

A swimming coach needs to assign a set of swimmers ( $N$ ) to a set of strokes ( $S$ ) in order to participate in a set of medley relay competitions. The swimmers' times for each stroke are given by  $t_{ij}$ , which indicates the best time of swimmer  $i \in N$  in stroke  $j \in S$ . A swimmer can participate in at most one competition. How can the coach assign swimmers to strokes to minimize the total relay swim times? Table 1 shows the information available for 12 swimmers.

1. Suppose that you are looking to create only one relay team with the swimmers in Table 1. Somebody tells you that the best way to do it is to proceed as follows:
  - Select the best swimmer per stroke. In case of a tie, choose randomly.
  - If a swimmer has been already selected, select the second best. If the second-best swimmer has been already selected, select the third best, and so on.
  - a) Report the solution of this assignment if the swimmers' times are examined in the sequence back, breast, fly, and free.
  - b) Explain why this procedure does not guarantee an optimal assignment.
  - c) Formulate an optimization problem to solve this problem. Specify parameters, decision variables, constraints, and objective function. Include constraints to indicate that a swimmer can compete in at most one stroke and that each stroke needs exactly one swimmer.
  - d) Implement and solve this problem in AMPL. Use Gurobi as a solver. Report the details of your implementation (i.e., code and data used). Report a table summarizing swimmer, assigned stroke, and time.

Table 1: Best swimming time per swimmer and stroke

Name	Gender	Best time			
		Back	Breast	Fly	Free
Emily	F	29.6	26.4	21.8	28.1
Lisa	F	27.5	25.3	19.5	19.8
Elizabeth	F	19.3	23.7	20.5	23.7
Helen	F	21.6	19.1	22.7	17.8
Linda	F	23.8	22.7	19.4	18.8
Ellen	F	21.5	24.9	20.3	21.1
Sarah	F	22.4	21.9	20.3	21.0
Michael	M	20.4	21.1	17.5	19.3
David	M	21.0	23.9	22.0	19.6
William	M	19.8	26.1	18.6	19.3
John	M	19.7	20.6	17.0	17.1
Robert	M	22.6	19.5	17.4	18.5

2. Suppose that you have to create two relay teams, one for each gender.
  - a) What do you expect about the relay teams' times with respect to the solution in Part 1?
  - b) Use your model and implementation from Part 1(d) to create the teams. Report a table for each team summarizing swimmer, assigned stroke, and time.
3. The competitiveness of a relay team is assessed by comparing its swimming time with respect to a national standard time  $t$ . If the swimming time of a team is less than  $t$ , then the team is said to be competitive. The current coach wants to create 2 competitive teams with the swimmers in Table 1 (regardless of the gender). The national standard time is  $t = 75.5$ .
  1. Somebody suggests using an objective function that minimizes the sum of the swimming times for both teams. Explain why this is not a good idea. Provide numerical evidence of your arguments.

2. Formulate an optimization problem to solve this problem. Specify parameters, decision variables, constraints, and objective function.
3. Implement and solve this problem in AMPL. Use Gurobi as a solver. Report the details of your implementation (i.e., code and data used). Report a table summarizing swimmer, assigned stroke, team, and time.
4. Compare your solution with that in Part 2. Is this result expected? Explain.