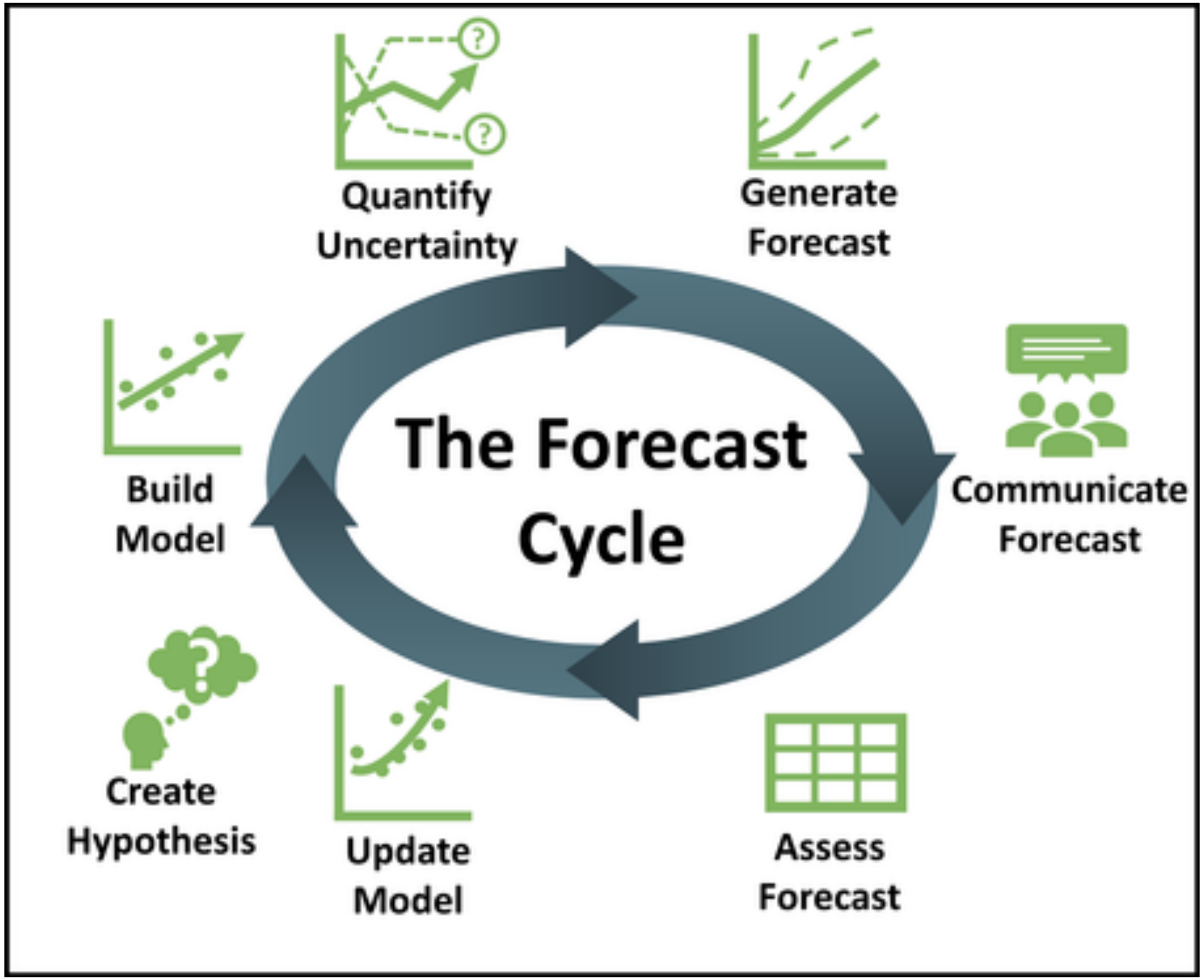
Autoregressive models

Flexible workhorse of time series forecasting



Moore, T.N., C.C. Carey, and R.Q. Thomas.

Autoregressive models

The current state is a linear combination of past states.
In other words a regression against itself.

AR(P) $$Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \beta_3 Y_{t-3} \dots + \beta_p Y_{t-p} + \varepsilon_t$$

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AR(1)

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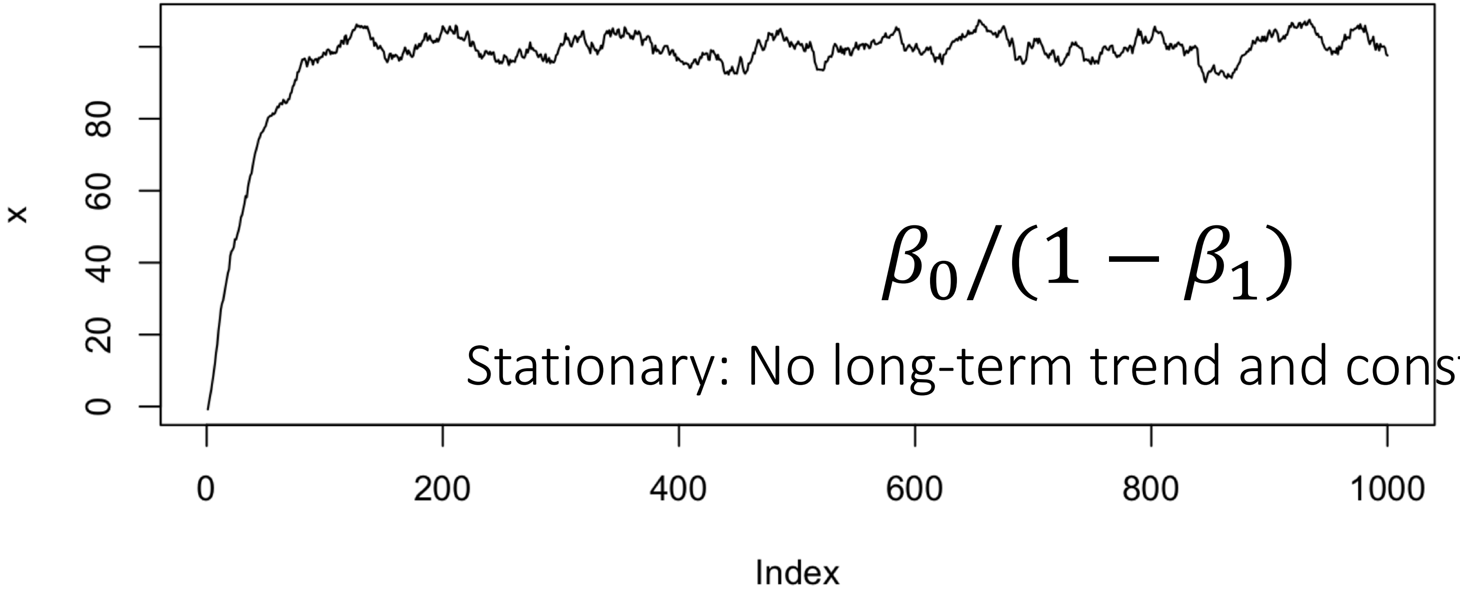
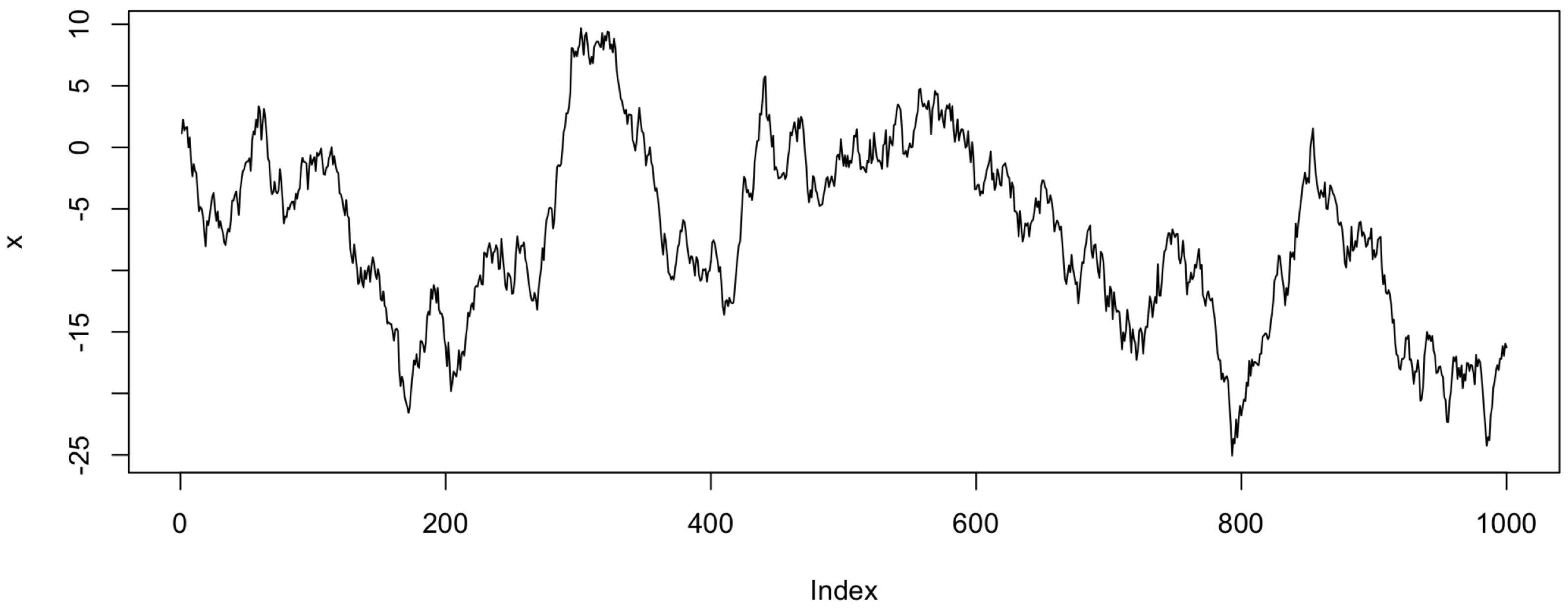


$$\beta_0/(1-\beta_1)$$

Stationary: No long-term trend and constant variance.

Random walk

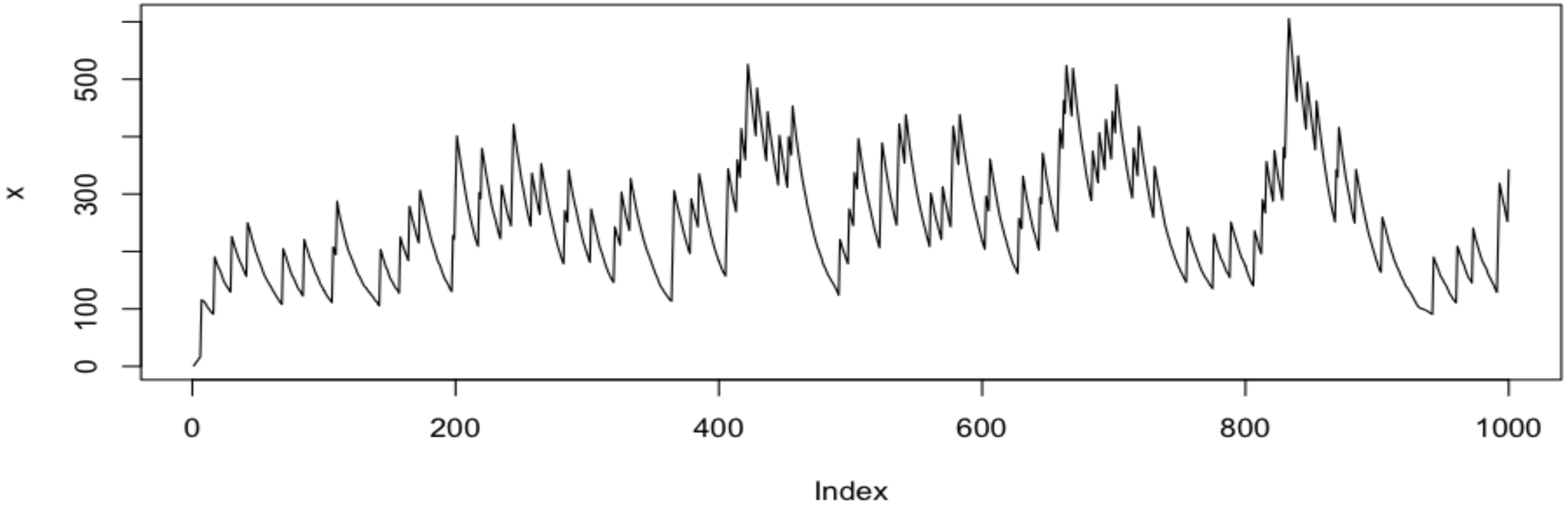
$$Y_t = 0 + 1 * Y_{t-1} + \varepsilon_t$$



What makes AR models so useful?

Very little data required, but can be made more complex

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 P_t + \varepsilon_t$$



Simple model for soil moisture dynamics

What makes AR models so useful?

Can also represent mechanistic models, e.g. Gompertz model

$$n_t = n_{t-1}e^{a+(b-1)\ln(n_{t-1})}$$

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$$\ln(n_t) = a + \mathrm{b} * \ln(n_{t-1})$$

a= low density growth rate

$a/(1-b)$= carrying capacity

Inverse and forward modeling

In todays lab, we will use both inverse and forward modeling

1) Inverse modeling to fit AR(1) model to NDVI data

2) Forward model to forecast NDVI given fit AR(1), where initial condition is last NDVI measurement