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Module :- 3

→ optical transition in Bulk Semiconductors
→ Absorption, spontaneous Emission & stimulated emission
→ Einstein Cofficients
→ Population Inversion
→ Applications in Semiconductor Laser
→ Joint density of states
→ Density of states of Phonons
→ Transition rates (fermi Golden Rule)
→ optical loss & Gain
→ Photovoltaic Effect
→ Exciton
→ Drude Model

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LASER

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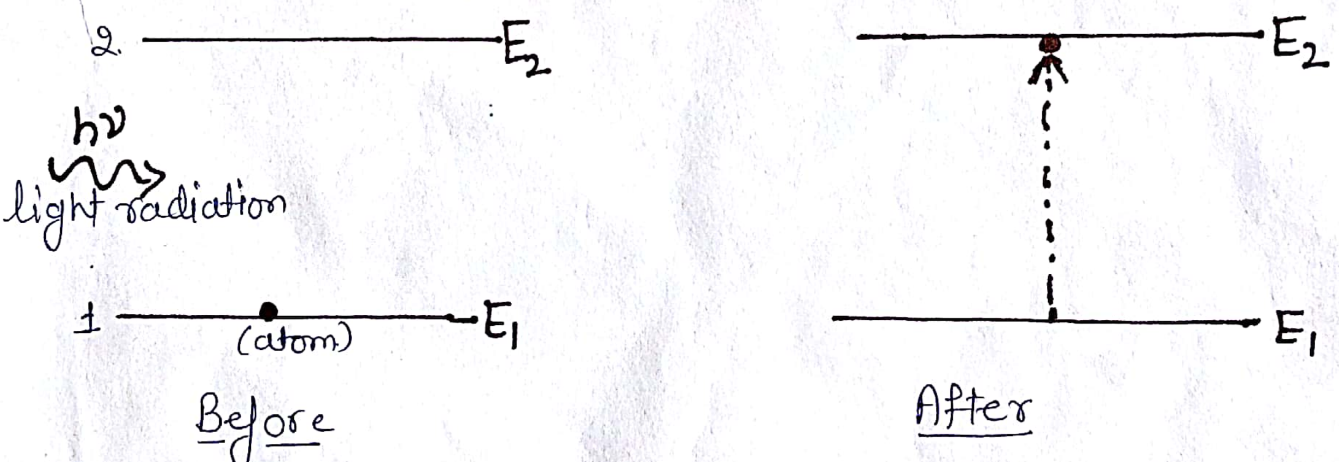
"Light Amplification by Stimulated Emission of Radiation"

→ Different Mechanisms

* Absorption
* Spontaneous emission
* Stimulated emission or Induced emission.

→ ABSORPTION

Let us consider two energy levels E_1 & E_2. Let the atom is initially in lower energy state E_1. Let atom is exposed to light radiation (photon) with energy $h\nu$. Atom get the energy from photon & jump to higher energy level E_2.



Now, $E_2 - E_1 = h\nu$ where h = Plank's Const.

$$\boxed{\nu = \frac{E_2 - E_1}{h}}$$

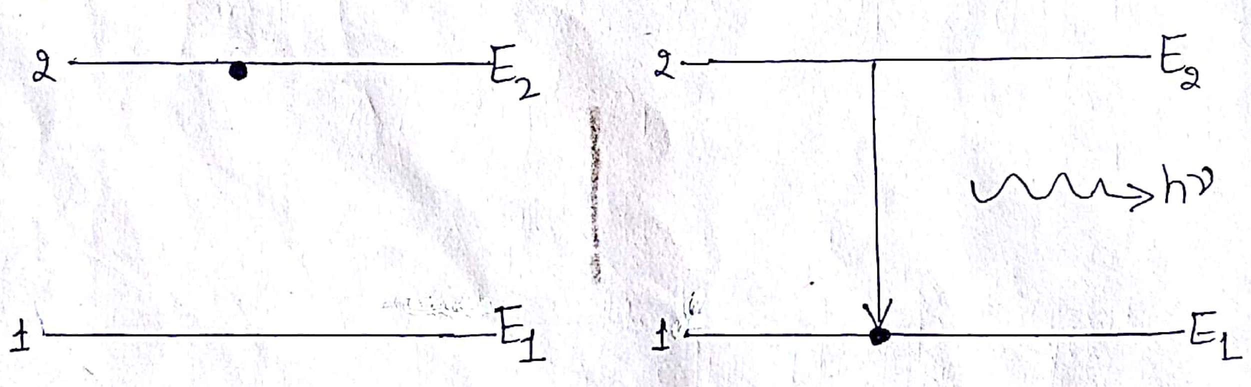
Now; probable rate of transition from 1 to 2 depends on properties of states 1 & 2 & energy density $u(\nu)$ i.e.

$$\boxed{P_{12} = B_{12}\, u(\nu)}$$

where B_{12} = constant of proportionality called Einstein cofficient of absorpion of radiation

→ SPONTANEOUS EMISSION

Let us consider two energy levels E_1 & E_2 ($E_1 < E_2$)
Let initially atom in higher energy level E_2.
An atom remains in excited state only for 10^{-8} sec. after this, its own jump to lower energy state E_1 by emitting a photon with frequency ν. This type of emission is called spontaneous emission of radiation



Intial state   Final State

$$\boxed{\nu = \frac{E_2 - E_1}{h}}$$

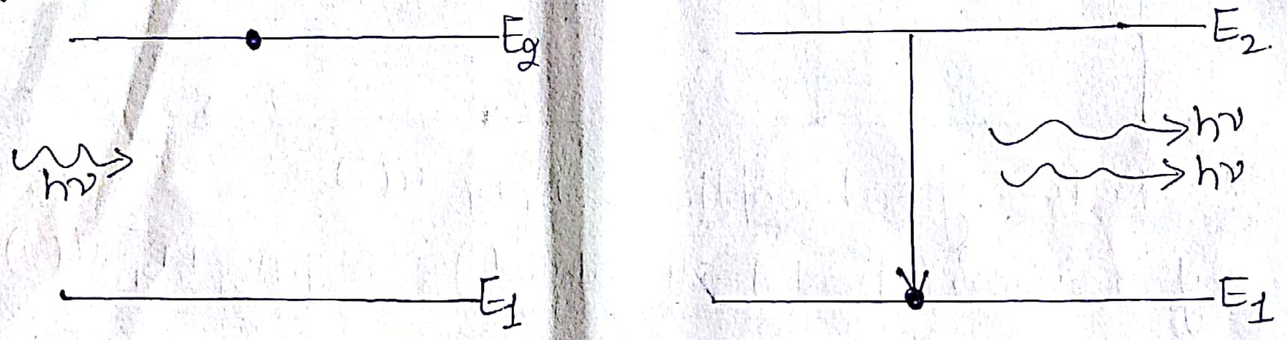
Now; probable rate of transition from $2 \to 1$ depends on properties of states of 1 & 2.
So.

$$\boxed{(P_{21})_{spont} = A_{21}}$$

where A_{21} = Einstein cofficient of spontaneous emission

STIMULATED OR INDUCED EMISSION

Let us Consider two energy levels E_1 & E_2 $(E_1<E_2)$ initially atom is in higher energy level E_2. If a photon of frequency ν is incident on it & cause the atom to jump to lower energy state E_1 by emitting an additional photon of frequency ν. This process is called stimulated emission of radiation



Intial State   final State

Now, probable rate of transition from $2 \rightarrow 1$ depends on energy density & as well as properties of states of 1 & 2.

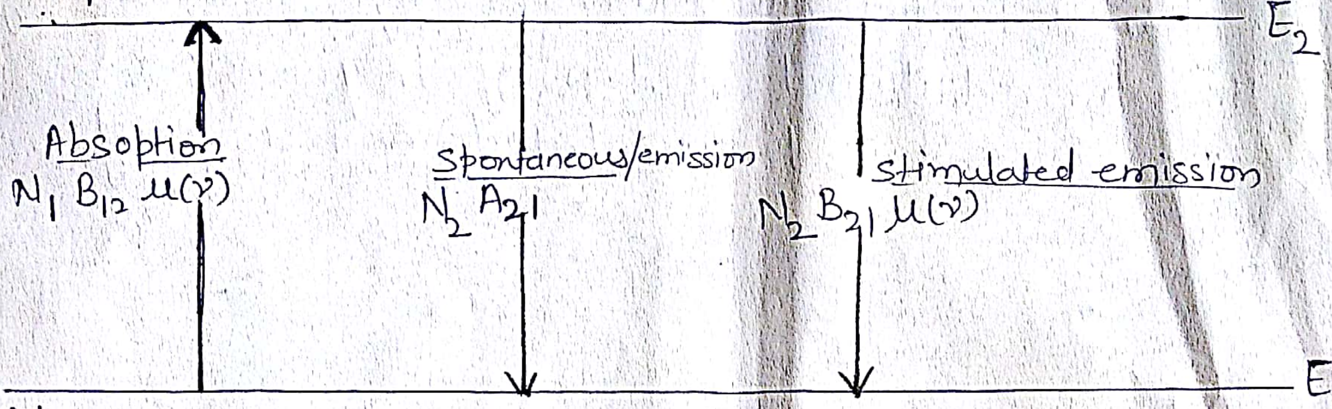
$$\boxed{(P_{21})_{stimulated} = B_{21}\, u(\nu)}$$

→ Difference b/w spontaneous & stimulated emission

SPONTANEOUS EMISSION	STIMULATED EMISSION
→ Emission has broad spectrum i.e. many wavelength	Emission has monochromatic radiation
→ It gives incoherent radiation	It gives coherent radiations
→ Less intense	highly intense
→ This emission gives radiations with less directionality & more angular spread.	This emission gives radiations with high directionality & less angular spread
eg:- Ordinary Source of light	eg:- Laser

→ Einstein Cofficients & Relations.

Let us Consider two energy level E_1 & E_2 ($E_1 < E_2$)
Let N_1 & N_2 number of atoms in energy states 1 & 2 respectively.



Now; Rate of absorption $= N_1 B_{12} u(\nu)$

Rate of emission (spont emission + Stimulated emission) $= N_2 [A_{21} + B_{12} u(\nu)]$

AT equilibrium Rate of absorption = Rate of emission

So;

$$N_1 B_{12} u(\nu) = N_2 [A_{21} + B_{21} u(\nu)]$$

$$N_1 B_{12} u(\nu) - N_2 B_{21} u(\nu) = N_2 A_{21}$$

$$u(\nu) = \frac{N_2 A_{21}}{N_1 B_{12} - N_2 B_{21}}$$

$$u(\nu) = \frac{A_{21}}{\frac{N_1}{N_2} B_{12} - B_{21}}$$ [dividing Numerator & Denominator by N_2]

$$u(\nu) = \frac{A_{21}}{B_{21}\left[\frac{N_1}{N_2}\frac{B_{12}}{B_{21}} - 1\right]} \quad \text{———— (1)}$$

According to Boltzmann distribution law, number of atoms N_1 & N_2 in Energy state E_1 & E_2 at temp T

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$$N_2 = N_1 e^{-h\nu/kT}$$ where h = Planks Const. (3)

$$\frac{N_2}{N_1} = e^{-h\nu/kT}$$

So, $$\frac{N_1}{N_2} = e^{h\nu/kT}$$

Put the value of $\frac{N_1}{N_2}$ in Eqn (1.)

$$u(\nu) = \frac{A_{21}}{B_{21}} \frac{1}{e^{h\nu/kT} \frac{B_{12}}{B_{21}} - 1}$$

Now, Comparing it by Plank's radiation law

$$u(\nu) = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1}$$

we get, $$\boxed{\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}} \quad \& \quad \boxed{B_{12} = B_{21}}$$

$\because B_{21} \propto \frac{1}{\nu^3}$, So rate of stimulated emission is higher at lower frequencies

$B_{12} = B_{21}$ means probability of stimulated emission is same as that of absorption

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Ques What are the units of Einstein Cofficients?

Answer:- Energy density $u(\nu) = \frac{\text{Energy}}{\text{Volume} \times \text{freq.}}$

Energy density $u(\nu) = \frac{\text{Energy} \times \text{Time}}{\text{Volume}} = \frac{JS}{m^3}$

Units of $\left[(P)_{12}, (P_{21})_{\substack{spot \\ Emission}}, (P_{21})_{\substack{Stimulated \\ Emission}}\right] = \frac{Number}{Time} = s^{-1}$

Now for absorption

$$P_{12} = B_{12} N_1 u(\nu)$$

$$\Rightarrow \quad B_{12} = \frac{P_{12}}{N_1 u(\nu)} = \frac{s^{-1} m^3}{J s} = m^3 J^{-1} s^{-2}$$

Similarly for stimulated Emission

$$\Rightarrow B_{21} = m^3 J^{-1} s^{-2}$$

for spontaneous Emission

$$(P_{21})_{\substack{spot \\ Emission}} = A_{21}$$

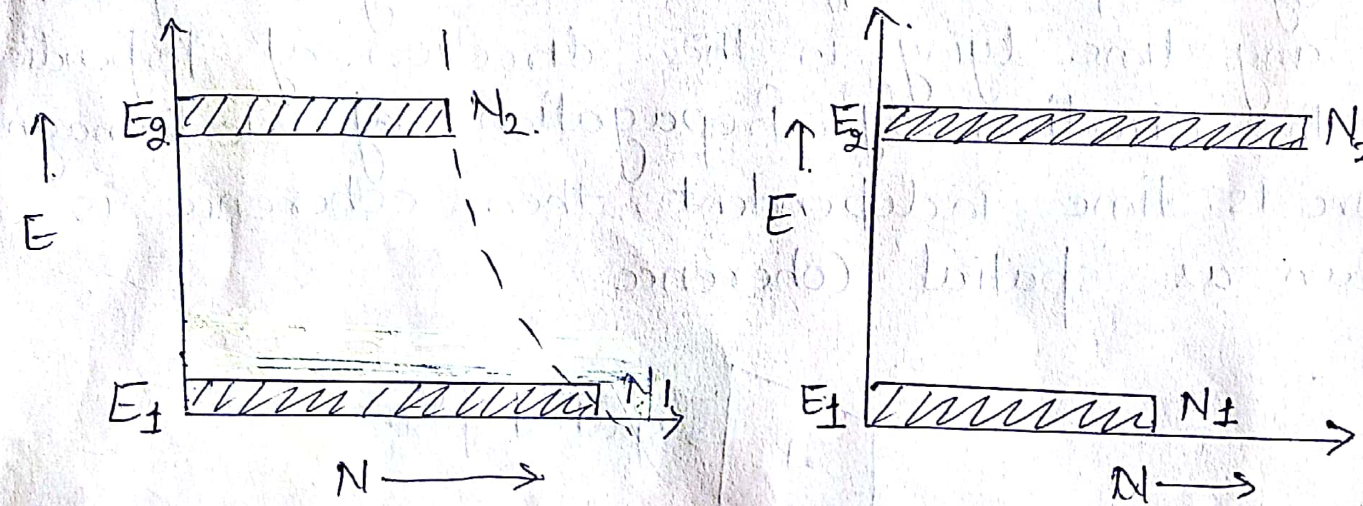
$$A_{21} = s^{-1}$$

Population Inversion

Let us consider two Energy levels (E_1 & E_2) where ($E_1 < E_2$). Let N_1 & N_2 be the number of atoms in energy levels E_1 & E_2 respectively.

Under Ordinary Conditions of.

thermal equillibrium no. of atoms in Energy level E_1 is greater than E_2. i.e. $(N_1 > N_2)$ But the condition in which no. of atoms in energy level E_2 is greater than E_1 i.e $(N_2 > N_1)$ is known as population Inversion. It is necessary & essential condition for Laser action to takes place.



Pumping

The process through which state of Population Inversion is achieved is known as Pumping. There are various types of Pumping like.

(1) optical Pumping (used in Ruby Laser)
(2) Electric discharge (used in He-Ne Laser)
(3) Inelastic atom-atom collision
(4) Direct Converstion (used in SemiConductor Laser)
(5) chemical reaction (used in CO_2 Laser)

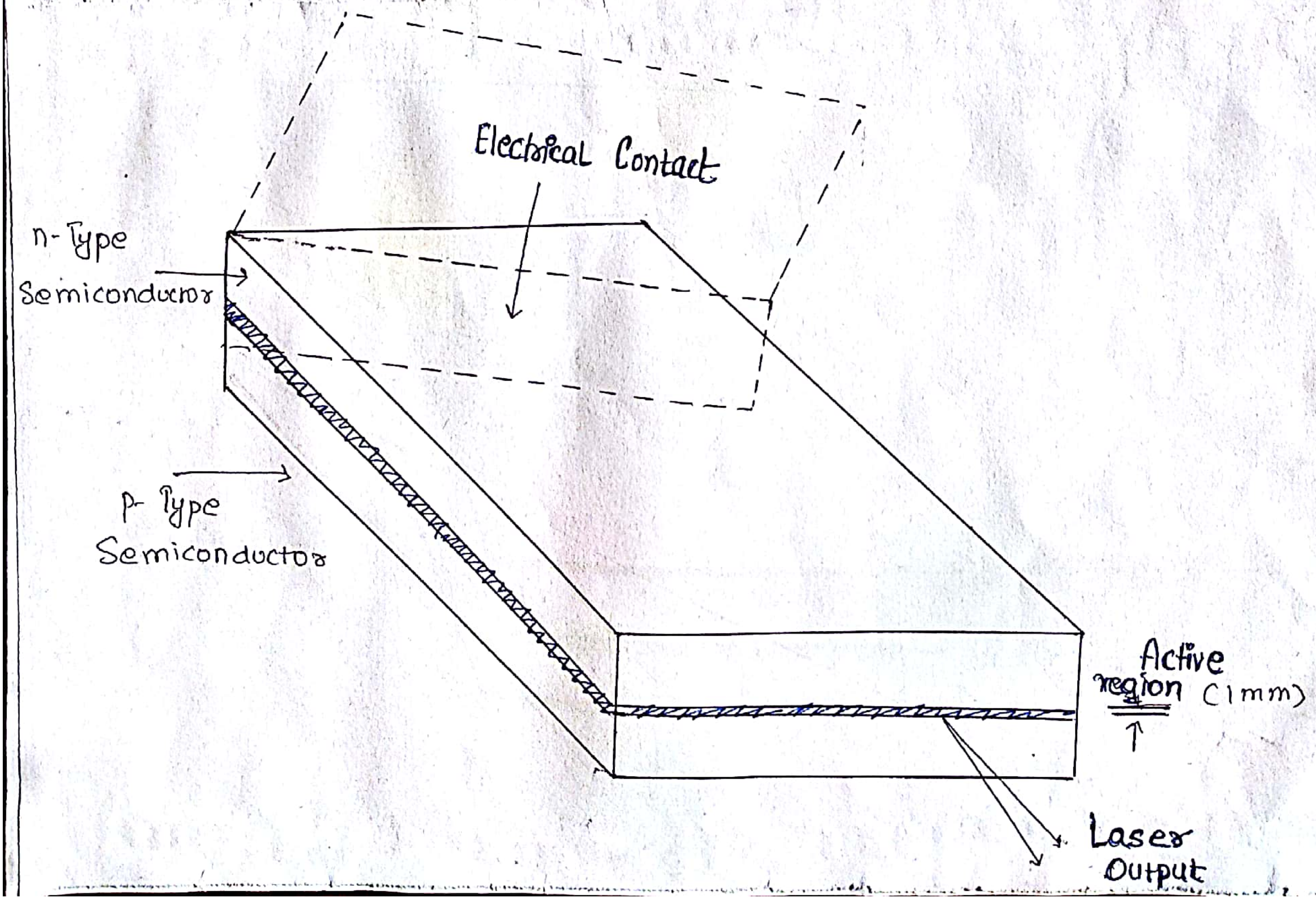
SemiConductor Laser.

SemiConductor Lasers are also known as Laser diode. It is pn junction device which emit Coherent light in forward bias condition.

Laser diode is very popular for their compactness & operational efficiency.

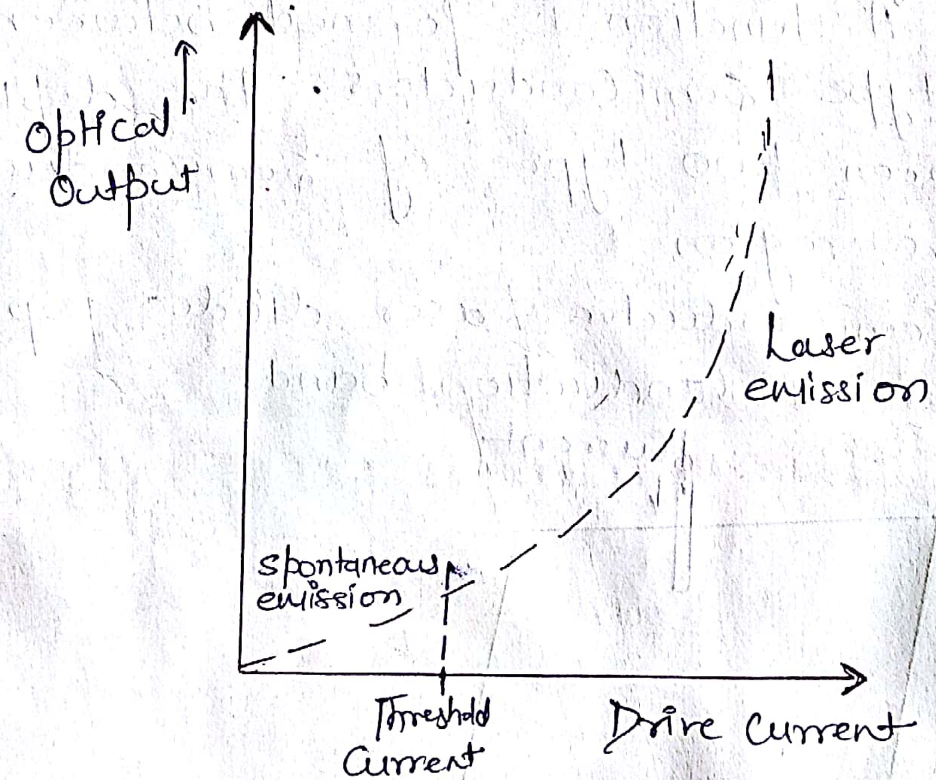
The SemiConductor Laser diode are made up of direct band SemiConductors like GaAs. A Junction is formed between P-type & N-type SemiConductors. The depletion region between two types of SemiConductors act as active region

In SemiConductor Laser diodes population inversion occur in Conduction band.



when laser diode is in forward bias electron from n-side & holes from p-side diffuse into depletion region & both carriers combine & energy is released in form of light photons

Under forward biased condition when drive current through electrical connection reaches threshold level, population inversion results & laser emission will takes place.



Semi Conductor Laser is different from Solid Gas & liquid Laser as

(1) Juction laser is quite small (approx 0.1 x 0.1 x 0.3 mm)

(2) Hight efficiency

(3) Laser output is easily modulated by.

controlling the Junction Current.

(4) Semiconductor Lasers operate at low power compared to ruby or CO_2 Laser.

(5) Output power is Higher than He-Ne Laser.

(6) These Lasers are portable also

Optical Transitions in Bulk Semiconductors

A number of mechanisms can lead to the absorption & emission of photons in bulk semiconductors. The most important of these are

→ Band to Band (Interband Transition)

→ Impurity to band transitions

→ free Carriers (Intraband Transitions)

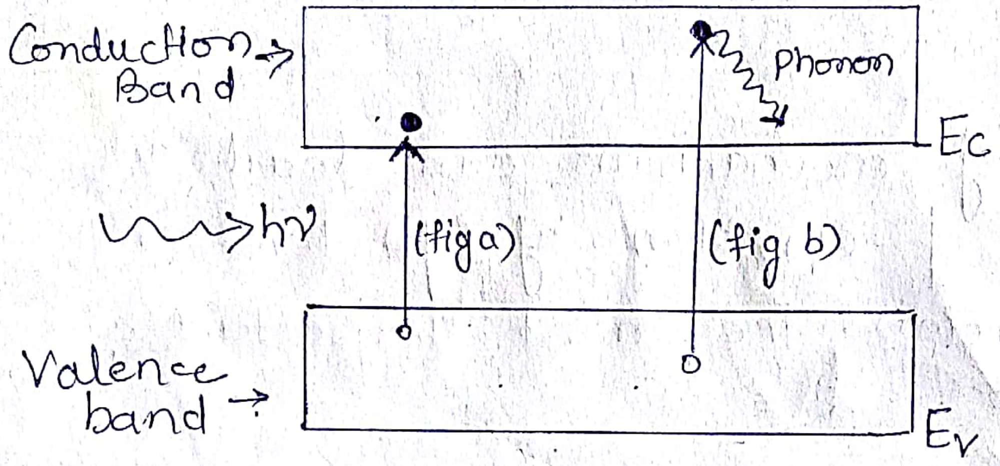
→ Phonon Transitions

→ Excitonic Transitions

1. Band to Band (Interband Transitions)

When a semiconductor is illuminated, photons are absorbed to create $e^{\ominus}$-hole pair. as shown is fig (a) for a photon of energy $h\nu = E_g$.

If a incident photon having Energy ($h\nu$) is greater than E_g, again $e^{\ominus}$ hole is generated & the excess Energy ($h\nu - E_g$) is dissipated through phonons as heat. as shown in fig(b) Both process is called intrinsic transitions or band to band transitions.



It is true for reverse situation also. for eg, an $e^{\ominus}$ at Conduction band edge (E_c), Combining with hole at Valence band edge (E_v), will gives emission of photon with Energy ($h\nu$) equal to E_g i.e

$$h\nu = E_g.$$

for Absorption & Emission of photons

$$h\nu > E_g.$$

The minimum frequency ν_{min} necessary for transition is

$$\boxed{\nu_{min} = \frac{E_g}{h}}$$

& corresponding wavelength (λ_{max}) called cut off wavelength is

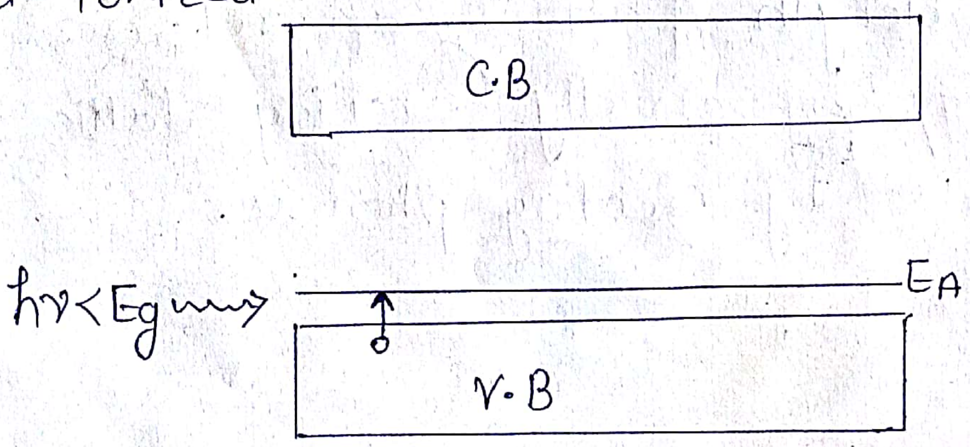
$$\boxed{\lambda_{max} = \frac{c}{\nu_{min}} = \frac{ch}{E_g} \simeq \frac{1.24}{E_g\,(eV)}\ (\mu m)}$$

2. Impurity to band Transition

If $h\nu < E_g$, a photon will be absorbed only if there are available energy states within forbidden gap (like donor level or Acceptor level) due to impurities.

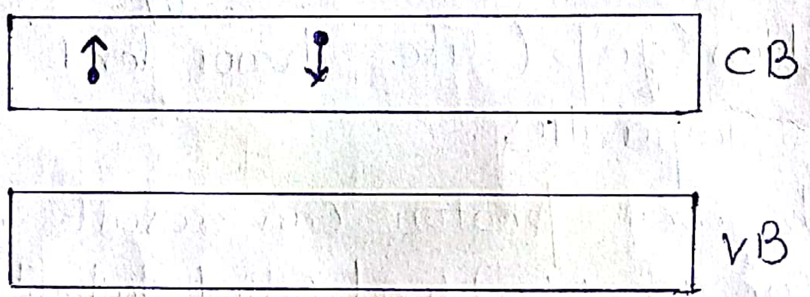
An absorbed photon can result in transition b/w donor (or acceptor) level called Impurity to band transition.

eg:- A low energy photon lift an electron from valence band of p-type semiconductor to the acceptor level, where it get trapped inside the inside the acceptor atom. In this process hole is created in valence band & acceptor atom get ionized.



3. free Carrier (Intraband Transition)

An absorbed photon Can impart its energy to the electron in a given band, Causing it to move higher with that of band. for example An electron in Conduction band Can absorb a photon & move to the higher Energy level within the Conduction band.



This will followed by the process of thermalization, where the electron relax down the the bottom of Conduction band while releasing its Energy in form of lattice Vibrations.

4. Phonon Transition

long Wavelength photons Can release their Energy by directly exciting lattice vibrations i.e. by creating phonons.

5. Excitonic Transitions

These transitions appear only in pure Semiconductors. The pairing of electron hole pair produced near the band gap is called exciton. (It is very much likely the hydrogen atom where electron & proton pair with a definite binding energy). The excitons appear in pure Semiconductors because of their low binding Energy & they are strongly temperature dependent.

Optical Joint Density of States OR Density of States for photons

The quantity $D(\nu)$ which incorporates the density of states in both Conduction & Valence band is called optical Joint density of states because a photon is emitted only when an electron Jump from one of the state of Conduction band to one of the State in Valence band

We know density of states in Conduction band is given by.

$$D(E) = N(E) = \frac{4\pi}{h^3} (2m_e^*)^{3/2} (E-E_c)^{1/2}$$ {from unit-2}

when $E > E_c$

E_c = energy at bottom of conduction band

$$D(E) = N(E) = 4\pi \left(\frac{2m_e^*}{h^2}\right)^{3/2} (E-E_c)^{1/2} \quad \text{--- (1)}$$

we know

$$E = E_c + \frac{m_r}{m_e^*}(h\nu - E_g)$$

so;

$$E - E_c = \frac{m_r}{m_e^*}(h\nu - E_g) \quad \text{--- (2)}$$

Put Eqⁿ (2) in (1)

$$D(E) = N(E) = 4\pi \left(\frac{2m_e^*}{h^2}\right)^{3/2} \left(\frac{m_r}{m_e^*}\right)^{1/2} (h\nu - E_g)^{1/2} \quad \text{--- (3)}$$

Now; density of states for photons is

$$D(\nu) = \frac{dE}{d\nu} D(E)$$

$$D(\nu) = \left(\frac{m_r h}{m_e^*}\right) 4\pi \left(\frac{2m_e^*}{h^2}\right)^{3/2} \left(\frac{m_r}{m_e^*}\right)^{1/2} (h\nu - E_g)^{1/2}$$

$$\boxed{D(\nu) = \frac{4\pi}{h^2} (2m_r)^{3/2} (h\nu - E_g)^{1/2}}$$

Fermi Golden Rule

Fermi Golden rule is a formula that describe the transition rate from one energy state into another energy state.
In Genral, the transition rate (or the transition probability per unit time) depends on

(1) Strength of Coupling b/w intial & final State
(2) the number of ways the transition Can happens i.e. density of final State.

The transition probability per unit has the form

$$\lambda_{if} = \frac{2\pi}{\hbar} |M_{if}|^2 \rho_F$$

where $\lambda_{if} \Rightarrow$ Transition probability Per unit time
$|M_{if}| \Rightarrow$ Coupling term b/w intial & final State
$\rho_F \Rightarrow$ Density of final State

The transition is high if the Coupling b/w intial & final State is stronger. The Coupling term is traditionally called "the matrix" element for the transition, is given by

$$M_{if} = \int \psi_f^* \, V \, \psi_i \, dv$$

where ψ_i = wave function of intial state

V = operator for the physical interaction which couples the intial & final state.

ψ_f^* = wave function of final state

The transition rate b/w two energy states is given by

$$\lambda_{if} = \frac{2\pi}{\hbar} |H'_{f}|^2 \delta(E_f - E_i - \hbar\omega) + \frac{2\pi}{\hbar} |H'_{if}|^2 \delta(E_f - E_i + \hbar\omega)$$

wher first terms corresponds to the absorption of photon where second term corresponds to emission of photon.

Optical Gain & Loss in two level System

Let us consider a two level System. Suppose a coherent monochromatic radiation of unit cross-sectional area passing through the lasing medium.

Inside the medium, absorption of radiation takes places so Intensity varies as

$$I = I_0 e^{-\alpha x} \quad \text{——— (1)}$$

(where I_0 = intensity of Incident radiation $(x=0)$)
α = absorption cofficient

Now, differentiating.

$$\frac{dI}{dx} = -\alpha I_0 e^{-\alpha x}$$

$$\frac{dI}{dx} = -\alpha I \quad \text{——— 1(a)} \; \{\text{using Eq}^n \ 1\}$$

If N_1 & N_2 no. of electrons in Energy level E_1 & E_2 respectively then Net rate of loss of photon density N_p is

$$-h\nu \frac{dN_p}{dt} = N_1 B_{12} u(\nu) - N_2 B_{21} u(\nu) \quad \text{——— (2)}$$

as $B_{12} = B_{21}$ {relation b/w Einstein cofficients}

as Eq^n (2) becomes

$$-h\nu \frac{dN_p}{dt} = (N_1 - N_2) B_{21} u(\nu) \quad \text{——— (3)}$$

Now; Energy density is

$$u(\nu) = N_p h\nu \quad \text{---} (4)$$

(where N_p = number of photons per unit volume having freq ν)

So, for photons of frequency ν is

$$I = u(\nu) \frac{c}{\eta_r} \quad \text{---} 4(a)$$

(where η_r = refractive index of medium)

$$I = \frac{N_p h\nu c}{\eta_r}$$

$$N_p = \frac{I \eta_r}{h\nu c}$$

If incident beam is passes through the medium b/w x & $x+\Delta x$, then change in photon density dN_p is

$$-dN_p = \frac{[I(x) - I(x+\Delta x)] \eta_r}{h\nu c}$$

$$-dN_p = -\frac{dI}{dx} \frac{\eta_r \Delta x}{h\nu c} \quad \text{---} (5)$$

Now, photons travel the distance Δx in time dt is $dt = \frac{\Delta x}{c/\eta_r}$.

So Eqⁿ (5) becomes

$$-dN_p = -\frac{dI}{dx} \cdot \frac{1}{h\nu} dt$$

$$-\frac{dN_p}{dt} = -\frac{dI}{dx} \cdot \frac{1}{h\nu} \quad \text{------------} (6)$$

Now, put the $\frac{dI}{dx}$ using Eq^n (1a) in above. Eq^n.

$$-\frac{dN_p}{dt} = \frac{\alpha I}{h\nu}$$

$$\frac{dN_p}{dt} = \frac{-\alpha I}{h\nu}$$

Now put the value of I using 4(a)

$$\frac{dN_p}{dt} = -\alpha\, u(\nu) \frac{c}{\eta_r} \frac{1}{h\nu}$$

$$-h\nu \frac{dN_p}{dt} = \alpha\, u(\nu) \frac{c}{\eta_r} \quad \text{------------} (7)$$

Now Compare Eq^n (3) & (7) we get

$$(N_1 - N_2) B_{21} \cancel{u(\nu)} = \alpha \cancel{u(\nu)} \frac{c}{n_r}$$

$$\boxed{\alpha = \frac{(N_1 - N_2) B_{21}\, \eta_r}{c}}$$

If $N_1 > N_2$ so α is positive. which represent the medium with loss whereas if $N_2 > N_1$ then α become negative. It represent optical Gain.

Photovoltaic Effect

In 1839, Becqurel discovered that when a pair of electrodes is dipped into electroyte & light is allowed to incident on one of the electrodes, a potential difference is created b/w these electrodes. The phenomenon is known as photovoltaic effect.

When photons are absorbed by photovoltaic cell, the energy from photon is transferred to an electron in an atom. The energized $e^{\ominus}$ is then escape its bond with in atom & generates an electric current

Drude-Model

According to drude Model optical Conductivity & dielectric constant can be determined by considering the motion of free electrons. In presence of collisions, the Equation of free $e^{\ominus}$ in Electric field is

$$m\left[\frac{d^2x}{dt^2}\right] + \underbrace{\frac{m}{\tau}\left[\frac{dx}{dt}\right]}_{} = -eE \quad \text{———} (1)$$

Collision term which is proportional to velocity also called friction term

where m = mass of $e^{\ominus}$
x = displacement
τ = relaxation time

Now if

$$x = x_0 e^{-i\omega t} \quad \text{———} (2)$$

then differentiate above Eq^n.

$$\frac{dx}{dt} = -i\omega x_0 e^{-i\omega t}$$

$$\frac{dx}{dt} = -i\omega x \qquad \{\text{using Eq}^n\ 2\}$$

again differentiate Eq^n (2)

$$\frac{d^2x}{dt^2} = -i\omega x - i\omega x_0 e^{-i\omega t}$$

$$\frac{d^2x}{dt^2} = -\omega^2 x \qquad \{\text{using Eq}^n \ 2\}$$

Now put the value of $\frac{dx}{dt}$ & $\frac{d^2x}{dt^2}$ from above Eq^n in Eq^n (1)

$$-m\omega^2 x - \frac{i\omega m x}{\tau} = -eE$$

$$-x\left[m\omega^2 + \frac{i\omega m}{\tau}\right] = -eE$$

$$x = \frac{eE}{m\omega^2 + \frac{i\omega m}{\tau}} \quad \text{———— (3)}$$

To obtained optical Conductivity, we consider $x = v\tau$ — (4) where v = mean velocity of $e^{\ominus}$

So Current density

$$J = nev = \frac{nex}{\tau}$$

Now put the value of x from Eq^n (3)

$$J = \frac{ne}{\tau}\left[\frac{eE}{m\omega^2 + \frac{i\omega m}{\tau}}\right] \quad \text{———— (4)}$$

we know $J = \sigma E$

$$\sigma = \frac{J}{E}$$

$$\sigma = \frac{ne^2}{\tau\left[m\omega^2 + \frac{i\omega m}{\tau}\right]} \qquad \{\text{using Eq}^n \ 4\}$$

$$\boxed{\sigma = \frac{ne^2}{m\omega\tau\left[\omega + \frac{i}{\tau}\right]}}$$

when collision frequency $\left(\frac{1}{\tau}\right)$ is small as compared with frequency ω of light (i.e. in absence of collision above Eqⁿ reduces to

$$\boxed{\sigma = \frac{ne^2}{m\omega\tau}} \quad \& \quad \boxed{x = \frac{eE}{m\omega^2}}$$

This is known as drude formula for optical conductivity.