$$1- \oint_C |z^2|\, dz = \oint_C |z|^2 dz = \oint_C \sqrt{(x^2+y^2)}^{\,2} dz = \oint_C (x^2+y^2)\, dz$$

Parametrizando C para $0 \leq x \leq 1$ e $y=0$

$$\left.\begin{matrix} x=t \\ y=0 \end{matrix}\right\} 0 \leq t \leq 1, \quad \frac{dz}{dt} = 1$$

Parametrizando C para $x=1$ e $0 \leq y \leq 1$

$$\left.\begin{matrix} x=1 \\ y=t \end{matrix}\right\} 0 \leq t \leq 1, \quad \frac{dz}{dt} = i$$

Parametrizando C para $0 \leq x \leq 1$ e $y=1$

$$\left.\begin{matrix} x=1-t \\ y=1 \end{matrix}\right\} 0 \leq t \leq 1, \quad \frac{dz}{dt} = -1$$

Parametrizando C para $x=0$ e $0 \leq y \leq 1$

$$\left.\begin{matrix} x=0 \\ y=1-t \end{matrix}\right\} 0 \leq t \leq 1, \quad \frac{dz}{dt} = -i$$

Onde $z = x+iy$

$$\oint_C |z^2|\, dz = \int_0^1 (t^2+0^2)\,dt + \int_0^1 (1^2+t^2)\,dt + \int_0^1 [(1-t)^2+1^2]\,dt + \int_0^1 [0^2+(1-t)^2]\,dt$$

$$\oint_c |z^2|\, dz = \int_0^1 (t^2+0^2)\,dz + \int_0^1 (1^2+t^2)\,dz + \int_0^1 [(1-t)^2+1^2]\,dz + \int_0^1 [0^2+(1-t)^2]\,dz$$

$$\int_0^1 (t^2+0^2)\,dz = \int_0^1 (t^2+0^2)\,dt = \frac{t^3}{3}\Big|_0^1 = \frac{1}{3}$$

$$\int_0^1 (1^2+t^2)\,dz = \int_0^1 (1+t^2)\,i\,dt = \frac{(3t+t^3)\,i}{3}\Big|_0^1 = \frac{4i}{3}$$

$$\int_0^1 (1-t)^2+1\,dz = \int_0^1 ((1-t)^2+1)(-1)\,dt = 2t - \frac{2t^2}{2} + \frac{t^3}{3}\Big|_0^1 (-1) = -\frac{4}{3}$$

$$\int_0^1 0^2+(1-t)^2\,dz = \int_0^1 (1-2t+t^2)(-i)\,dt = t - t^2 + \frac{t^3}{3}\Big|_0^1 (-i) = \frac{-1}{3}\,i$$

$$\oint_c |z^2|\,dz = \frac{1}{3} + \frac{4}{3}i - \frac{4}{3} - \frac{1}{3}i = -1+i$$

2- $\oint_C Log(z+2)\,dz$

$$Log(z+2) = \ln|(x+2)+yi| + i\,arctg\left(\frac{y}{x+2}\right) = U_{(x,y)} + iV_{(x,y)}$$

$$U_{(x,y)} = \ln\left[\sqrt{(x+2)^2+y^2}\right] = \frac{1}{2}\ln[(x+2)^2+y^2]$$

$$V_{(x,y)} = arctg\left(\frac{y}{x+2}\right)$$

$$U_x = \frac{1}{2}\,\frac{1}{(x+2)^2+y^2}\,(2x+4) = \frac{x+2}{(x+2)^2+y^2} \qquad U_y = \frac{y}{(x+2)^2+y^2}$$

$$V_x = \frac{1}{1+\left[\frac{y}{(x+2)}\right]^2}\cdot\frac{-y}{(x+2)^2}\cdot 1 = \frac{-y}{(x+2)^2+y^2} \qquad V_y = \frac{1}{1+\left[\frac{y^2}{(x+2)^2}\right]}\,\frac{1}{x+2} = \frac{x+2}{(x+2)^2+y^2}$$

$U_x = V_y$ ✓

$V_x = -V_y$ ✓

$$3-\quad sen(w) = \frac{1}{2i}\left(e^{iw} - e^{-iw}\right)$$

$$\cos(w) = \frac{1}{2}\left(e^{iw} + e^{-iw}\right)$$

$$w = \operatorname{arctg}(z) = ?$$

$$z = tg(w) = \frac{sen(w)}{\cos(w} = \frac{2}{2i}\,\frac{\left(e^{iw} - e^{-iw}\right)}{\left(e^{iw} + e^{-iw}\right)} = \frac{1}{i}\,\frac{e^{i2w} - 1}{e^{i2w} + 1}$$

$$z = \frac{1}{i}\left(\frac{e^{i2w} - 1}{e^{i2w} + 1}\right) \Rightarrow iz\left(e^{i2w} + 1\right) = e^{i2w} - 1$$

$$ize^{i2w} + iz - e^{i2w} + 1 = 0$$

$$e^{i2w}(iz - 1) + iz + 1 = 0$$

$$e^{i2w} = \frac{-(iz + 1)}{(iz - 1)} = \frac{(1 + iz)}{(1 - iz)}$$

$$i2w = \operatorname{Log}\frac{(1 + iz)}{(1 - iz)}$$

$$w = \frac{1}{2i}\operatorname{Log}\left[\frac{(1+iz)}{(1-iz)}\right] = \frac{-i}{2}\operatorname{Log}\left[\frac{(1+iz)}{(1-iz)}\right] = \frac{i}{2}\operatorname{Log}\left[\frac{(1+iz)}{(1-iz)}\right]^{-1} = \frac{i}{2}\operatorname{Log}\left[\frac{(1-iz)}{(1+iz)}\right]$$

$$w' = \frac{i}{2}\,\frac{(1+iz)}{(1-iz)} \cdot \frac{(-i)(1+iz) - (i)(1-iz)}{(1+iz)^2} =$$

$$= \frac{i}{2}\,\frac{1}{1-iz} \cdot \frac{(-i+z) - (i+z)}{(1+iz)} = \frac{\cancel{i}}{\cancel{2}}\,\frac{\cancel{-2i}}{(1-iz)(1+iz)} = \frac{1}{1+z^2}$$

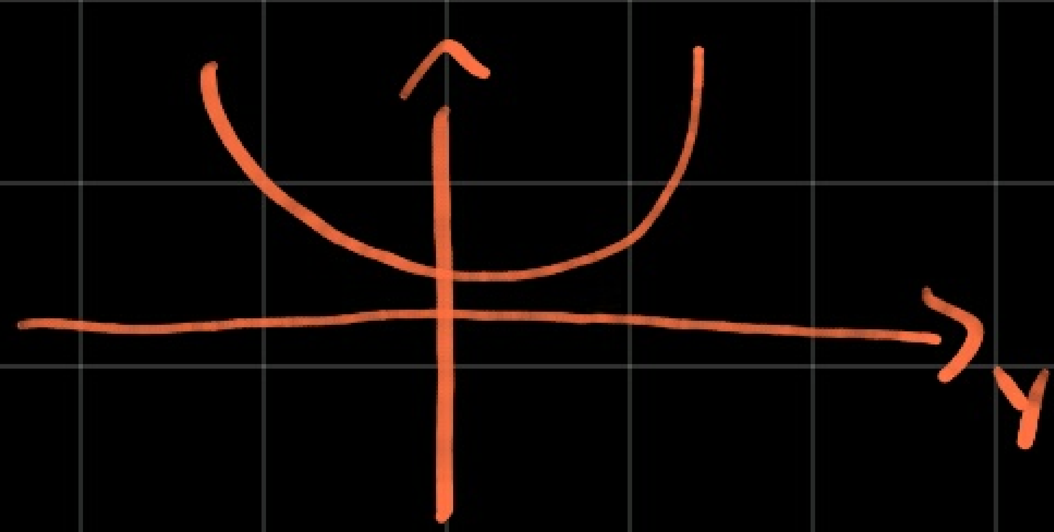
4- a)

$$\left|\oint_C \frac{sen(z)}{z^2}\,dz\right| \le 2\pi \cdot e$$

$$\oint_C \left|\frac{sen(z)}{z^2}\right| dz = \oint_C \frac{|sen(z)|}{|z|^2}\,dz \le M.L = 2\pi \cdot \frac{|sen(z)|}{\underbrace{|z|^2}_{|z|=1}}$$

$$\left|\oint_C \frac{sen(z)}{z^2}\,dz\right| \le 2\pi\,|sen(z)| = \frac{2\pi}{2i}\,|e^{iz}-e^{-iz}| \underbrace{\le}_{\text{Desigualdade triangular}} \frac{2\pi}{2i}\,|e^{iz}|+|e^{-iz}|$$

$$\left|\oint_C \frac{sen(z}{z^2}\right| dz \le 2\pi\,|sen(z)| \le \frac{2\pi}{2}\underbrace{(e^{y}+e^{-y})}_{=\cosh(y)} = 2\pi\,\frac{(e^{1}+e^{-1})}{2}$$



Dentro da região de interesse $|z|=1$ o maior valor de $\cosh(y)$ se dá em $y=1$

$$\left|\oint_C \frac{sen(z)}{z^2}\,dz\right| \le 2\pi\,|sen(z)| \le 2\pi\,\frac{e+1/e}{2} \le 2\pi e$$

$$\left|\oint_C \frac{sen(z)}{z^2}\,dz\right| \le 2\pi e \quad ✓$$

b) $\oint_C \frac{sen(z)}{z^2}\,dz = \oint_C \frac{sen(z)\,dz}{(z-z_0)^2}$, $z_0 = 0$, $f(z) = sen(z)$

$f^{(n)}(z_0) = \frac{n!}{2\pi i}\oint_C \frac{f(z)\,dz}{(z-z_0)^{n+1}} \Rightarrow f'(z) = \cos(z)$

$$2\pi i\cos(0) = \oint_C \frac{sen\,z\,dz}{z^2} = 2\pi i$$

5- $f(z)=(z+i)^{-1}$ $\qquad \dfrac{2\pi i}{1+i} = \displaystyle\oint_c \frac{1}{(z-1)(z+i)}\,dz$

$z_0 = 1$

$f(1) = (1+i)^{-1}$

$$\oint_{c_1} \frac{1}{(z+i)(z-1)} = \frac{2\pi i}{(1+i)} \cdot \frac{(1-i)}{(1-i)} = \frac{2\pi + 2\pi i}{1-i^2} = \pi + \pi i$$

$h(z) = (z-1)^{-1}$ $\quad h(-i) = \dfrac{1}{-i-1} = -\dfrac{1}{1+i}$

$z_1 = -i$

$$\oint_{c_2} \frac{h(z)}{z-z_1} = -\frac{2\pi i}{1+i} \quad \frac{(1-i)}{(1-i)} = -\pi i + \pi = -\pi - \pi i$$

$$\oint_{c_1} \frac{f(z)\,dz}{z-z_0} + \oint_{c_2} \frac{h(z)}{z-z_1}\,dz = \pi + \pi i + (-\pi - \pi i) = 0$$