5- $\cos(z) = \cos(x)\cosh(y) - i\,\text{sen}(x)\,\text{senh}(y)$

$\text{sen}(z) = \text{sen}(x)\cosh(y) + i\cos(x)\,\text{senh}(y)$

$$|\cos(z)| = \sqrt{\cos^2(x)\cosh^2(y) + \text{sen}^2(x)\,\text{senh}^2(y)}$$

$$|\cos(z)|^2 = \cos^2(x)\cosh^2(y) + \text{sen}^2(x)\,\text{senh}^2(y)$$

$$\boxed{\begin{aligned}&\cos^2(x) + \text{sen}^2(x) = 1\\ &\text{sen}^2(x) = 1 - \cos^2(x)\end{aligned}}$$

$$|\cos(z)|^2 = \cos^2(x)\cosh(y) + [1 - \cos^2(x)]\,.\,\text{senh}^2(y)$$

$$|\cos(z)|^2 = \cos^2(x)\cosh^2(y) + \text{senh}^2(x) - \cos^2(x)\,\text{senh}^2(y)$$

$$|\cos(z)|^2 = \cos^2(x)\,.\left[\cosh^2(y) - \text{senh}^2(y)\right] + \text{senh}^2(y)$$

$$\cosh(y) = \frac{e^{y} + e^{-y}}{2} \Rightarrow \cosh^2(y) = \left(\frac{e^{y} + e^{-y}}{2}\right)^2 = \frac{e^{2y} + 2e^{y}e^{-y} + e^{-2y}}{4} = \frac{e^{2y} + e^{-2y} + 2}{4} = \frac{e^{2y} + e^{-2y} + 2}{4}$$

$$\text{senh}(y) = \frac{e^{y} - e^{-y}}{2} \Rightarrow \text{senh}^2(y) = \left(\frac{e^{y} - e^{-y}}{2}\right)^2 = \frac{e^{2y} - 2e^{y}e^{-y} + e^{-2y}}{4} = \frac{e^{2y} + e^{-2y} - 2}{4}$$

$$|\cos(z)|^2 = \cos^2(x)\left[\frac{\cancel{e^{2y}} + \cancel{e^{-2y}} + 2 - (\cancel{e^{2y}} + \cancel{e^{-2y}} - 2)}{4}\right] + \text{senh}^2(y)$$

$$|\cos(z)|^2 = \cos^2(x)\left[\frac{+2 - (-2)}{4}\right] + \text{senh}^2(y)$$

$$|\cos(z)|^2 = \cos^2(x)\,.\,1 + \text{senh}^2(y)$$

$$|\cos(z)|^2 = \cos^2(x) + \text{senh}^2(y)$$

$$|\text{sen}(z)| = \sqrt{\text{sen}^2(x)\cosh^2(y) + \cos^2(x)\,\text{senh}^2(y)}$$

$$|\text{sen}(z)|^2 = \text{sen}^2(x)\cosh^2(y) + (1 - \text{sen}^2(x))\,\text{senh}^2(y)$$

$$|\text{sen}(z)|^2 = \text{sen}^2(x)\left[\cancel{\cosh^2(y) - \text{senh}^2(y)}^{\,1}\right] + \text{senh}^2(y)$$

$$|\text{sen}(z)|^2 = \text{sen}^2(x) + \text{senh}^2(y)$$

$$6\text{-a)}\ |w| = \sqrt{1^2 + \sqrt{3}^2} = 2$$

$$\left.\begin{array}{l} |w|\cos(\theta_w) = 1 \\ \cos(\theta_w) = 1/2 \\ |w|\,\mathrm{sen}(\theta_w) = \sqrt{3} \\ \mathrm{sen}(\theta_w) = \sqrt{3}/2 \end{array}\right\} \theta_w = \pi/3 + 2k\pi$$

$$e^z = e^x e^{yi} = 2\, e^{i(\pi/3 + 2k\pi)}$$

$$x = \ln 2$$

$$y = (\pi/3) + 2k\pi$$