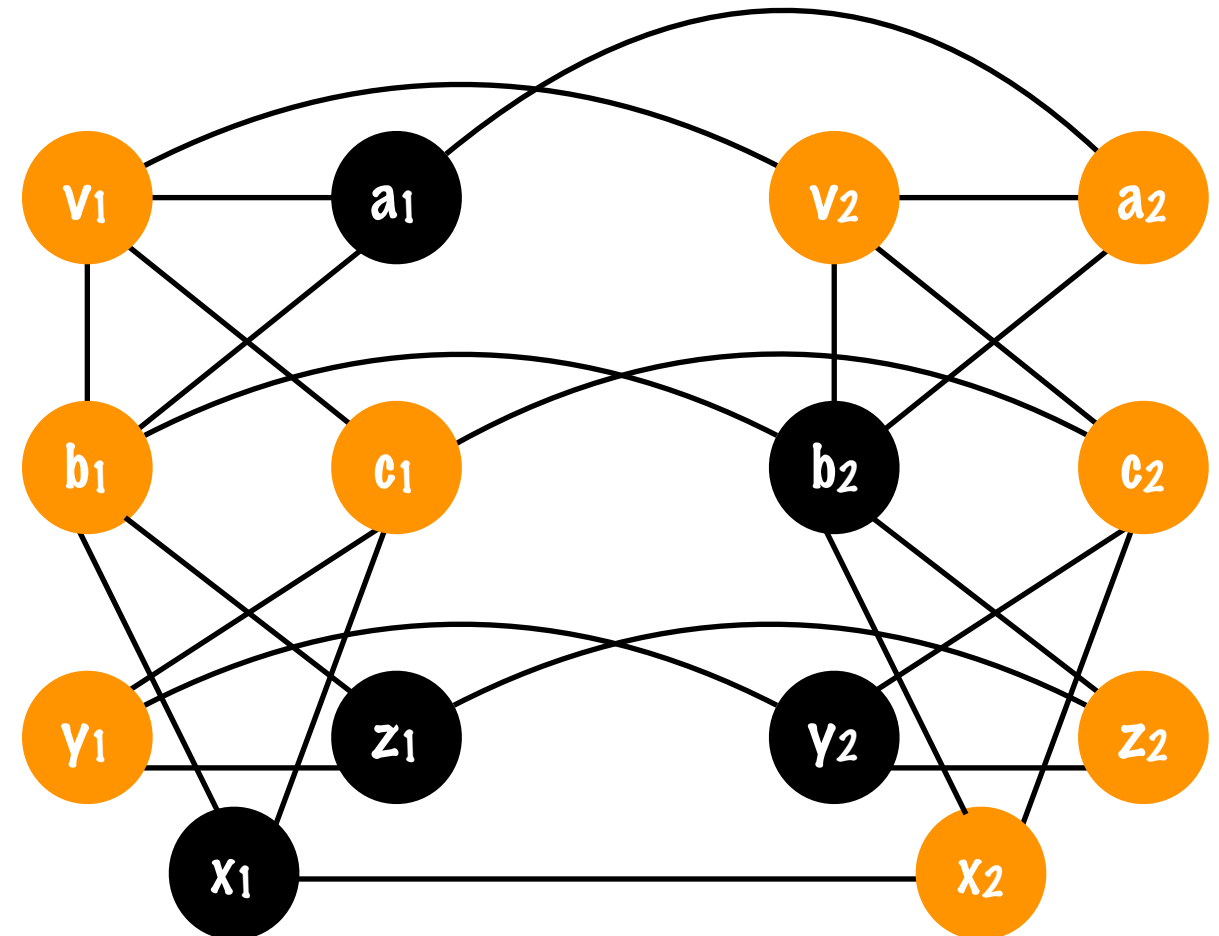
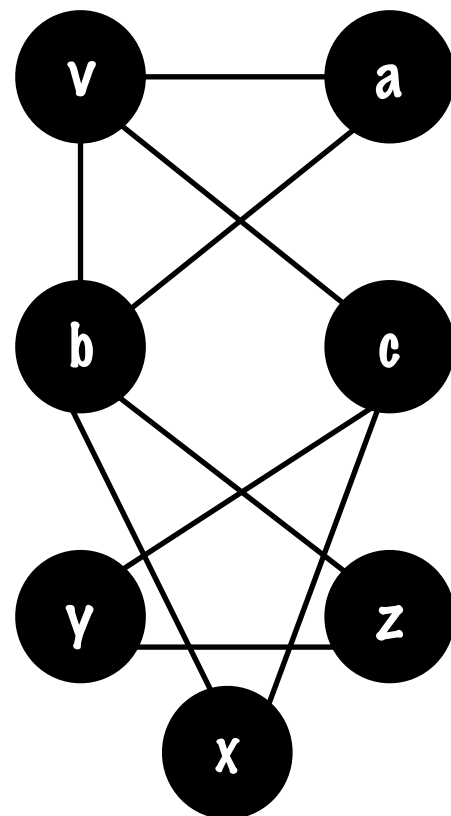


$H$  has an IS of size  $|V(G)| - k$

$H$  has a VC of size  $|V(G)| + k$

# OCT Reduces to VC Above LP

Suppose  $H$  has a VC of size  $|V(G)|+k$

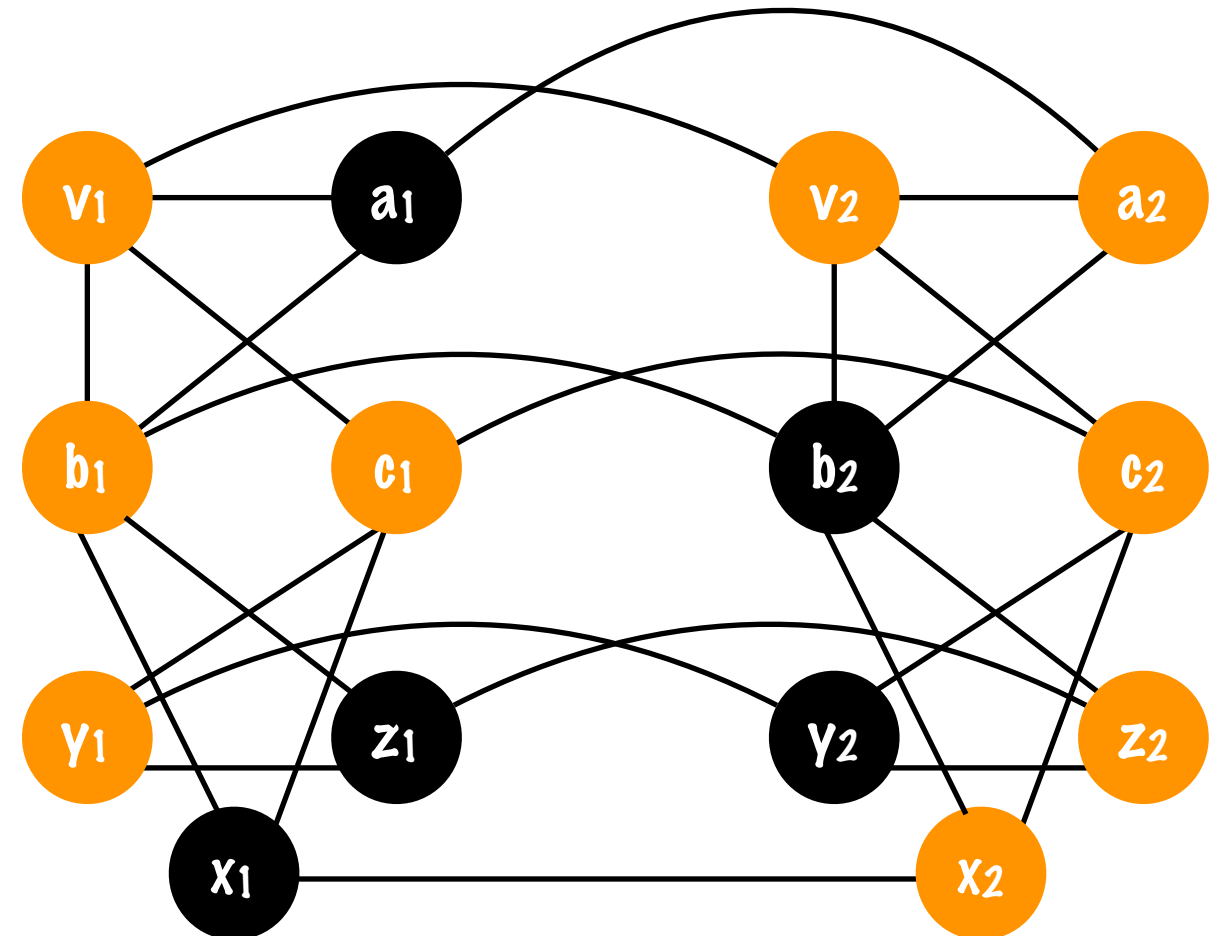
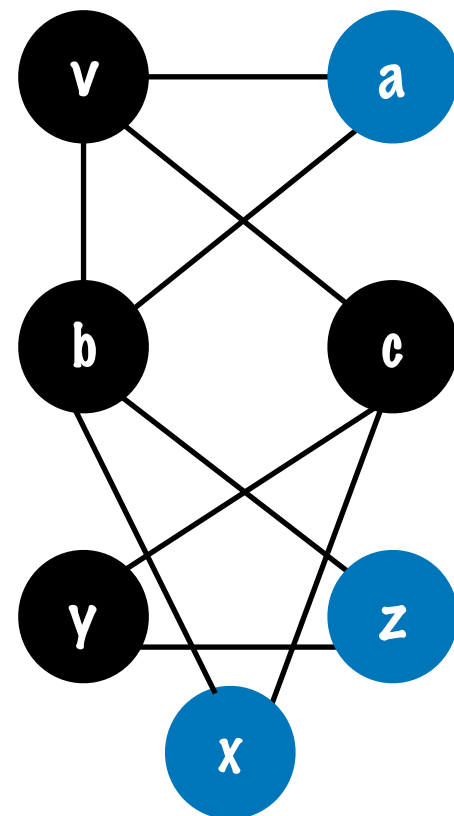


$H$  has an IS of size  $|V(G)|-k$



# OCT Reduces to VC Above LP

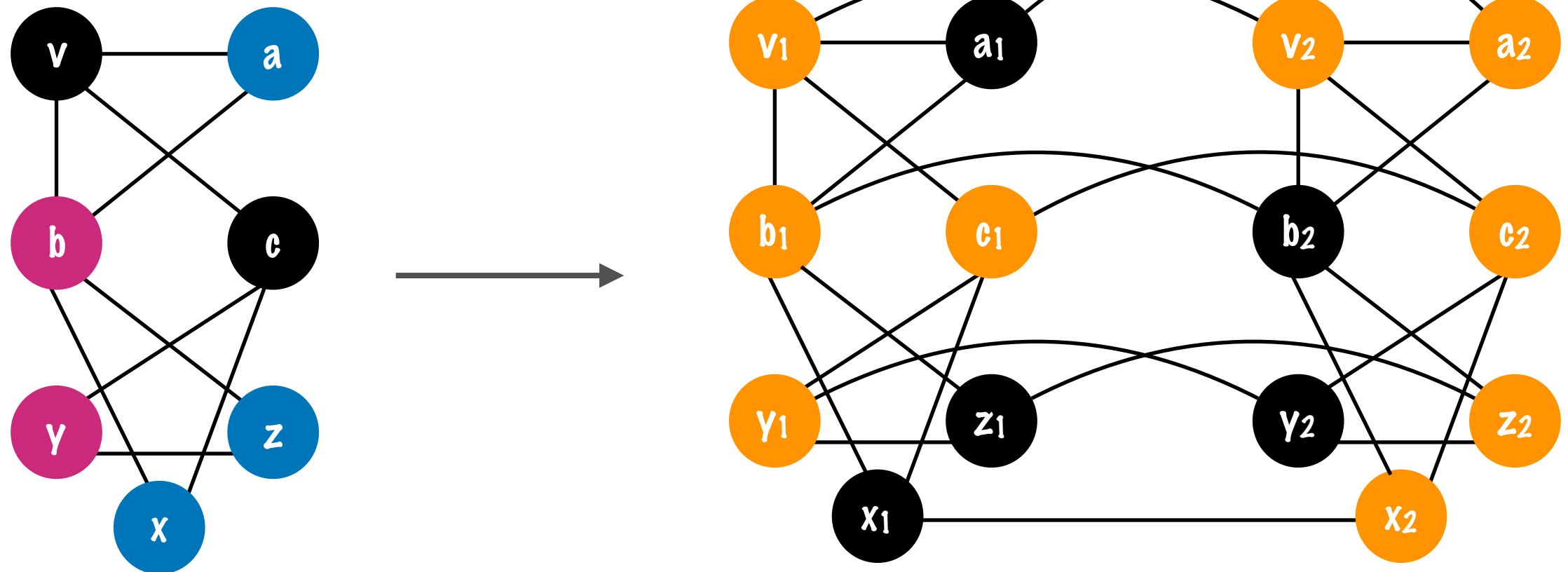
Suppose  $H$  has a VC of size  $|V(G)|+k$



$H$  has an IS of size  $|V(G)|-k$

# OCT Reduces to VC Above LP

Suppose  $H$  has a VC of size  $|V(G)|+k$



$G$  has a bipartite graph of size  $|V(G)|-k$

$G$  has an OCT of size  $k$

# OCT Reduces to VC Above LP



$G$  has an OCT of size  $k$  iff  $H$  has a VC of size  $|V(G)|+k$

- \* To determine if  $G$  on  $n$  vertices has an OCT of size  $k$ ,
  - \* Construct  $H$  from  $G$
  - \* Matching  $M$  of size  $n$  in  $H$
  - \* Determine if  $H$  has a VC of size  $n+k$ 
    - \* Use the Vertex Cover Above LP algorithm on  $H$  that has matching  $M$ 
      - \*  $(4^{n+k-|popt(H)|}) n^{O(1)}$  time which is  $4^{n+k-|M|=k} n^{O(1)}$  time)

# Iterative Compression

Does  $G$  have an odd cycle transversal of size at most  $k$ ?

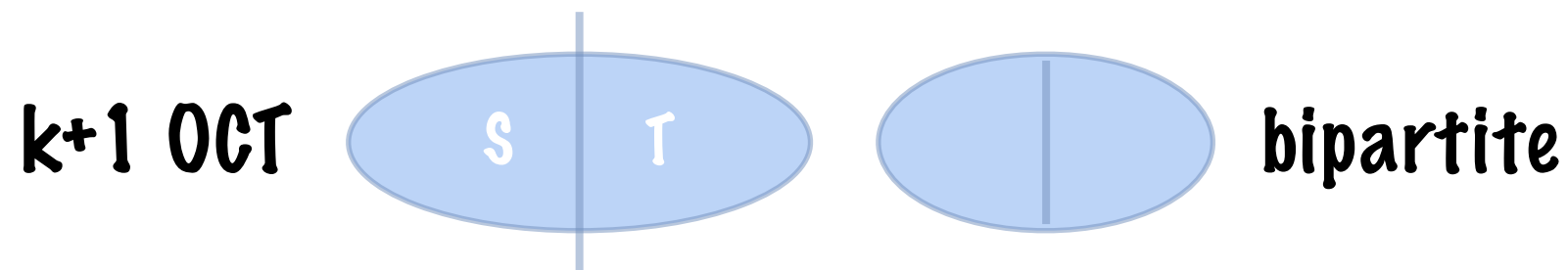
$O^*(f(k))$  algorithm

reduces to  
in  $2^{k+1}$  time

Given an odd cycle transversal  $T$  of  $G$ , find a smaller disjoint odd cycle transversal

$O^*(f(|T|))$  algorithm

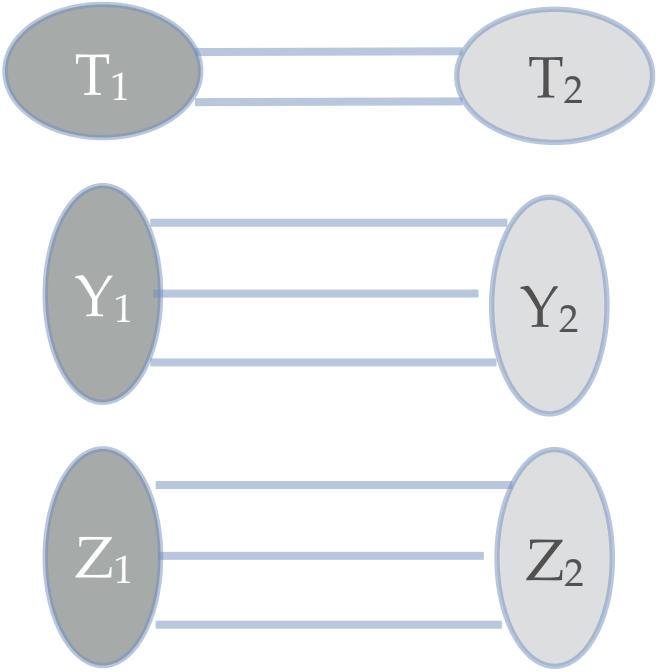
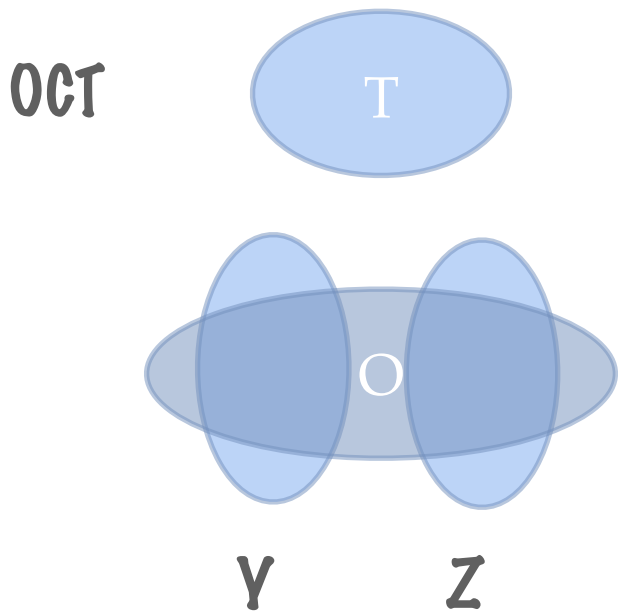
- \* Assume by induction that  $G$  has an OCT of size  $k+1$ 
  - \* Base case: any subgraph on  $k+3$  vertices has an OCT of size  $k+1$
- \* Guess its intersection  $S$  with a smaller solution ( $2^{k+1}$  choices)



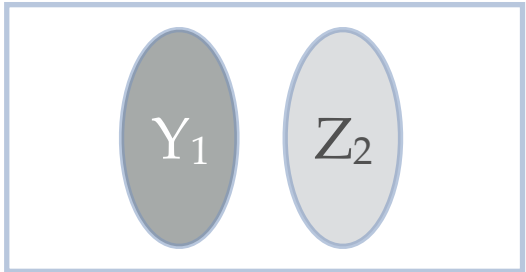
Solve disjoint compression step: given an OCT, find a smaller disjoint OCT

# Disjoint Compression for OCT

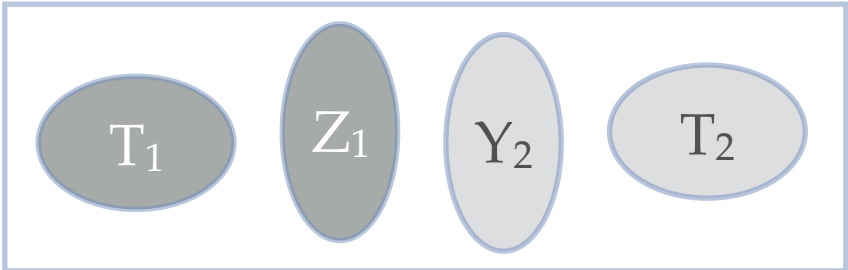
Given an OCT  $\mathcal{T}$ , find a smaller disjoint OCT  $\mathcal{O}$



$\cong$



IS

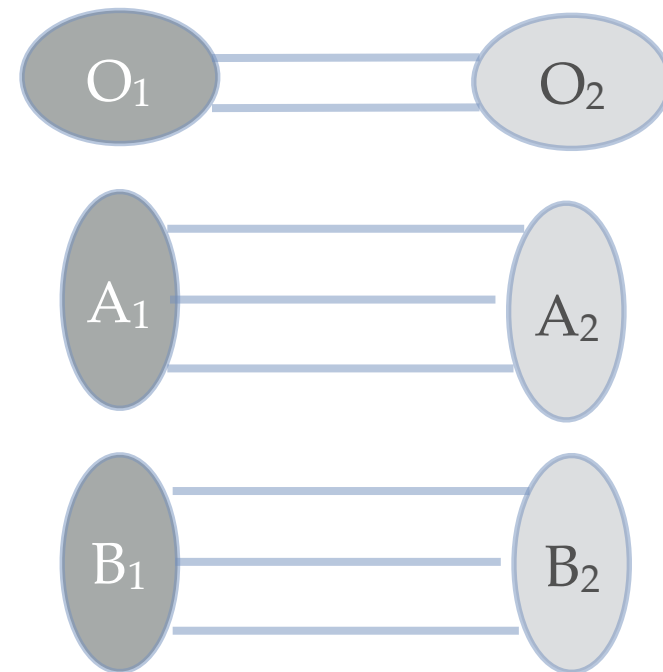
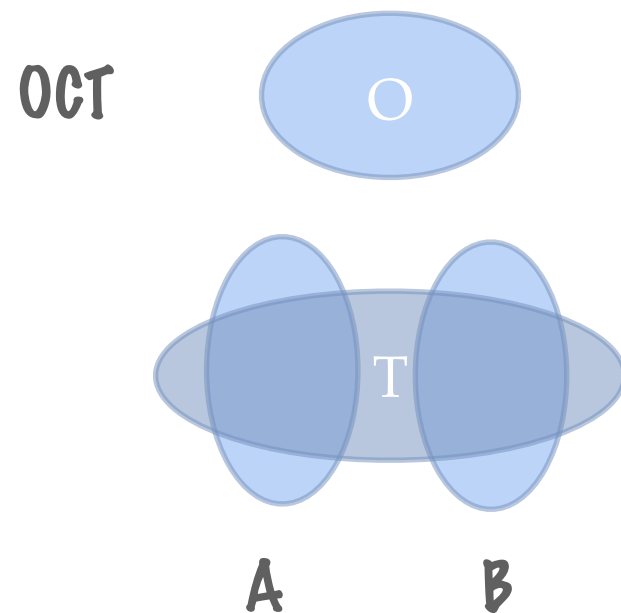


VC

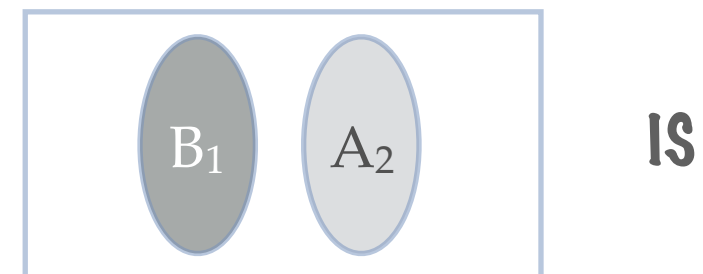
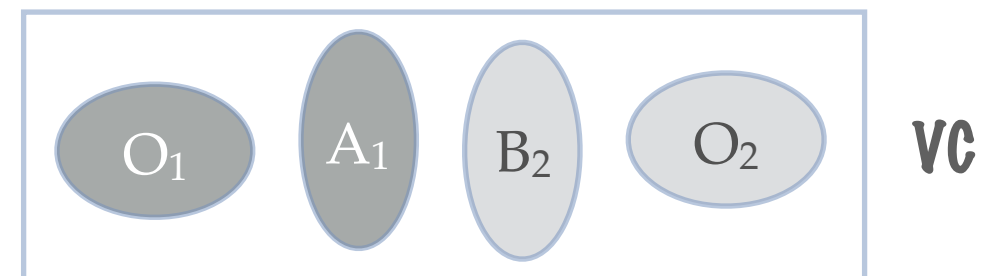
$\mathcal{T}$  is the set of those vertices whose both copies are in the vertex cover

# Disjoint Compression for OCT

Given an OCT  $T$ , find a smaller disjoint OCT  $O$



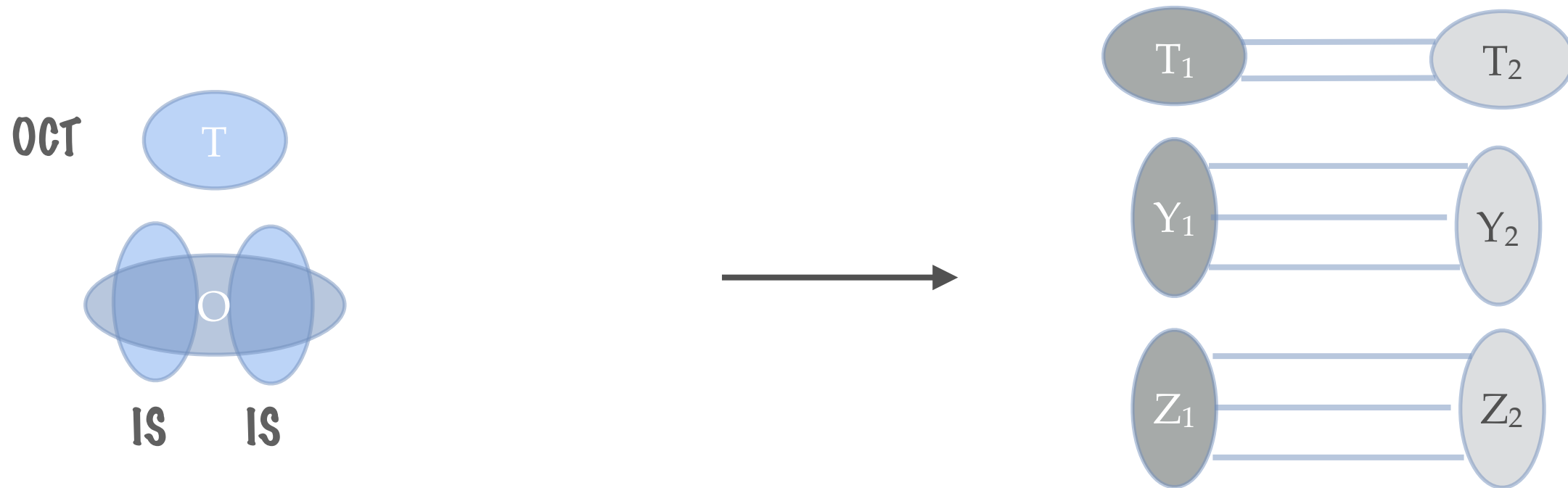
$\cong$



$O$  is the set of those vertices whose both copies are in the vertex cover

# Disjoint Compression for OCT

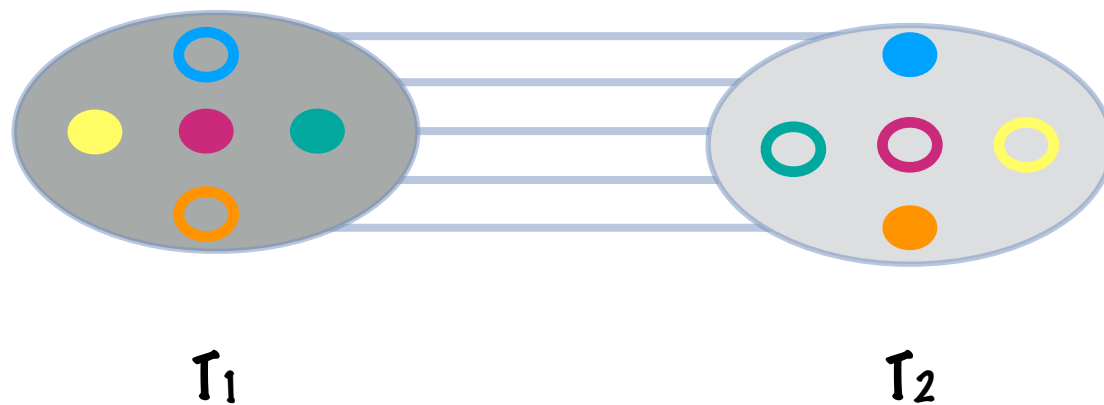
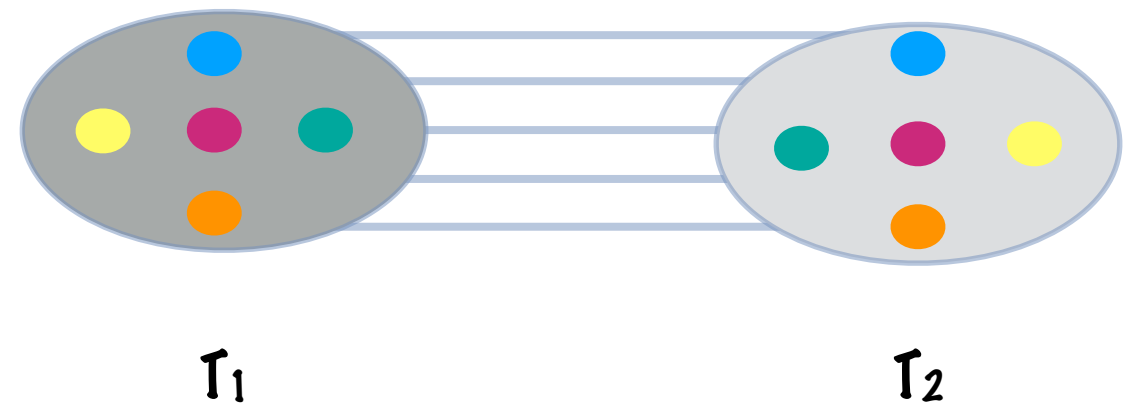
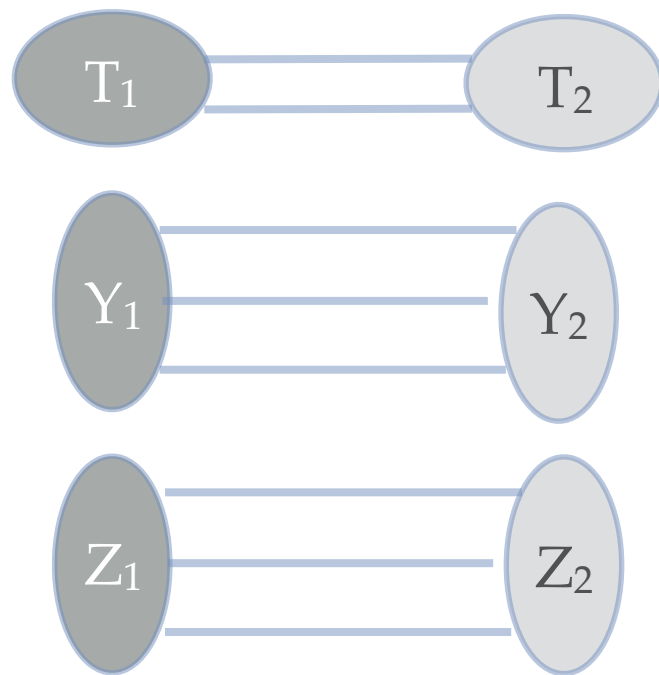
Given an OCT  $T$ , find a smaller disjoint OCT  $O$



Find a min vertex cover that covers the edges across  $T_1$  and  $T_2$  exactly once

# Disjoint Compression for OCT

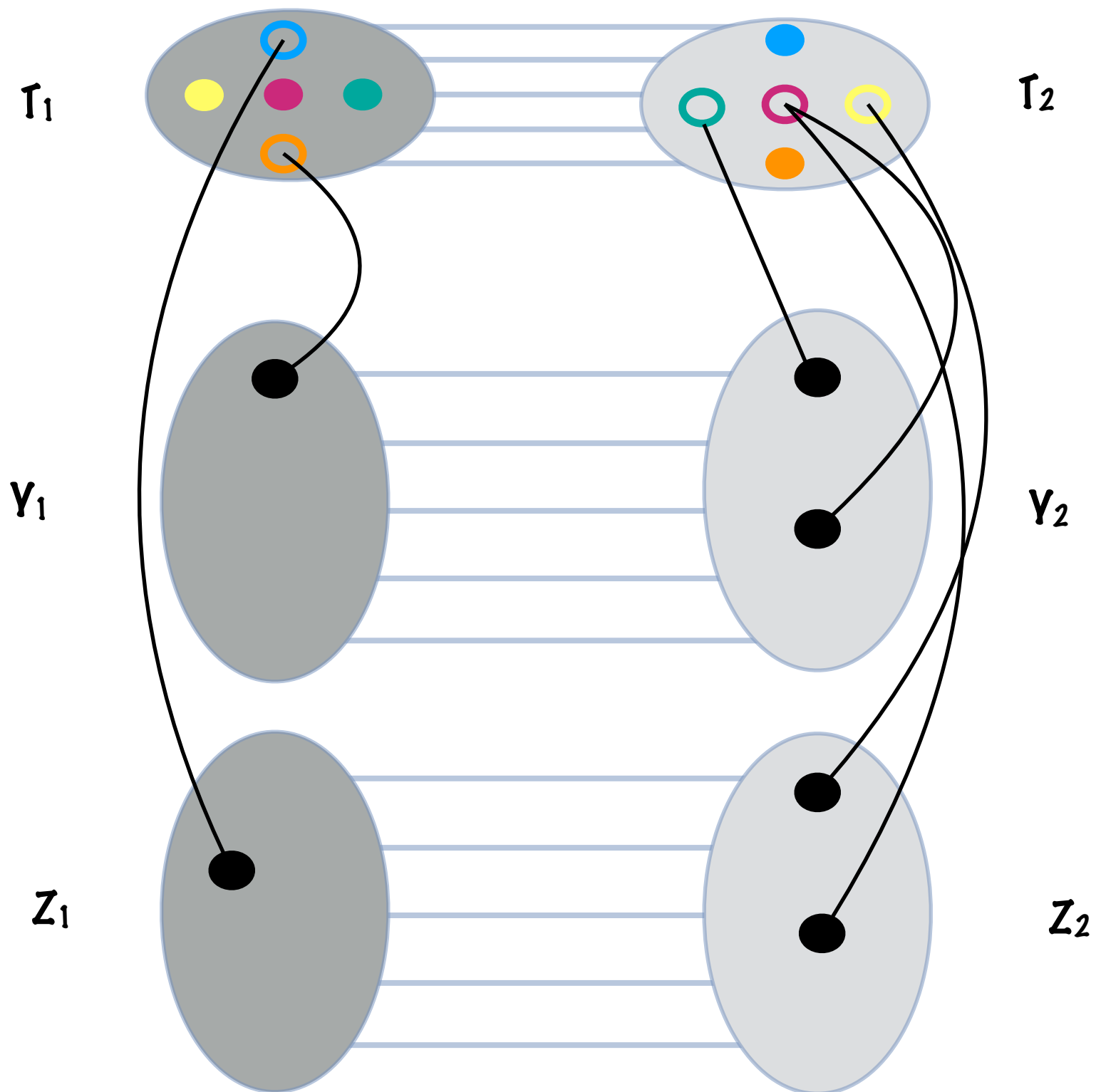
Find a min vertex cover that covers the edges across  $T_1$  and  $T_2$  exactly once



$2^{|T_1|}$  choices

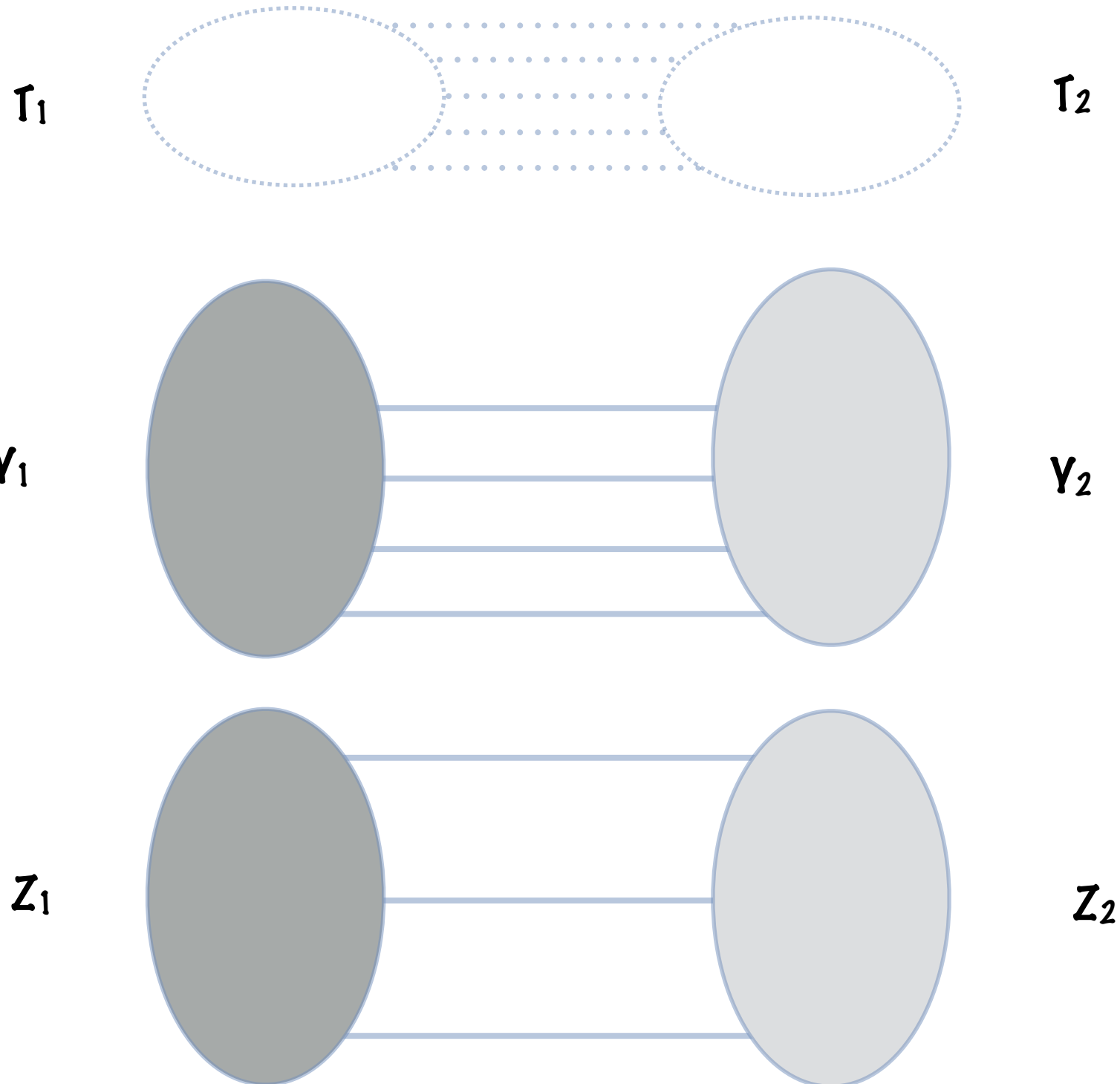


# Disjoint Compression for OCT



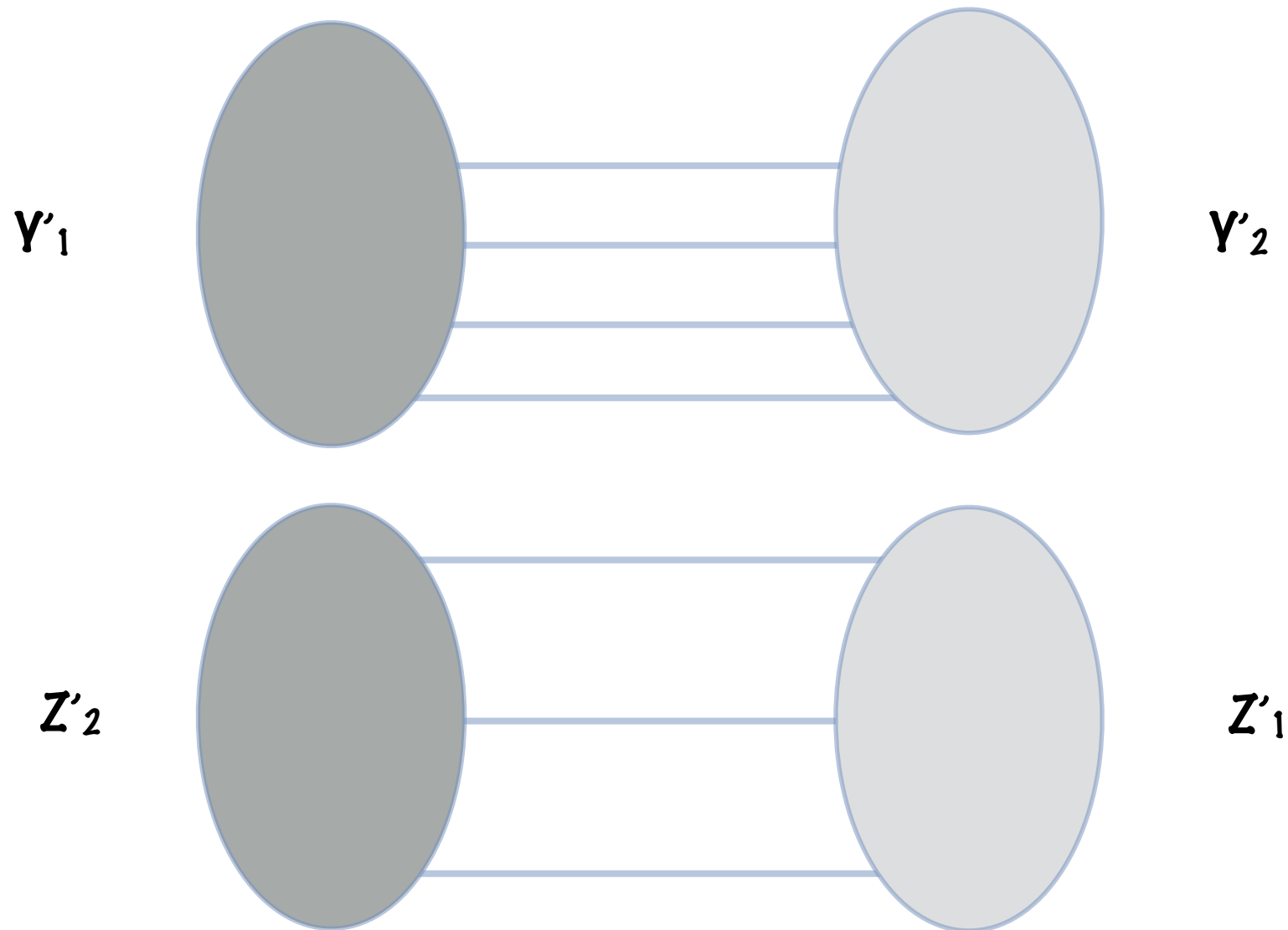
# Disjoint Compression for OCT

Find a min vertex cover



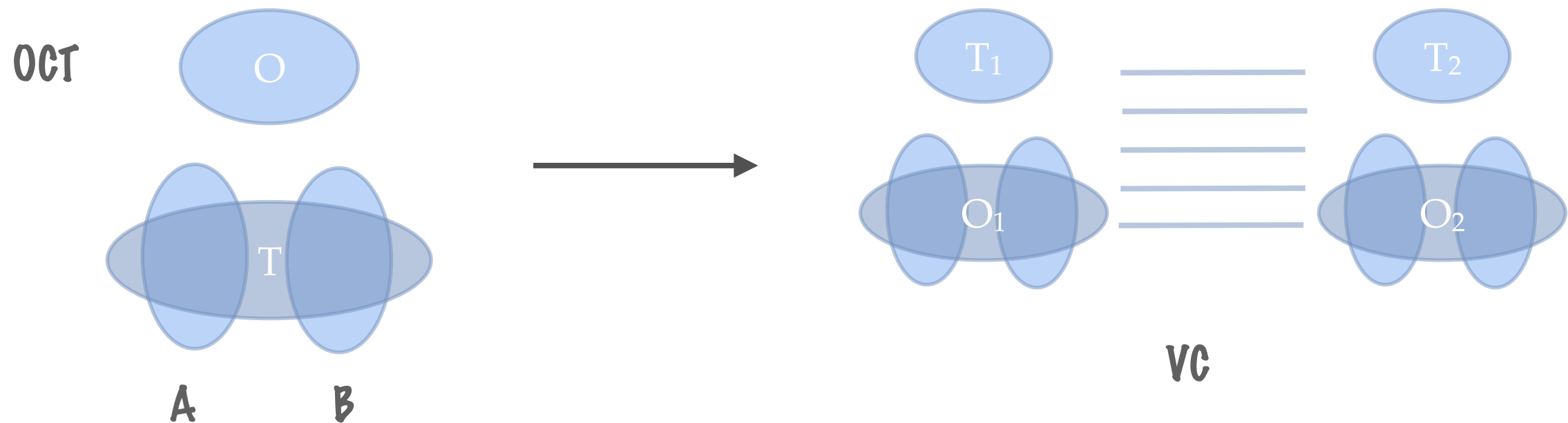
# Disjoint Compression for OCT

Find a min vertex cover in a bipartite graph

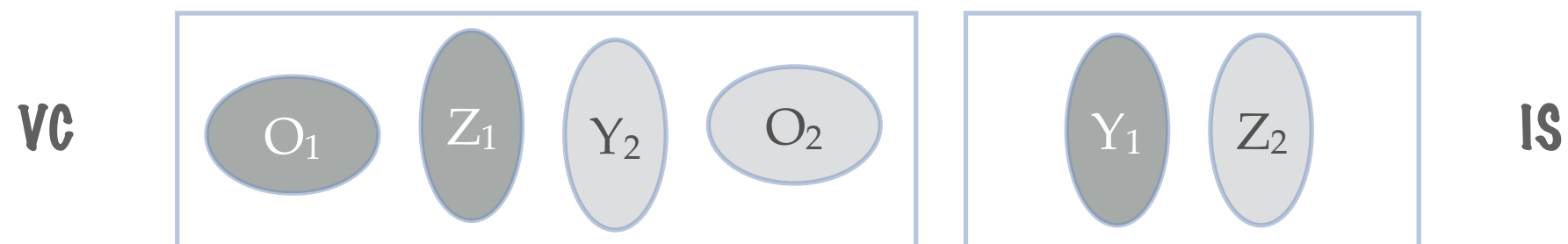


Polynomial time

# Disjoint Compression for OCT



Find a min vertex cover that covers the edges across  $T_1$  and  $T_2$  exactly once



$O^*(3^k)$  algorithm for OCT