

CS 5003: Parameterized Algorithms

Lecture 7

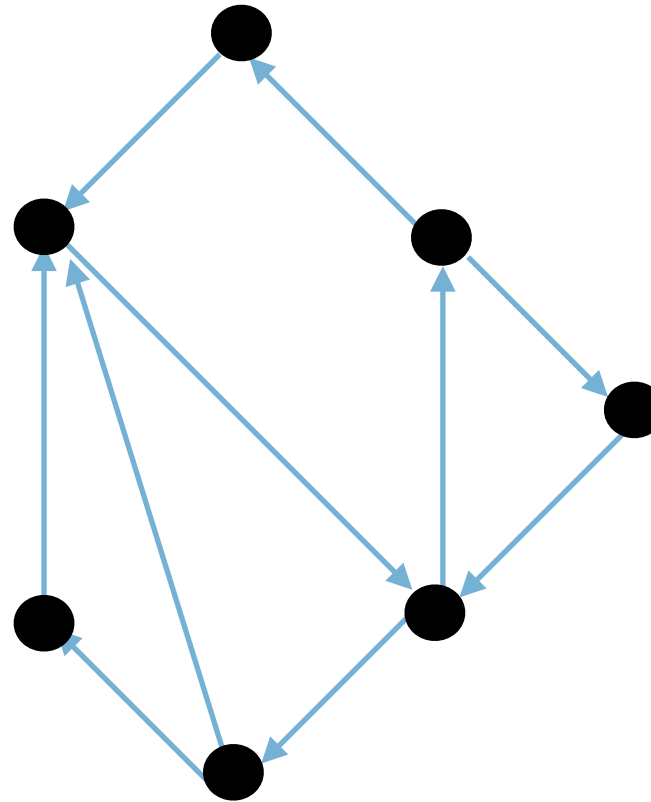
Krithika Ramaswamy

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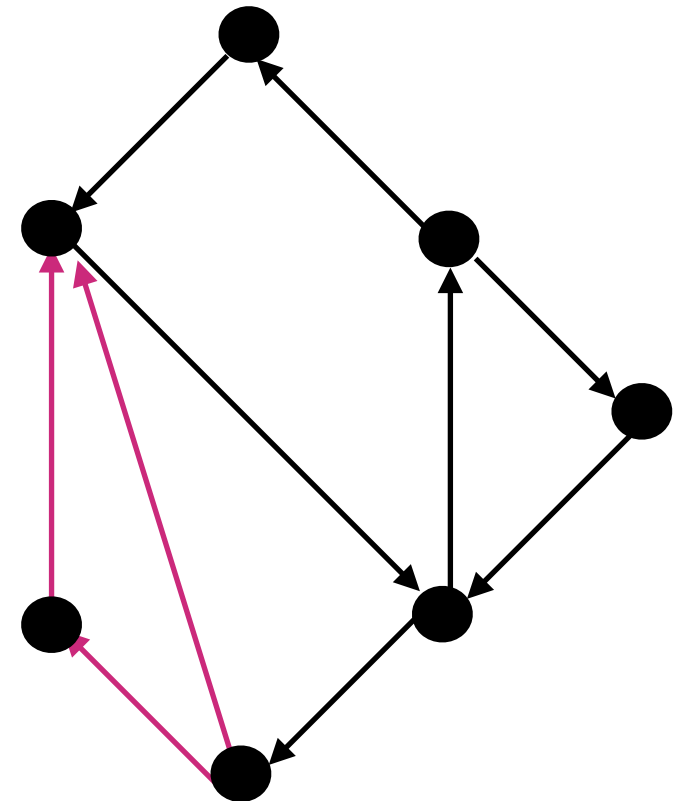
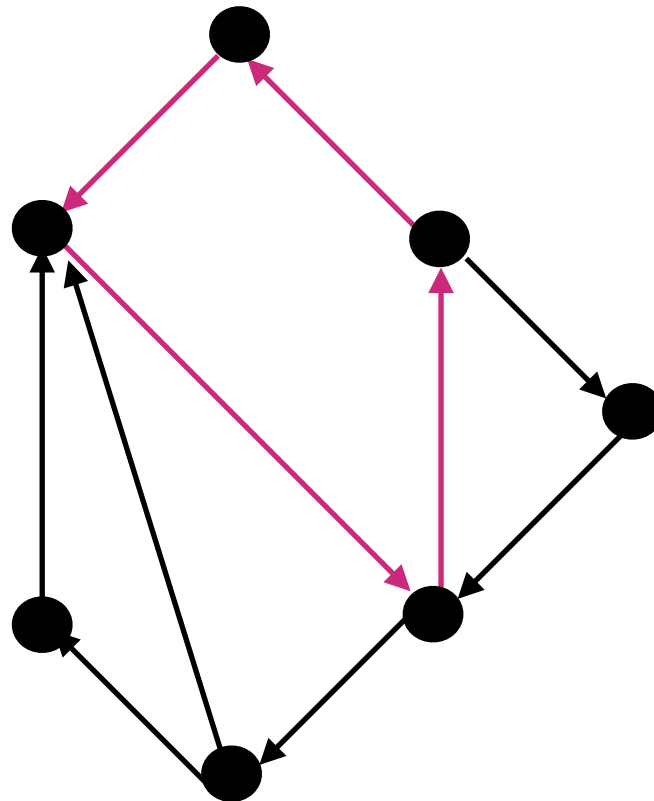
Reference Books: Parameterized Algorithms by Cygan et al. and Kernelization by Fomin et al.

Directed Graphs

Every edge has direction: **arc**

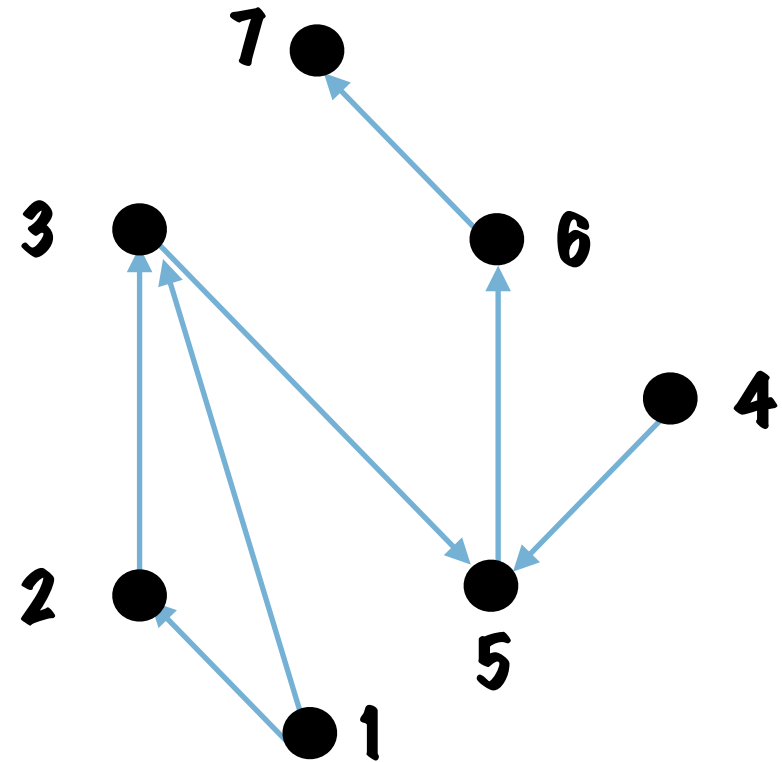


Directed cycle

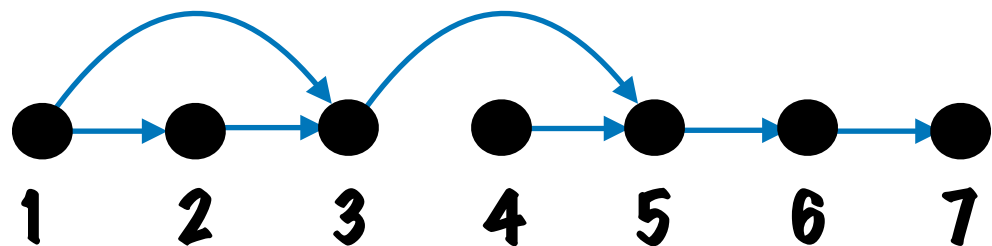


Directed Acyclic Graphs

Directed graphs with no directed cycle



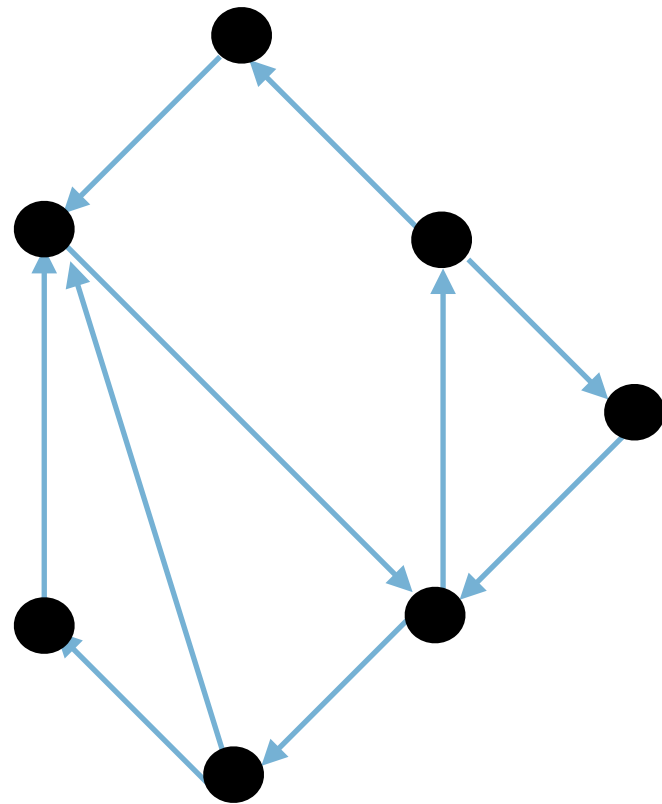
Lemma: A digraph is a DAG iff it has a topological ordering



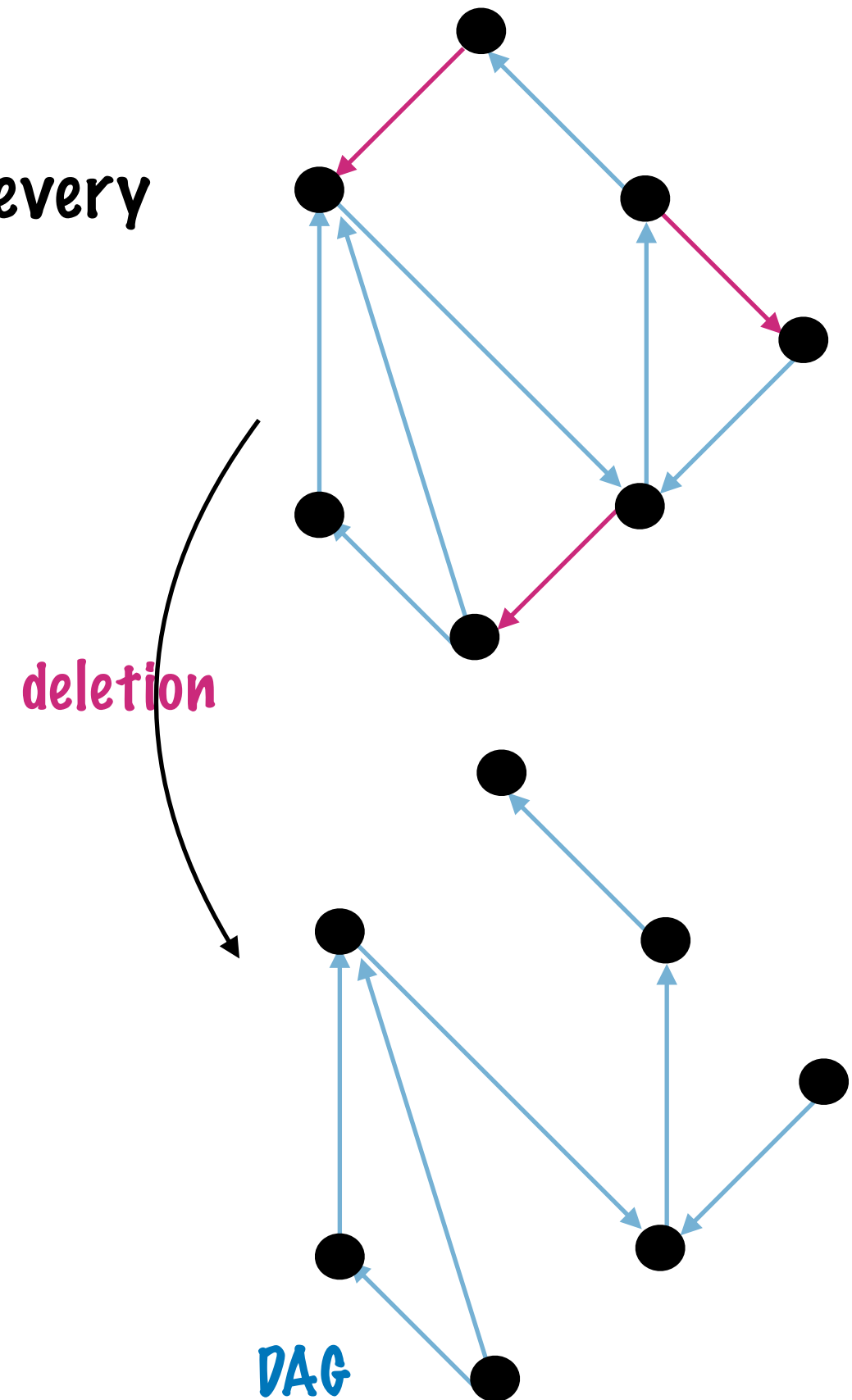
Topological ordering

Feedback Arc Set

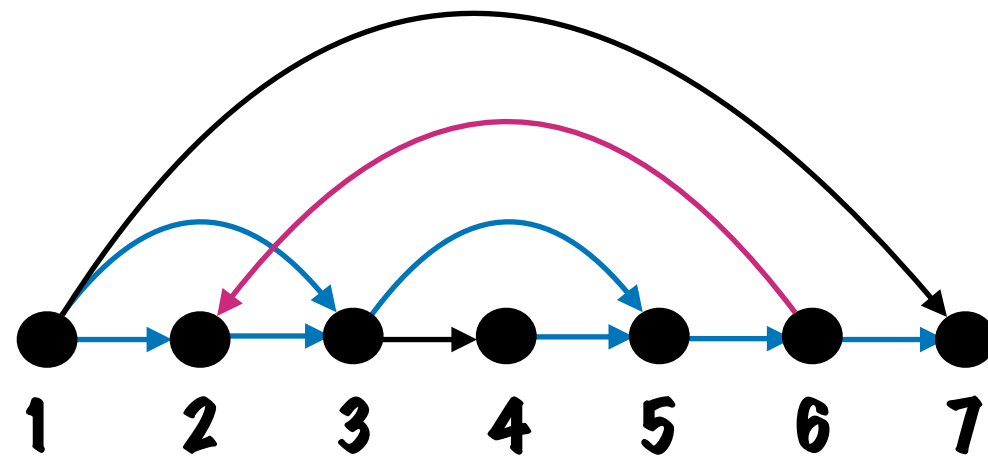
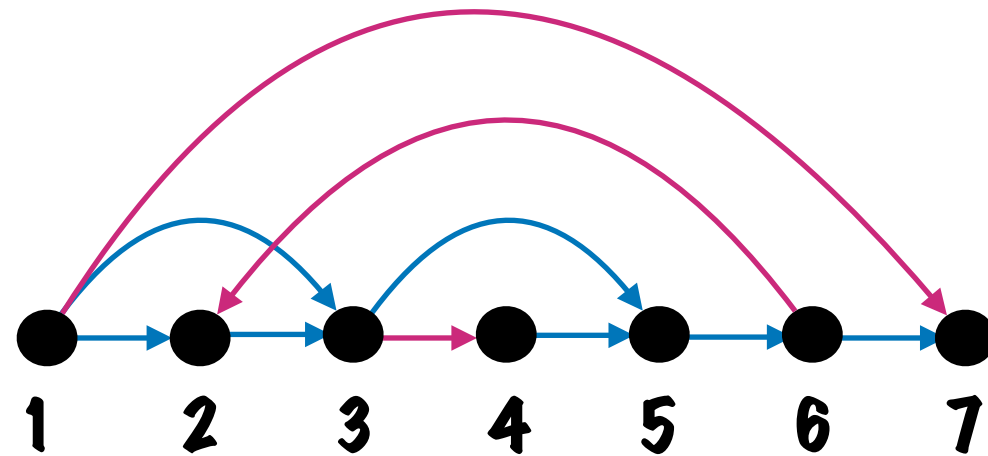
FAS - set of arcs that has at least one arc of every directed cycle



Digraph



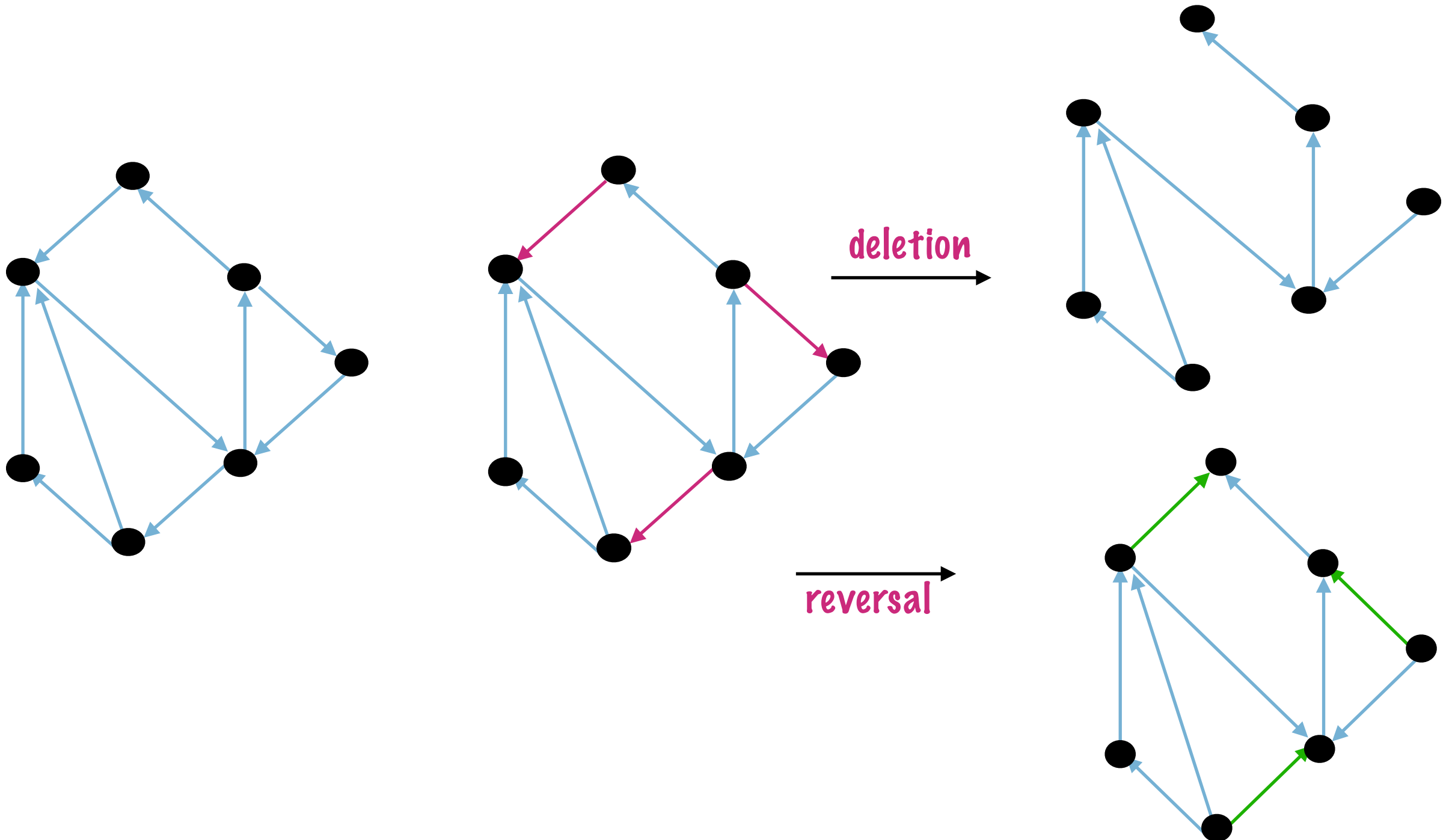
Minimal Feedback Arc Set



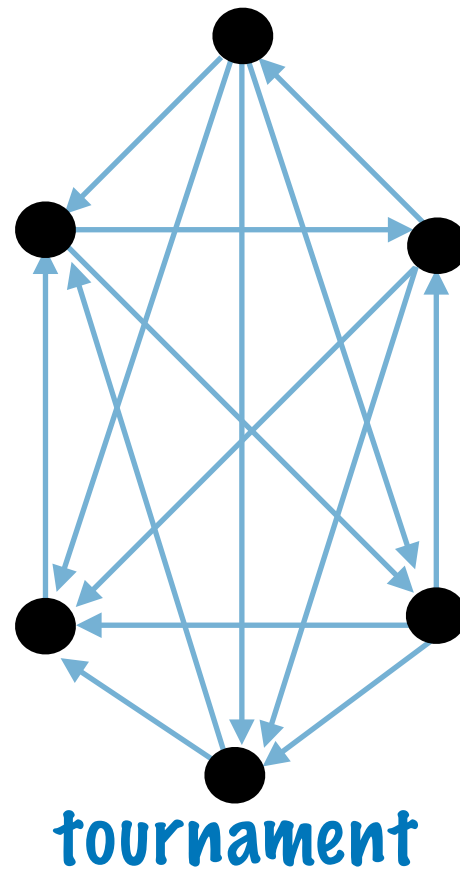
Lemma: A set of arcs F is a minimal FAS iff it is a minimal set of arcs whose reversal results in a DAG

Feedback Arc Set: Reversal and Deletion

Lemma: A set of arcs F is a minimal FAS iff it is a minimal set of arcs whose reversal results in a DAG

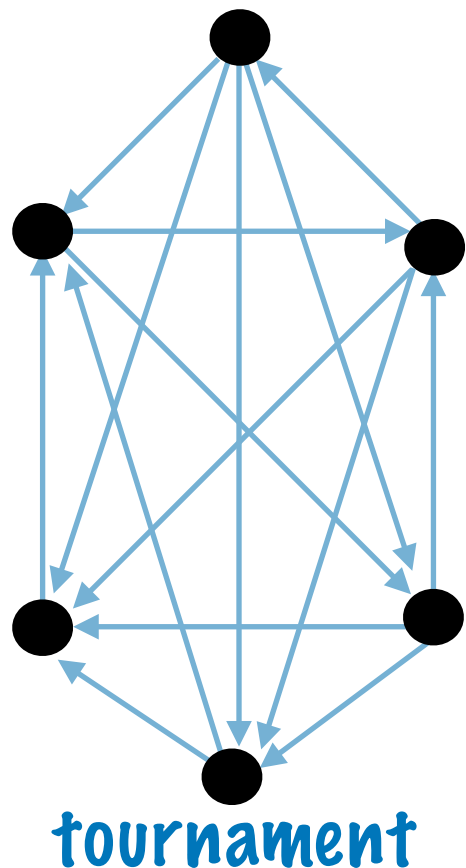


Tournaments



Lemma: Acyclic tournaments have unique topological ordering

Feedback Arc Set in Tournaments



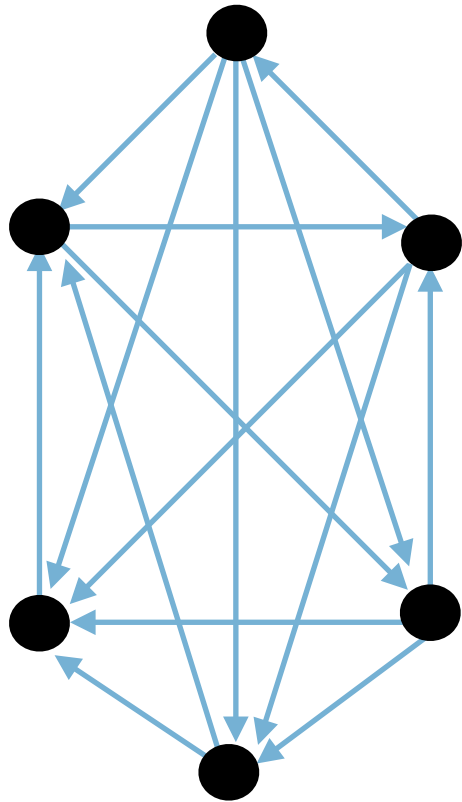
Feedback Arc Set in Tournaments

Instance: A tournament T and an integer k

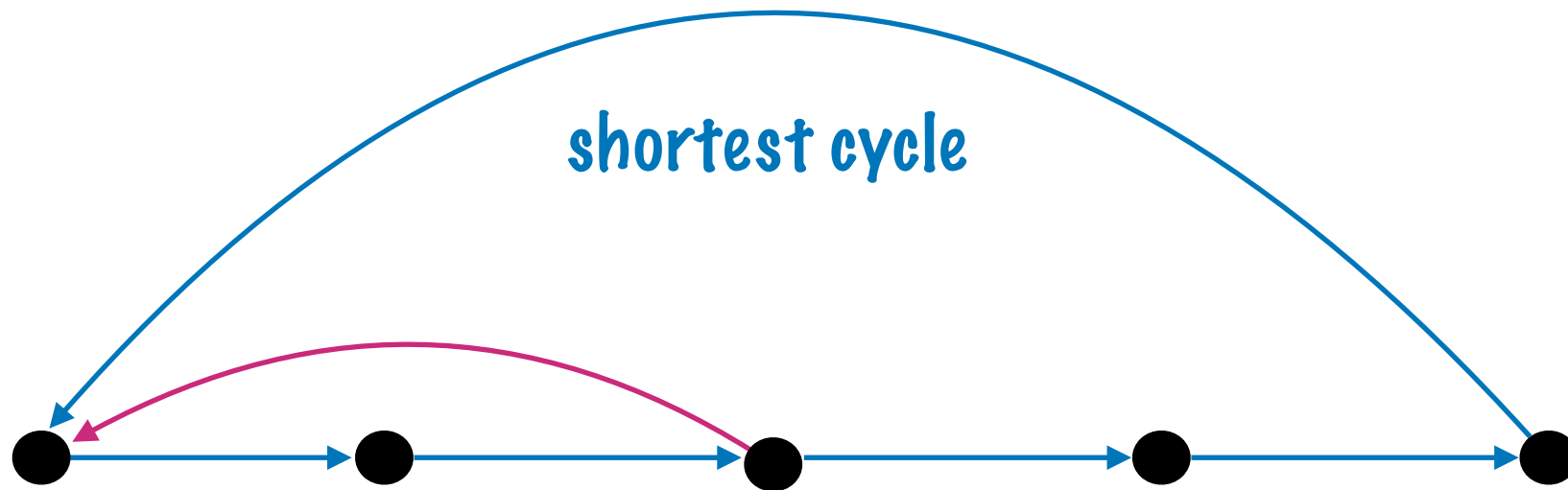
Question: Does there exist a feedback arc set of T of size at most k ?

Parameter: k

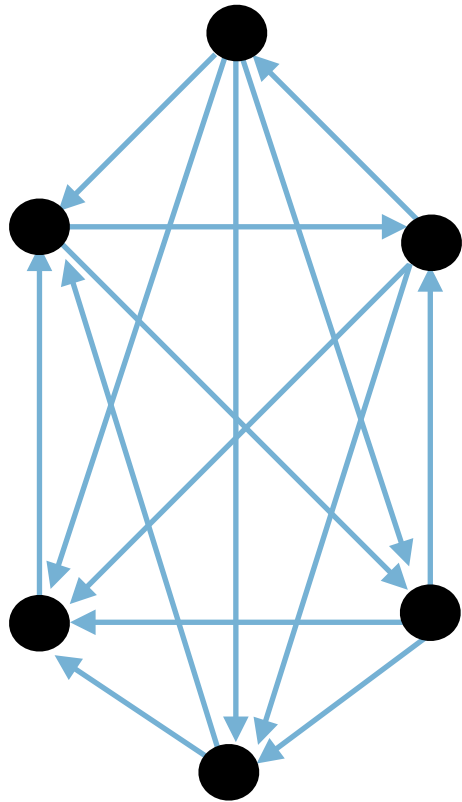
Feedback Arc Set in Tournaments



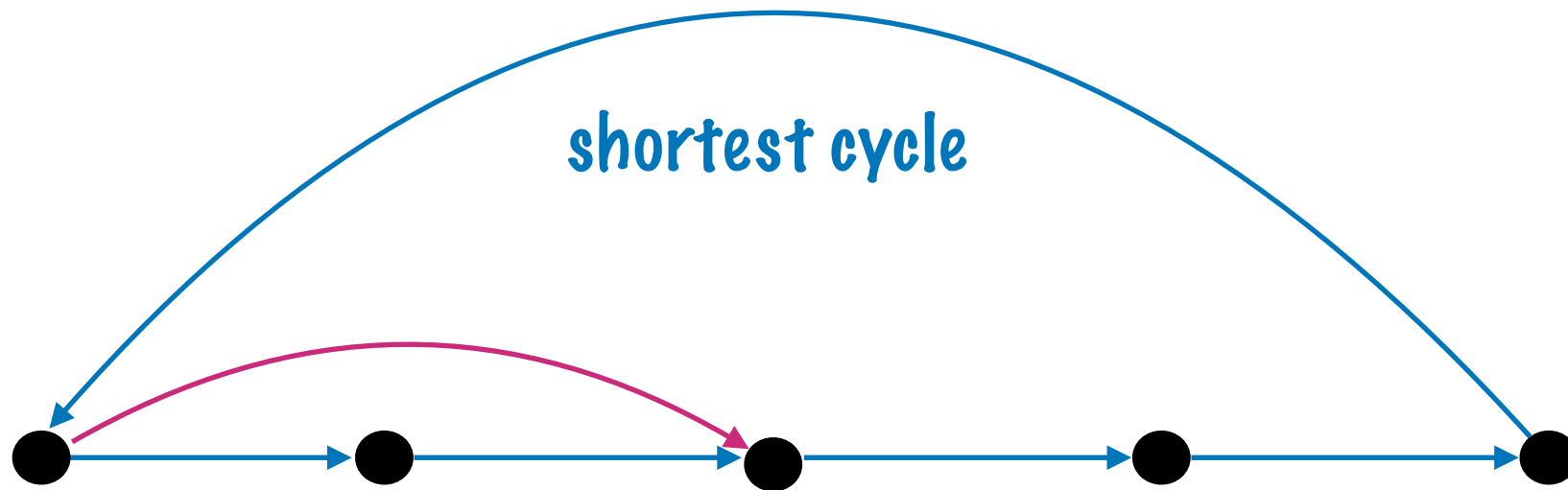
Lemma: A tournament is acyclic iff it has no triangle



Feedback Arc Set in Tournaments

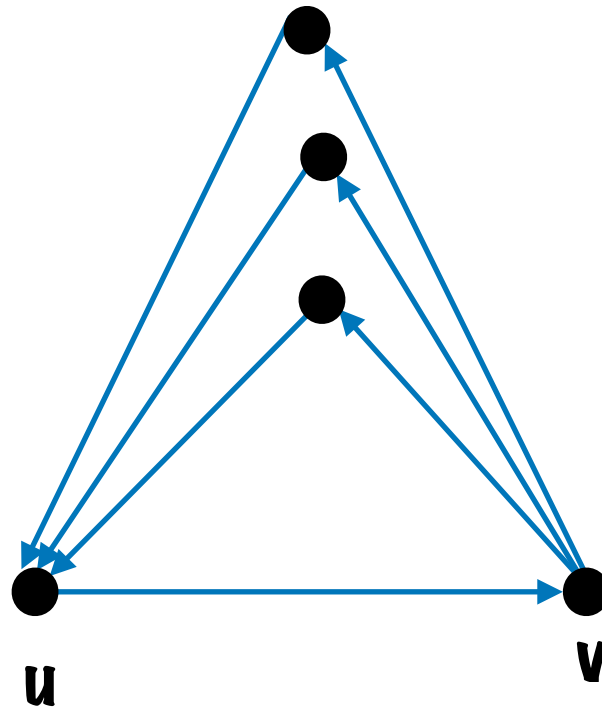


Lemma: A tournament is acyclic iff it has no triangle



Feedback Arc Set in Tournaments: Quadratic Kernel

- * **Reduction Rule 1:** If an arc $e=(u,v)$ is in $k+1$ triangles, reverse it and reduce k by 1



Resulting digraph is still a tournament

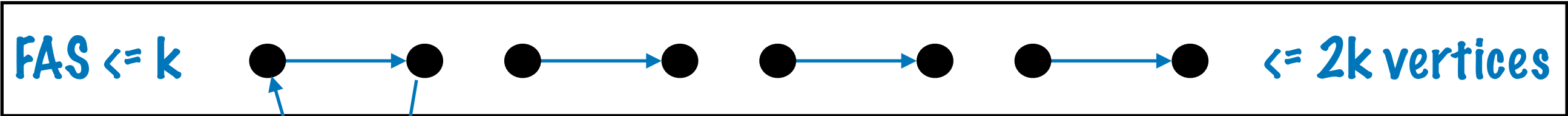
(T, k) is a yes-instance iff $(T', k-1)$ is a yes-instance

- * **Reduction Rule 2:** Delete vertices not in any triangle

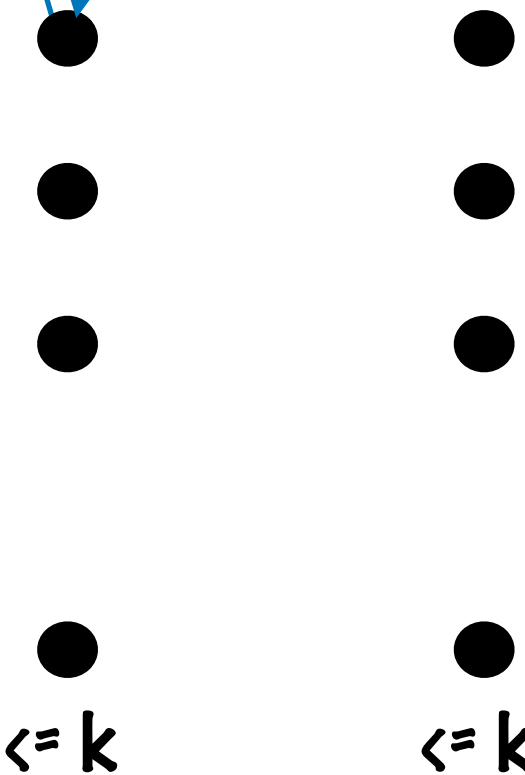
(T, k) is a yes-instance iff $(T-v, k)$ is a yes-instance

Feedback Arc Set in Tournaments: Quadratic Kernel

Suppose (T, k) is a yes-instance



How many more vertices can be there?



- * Every vertex is a part of a triangle
- * Every triangle has an arc in FAS

... $k(k) + 2k$ vertices

Quadratic vertex kernel

Feedback Arc Set in Tournaments: $O^*(3^k)$ Algorithm

Lemma: A tournament is acyclic iff it has no triangle

Branching Algorithm?

What are the recursive subproblems?

$O^*(3^k)$ algorithm