CS 5003: Parameterized Algorithms

Lectures 5-7

Krithika Ramaswamy

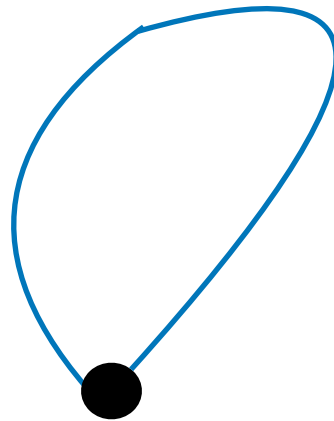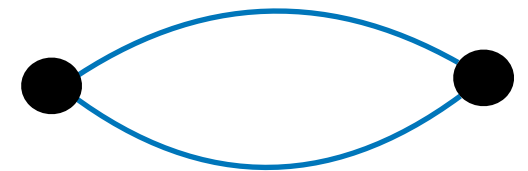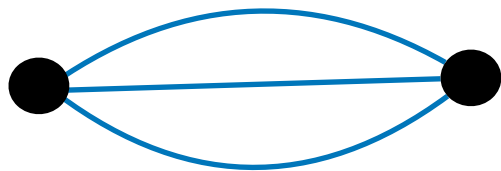## IIT Palakkad

Feedback Vertex Set

Assume graph is a multigraph

* **Reduction Rule 1:** Delete isolated vertices
* **Reduction Rule 2:** Delete degree-1 vertices
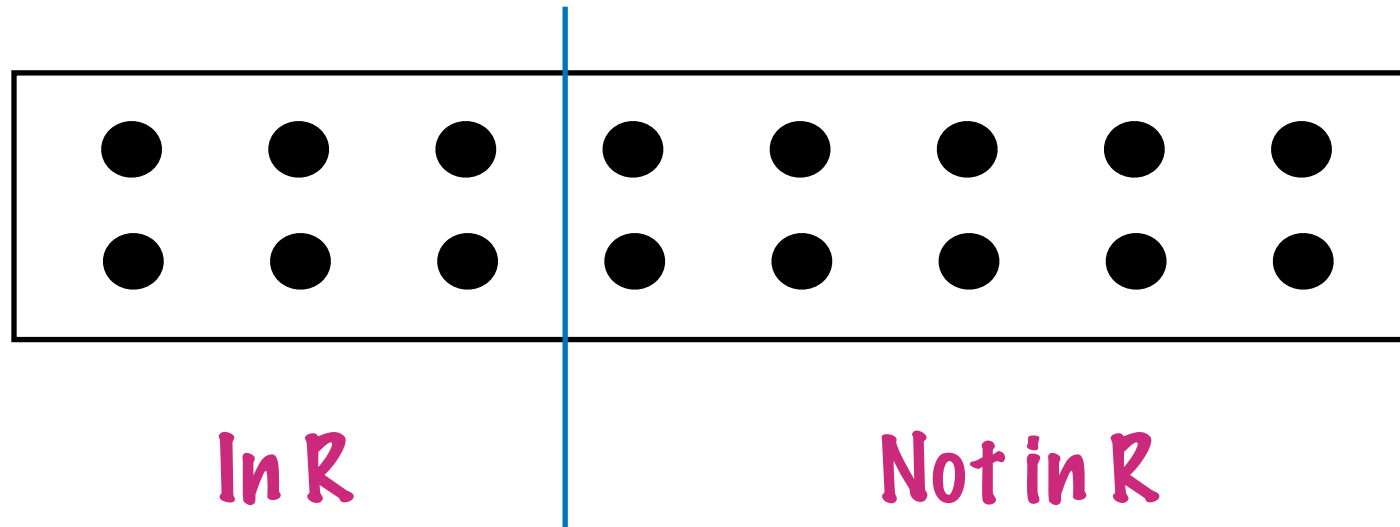* **Reduction Rule 3:** If there is a loop at a vertex v, delete v from the graph and reduce the parameter by 1

* **Reduction Rule 4:** If there is an edge with multiplicity > 2, reduce it to 2

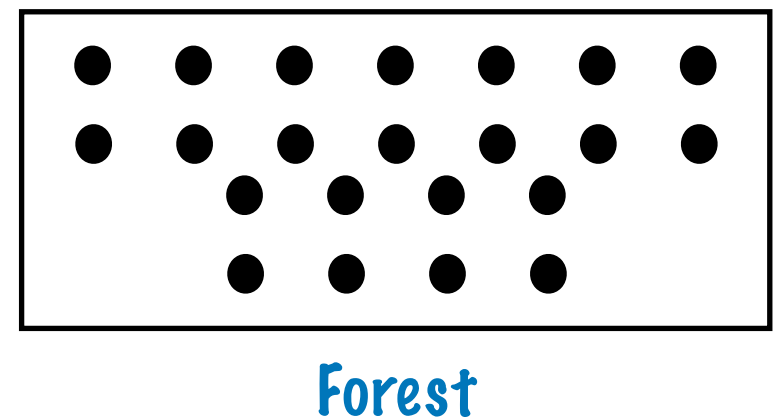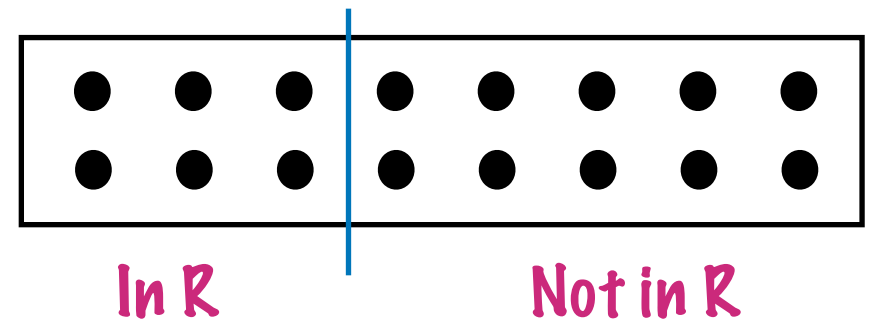# Feedback Vertex Set
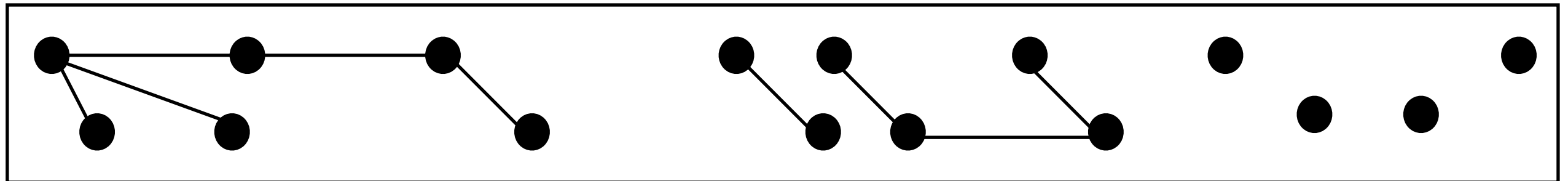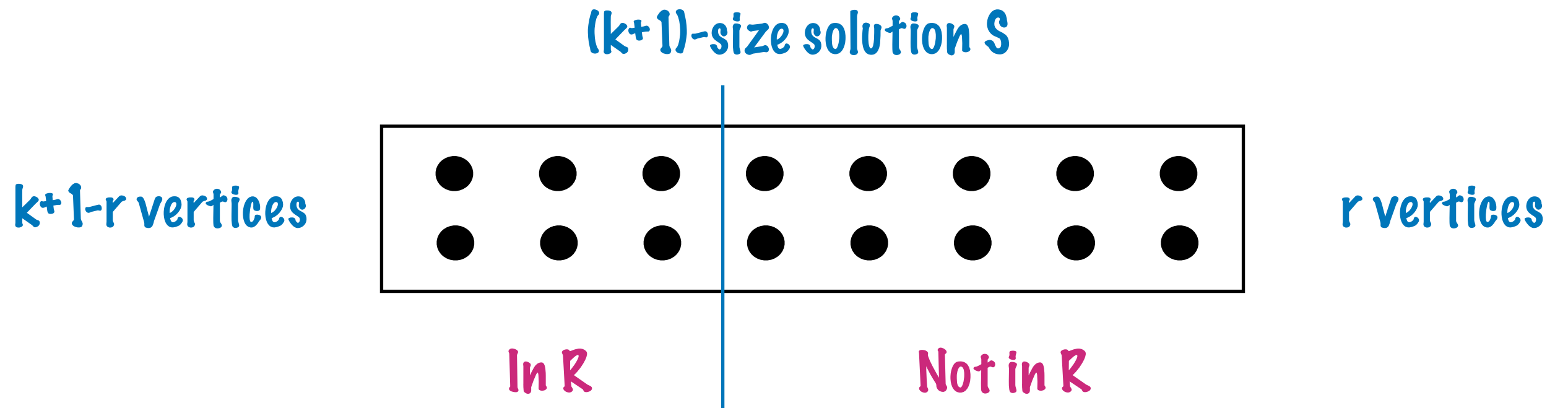
Suppose we have a (k+1)-size solution S

In R   Not in R

* We want <= k size solution R

* Suppose we know $S \cap R$

  * If we don't know $S \cap R$, guess!

    * 2^{k+1} choices

In R   Not in R

Forest

Feedback Vertex Set

(k+1)-size solution S
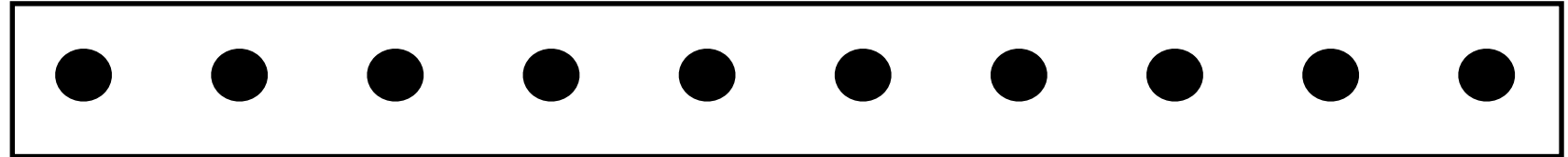
k+1-r vertices

r vertices

In R

Not in R

Forest

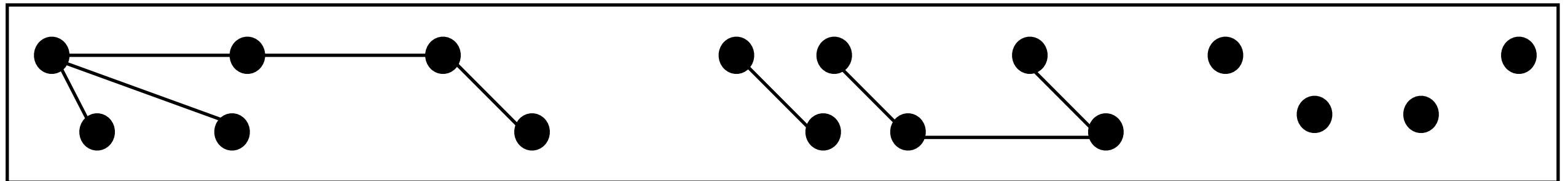To find a set of <= r-1 vertices here

Feedback Vertex Set
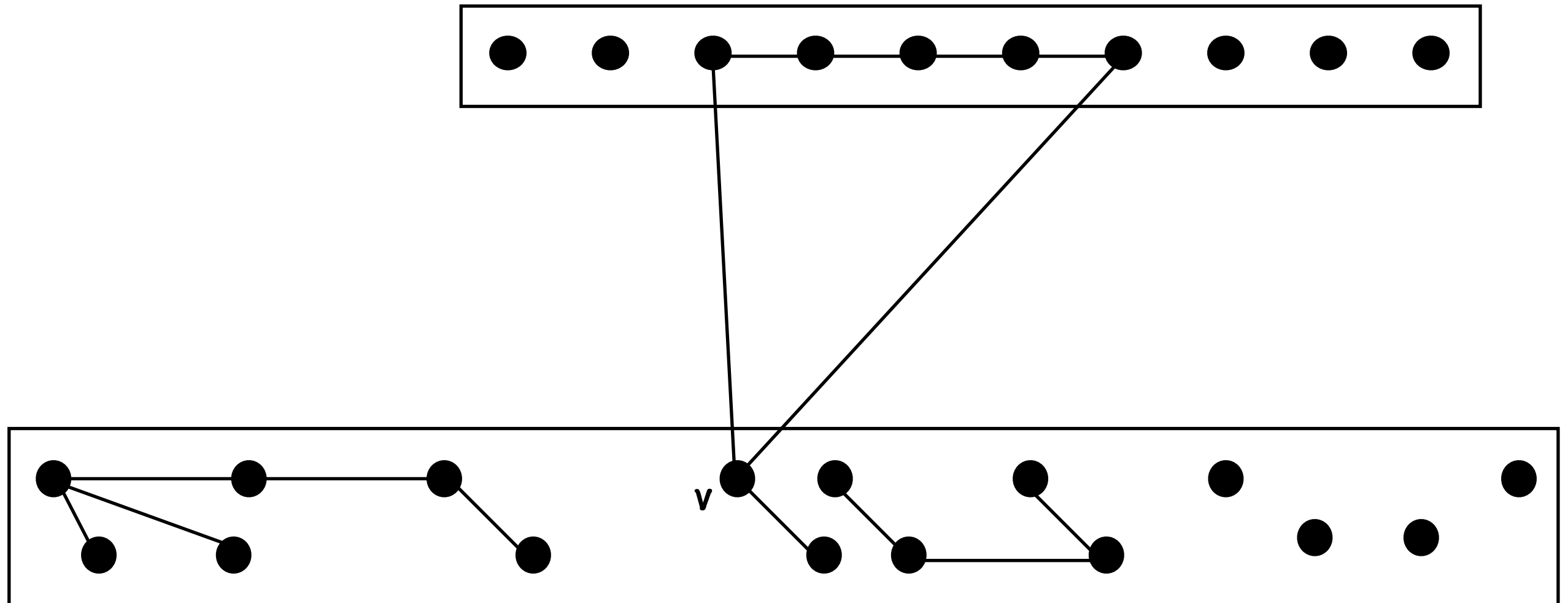
r-size solution

Forest

Forest

Find a disjoint (r-1) size solution

Disjoint Compression

Feedback Vertex Set

Disjoint Compression

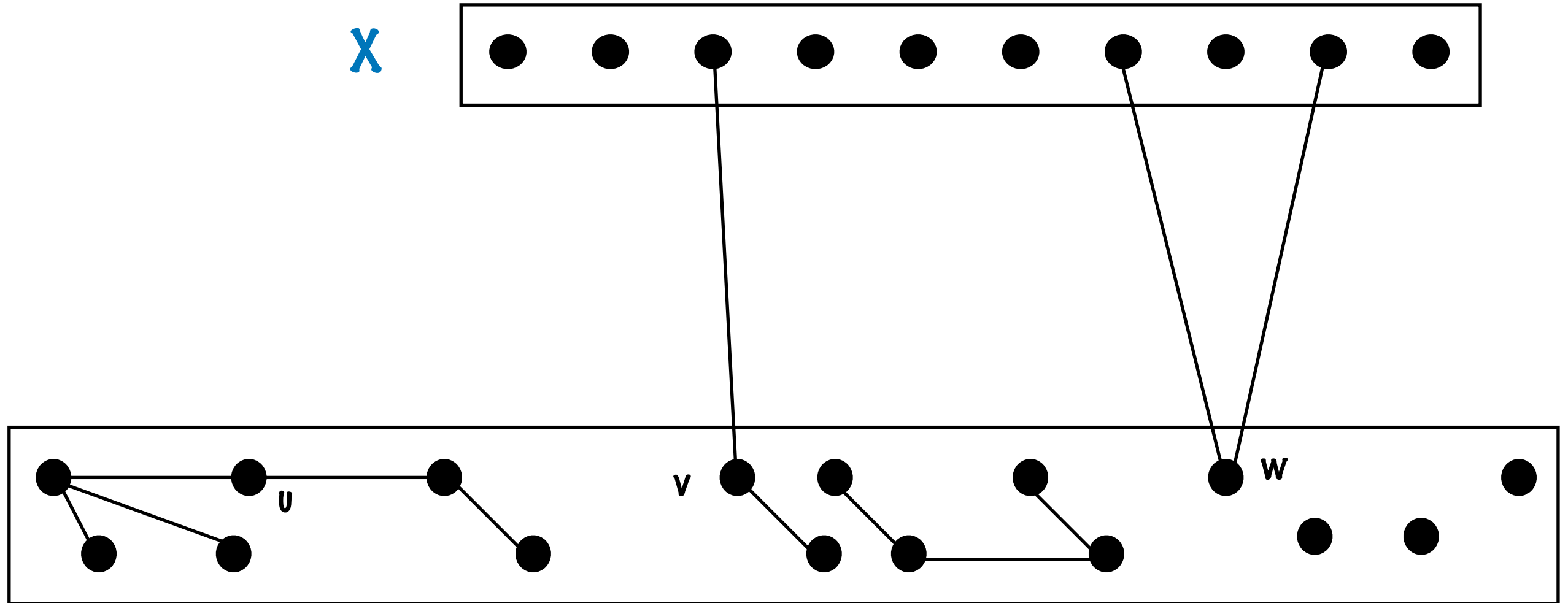Add v into solution and reduce parameter by 1

Feedback Vertex Set

Disjoint Compression

Short circuit some deg 2 vertices
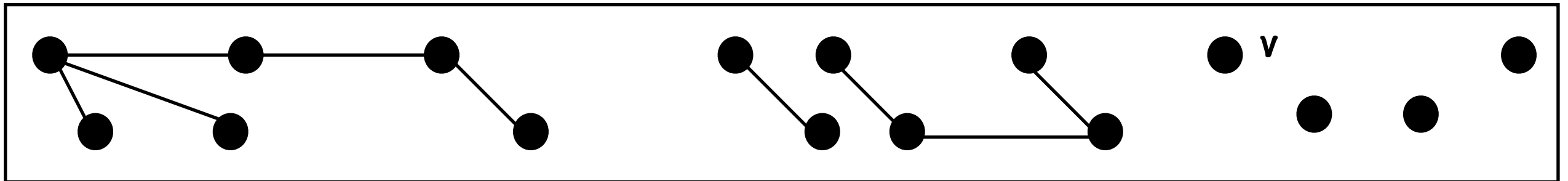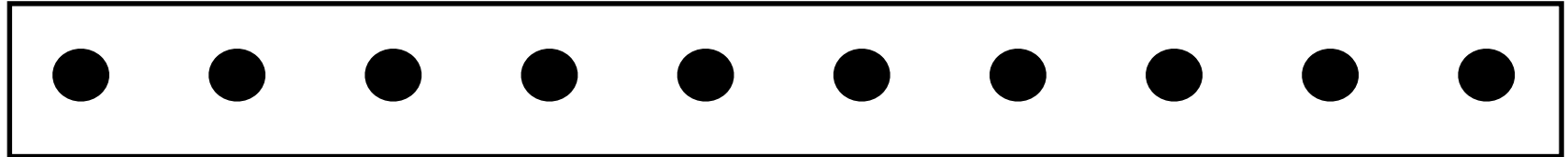
Feedback Vertex Set

Disjoint Compression

Instance: (X,Y,r-1)

v : vertex of degree (in Y) <=1

Feedback Vertex Set

Instance: (X,Y,r-1)

X

Y

v

v has >=2 neighbours in X
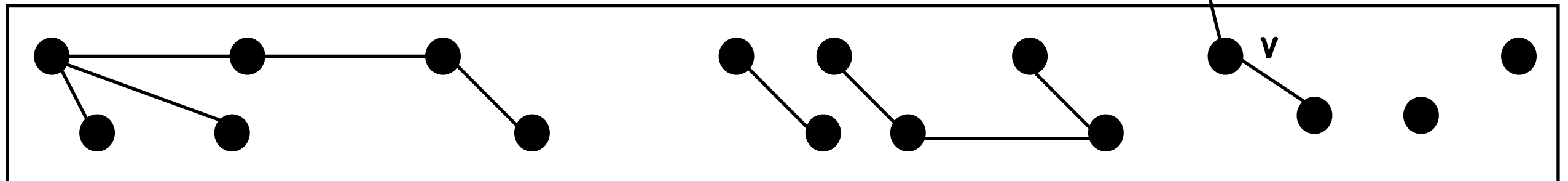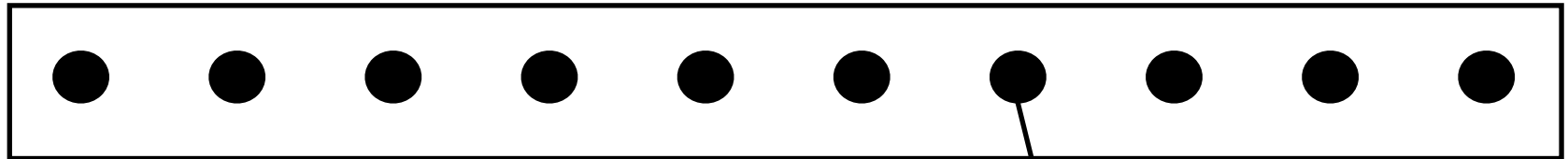
Feedback Vertex Set

Disjoint Compression

Instance: (X,Y,r-1)

X

Y

v

v has >=2 neighbours in X

Feedback Vertex Set

Disjoint Compression

Instance: (X,Y,r-1)

X

v has >=2 neighbours in X

Feedback Vertex Set

Instance: (X,Y,r-1)

X

a

b

v

Y
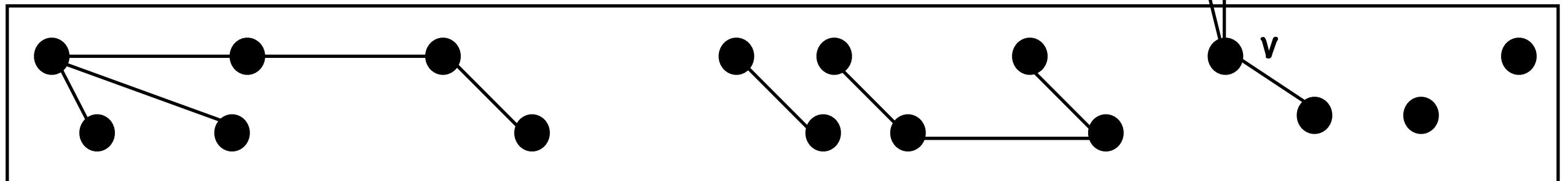
v has >=2 neighbours in X

Feedback Vertex Set

Disjoint Compression

Instance: (X,Y,r-1)

a and b are in different components

Feedback Vertex Set
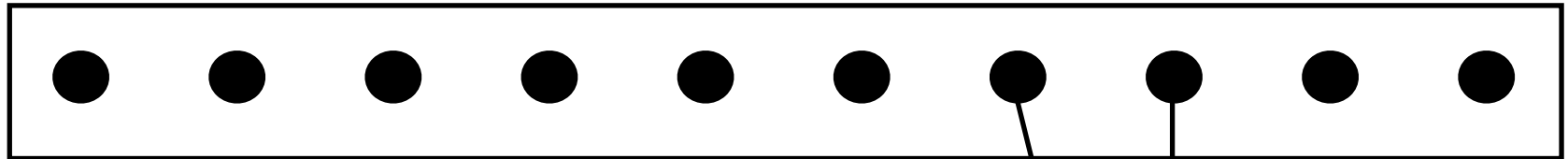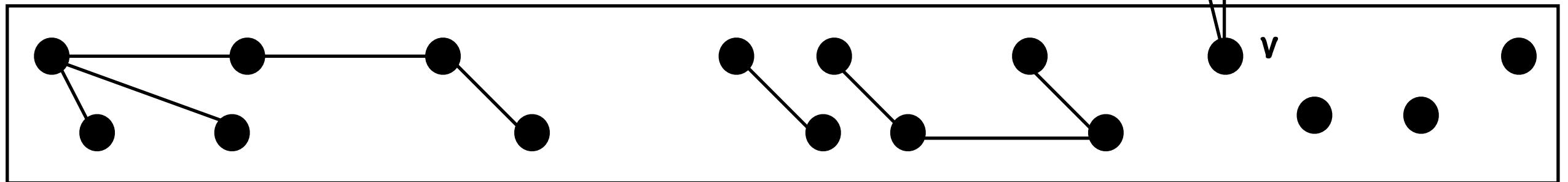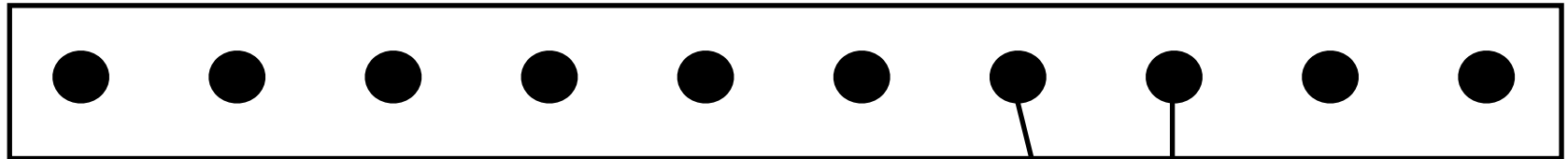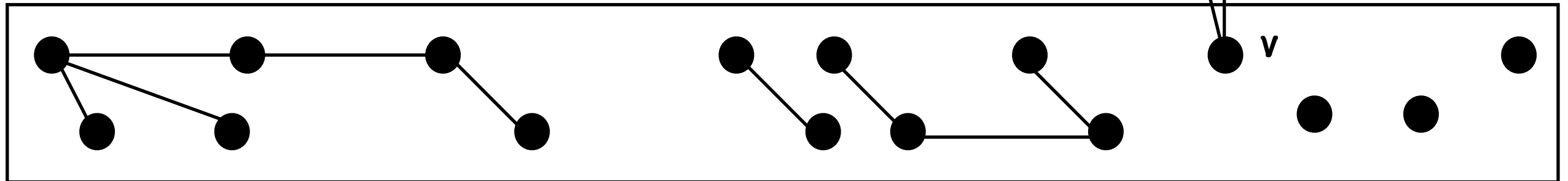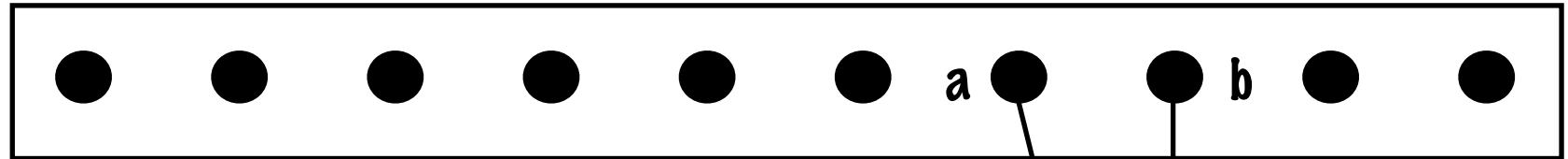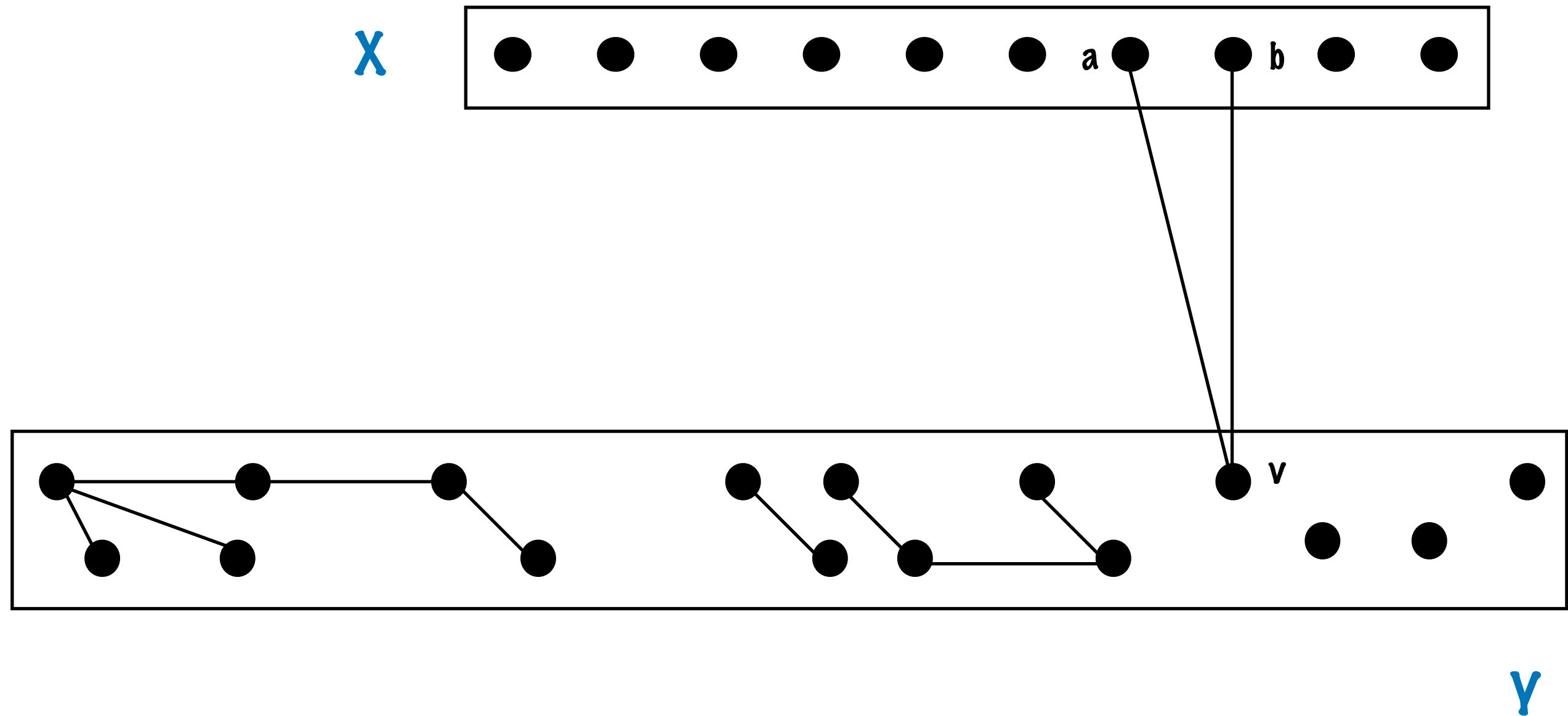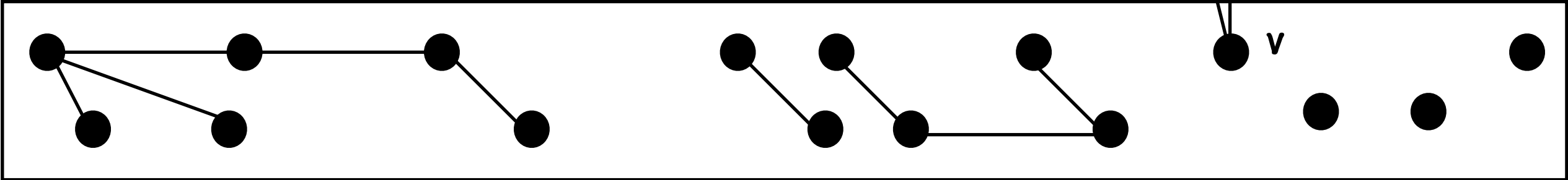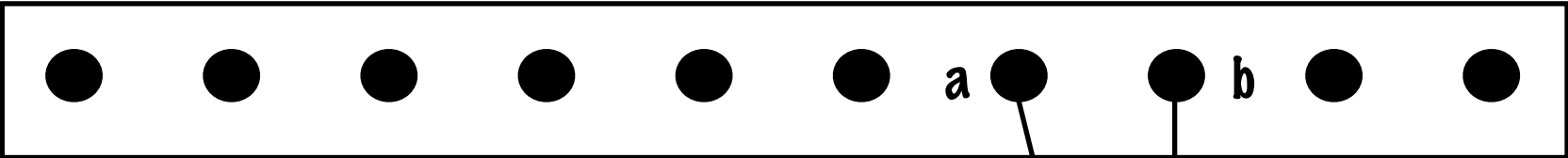
Disjoint Compression

Instance: (X,Y,r-1)

X

a b

v

Branch 1: v in the solution

Instance: (X,Y-{v},r-2)

Y

Feedback Vertex Set

Disjoint Compression   **Instance: (X,Y,r-1)**

X

a   b
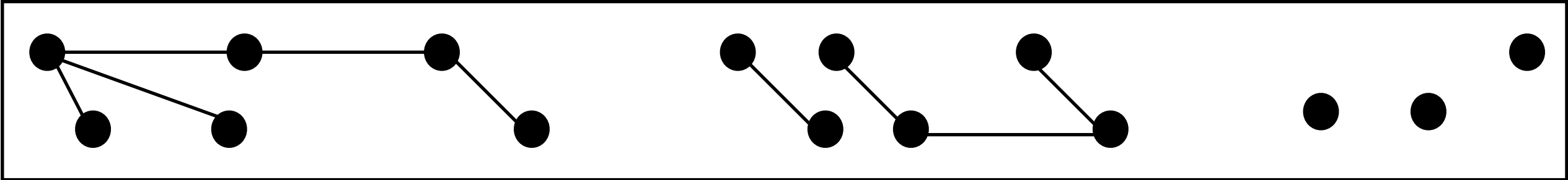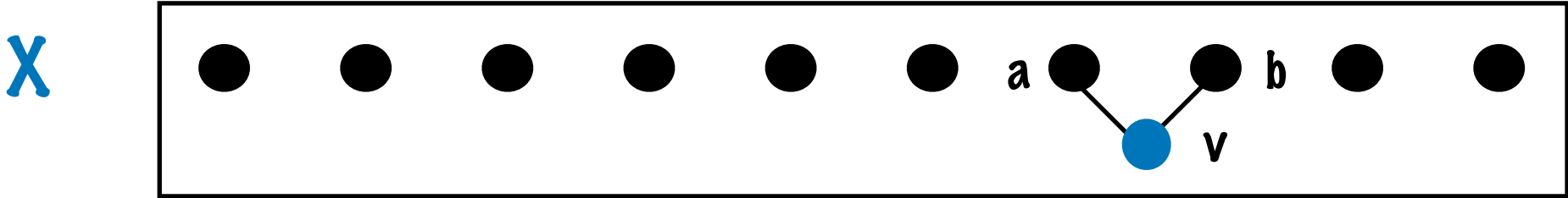
v

Branch 2: v not in the solution

Y

Instance: (X∪{v}, Y-{v}, r-1)

Feedback Vertex Set

Instance: (X,Y,r-1)

Branch 2: v not in the solution

Instance: (X∪{v}, Y-{v}, r-1)

Feedback Vertex Set

Disjoint Compression

$$(X,Y,r\text{-}1)$$

v

r drops by 1 $(X,Y\text{-}\{v\},r\text{-}2)$ $(X\cup\{v\},\ Y,\ r\text{-}1)$ #C(X) drops by 1

. . . .
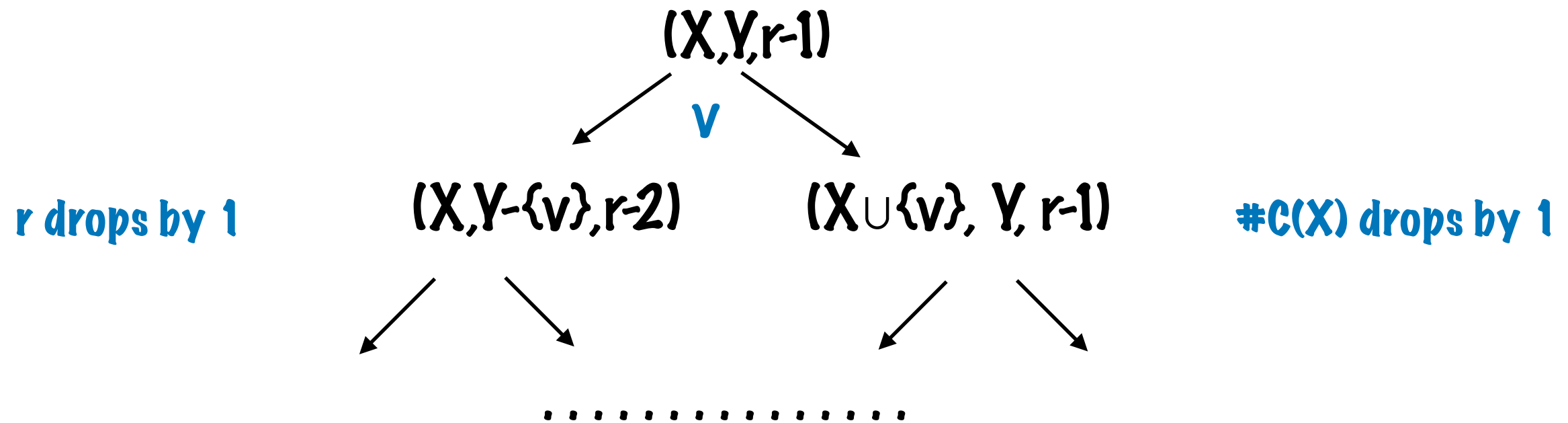
r + #C(X) <= k+1 + k <= 2k+1

Feedback Vertex Set

Disjoint Compression

$$(X, Y, r-1, l)$$

$$v$$

l drops by 1

$$(X, Y-\{v\}, r-2, l-1) \quad (X \cup \{v\}, Y, r-1, l-1)$$

l drops by 1

. . . .
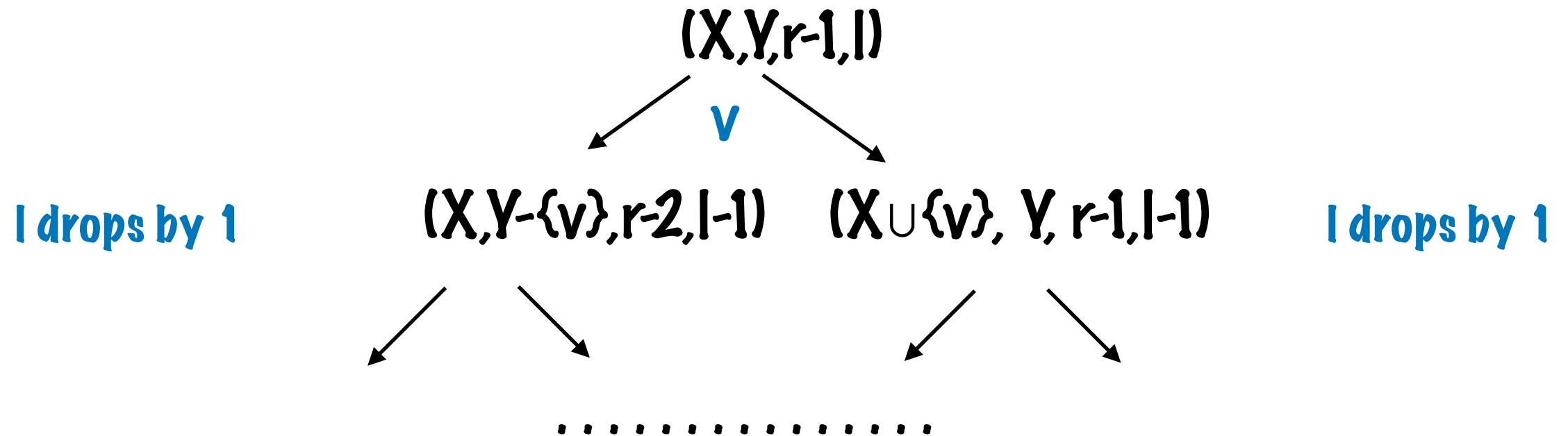
$$l = r + \#C(X) <= k + 1 + k <= 2k+1$$

Let $T(l)$ denote the no. of leaves in the tree rooted at instance with measure l

$$T(l) <= 2T(l-1) \text{ if } l >= 1$$
$$1 \quad \text{otherwise}$$

$$T(l) <= 2^l$$

$$O^*(2^{2k}) \text{ time for Disjoint Compression}$$

Feedback Vertex Set

(k+1)-size solution S

In R | Not in R
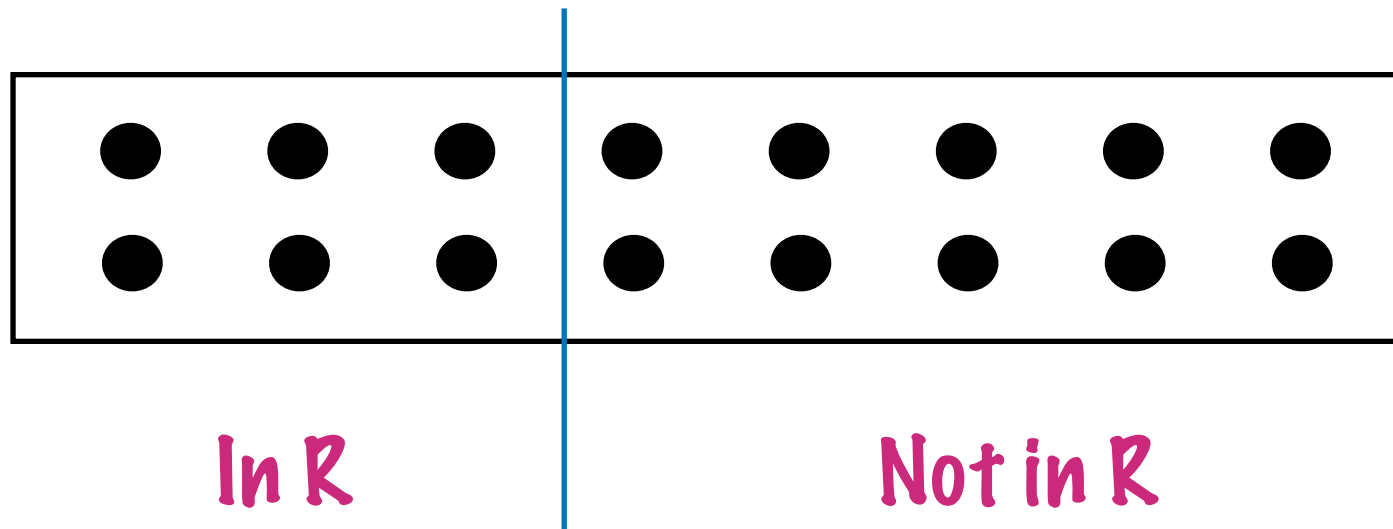
* We want <= k size solution R

* Guess S ∩ R (2^{k+1} choices)

  * Solve Disjoint Compression in $O^*(4^k)$ time
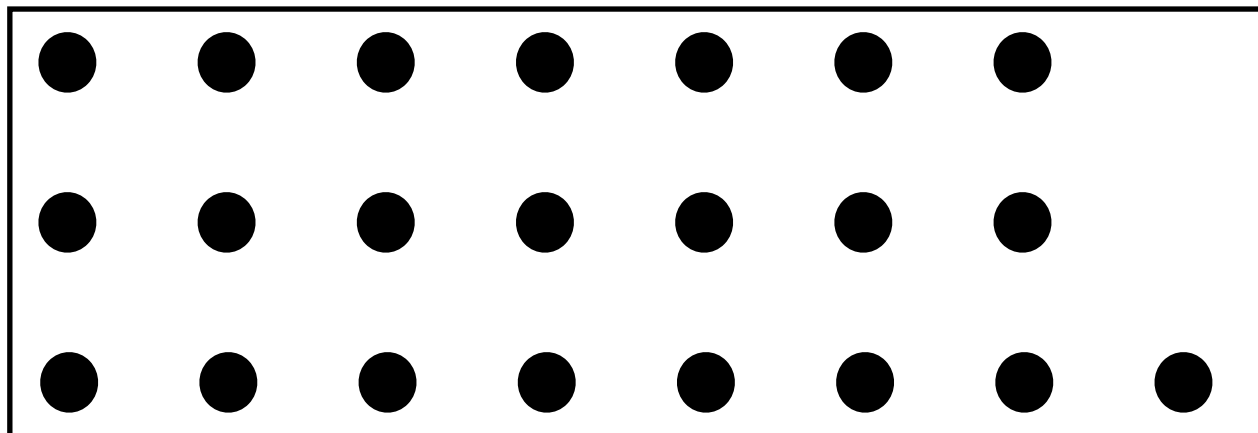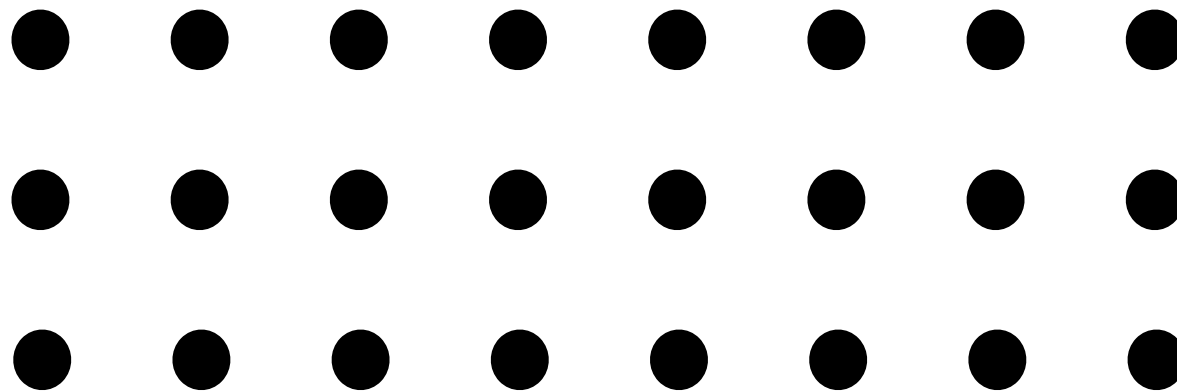
$O^*(5^k)$ algorithm
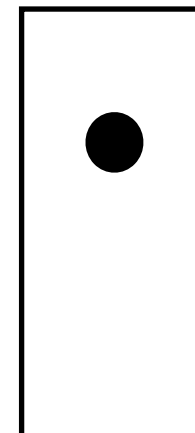
Feedback Vertex Set

How to get a (k+1)-size solution S?
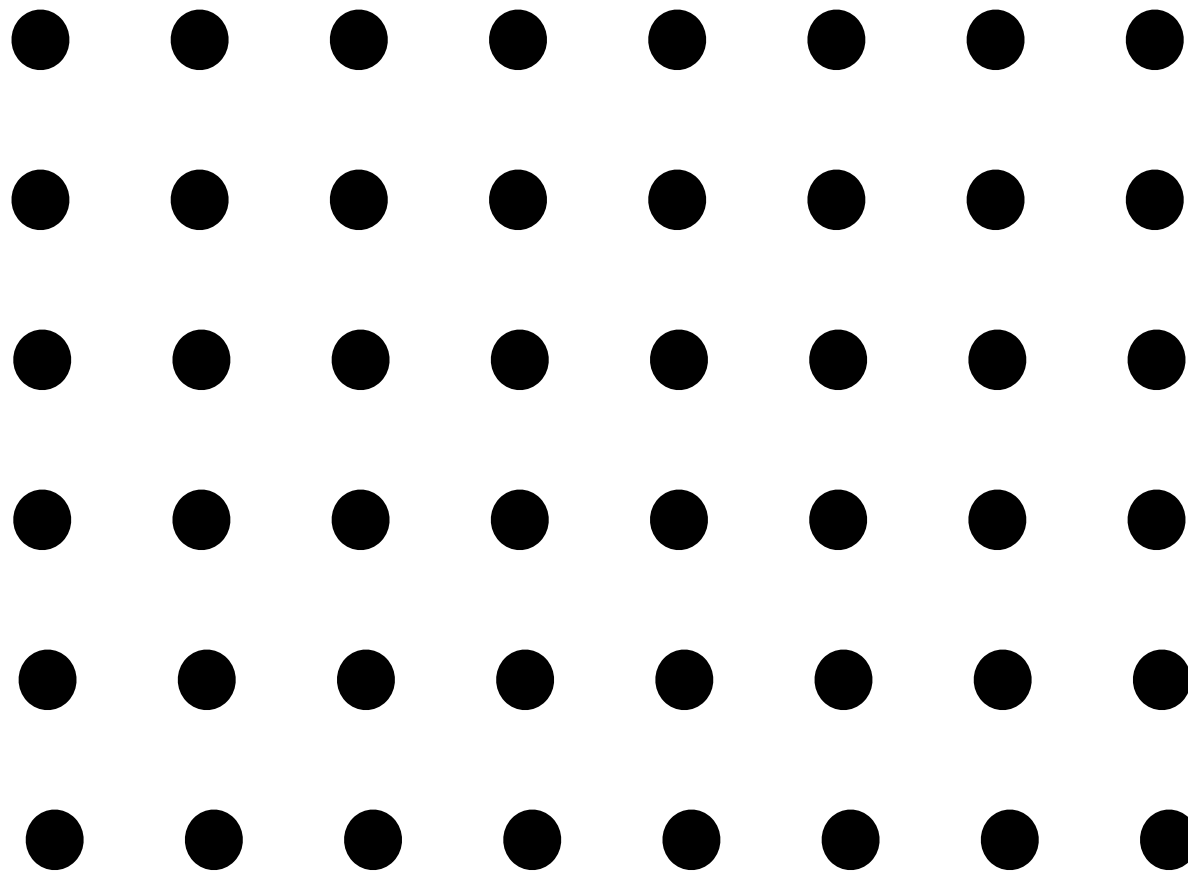
Graph G

Feedback Vertex Set
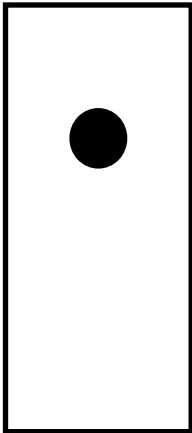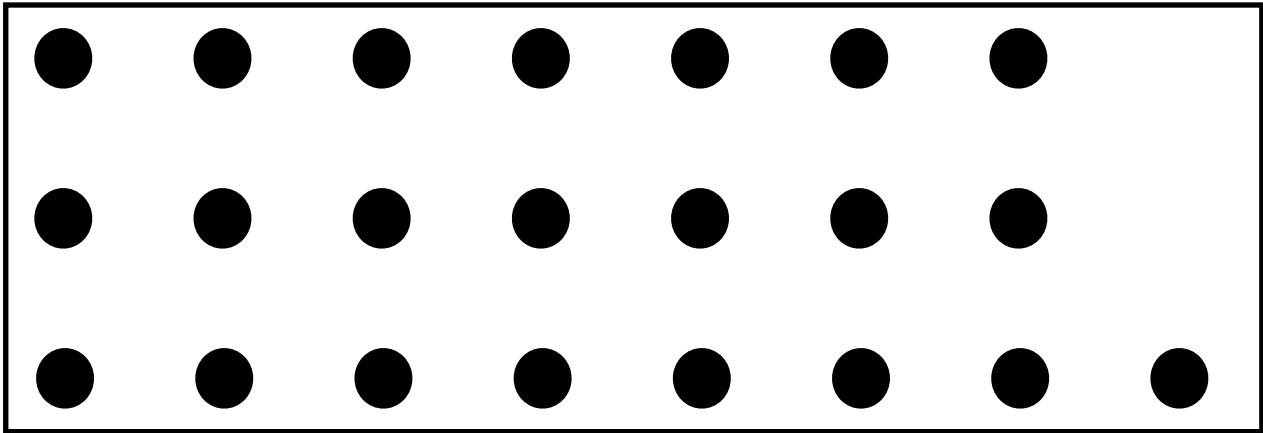
Consider any k+2 vertices of G

k+1 solution

Forest

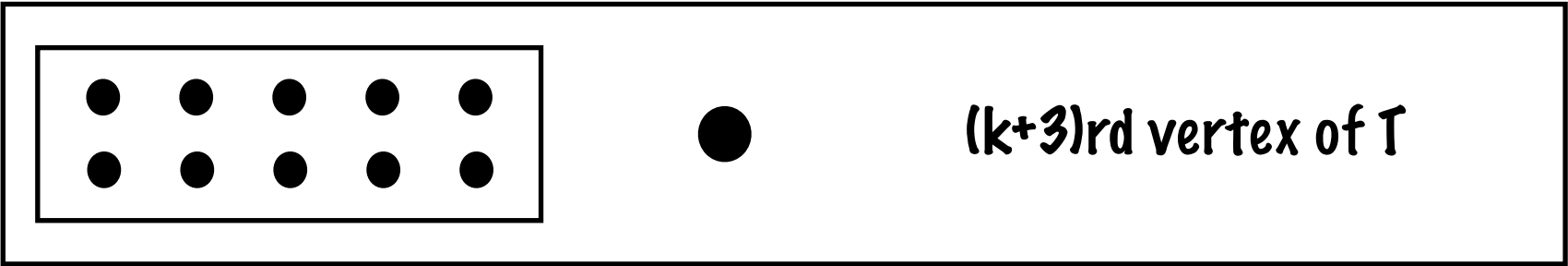# Feedback Vertex Set

k+1 solution for subgraph on k+2 vertices

Compress in $O^*(5^k)$ time

No k solution

<= k solution

(G,k) is a no-instance

(k+3)rd vertex of T

k+1 solution for subgraph on k+3 vertices

Iterative Compression