Root Locus

Step 1) Write characteristic equation in the form: $1+kG(s)$ with $G(s)=\frac{Q(s)}{P(s)}$

Example: $\frac{s^2+s+1}{s^3+4s+ks+1}$

characteristic equation

Group k terms:

$$(s^3+4s+1)+ks=0$$

Divide equation by non-k terms:

$$\cancel{\frac{(s^3+4s+1)}{(s^3+4s+1)}}^{1}+\frac{ks}{(s^3+4s+1)}=1+K\boxed{\frac{s}{s^3+4s+1}}=0$$

$G(s)$

$P(s)=s^3+4s+1$

$Q(s)=s$

Step 2: Sketch poles and zeroes of $G(s)$ in the s-plane $j\omega \times \sigma$

As k increases from 0 to ∞ the roots move from the poles of $G(s)$ to the zeroes of $G(s)$

If there are more poles than zeroes, consider new zeroes in ∞. If there are more zeroes than poles, add poles in ∞.

Step 3: Identify sections of the real axis that make up the root loci

From $+\infty$ to $-\infty$, the root locus exists to the left of every odd number of open loop zeroes + openloop poles

Example:

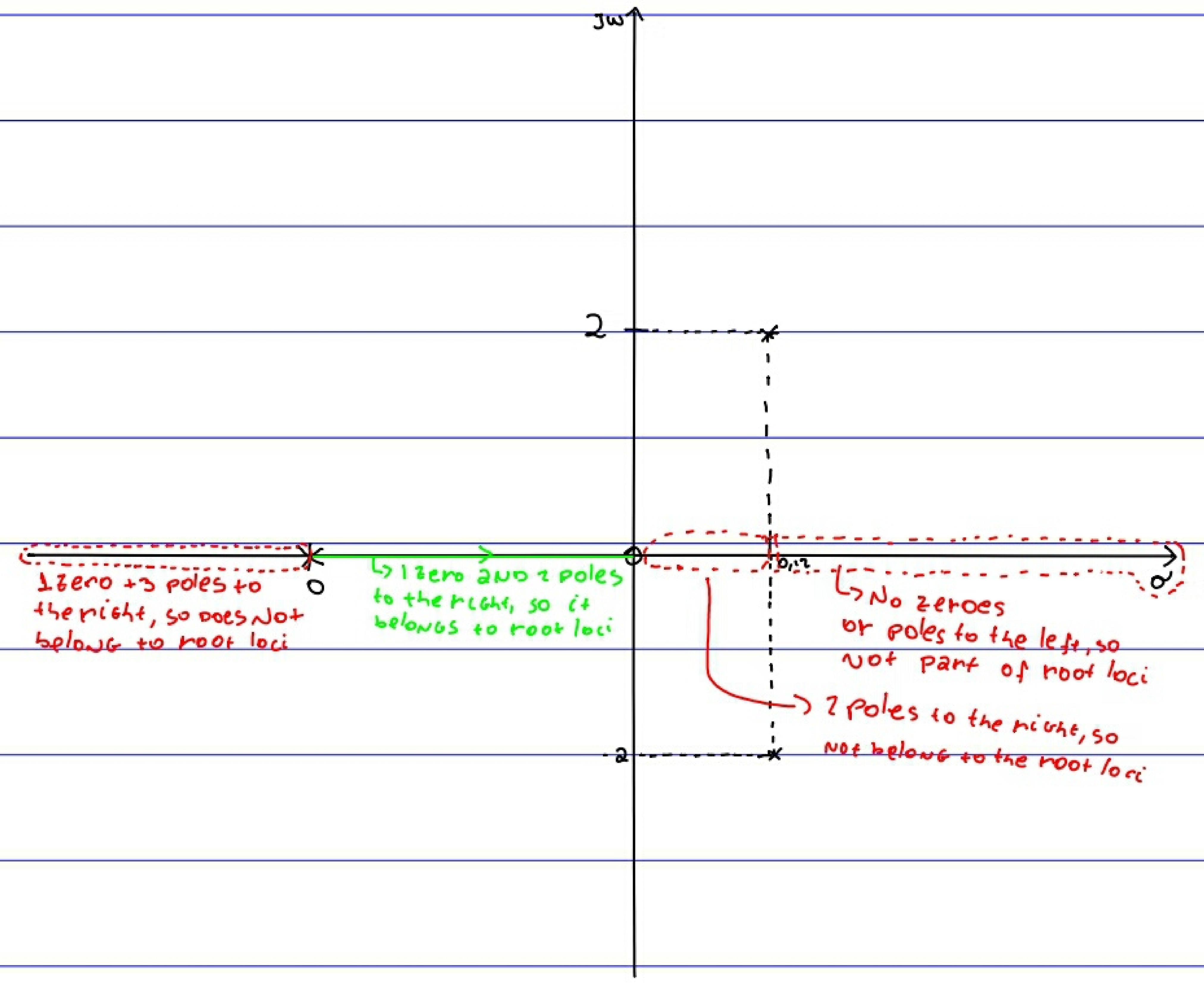
$G(s)=\frac{s}{s^3+4s+1}$

$z=0$

$P_1=-0{,}246$

$P_2=0{,}123-2{,}011$

$P_3=0{,}123+2{,}011$



Step 4: Identify the assymptotes:

The assymptote is defined by 2 equations:

x-axis crossing: $\sigma_a = \dfrac{\sum \text{finite poles} - \sum \text{finite zeros}}{\#\text{finite poles} - \#\text{finite zeros}}$

angle of assymptote: $\theta_a = \dfrac{(2k+1)\pi}{\#\text{finite poles} - \#\text{finite zeroes}}$

for our Example, there are 2 non-paired poles.

$z = 0$
$P_1 = -0{,}246$
$P_2 = 0{,}123 + 2i$
$P_3 = 0{,}123 - 2i$

$$\sigma_a = \frac{0{,}123 + 0{,}123 - 0{,}246}{3-1} = 0$$

$$\theta_{a0} = \frac{(2 \cdot 0 + 1)\pi}{3-1} = \pi/2$$

$$\theta_{a1} = \frac{(2 \cdot 1 + 1)\pi}{3-1} = \frac{3\pi}{2} = -\pi/2$$