Variáveis Complexas

Lista 1:

1-

a) $(2+3i)\cdot(7+i)$

$14+2i+3i^2+21i$

$(14-3)+i(2+21)$

$11+23i$

b) $(8+6i)^2$

$(8+6i)(8+6i)$

$64+2\cdot48i+36i^2$

$64+96i-36$

$28+96i$

c) $\frac{5}{-3+4i}$

$\frac{5(-3+4i)}{(-3+4i)(-3-4i)}$

$\frac{-15+20i}{9+12i-12i+16}$

$\frac{-15}{25}+\frac{20}{25}i$

$\frac{-3}{5}+\frac{4}{5}i$

D) $\frac{4+i}{2-3i} = \frac{4+i}{2-3i} \cdot \frac{(2+3i)}{2+3i}$

$\frac{8+12i+2i-3}{4+9} = \frac{5}{13} + \frac{14i}{13}$

e) $\frac{-1+3i}{2-i} = \frac{-1+3i}{2-i} \cdot \frac{(2+i)}{(2+i)}$

$\frac{-2-i+6i-3}{4+1} = \frac{-5}{5} + \frac{5i}{5}$

$-1+i$

f) $\left(\frac{1}{i} + \frac{3}{(1+i)}\right)^2$

$\left(\frac{1+i+3i}{i-1}\right)^2$

$\frac{(1+4i)^2}{(i-1)^2}$

$\frac{1+8i-16}{1-2i-1}$

$\frac{-15+8i}{-2i}$

$-4 - \frac{15}{2}i$

c) $\dfrac{5}{(1-i)(2-i)(3-i)} = \dfrac{5}{(2\ \ i-2i\ \ 1)(3-i)}$

$\dfrac{5}{(1-3i)(3-i)} = \dfrac{5}{3-i-9i-3} = \dfrac{5}{10i} = \dfrac{-1}{2}i$

2- a) $(x+yi)^2$

$x^2+2xyi-y^2$

x^2-y^2+2xyi

b) $(x+yi)^3$

$(x+yi)^2(x+yi)$

$(x^2+2xyi-y^2)\ \ x+yi)$

$x^3+2x^2yi-2xy^2-y^3i$

$(x^3-2xy^2)+i(2x^2y-y^3)$

c) $\dfrac{1}{z} = \dfrac{1}{x+yi} = \dfrac{x-yi}{x^2-y^2}$

D) $1/z^2$

$(x+yi)^{-2}$

$(x^2+2xyi-y^2)^{-1}$

$\left[(x^2-y^2)+i(2xy)\right]^{-2}$

$\dfrac{\left[(x^2-y^2)-i(2xy)\right]}{(x^2-y^2)^2+(2xy)^2}$

$\dfrac{\left[(x^2-y^2)-i(2xy)\right]}{x^4+2x^2y^2+y^4}$

e) $\frac{z-1}{z+1}$

$$\frac{x+yi-1}{x+yi+1}$$

$$\frac{(x-1)+yi}{(x+1)+yi} \cdot \frac{(x+1)-yi}{(x+1)-yi}$$

$$\frac{x^2-1-xyi+yi+xyi+yi+y^2}{(x+1)^2+y^2}$$

$$\frac{x^2-1+2yi+y^2}{x^2+2x+1+y^2}$$

4.

a) $-1+i\sqrt{3}$

$|z|=\sqrt{(-1)^2+\sqrt{3}^2}$

$|z|=\sqrt{1+3}=2$

$z=a+bi$

$a=-1=2\cos(\theta)$

$\theta=a\cos(-1/2)$

$b=\sqrt{3}=2\operatorname{sen}(\theta)$

$\theta=a\operatorname{sen}\left(\frac{\sqrt{3}}{2}\right)$

$\theta=\pi-\frac{\pi}{3}+2k\pi$

$\theta=\frac{2\pi}{3}+2k\pi$

b) $|z| = \sqrt{(-1/2)^2 + (\sqrt{3}/2)^2}$

$|z| = \sqrt{1/4 + 3/4} = 1$

$\left.\begin{array}{l} a = -1/2 = \cos\theta \\ b = \sqrt{3}/2 = \operatorname{sen}\theta \end{array}\right\}$ $\theta = \pi - \frac{\pi}{3} + 2k\pi$

$\theta = \frac{2\pi}{3} + 2k\pi$

c) $-\cos\left(\frac{\pi}{7}\right) + i\operatorname{sen}\left(\frac{\pi}{7}\right)$

$1\angle \pi - \frac{\pi}{7} + 2k\pi = 1\angle 6\pi/7$

D) $|z| = \sqrt{(3/2)^2 + (\sqrt{3}/2)^2}$

$|z| = \sqrt{(9/4) + (3/4)}$

$|z| = \sqrt{12/4} = \sqrt{3}$

$a = 3/2 = \sqrt{3}\cos(\theta)$

$b = -\sqrt{3}/2 = \sqrt{3}\operatorname{sen}(\theta)$

$\left.\begin{array}{l} \cos\theta = \sqrt{3}/2 \\ \operatorname{sen}\theta = -1/2 \end{array}\right\}$ $\theta = -\frac{\pi}{6} + 2k\pi$

5-

a) $z^3 = (1+i)^3$

$|z| = \sqrt{2}$

$\theta = \pi/4$

$z^3 = \sqrt[2]{2^3}\left[\cos(3\pi/4) + i\operatorname{sen}(3\pi/4)\right]$

$2\sqrt{2}\left(-\sqrt{2}/2 + i\sqrt{2}/2\right)$

$z^3 = -2 + 2i$

b) $z = (1+i)^{25}$

$|z| = \sqrt{2^{25}} = \sqrt{2^{24} \cdot 2} = 2^{12}\sqrt{2} = 4096\sqrt{2}$

$z = 4096\,[\cos(25\pi/4) + i\,\mathrm{sen}(25\pi/4)]\sqrt{2}$

$z = 4096\sqrt{2}\left[\cos\left(\frac{24\pi}{4} + \frac{\pi}{4}\right) + i\,\mathrm{sen}\left(\frac{24\pi}{4} + \frac{\pi}{4}\right)\right.$

$z = 4096\sqrt{2}\,[\cos(6\pi + \pi/4) + i\,\mathrm{sen}\,(6\pi + \pi/4)$

$z = 4096\sqrt{2}\,(\sqrt{2}/2 + i\sqrt{2}/2)$

$z = 4096 + i\,4096$

c) $z = (-1+i)^7$

$|z| = \sqrt{2^7} = \sqrt{2^6 \cdot 2} = 8\sqrt{2}$

$\theta = 7\pi/4 = 8\pi/4 - \pi/4$

$z = 8\sqrt{2}\,[\cos(-\pi/4) + i\,\mathrm{sen}(-\pi/4)$

$z = 8\sqrt{2}\,(\sqrt{2}/2 - i\sqrt{2}/2)$

$z = 8 - 8i$

D) $z^{99} = (1 + i\sqrt{3})^{99}$

$|z| = \sqrt{1+3} = 2$

$\left.\begin{array}{l} 2\cos(\theta_z) = 1 \\ 2\,\mathrm{sen}\,(\theta_z) = \sqrt{3} \end{array}\right\}\ \theta_z = \pi/3$

$z^{99} = 2^{99}\,[\cos(99\pi/3) + i\,\mathrm{sen}(99\pi/3)$

$z^{99} = 2^{99}\,[\cos(32\pi + \pi) + i\,\mathrm{sen}\,(32\pi + \pi)]$

$z^{99} = 2^{99}\,(-1 + i \cdot 0)$

$z^{99} = -2^{99}$

e) $\dfrac{(1+i)^9}{(1-i)^7} = \dfrac{z_1^{\,9}}{z_2^{\,7}} = \dfrac{\sqrt{2}^{\,9}}{\sqrt{2}^{\,7}}\left[\dfrac{\cos(9\pi/4) + i\,\mathrm{sen}\,(9\pi/4)}{\cos(7\pi/4) + i\,\mathrm{sen}(7\pi/4)}\right]$

$|z_1| = |z_2| = \sqrt{2}$

$\theta_1 = \pi/4$

$\theta_2 = -\pi/4$

$2\,\dfrac{\cos(\pi/4) + i\,\mathrm{sen}\,(\pi/4)}{\cos(-\pi/4) + i\,\mathrm{sen}(-\pi/4)} = 2\,\dfrac{\sqrt{2}/2 + i\sqrt{2}/2}{\sqrt{2}/2 - i\sqrt{2}/2}$

$$\frac{2\sqrt{2}/2+i\sqrt{2}/2}{\sqrt{2}/2-i\sqrt{2}/2}=\frac{2\left(\sqrt{2}/2+i\sqrt{2}/2\right)^2}{1/2+1/2}=2\left(\cancel{1/2}+2i\cdot 1/2-\cancel{1/2}\right)=2i$$

f) $$z^{30}=\left(\frac{1+i\sqrt{3}}{1-i}\right)^{30}$$

$$z=\frac{1+i\sqrt{3}}{1-i}\Rightarrow\begin{cases}z_1=1+i\sqrt{3}\\z_2=1-i\end{cases}\Rightarrow\begin{cases}|z_1|=2, & \theta_1=\pi/3\\|z_2|=\sqrt{2}, & \theta_2=-\pi/4\end{cases}$$

$$z=\frac{2}{\sqrt{2}}\left[\frac{\cos(\pi/3)+i\,\mathrm{sen}(\pi/3)}{\cos(-\pi/4)+i\,\mathrm{sen}(-\pi/4)}\right]$$

$$z^{30}=\left(\frac{2^{10}}{2^5}\right)^3\left[\frac{\cos(10\pi+i\,\mathrm{sen}(10\pi)}{\cos(\pi/2)+i\,\mathrm{sen}(\pi/2)}\right]=2^{15}\cdot\frac{1}{i}=-i2^{15}=-i\,32768$$

6- a) $|z|-z=1+2i$

$$\sqrt{a^2+b^2}-a-bi=1+2i$$

$$b=2$$

$$\sqrt{a^2+4}-a=1$$

$$\sqrt{a^2+4}=1+a$$

$$a^2+4=(1+a)^2$$

$$a^2+4=(1+a)^2$$

$$\cancel{a^2}+4=\cancel{a^2}+2a+1$$

$$2a=3$$

$$a=3/2$$

b) $z^2-2z+2=0$

$$z=\frac{2\pm\sqrt{4-8}}{2}=\frac{2\pm 2i}{2}=1\pm i$$

c) $z^2 = \bar{z}$

$(a+bi)^2 = a - bi$

$a^2 + 2abi - b^2$

$a^2 - b^2 = a$

$2ab = -b$

$2a = -1$

$a = -1/2$

$\left(\frac{-1}{2}\right)^2 - b^2 = \frac{-1}{2}$

$\frac{1}{4} + \frac{1}{2} = b^2$

$b = \frac{\sqrt{3}}{2}$

D) $z^3 = \bar{z}$

$r^3 e^{i3\theta} = r e^{-i\theta}$

$r^2 \frac{e^{i3\theta}}{e^{-i\theta}} = 1$

$r^2 e^{i4\theta} = 1$

$r^2(\cos 4\theta + i \operatorname{sen} 4\theta) = 1$

$r^2 \cos(4\theta) = 1$

$r^2 \operatorname{sen}(4\theta) = 0$

$\operatorname{sen}(4\theta) = 0 \Rightarrow \theta = k\pi/4$

$r^2 \cos(4\theta) = 1$

$r^2 \cos(k\pi) = 1$

$r^2 > 0$

$k = 2m, \ m \in \mathbb{Z} \mid m \geq 0$

e) $z^2+1=0$

$z^2=-1$

$z=\pm\sqrt{-1}$

$z=\pm i$

f) $z^2-2i=0$

$z^2=2i$

$z^2=r^2[\cos(2\theta)+i\,sen(2\theta)]=2i$

$r^2=2$

$2\cos(2\theta)=0$

$2sen(2\theta)=2$

$sen(2\theta)=1$

$2\theta=\frac{\pi}{2}+2k\pi$

$\theta=\frac{\pi}{4}+k\pi$

g) $z^3+1=0$

$z^3=-1$

$r^3[\cos(3\theta)+i\,sen(3\theta)]=-1$

$r=1$

$r^3=1$

$\cos(3\theta)=-1$

$3\theta=\pi+2k\pi$

$\theta=\frac{\pi}{3}+\frac{2}{3}k\pi$

$\theta_0=\frac{\pi}{3}$

$\theta_1=\frac{\pi}{3}+\frac{2}{3}\pi=\pi$

$\theta_2=\frac{\pi}{3}+\frac{4}{3}\pi=\frac{5}{3}\pi$

h) $z^3 + i = 0$

$z^3 = -i$

$z^3 = -i$

$r^3 \cos(3\theta) = 0$

$\cos(3\theta) = 0$

$3\theta = \frac{\pi}{2} + k\pi$

$\theta = \pi/6 + k\pi/3$

$r^3 \operatorname{sen} 3\theta = -1$

$r^3 \operatorname{sen}[3(\pi/2 + k\pi)/3] = -1$

$r^3 \operatorname{sen}(\pi/2 + k\pi) = -1$

since $r^3 > 0$

$\operatorname{sen}(\pi/2 + k\pi) = -1$, for this to be true

$k = (2n+1)$

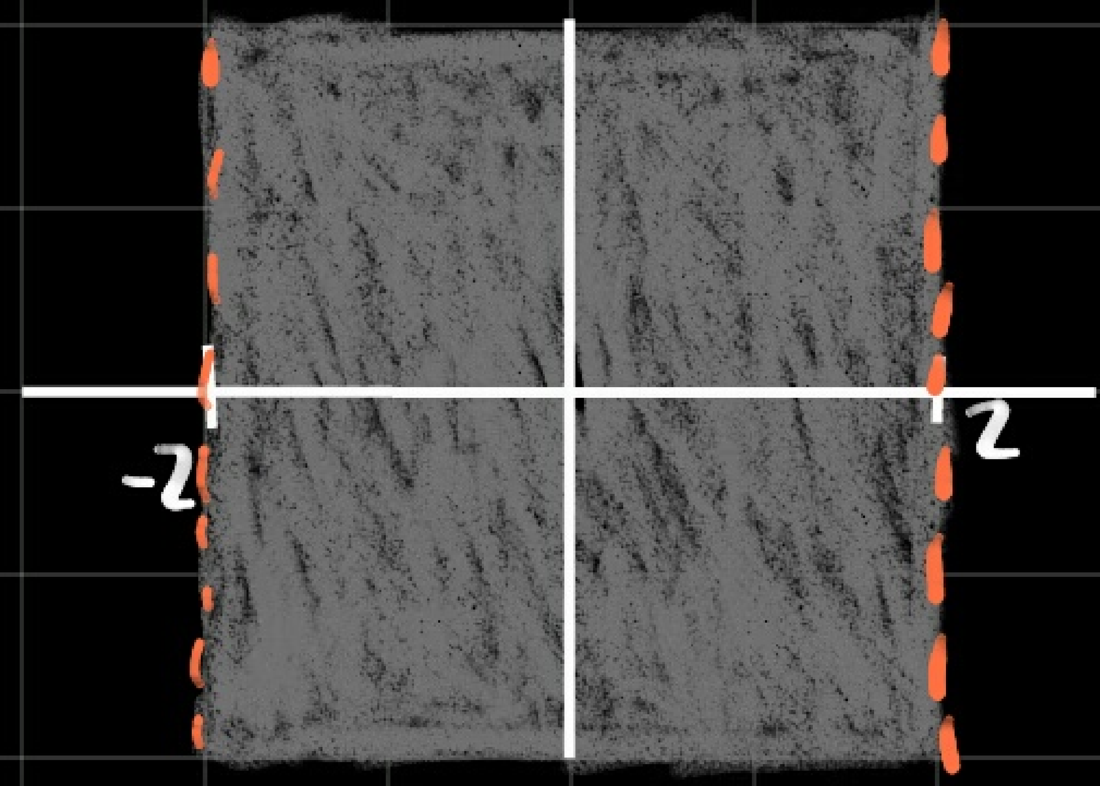
$z^3 - i = 0$

$z^3 = i$

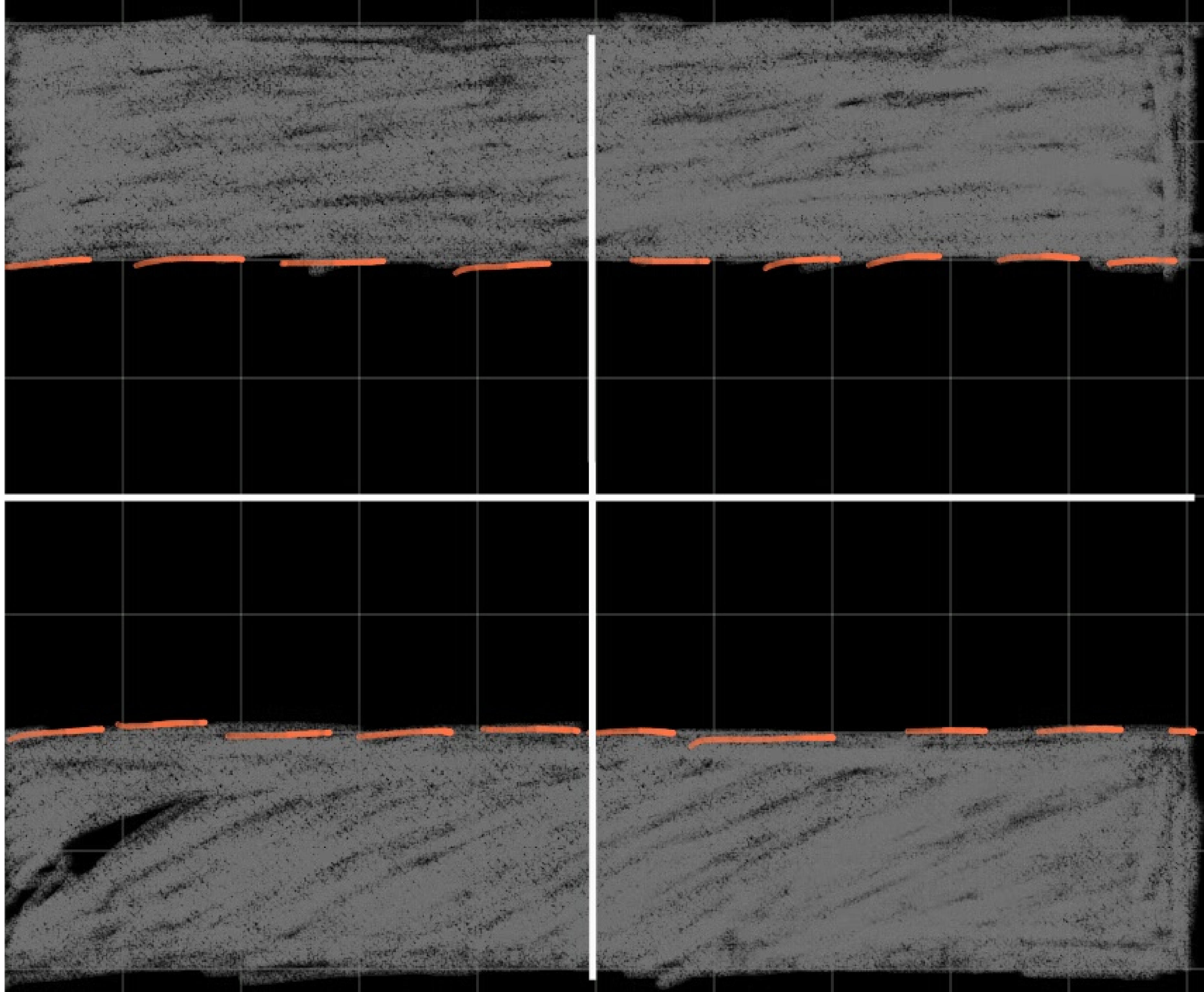
$r^3 \cos(3\theta) = 0$	$r^3 \operatorname{sen}(3\theta) = 1$
$r^3 \neq 0$	$r^3 \operatorname{sen}(\pi/2 + k\pi) = 1$
$\cos(3\theta) = 0$	$r^3 > 0$
$3\theta = \pi/2 + k\pi$	$\operatorname{sen}(\pi/2 + k\pi) = 1$
$\theta = \pi/6 + k\pi/3$	$k = 2m,\ m \in \mathbb{Z} \geq 0$
	$\theta = \pi/6 + k\pi/3$
	$r^3 = 1$
	$r = 1$

8.

a) $\{z \in \mathbb{C} \mid |Re(z)| <$



b) $\{z \in \mathbb{C} \mid |Im(z)| > 3\}$



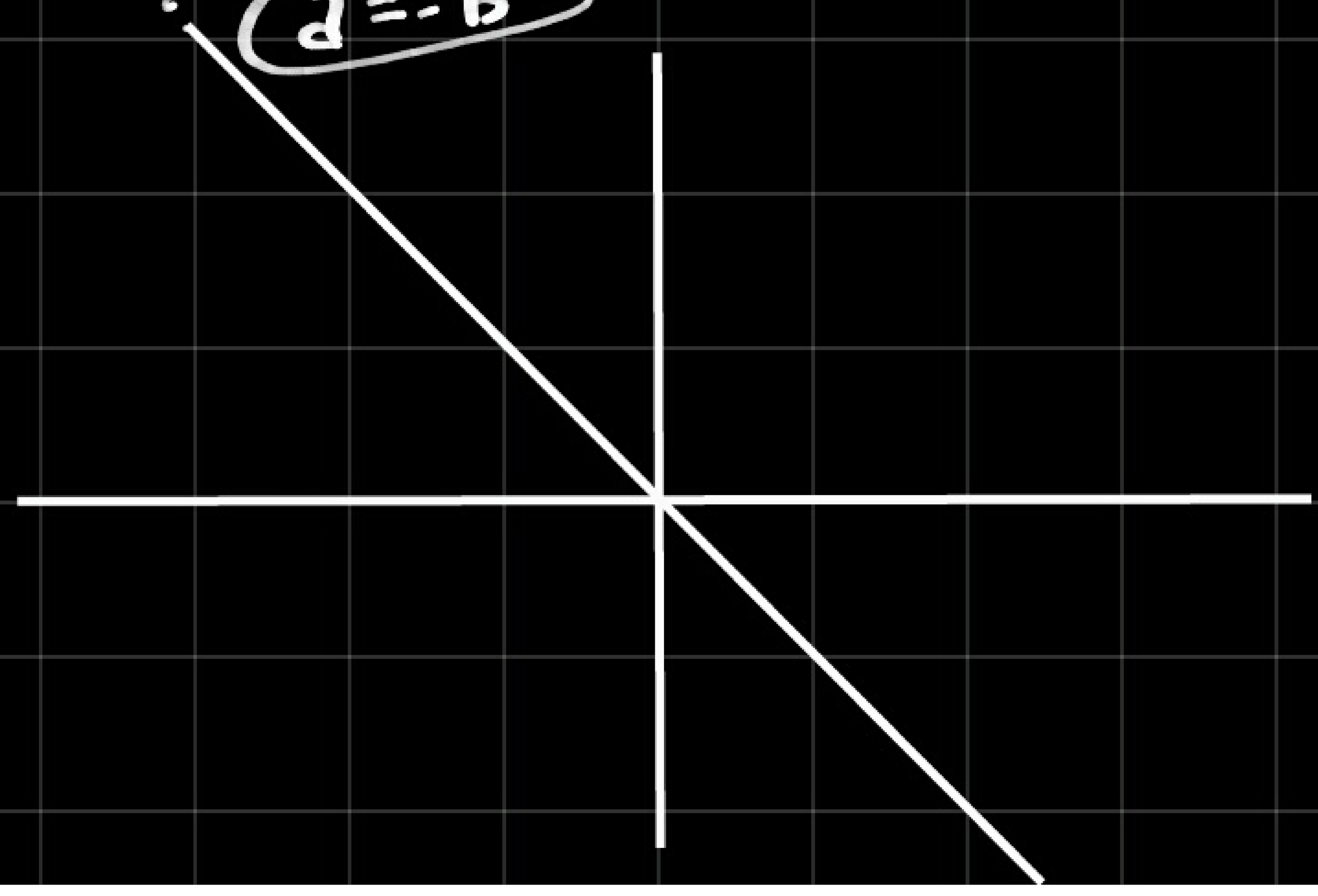
c) $\{z \in \mathbb{C} \mid |z-1| = |z+i|\}$

$$\sqrt{(a-1)^2+b^2} = \sqrt{a^2+(b+1)^2}$$

$$\cancel{a^2} - 2a + \cancel{1} + \cancel{b^2} = \cancel{a^2} + \cancel{b^2} + 2b + \cancel{1}$$

$$-a = b$$

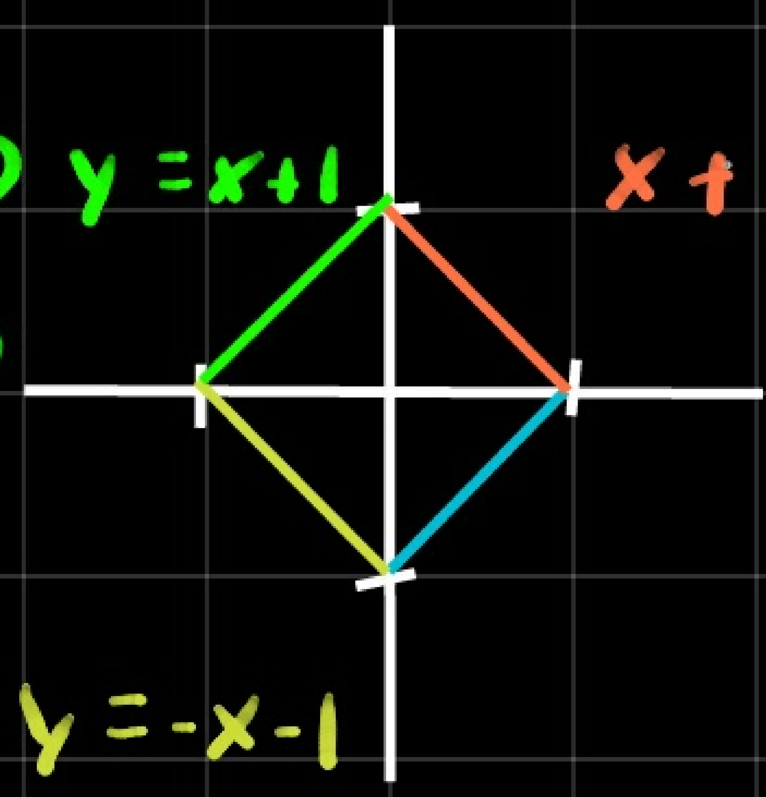
$$\therefore\ a = -b$$



d) $\{z \in \mathbb{C};\ |Re(z)| + |Im(z)| = 1$

$-x+y=1 \Rightarrow y = x+1$

$x<0;\ y>0$



$x+y=1 \Rightarrow y=-x+1;\ x>0$ e $y>0$

$-x-y=1 \Rightarrow y=-x-1$

$x<0;\ y<0$

$x-y=1 \Rightarrow y=x-1;\ x>0;\ y<0$

e) $\{z \in \mathbb{C};\ |Re(z)| + |Im(z)| < 1$

$-x+y=1 \Rightarrow y = x+1$

$x<0;\ y>0$

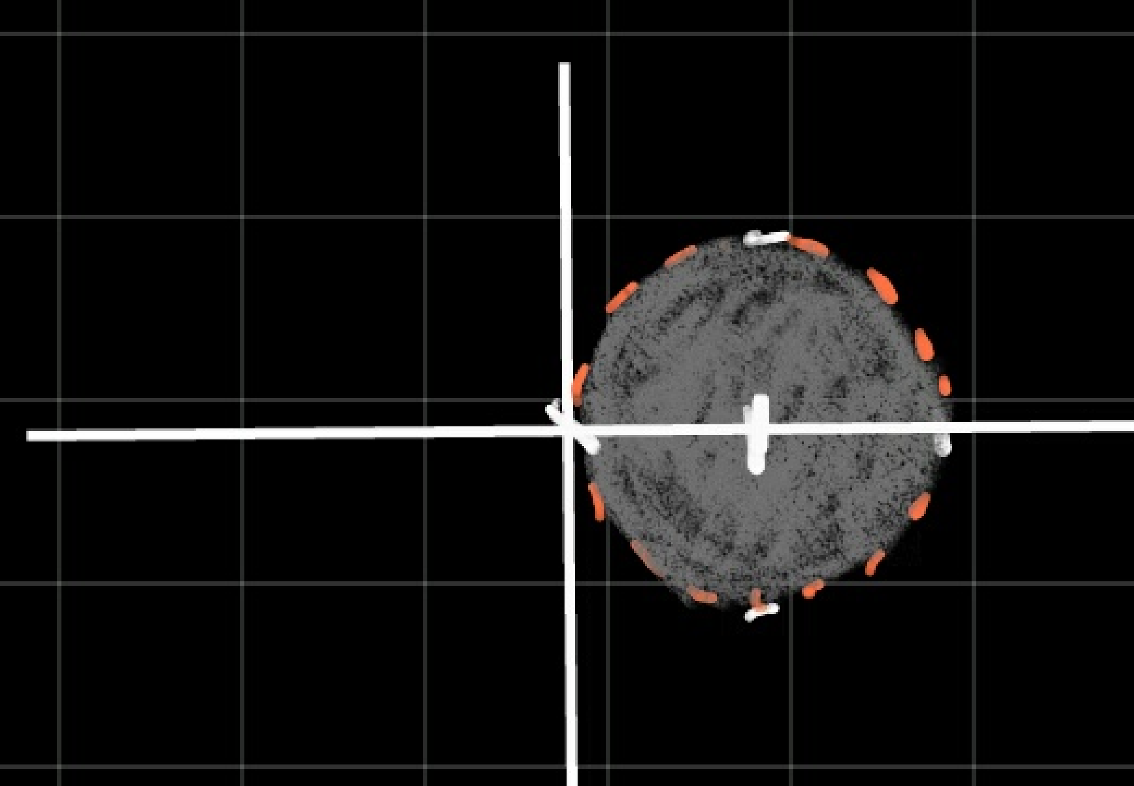
$x+y=1 \Rightarrow y=-x+1 \cdot x>0$ e $y>0$

$-x-y=1 \Rightarrow y=-x-1$

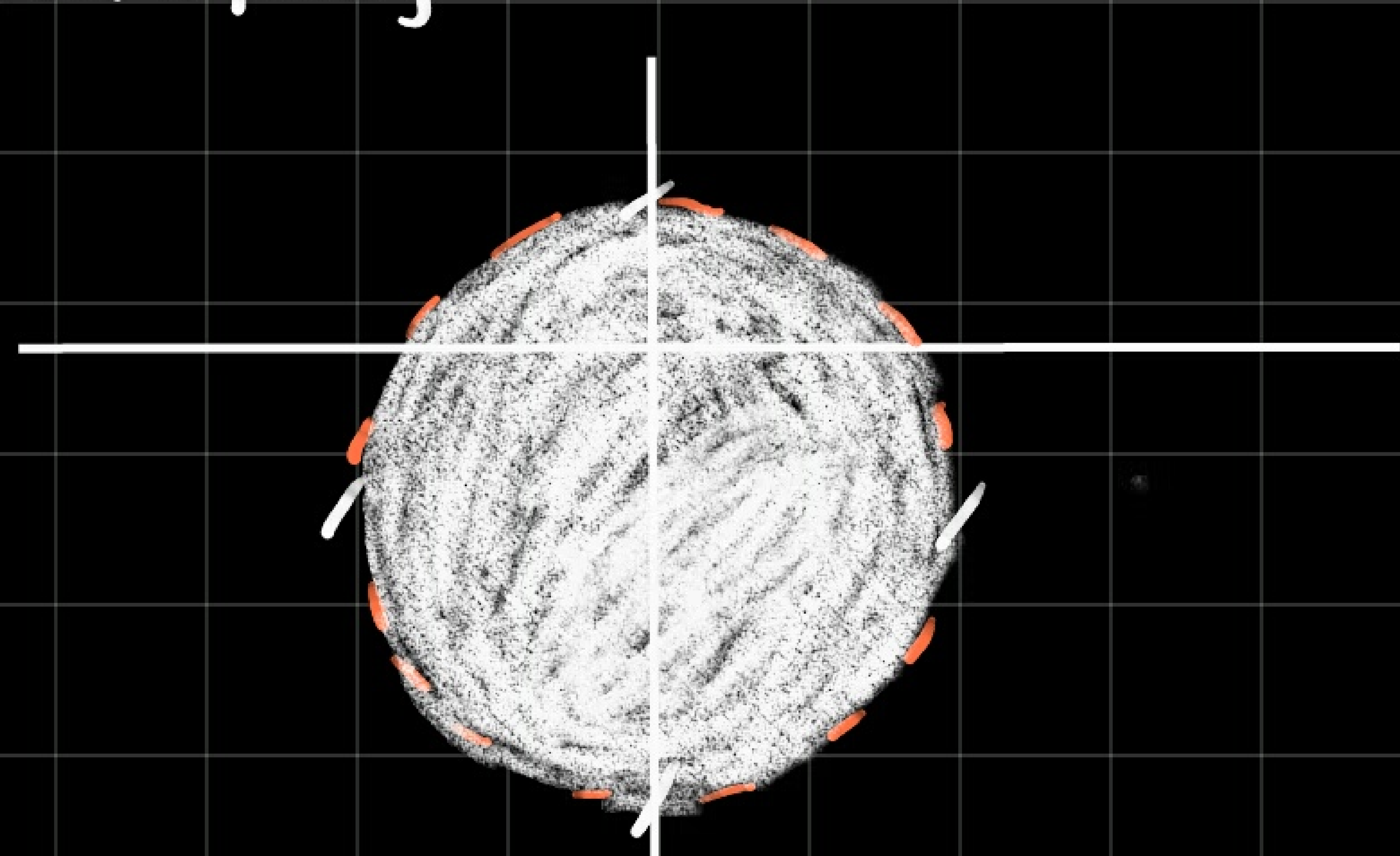
$x<0;\ y<0$

$x-y=1 \Rightarrow y=x-1;\ x>0;\ y<0$

f) $z \in \mathbb{C};\ |z-1| < 1$



g) $\{z \in \mathbb{C};\ |z+i| < 2\}$



SIMULADO P_2

1- $V_{(x,y)} = \operatorname{arctg}\left(\frac{y}{x}\right)$

$$V_y = \frac{\partial}{\partial u} \operatorname{arctg}(u) \cdot \frac{\partial u}{\partial y} \; ; \; u = y/x$$

$$V_y = \frac{1}{1+u^2} \cdot \frac{1}{x}$$

$$V_y = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{1}{x} = \frac{1}{x+\frac{y^2}{x}} = \frac{1}{\frac{x^2+y^2}{x}} = \frac{x}{x^2+y^2}$$

$$V_{yy} = \frac{\partial}{\partial y} \frac{x}{x^2+y^2}$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$f' = \frac{\partial}{\partial y} x = 0 \qquad g' = \frac{\partial}{\partial y} x^2+y^2 = 2y$$

$$V_{yy} = \frac{0 \cdot (x^2+y^2) - x\,2y}{(x^2+y^2)^2} = \frac{-2xy}{(x^2+y^2)^2}$$

$$V_x = \frac{\partial}{\partial x} \operatorname{arctg}\left(\frac{y}{x}\right) = \frac{\partial}{\partial u} \operatorname{arctg}(u) \cdot \frac{\partial}{\partial x} \frac{y}{x}$$

$$V_x = \frac{1}{1+u^2} \cdot \left(\frac{-y}{x^2}\right) = \frac{-y}{x^2+y^2}$$

$$V_{xx} = -y \frac{\partial}{\partial x} \frac{1}{x^2+y^2} = -y \frac{\partial}{\partial u} \frac{1}{u} \cdot \frac{\partial}{\partial x} x^2+y^2$$

$$V_{xx} = \frac{y}{u^2} \cdot 2x = \frac{2xy}{(x^2+y^2)^2} \; ; \; V_{xx} + V_{yy} = 0$$

Para encontrarmos a conjugada harmônica, precisamos de uma função $U_{(x,y)}$ que atenda as equações de Cauchy-Riemman

$$U_x = V_y$$
$$U_y = -V_x$$

$$\int V_y\,dx = \int \frac{x}{x^2+y^2}\,dx = \int \frac{\cancel{x}}{m}\,\frac{1}{2\cancel{x}}\,dm = \frac{1}{2}\int \frac{1}{m}\,dm =$$

$$\boxed{\begin{aligned} x^2+y^2 &= m \\ \frac{dm}{dx} &= 2x \\ dx &= \frac{1}{2x}\,dm \end{aligned}}$$

$$\int V_y\,dx = \frac{1}{2}\ln(x^2+y^2)+c = U_{(x,y)}$$

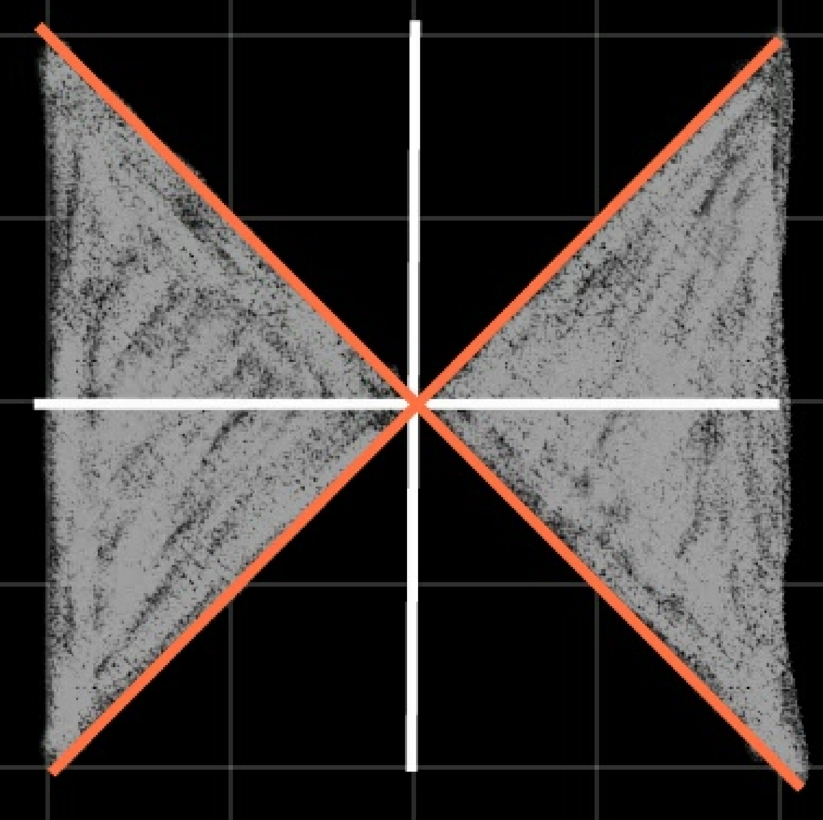
$$U_{(\sqrt{e},0)} = 1$$

$$\frac{1}{2}\ln(\sqrt{e}^2+0^2)+c=1$$

$$\ln(e) = 2(1-2c)$$
$$\ln(e) = 2.C_1$$
$$2C_1 = 1$$
$$C_1 = 1/2$$

2- $A = \{z \in \mathbb{C} / Re(z^2) > 0$

$$z^2 = x^2+2xyi-y^2$$
$$Re(z^2) = x^2-y^2 > 0$$
$$x^2+y^2 = 0$$
$$y^2 = x^2$$

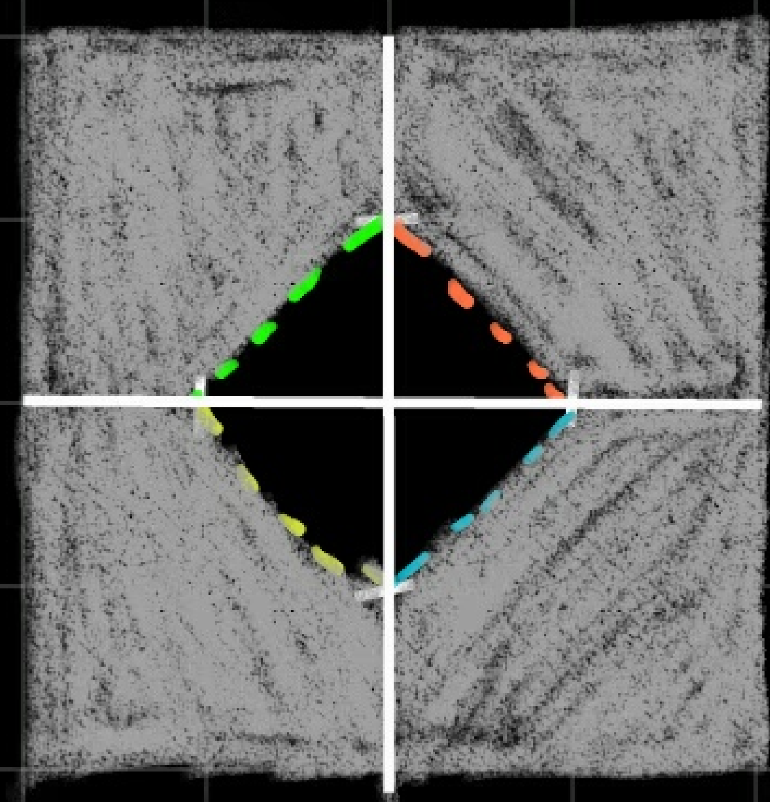


b) $\{z \in \mathbb{C};\ |Re(z)| + |Im(z)| < 1$

$x+y=1 \Rightarrow y=-x+1 \cdot x>0$ e $y>0$

$-x+y=1 \Rightarrow y=x+1$

$x<0; y>0$



$-x-y=1 \Rightarrow y=-x-1$

$x<0;\ y<0$

$x-y=1 \Rightarrow y=x-1;\ x>0;\ y<0$

3- a) $P(z)=z^2+(2i-3)z+(5-i)$

$P(z)=(z-z_0).(z-z_1)$

$z^2 - z.z_1 - z.z_0 + z_0 z_1$

$z^2 - z(z_1+z_0) + z_0 z_1$

$2i-3 = z_1+z_0$

$z_0 z_1 = 5-i$

$a+c=-3 \qquad (a+bi)(c+di)=ac+iad+ibc-bd$

$b+d=2 \qquad ac-bd=5$

$a=-(3+c) \qquad ad+bc=-1$

$b=2-d \qquad -(3+c)c-(2-d).d=5$

$-3c-c^2-2d-d^2=5$

$-3d-cd+2c-cd=-1$

$-3d+2cd+2c=-1$

$$5-\ \cos(z) = \cos(x)\cosh(y) - i\,\text{sen}(x)\,\text{senh}(y)$$
$$\text{sen}(z) = \text{sen}(x)\cosh(y) + i\cos(x)\,\text{senh}(y)$$

$$|\cos(z)| = \sqrt{\cos^2(x)\cosh^2(y) + \text{sen}^2(x)\,\text{senh}^2(y)}$$
$$|\cos(z)|^2 = \cos^2(x)\cosh^2(y) + \text{sen}^2(x)\,\text{senh}^2(y)$$

$$\boxed{\begin{array}{l}\cos^2(x) + \text{sen}^2(x) = 1 \\ \text{sen}^2(x) = 1 - \cos^2(x)\end{array}}$$

$$|\cos(z)|^2 = \cos^2(x)\cosh(y) + [1 - \cos^2(x)]\,.\,\text{senh}^2(y)$$
$$|\cos(z)|^2 = \cos^2(x)\cosh^2(y) + \text{senh}^2(x) - \cos^2(x)\,\text{senh}^2(y)$$
$$|\cos(z)|^2 = \cos^2(x)\,.\left[\cosh^2(y) - \text{senh}^2(y)\right] + \text{senh}^2(y)$$

$$\cosh(y) = \frac{e^{y}+e^{-y}}{2} \Rightarrow \cosh^2(y) = \left(\frac{e^{y}+e^{-y}}{2}\right)^2 = \frac{e^{2y}+2e^{y}e^{-y}+e^{-2y}}{4} = \frac{e^{2y}+e^{-2y}+2}{4} = \frac{e^{2y}+e^{-2y}+2}{4}$$

$$\text{senh}(y) = \frac{e^{y}-e^{-y}}{2} \Rightarrow \text{senh}^2(y) = \left(\frac{e^{y}-e^{-y}}{2}\right)^2 = e^{2y} - 2e^{y}e^{-y} + e^{-2y} = \frac{e^{2y}+e^{-2y}-2}{4}$$

$$|\cos(z)|^2 = \cos^2(x)\left[\frac{\cancel{e^{2y}}+\cancel{e^{-2y}}+2-(\cancel{e^{2y}}+\cancel{e^{-2y}}-2)}{4}\right] + \text{senh}^2(y)$$

$$|\cos(z)|^2 = \cos^2(x)\left[\frac{+2-(-2)}{4}\right] + \text{senh}^2(y)$$

$$|\cos(z)|^2 = \cos^2(x)\,.\,1 + \text{senh}^2(y)$$

$$|\cos(z)|^2 = \cos^2(x) + \text{senh}^2(y)$$

$$|\text{sen}(z)| = \sqrt{\text{sen}^2(x)\cosh^2(y) + \cos^2(x)\,\text{senh}^2(y)}$$
$$|\text{sen}(z)|^2 = \text{sen}^2(x)\cosh^2(y) + (1 - \text{sen}^2(x))\,\text{senh}^2(y)$$
$$|\text{sen}(z)|^2 = \text{sen}^2(x)\overbrace{\left[\cosh^2(y) - \text{senh}^2(y)\right]}^{1} + \text{senh}^2(y)$$
$$|\text{sen}(z)|^2 = \text{sen}^2(x) + \text{senh}^2(y)$$

6 - a) $|w| = \sqrt{1^2 + \sqrt{3}^2} = 2$

$$\left.\begin{array}{l} |w|\cos(\theta_w) = 1 \\ \cos(\theta_w) = 1/2 \\ |w|\,\mathrm{sen}(\theta_w) = \sqrt{3} \\ \mathrm{sen}(\theta_w) = \sqrt{3}/2 \end{array}\right\} \theta_w = \pi/3 + 2k\pi$$

$$e^z = e^x e^{yi} = 2e^{i(\pi/3 + 2k\pi)}$$

$$e^x = e^{\ln 2} \Rightarrow x = \ln 2$$

$$y = (\pi/3) + 2k\pi$$

b) $e^z = 1 + i$

$$|w| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\theta_w = \pi/4 + 2k\pi$$

$$e^{x+yi} = \sqrt{2}\, e^{(\pi/4 + 2k\pi)i}$$

$$e^x = e^{\ln\sqrt{2}} \Rightarrow x = \frac{1}{2}\ln 2$$

$$y = \pi/4 + 2k\pi$$

c) $\cos(z) = 2$

$$\cos(z) = \cos(x)\cosh(y) - i\,\mathrm{sen}\,x\,\mathrm{senh}\,y = 2$$

$$\mathrm{sen}(x)\,\mathrm{senh}(y) = 0$$

$$\mathrm{sen}(x) = 0 \Rightarrow x = k\pi \quad \text{I}$$

$$\mathrm{senh}(y) = 0 \Rightarrow \frac{e^y - e^{-y}}{2} = 0 \Rightarrow e^y = e^{-y} \Rightarrow y = -y \therefore y = 0 \quad \text{II}$$

$$\cos(x)\cosh(y) = 2$$

Se I

$$\cos(x) \cdot \cancel{\left(\frac{e^0 + e^0}{2}\right)}^{1} = 2$$

$\cos(x) = 2 \Rightarrow$ ABSURDO

Se II

$$(-1)^k \cosh(y) = 2, \; k = 2n \mid n \in \mathbb{Z}$$

$$\cosh(y) = 2$$

$$\frac{e^y + e^{-y}}{2} = 2 \; ; \; e^{2y} - 4e^y + 1 = 0$$

$e^{2y} - 4e^{y} + 1 = 0$

$e^{y} = w$

$w^{2} - 4w + 1 = 0$

$w = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}$

$y = \ln(2 \pm \sqrt{3})$

D) $\operatorname{sen}(z) = \sqrt{3}$

$\operatorname{sen}(z) = \operatorname{sen}(x)\cosh(y) + i\cos(x)\operatorname{senh}(y) = \sqrt{3}$

$\cos(x)\operatorname{senh}(y) = 0$

$\operatorname{senh}(y) = 0 \Rightarrow y = 0$ (I)

$\cos(x) = 0 \Rightarrow x = \pi/2 + k\pi$

$\operatorname{sen}(x)\cosh(y) = \sqrt{3}$

Se I

$\operatorname{sen}(x)\left(\frac{e^{0} + e^{-0}}{2}\right) = \sqrt{3}$

$\operatorname{sen}(x) = \sqrt{3}$ ABSURDO

$\operatorname{sen}(\pi/2 + k\pi) \cdot \cosh y = \sqrt{3}$

$(-1)^{k} \cdot \frac{e^{y} + e^{-y}}{2} = \sqrt{3}$; $k = 2n$, $n \in \mathbb{Z}$

$e^{2y} - 2\sqrt{3}e^{y} + 1 = 0$

$w = e^{y}$

$w^{2} - 2\sqrt{3}w + 1 = 0$

$\frac{2\sqrt{3} \pm \sqrt{12-4}}{2} = \sqrt{3} \pm \sqrt{2}$

$y = \ln(\sqrt{3} \pm \sqrt{2}$