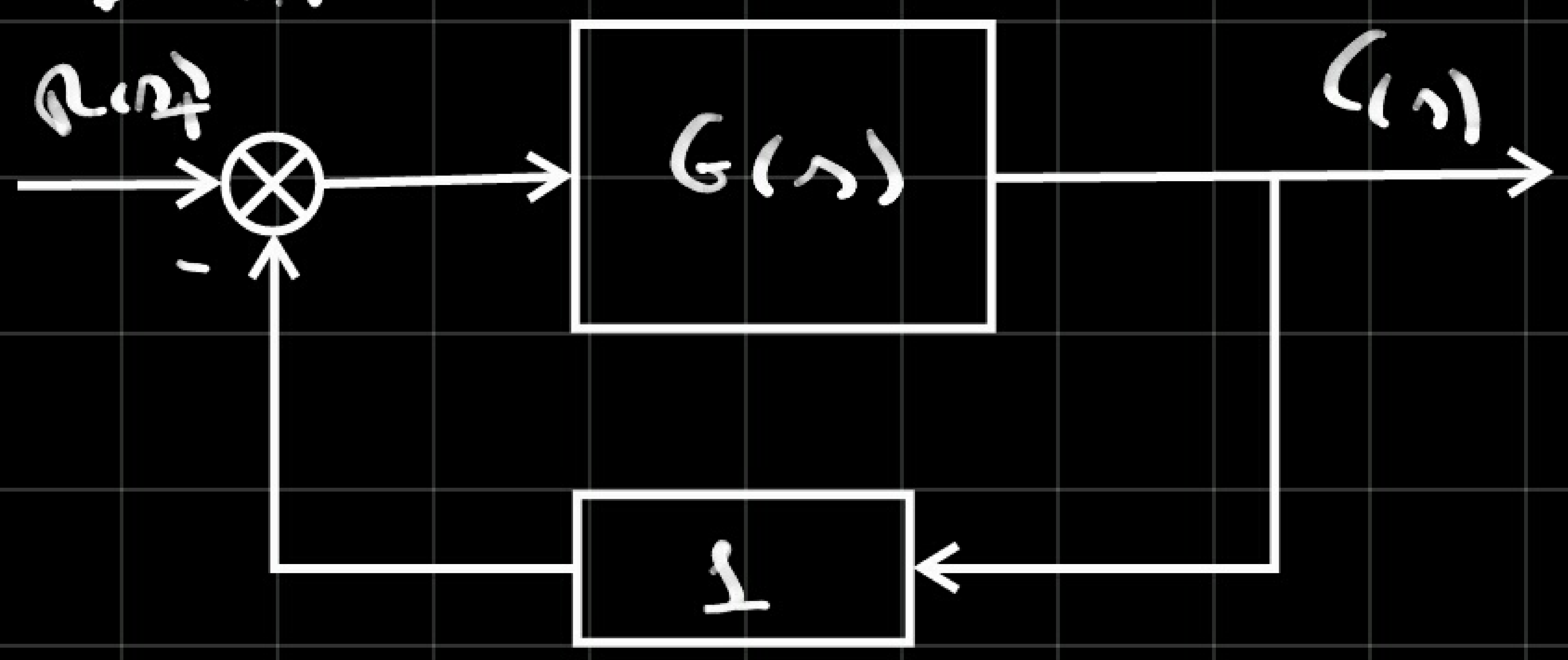
B-7.1-



$$G_{cl} = \frac{GH}{1+GH} = \frac{10/s+1}{1+(10/s+1)} = \frac{10}{s+1+10} = \frac{10}{s+11}$$

a) $\left|\frac{10}{j+11}\right| = \frac{10}{\sqrt{11^2+1}} = 0,9053$

$$\angle\left(\frac{10}{j+11}\right) = -5,1944$$

$$C_{ss} = 0,9053 \sin(t + 24,8^\circ)$$

b) $\left|\frac{10}{2j+11}\right| = 0,8944$

$$\angle\left(\frac{10}{2j+11}\right) = -10,3048$$

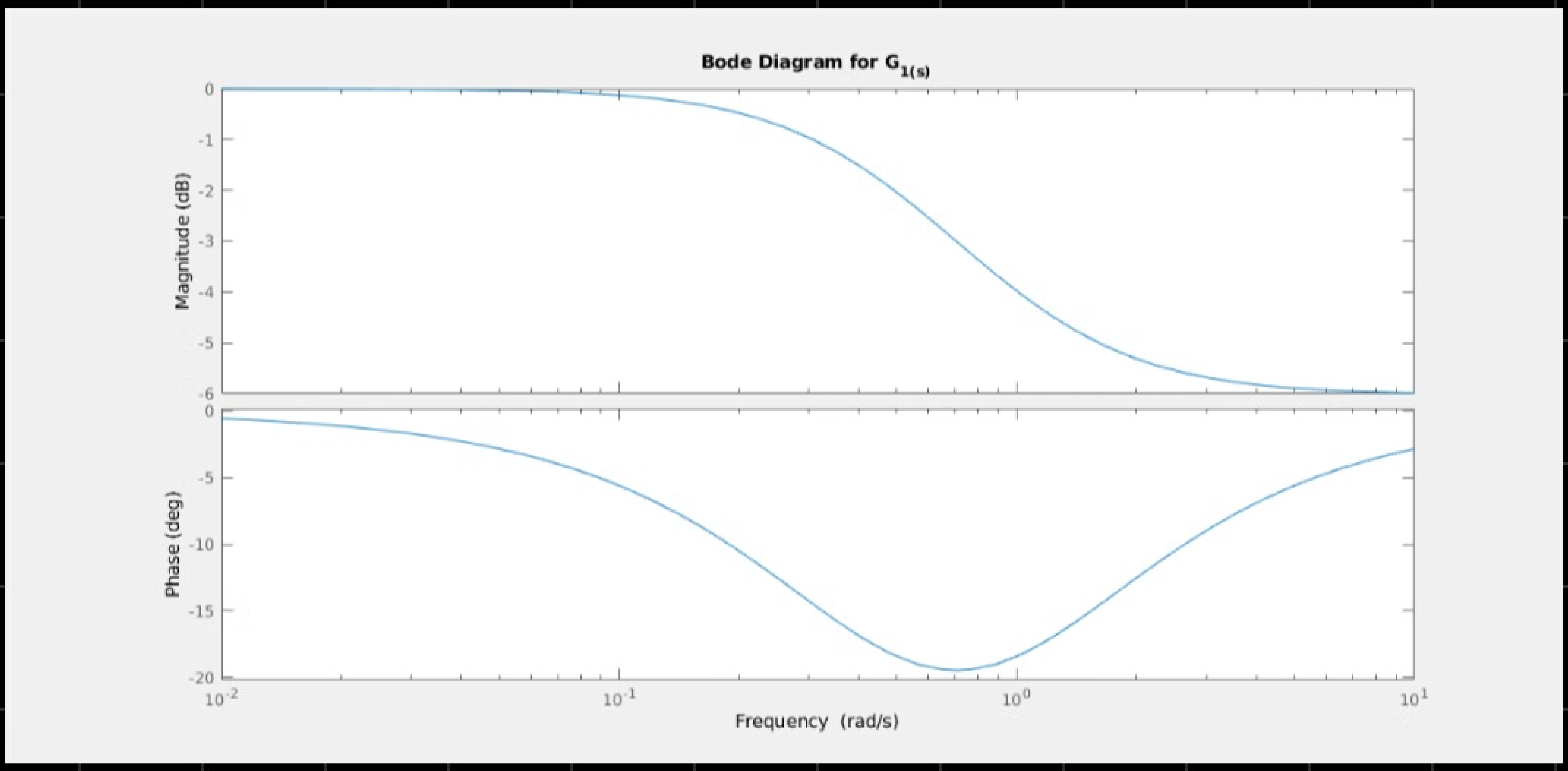
$$C_{ss} = 1,7888 \cos(2t - 55,3048)$$

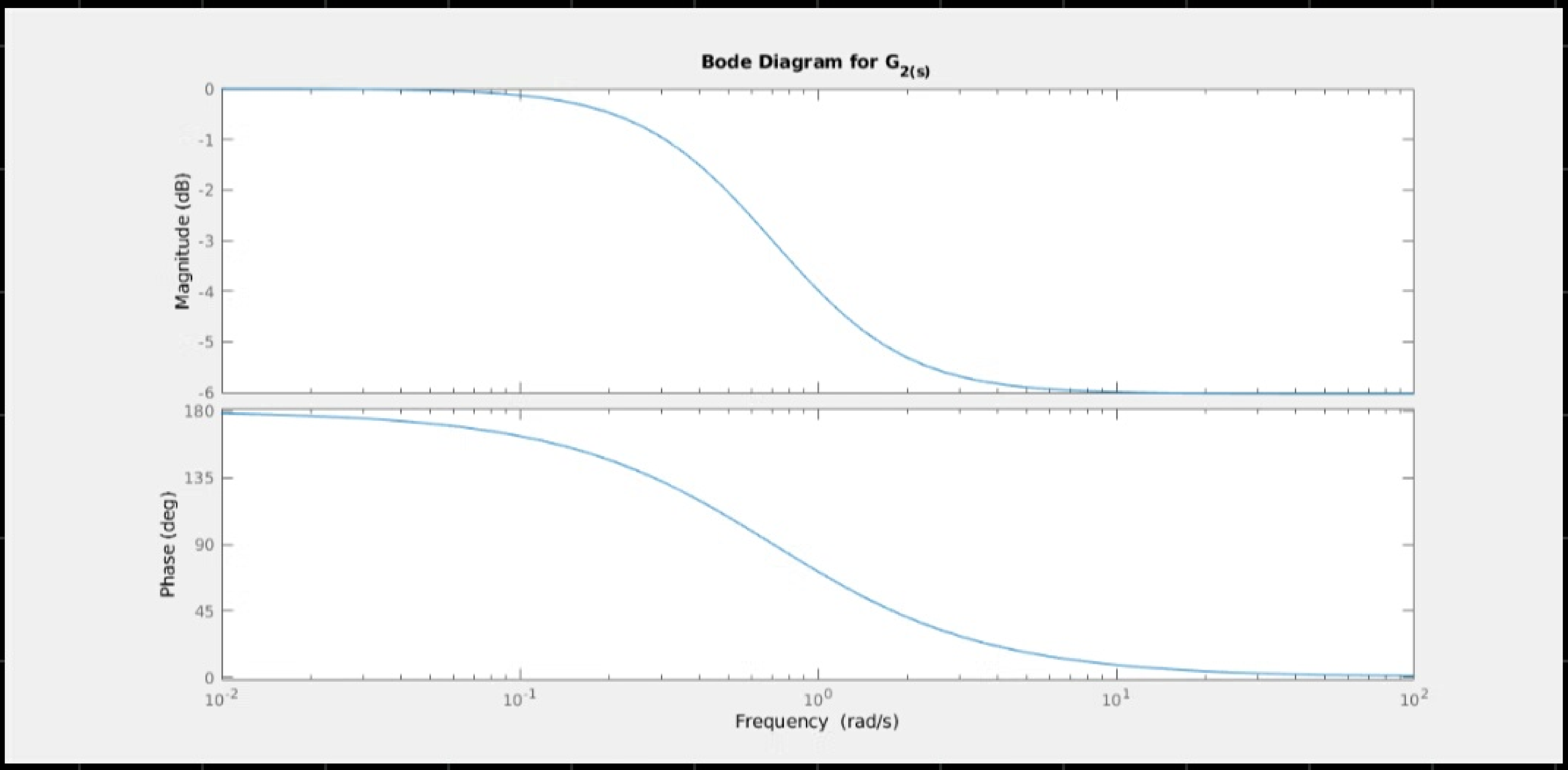
c) $C_{ss} = 0,9053 \sin(t + 24,8^\circ) - 1,7888 \cos(2t - 55,3048)$

B-7.2

$$C(s) = KR\sqrt{\frac{(\omega^2 T_2^2 + 1^2)}{(\omega^2 T_1^2 + 1^2)}}\ \sin\left[\omega t + tg^{-1}(\omega T_2) - tg^{-1}(\omega T_1)\right]$$

B-7.3

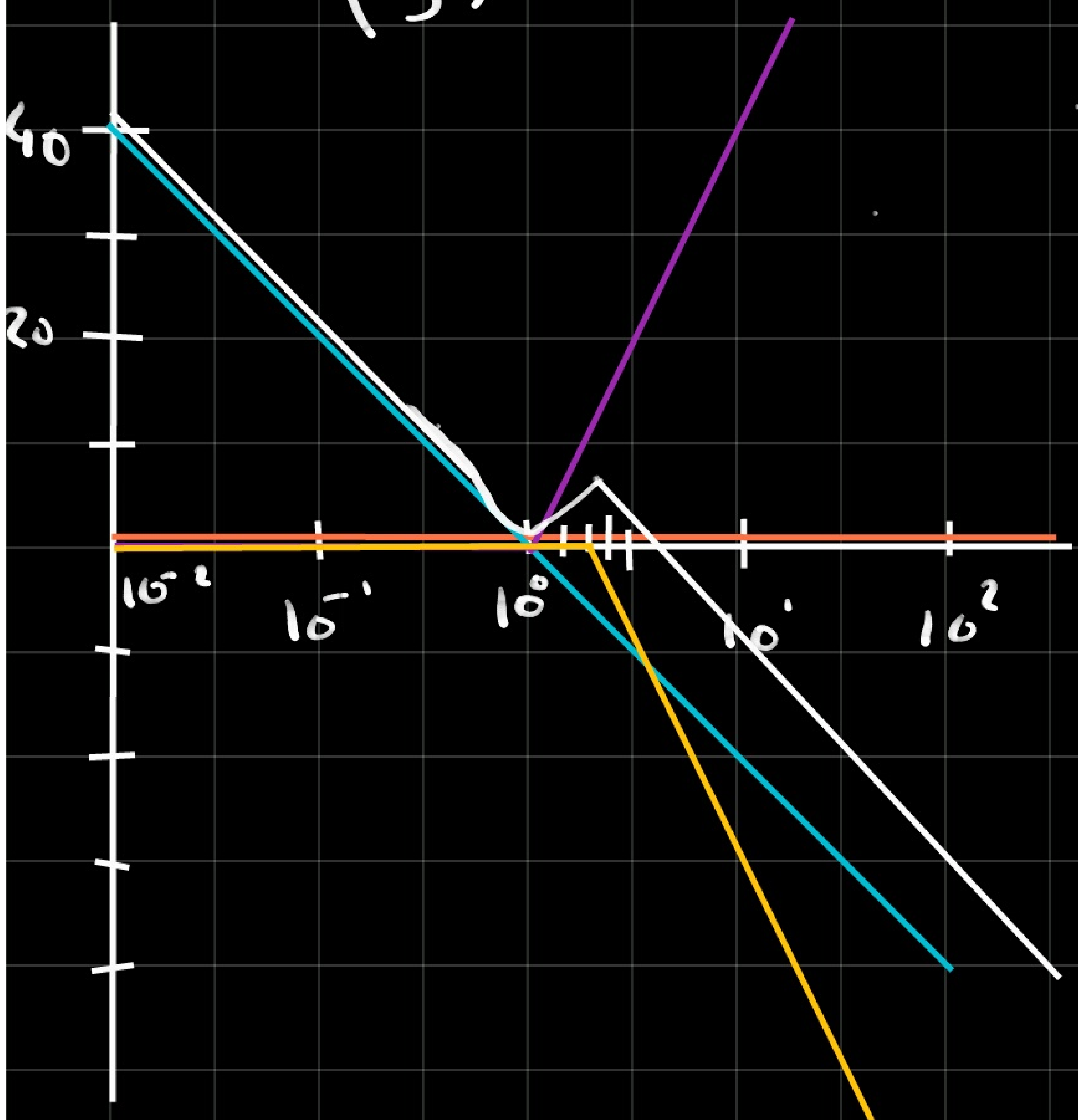





```
Gs1=tf([1 1],[2 1])
figure
bode(Gs1)
Gs2=tf([1 -1],[2 1])
title("Bode Diagram for G_{1(s)}")
figure
bode(Gs2)
title("Bode Diagram for G_{2(s)}")
```

7.4 $G(s) = \dfrac{10(s^2+0{,}4s+1)}{s(s^2+0.8s+9)} = \boxed{\dfrac{10}{9}} \dfrac{\left[1+(2 \cdot 0.2 j\omega/1)+(j\omega/1)^2\right]}{s\left[1+2 \cdot 0{,}4 j\omega/3+(j\omega/3)^2\right]}$

$G_0 = 20\log\left(\dfrac{10}{9}\right) = 0{,}915$



B-7.5

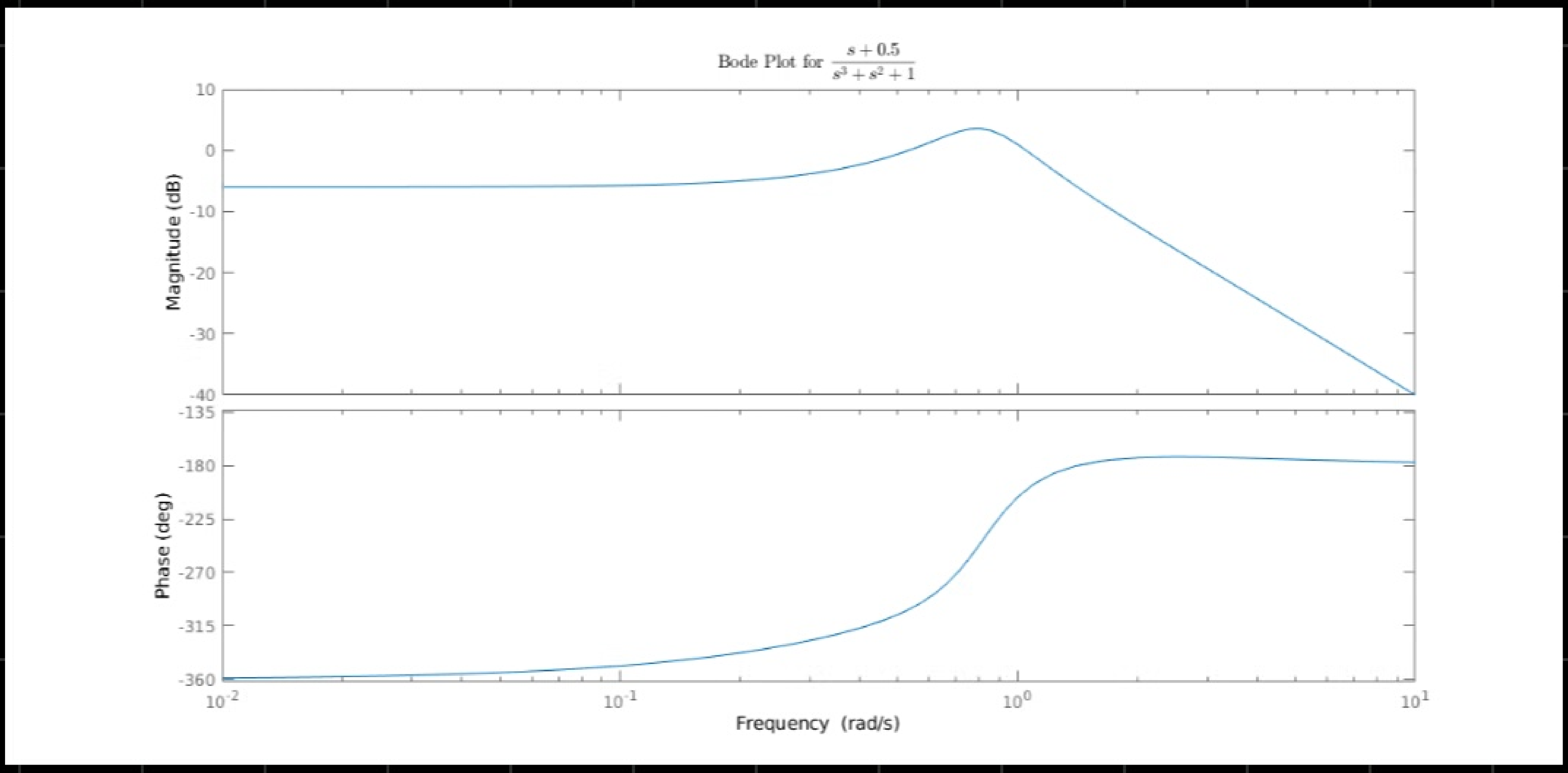
$$G(j\omega) = \frac{1}{(j\omega)^2 + 2\zeta\omega_n(j\omega) + \omega_n^2} = \frac{1}{\left(j\frac{\omega}{\omega_n}\right)^2 + 2\zeta\left(j\frac{\omega}{\omega_n}\right) + 1}$$

$$G(j\omega_n) = \frac{1}{-\frac{\omega_n}{\omega_n} + 2\xi j\frac{\omega_n}{\omega_n} + 1}$$

$$|G(j\omega_n)| = \left|\frac{1}{-1 + 2\xi j + 1}\right| = \frac{1}{2\xi}$$

B-7.6

```
Gs1=tf([1 0.5], [1 1 0 1])
figure
bode(Gs1)
```

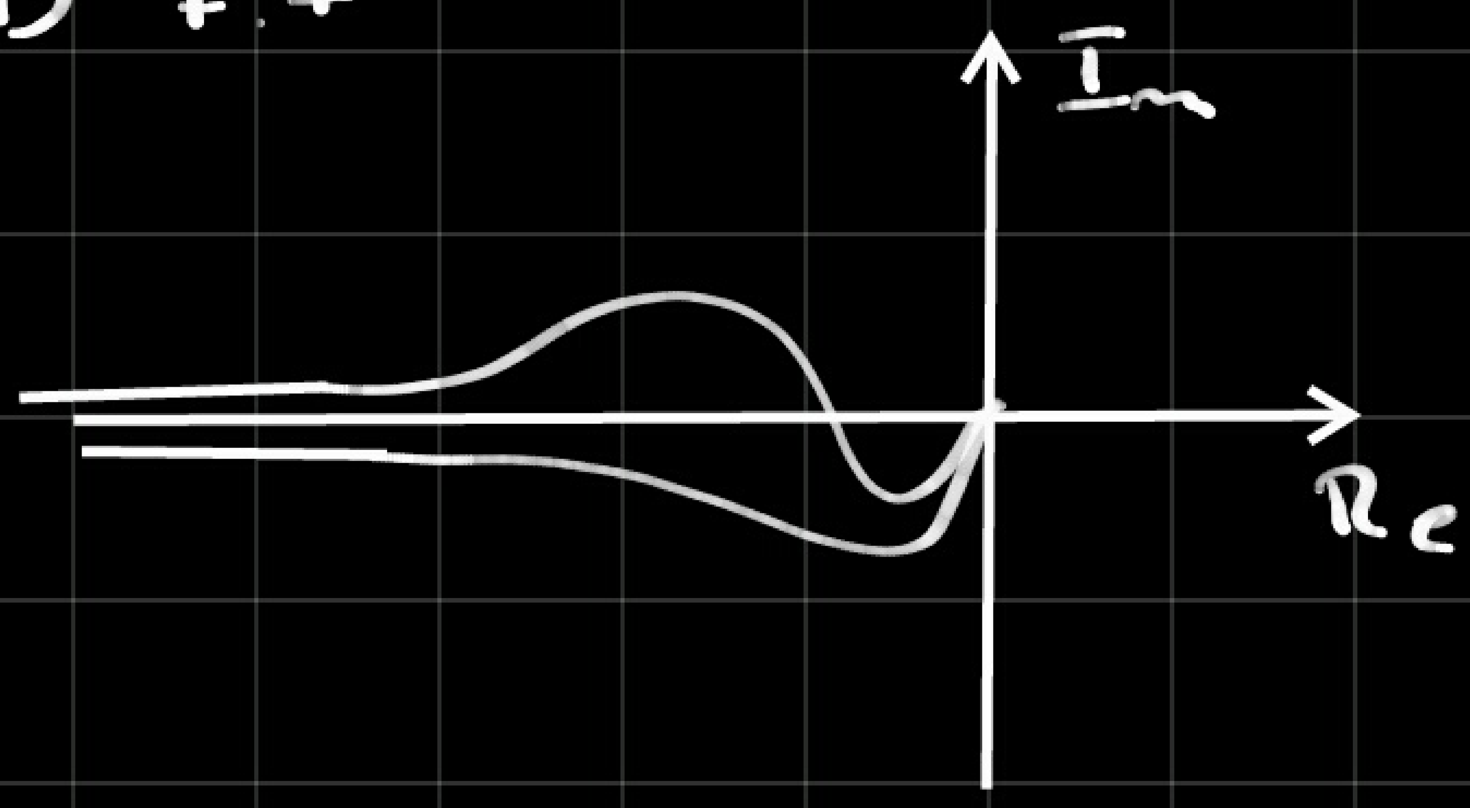


O ângulo de fase do sistema acima vai de 0 a 180° pois o sistema é de fase não mínima. Assim, calculando a fase do sistema para $\omega \to 0$ e $\omega \to \infty$, temos:

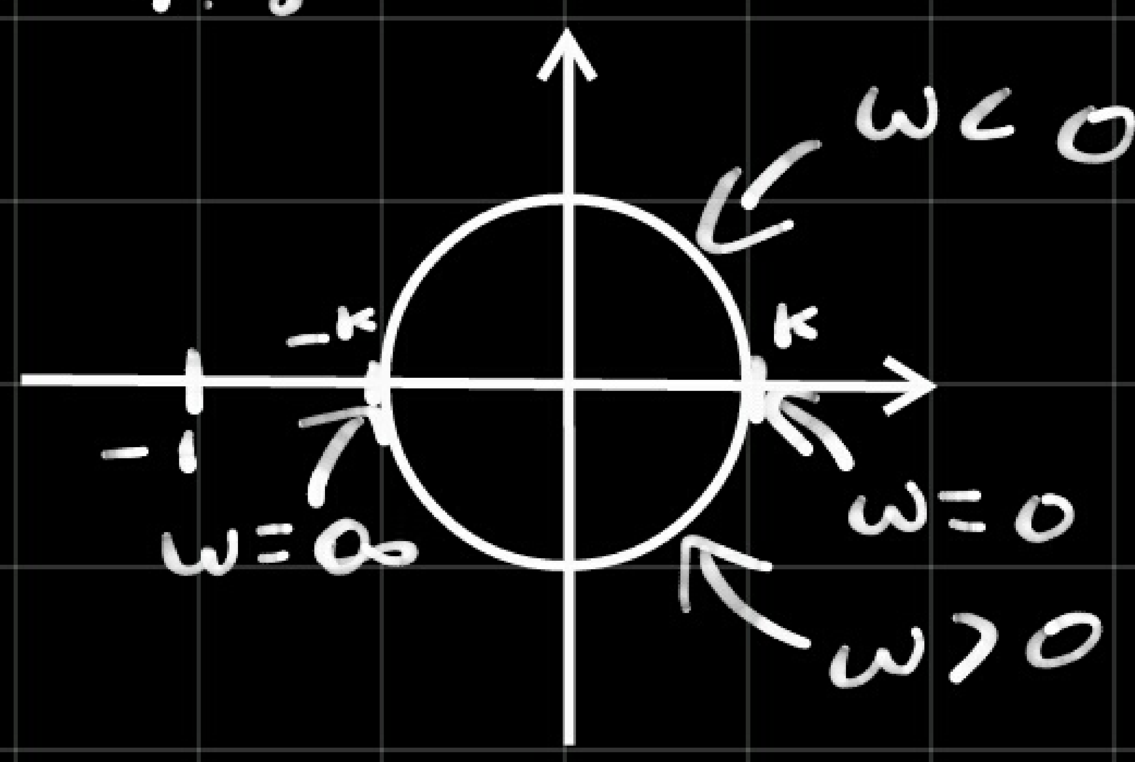
$$\angle G(j0) = \angle 0{,}5 - \angle 1{,}4656 - tg^{-1}\left(\frac{0{,}7926}{0{,}2328}\right) + tg^{-1}\left(\frac{0{,}7926}{0{,}2328}\right) = 0$$

$$\angle G(j\infty) = 90 - 90 - tg^{-1}\left(\frac{\infty}{-0{,}2328}\right) - tg^{-1}\left(\frac{\infty}{-0{,}2328}\right) = 180^\circ$$

B-7.7



B-7.8



B-7.10

$$G(s)H(s) = \frac{10K(s+0,5)}{s^2(s+2)(s+10)}$$

Para $\omega = 0$

$$\lim_{\omega \to 0^+} G(j\omega)H(j\omega) = -\infty - j0 \quad \Big\{ \quad \lim_{\omega \to 0^-} G(j\omega)H(j\omega) = -\infty + j0$$

Para $\omega = \infty$

$$\lim_{\omega \to \infty} G(j\omega)H(j\omega) = -0 + j0 \quad \Big\{ \quad \lim_{\omega \to \infty^-} G(j\omega)H(j\omega) = -0 - j0$$

Separando parte real e imaginária, temos:

$$G(j\omega)H(j\omega) = \frac{10K(j\omega+0,5)}{(j\omega)^2(j\omega+2)(j\omega+10)} = \frac{10K(j\omega+0,5)}{-\omega^2(j\omega+2)(j\omega+10)} = \frac{10K(j\omega+0.5)}{\omega^4 - 12i\omega^3 - 20\omega^2}$$

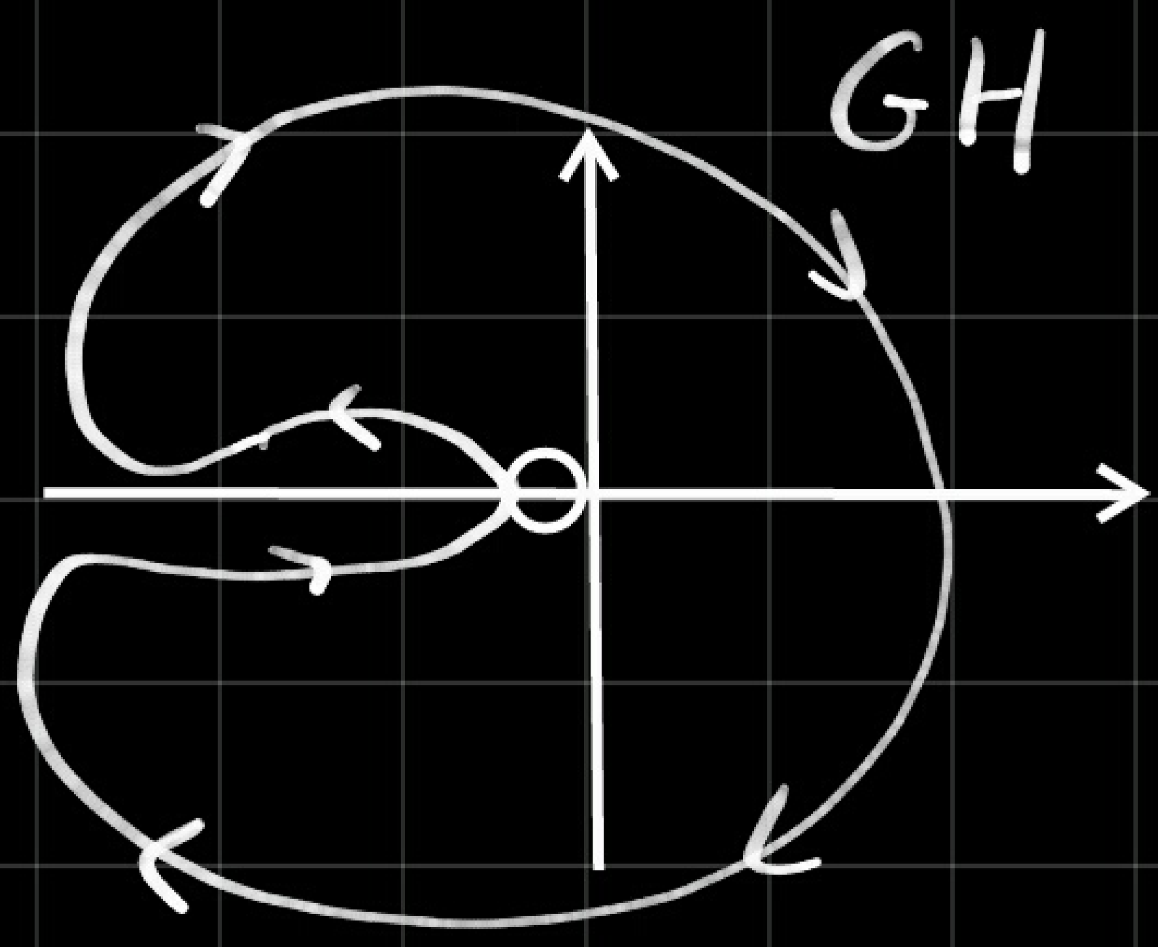
$$= \frac{10K(j\omega+0.5)}{(\omega^4-20\omega^2) - i(12\omega^3)} \cdot \frac{(\omega^4-20\omega^2)+i12\omega^3}{(\omega^4-20\omega^2)+i12\omega^3} =$$

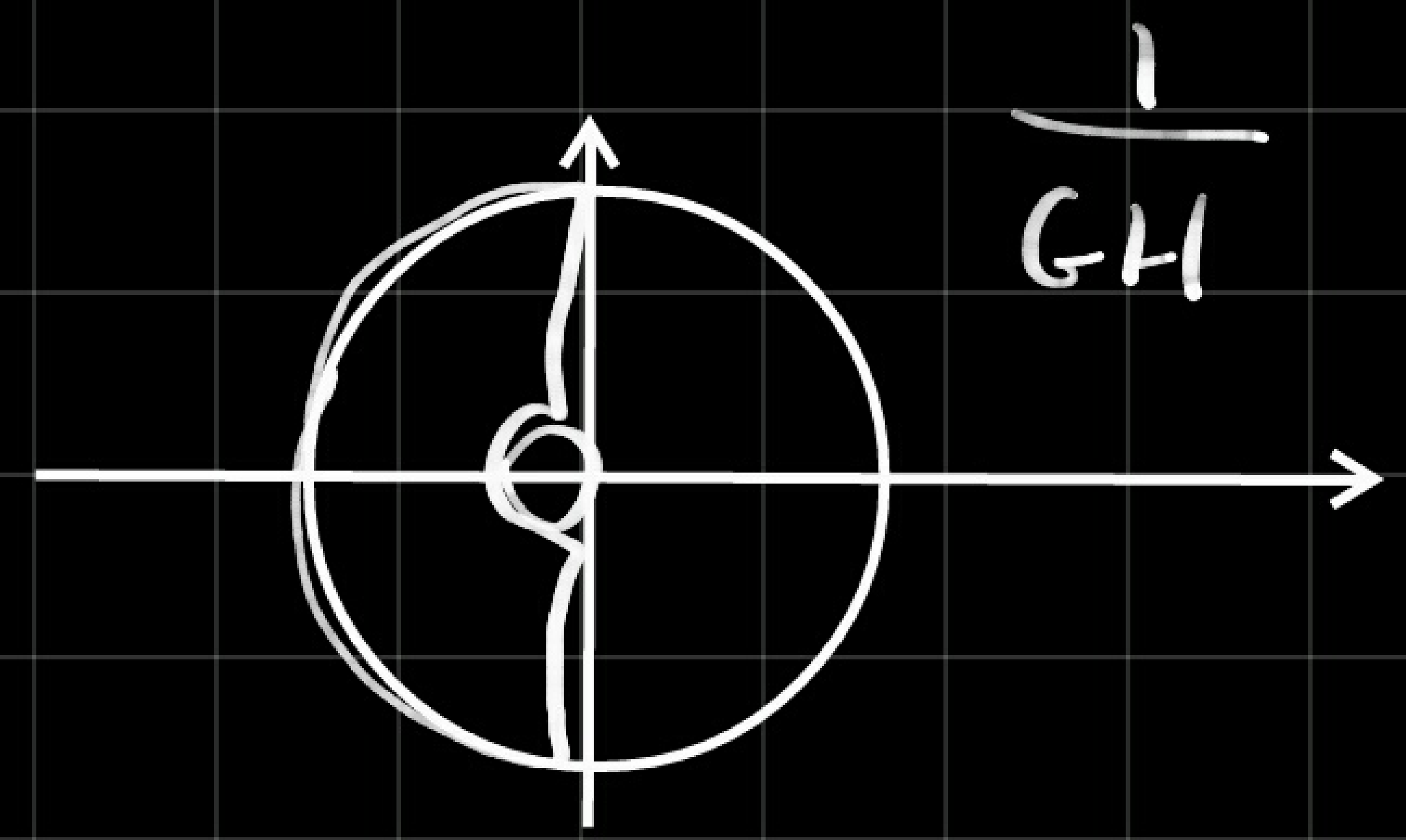
$$= \frac{-(115\omega^4+10\omega^2) + i(10\omega^5 - 140\omega^3)}{(\omega^4-20\omega^2)^2 + (12\omega^3)^2}$$

$$= \frac{-(115\omega^4+10\omega^2)+i(10\omega^5-140\omega^3)}{(\omega^4-20\omega^2)^2+(12\omega^3)^2 i}$$

$$\text{Real}\left(G(j\omega)H(j\omega)\right) = \frac{-(115\omega^4+10\omega^2)}{(\omega^4-20\omega^2)^2+(1\ \omega^3)^2} =$$

$$= \frac{-(115\omega^4+10\omega^2)+i(10\omega^5-140\omega^3)}{\omega^8-104\omega^6+400\omega^4} = \frac{-(115\omega^2+10)}{\omega^6-104\omega^4+400\omega^2}$$





B-7.11

$$G(j\omega)H(j\omega) = \frac{k\,e^{-2j\omega}}{j\omega}$$

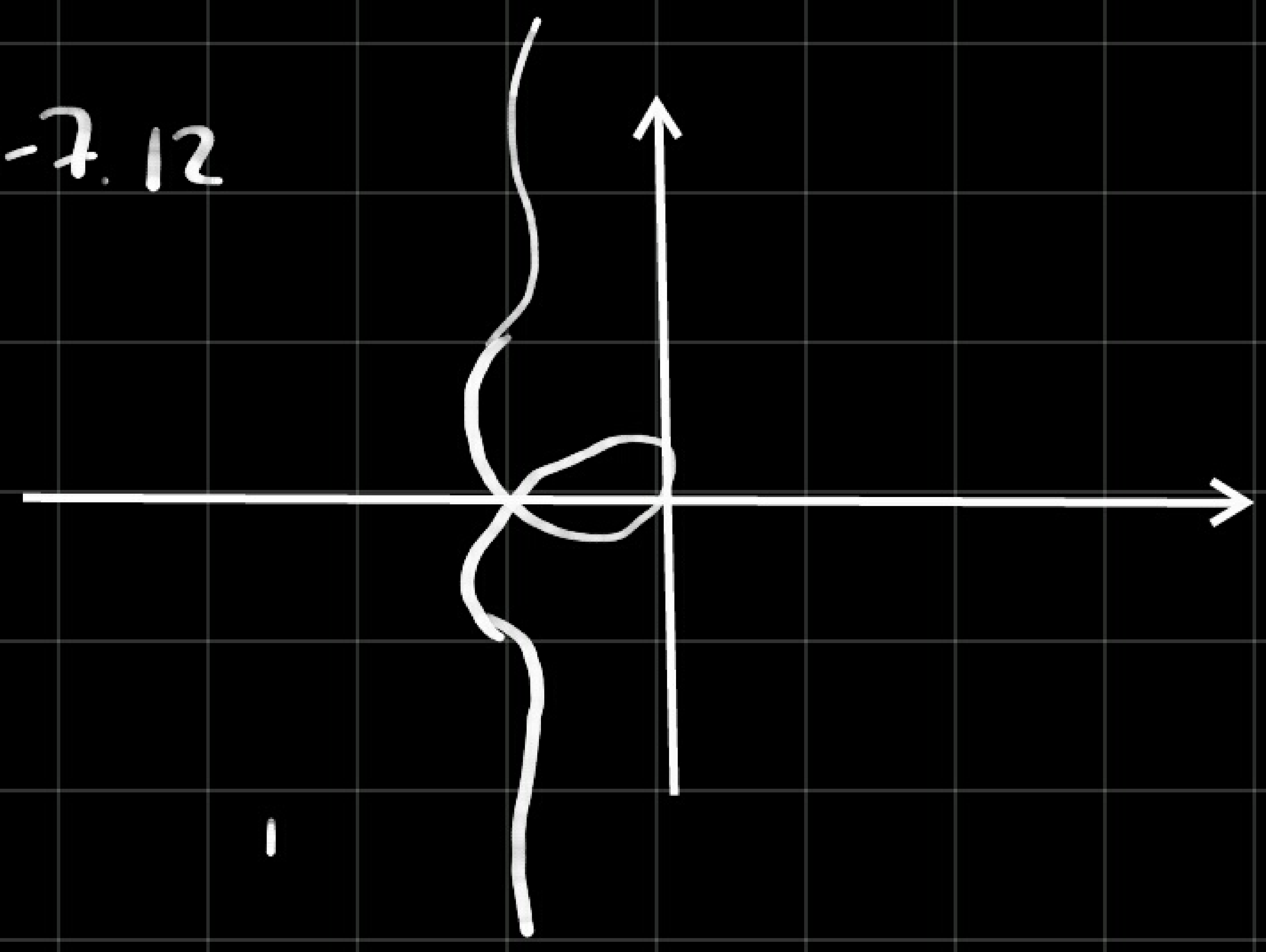
$$\angle G(j\omega)H(j\omega) = \angle[\cos(2\omega) - j\sin(2\omega)] - 90^\circ =$$
$$= -2\omega - 90^\circ$$

Para $2\omega = 90^\circ$, $\omega = 45^\circ$, o ângulo alcança 180°.

Com isso, precisamos que para $\theta = 45^\circ$, $|G(j\omega)H(j\omega)| < 1$

$$|G(j\omega)H(j\omega)| = \frac{k}{\omega}$$

B-7.12



B-7-14