Controle Dinâmico
Lista de Exercícios
Estabilidade

B-5-21.

$$s^4+2s^3+(4+k)s^2+9s+25=0$$

$$\begin{array}{llll} s^4 & 1 & 4+k & 25 \\ s^3 & 2 & 9 & 0 \\ s^2 & \frac{2k-1}{2} & 25 & \\ s^1 & \frac{18k-109}{2} & & 0 \\ s^0 & 25 & & \end{array}$$

Portanto, para garantir a estabilidade, é necessário que

$$\frac{2k-1}{2}>0 \quad e \quad \frac{18k-109}{2}>0$$

$$2k-1>0 \qquad 18k-109>0$$

$$2k>1 \qquad 18k>109$$

$$\underbrace{k>1/2 \qquad k>109/18}$$

$$K>6{,}056$$

B-5-22.

$$G(s)=\frac{k(s-2)}{(s+1)(s^2+6s+25)}$$

$$H(s)=1$$

$$C(s)=G(s)\cdot\left[R(s)-C(s)\cdot H(s)\right]$$

$$\frac{C(s)}{R(s)}\left[1+G(s)H(s)\right]=G(s) \Rightarrow \frac{C(s)}{R(s)}=\frac{G(s)}{1+G(s)H(s)}$$

$$\frac{\dfrac{k(s-2)}{(s+1)(s^2+6s+25)}}{1+\dfrac{k(s-2)}{(s+1)(s^2+6s+25)}} =$$

$$\frac{k(s-2)}{\cancel{(s+1)(s^2+6s+25)}} \cdot \frac{\cancel{(s+1)(s^2+6s+25)}}{[(s+1)(s^2+6s+25)]+k(s-2)} =$$

$$\frac{ks-2k}{s^3+7s^2+(31+k)s+(25-2k)}$$

$$\begin{array}{lll} s^3 & 1 & 31+k \\ s^2 & 7 & 25-2k \\ s^1 & \dfrac{9k+192}{7} & 0 \\ s^0 & 25-2k & \end{array}$$

$9k+192>0$

$9k>-192$

$k>-\dfrac{192}{9}=-21.33$

$25-2k>0$

$25>2k$

$k<12.5$

B-5-23

$$G(s) = \frac{K}{Js} \qquad H(s) = K_h$$

$$G_1(s) = \frac{K/Js}{1 + K_h K/Js} \qquad 1/s$$

$$G_1(s) = \frac{K}{\cancel{Js}} \cdot \frac{\cancel{Js}}{Js + K_h K} \cdot \frac{1}{s}$$

$$G_1(s) = \frac{K}{Js^2 + K K_h s}$$

$$G_2(s) = \frac{G_1(s)}{1 + G_1(s)}$$

$$G_2(s) = \frac{K/(Js^2 + K K_h s)}{1 + K/(Js^2 + K K_h s)}$$

$$G_2(s) = \frac{K}{Js^2 + K K_h s + K}$$

$$G_2(s) = \frac{K}{J(s^2 + K K_h s/J + K/J)}$$

$$G_2(s) = \frac{K/J}{s^2 + \frac{K}{J} K_h s + \frac{K}{J}}$$

Se $K/J = 4$

$$G_2(s) = \frac{4}{s^2 + 4K_h + 4} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\left.\begin{aligned} &2\zeta\omega_n = 4K_h \\ &\zeta = 0.6 \\ &\omega_n^2 = 4 \Rightarrow \omega_n = 2 \end{aligned}\right\} \quad \begin{aligned} &2 \cdot 0.6 \cdot 2 = 4K_h \\ &K_h = 0.6 \end{aligned}$$

B-5-24

$$G_1(s) = \frac{20K}{(s+1)(s+4)} \qquad G_2(s) = \frac{G(s)}{1 + G(s)H(s)}$$

$$H_1(s) = K_h \qquad G_2(s) = \frac{20K/\cancel{[(s+1)(s+4)]}}{1 + \{20K K_h/\cancel{[(s+1)(s+4)]}}$$

$$G_2(s) = \frac{20K}{20K K_h + (s+1)(s+4)} \Rightarrow G_3(s) = \frac{20K}{s^3 + 5s^2 + (4 + 20K K_h)s}$$

$$G(s) = \frac{G_3(s)}{1+G_3(s)} = \frac{20K}{s^3+5s^2+(4+20KK_h)s+20K}$$

$$\begin{array}{lll} s^3 & 1 & 4+20KK_h \\ s^2 & 5 & 20K \\ s^1 & 4+20KK_h-4K & 0 \\ s^0 & 20K & \end{array}$$

$4+20KK_h-4K > 0$ $\qquad$ $20K>0$

$20KK_h > 4K-4$ $\qquad$ $K>0$

$5KK_h > K-1$

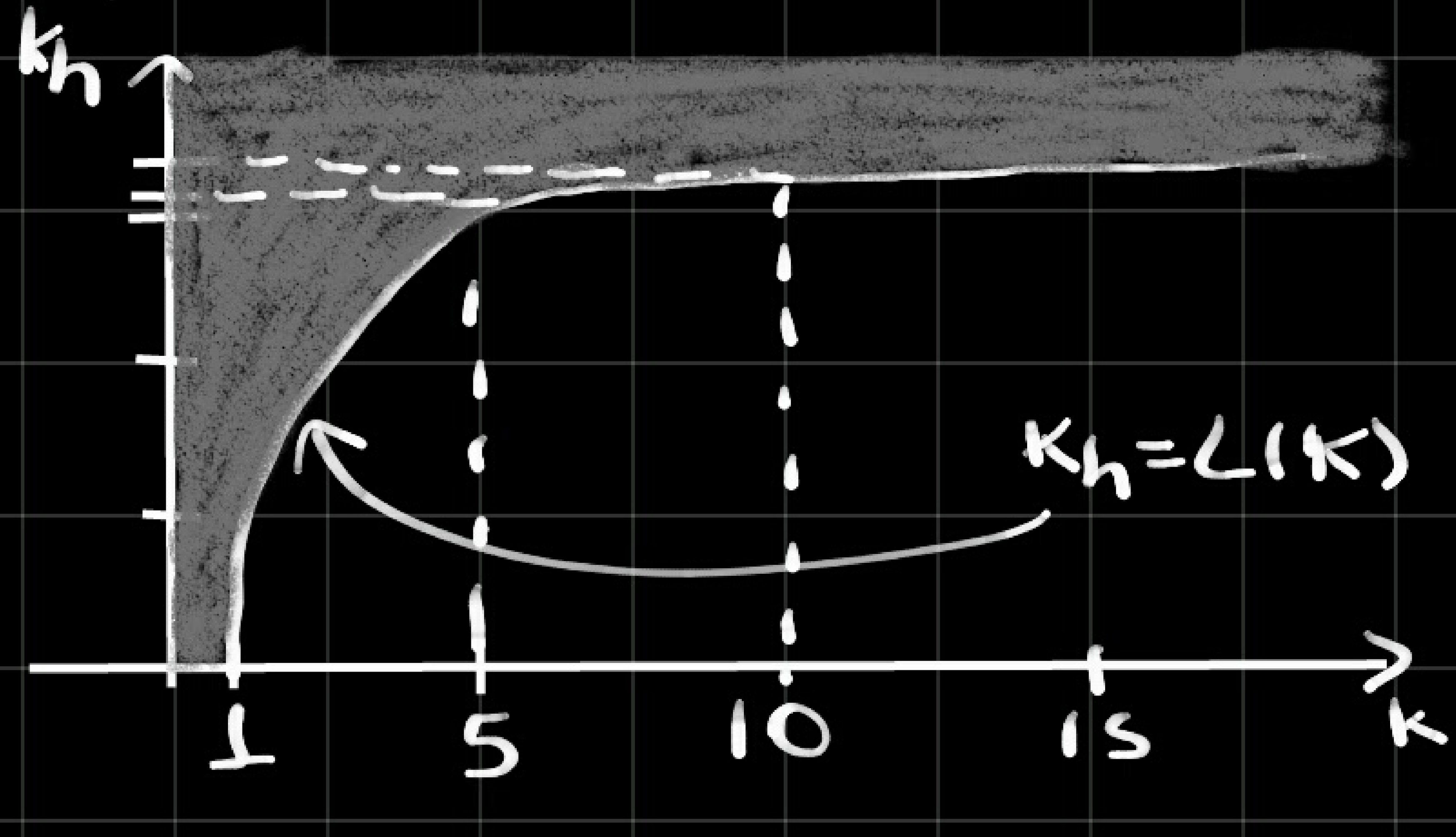
$L(K) = \frac{1}{5} - \frac{1}{5K}$ $\qquad$ $L(5) = \frac{1}{5} - \frac{1}{25}$

$L(K) = 0$ $\qquad$ $L(5) = \frac{4}{25} = 0{,}16$

$\frac{1}{5} - \frac{1}{5K} = 0$

$\frac{1}{5} = \frac{K}{5}$ $\qquad$ $L(10) = \frac{1}{5} - \frac{1}{50} = \frac{9}{50} = 0{,}18$

$K=1$ $\qquad$ $L(15) = \frac{1}{5} \; \frac{1}{75} = \frac{14}{5} = 0{,}186667$



B-5-25

$$|sI-A| = \begin{vmatrix} s & -1 & 0 \\ b_3 & s & -1 \\ 0 & b_2 & s+b_1 \end{vmatrix} = s^3 + b_1 s^2 + b_2 s + b_3 s + b_1 b_3 =$$

$$= s^3 + b_1 s^2 + (b_2 + b_3) s + b_1 b_3$$

s^3	1	$b_2 + b_3$
s^2	b_2	$b_1 b_3$
s^1	b_2	0
s^0	$b_1 b_3$	0