

A brewery produces lager and ale beers. The net profit obtained for a barrel of lager sold is \$70 and for a barrel of ale is \$30. Producing a barrel of lager requires 5 pounds of corn, 180 grams of yeast, and 2 pounds of hops. Producing a barrel of ale requires 2 pounds of corn, 120 grams of yeast, and 1 pound of hops. The brewery has 60 pounds of corn, 2800 grams of yeast, and 25 pounds of hops. What is the optimal production of each of the beers that maximizes the brewery's revenue? Assume that you can produce fractions of a barrel.

1. Formulate this problem mathematically (specifying parameters, variables, constraints, and objective function) and solve it in AMPL. Use Gurobi. What is the optimal production plan?
2. Formulate and solve the dual problem in AMPL. Use Gurobi. How can you interpret the optimal solution of the dual problem?

Answer the following questions using the sensitivity analysis tools in AMPL (i.e., using the options available in option gurobi\_options 'solnsens=1'). Perform your analyses with respect to the original problem.

3. What should be the profit of a barrel of ale beer so that the brewery increases its production? Use AMPL to find the answer. Report the code used.
4. What should be the profit of a barrel of ale beer so that the brewery only produces ale beer? Use AMPL to find the answer. Report the code used.
5. The brewery just realized that the corn available is about 60 pounds, but there is no certainty on its real amount. Perform a sensitivity analysis to determine the production plan for any possible available amount of corn. Using AMPL, construct a graph that shows the objective function value as a function of the corn available. Explain the piecewise nature of this function, the slope of each segment, and justify the changes of slope. Report the code used and the equation behind of each piecewise linear segment.
6. Local stores contacted the brewery to offer corn, yeast, and hops. The prices ranged from \$12 to \$15 per pound of corn, \$0.50 to \$0.75 per gram of yeast, and \$7 to \$11 per pound of hops. Is the brewery interested in buying additional resources at these prices? Use AMPL to find the answer. Report the code used.
7. To expand its market, the brewery is thinking on producing a pilsner beer. After analyzing the production requirements, the brewery determines that such beer has a net profit of \$50 a barrel, and requires 4 pounds of corn, 150 grams of yeast, and 2 pounds of hops. Does the production plan need to be updated to produce pilsner? Solve this problem analytically and confirm your answer by re-solving the problem in AMPL. Show all your calculations.
8. Under the conditions of Part 7, What is the required profit per barrel of pilsner that the brewery expects to start producing it? Use the sensitivity analysis tools in AMPL.