Solution:

(a) The apparent weight of the man is

$$F = w + \frac{w}{g}a = w\left(1 + \frac{a}{g}\right) ,$$

g being the acceleration of gravity.

(b) The man's rate of expenditure of energy is

$$Fv_t = w\left(1 + \frac{a}{g}\right)(V + v) .$$

Solution:

The observer must be on the equator of the earth. The orbit of the space station is a large circle in the equatorial plane with center at the center of the earth. The radius of the orbit can be figured out using the orbiting period of 24 hours* as follows. Let the radius of the orbit be R and that of the earth be R_0.

We have

$$\frac{mv^2}{R} = \frac{GMm}{R^2} ,$$

where v is the speed of the space station, G is the universal constant of gravitation, m and M are the masses of the space station and the earth respectively, giving

$$v^2 = \frac{GM}{R} .$$

As

$$mg = \frac{GMm}{R_0^2} ,$$

we have

$$GM = R_0^2 g .$$

Hence

$$v^2 = \frac{R_0^2 g}{R} .$$

For circular motion with constant speed v, the orbiting period is

$$T = \frac{2\pi R}{v} .$$

Hence

$$\frac{4\pi^2 R^2}{T^2} = \frac{R_0^2 g}{R}$$

and

$$R = \left(\frac{R_0^2 T^2 g}{4\pi^2}\right)^{\frac{1}{3}} = 4.2 \times 10^4 \text{ km} .$$

Solution:

Under the critical circumstance that the child just starts to slide,

$$mR\omega^2 = \mu mg .$$

Hence

$$R = \frac{\mu g}{\omega^2} = \frac{0.4 \times 9.8}{2^2} = 0.98 \text{ m} .$$

As the centrifugal force is proportional to the radius, this is the maximum radius for no-sliding.

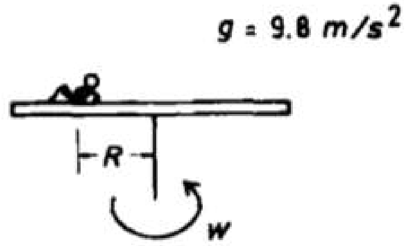

Fig. 1.1.

Solution:

Neglecting the moment of inertia of the pulley, we obtain the equations of motion

$$m_1\ddot{x} = m_1 g - F$$

and

$$m_2\ddot{x} = F - m_2 g .$$

Hence the tension of the cord and the acceleration are respectively

$$F = \frac{2m_1m_2g}{m_1 + m_2} = 77.2 \text{ N}$$

and

$$\begin{aligned}\ddot{x} &= \frac{(m_1 - m_2)g}{m_1 + m_2} = \frac{2g}{16} \\ &= 1.225 \text{ m/s}^2 \ .\end{aligned}$$

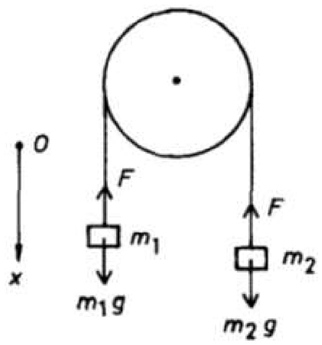

Fig. 1.2.

Solution:

Choose Cartesian coordinates as shown in Fig. 1.3. For $\dot{x} > 0$, the equation of the motion of the brick is

$$m\ddot{x} = -mg\sin\theta - \mu mg\cos\theta \ ,$$

giving

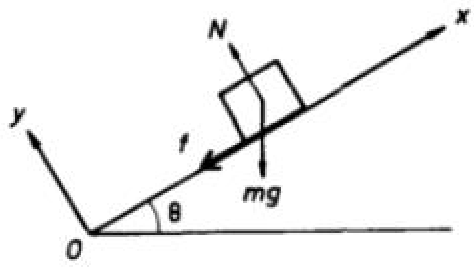

Fig. 1.3.

$$\ddot{x} = -g(\sin\theta + \mu\cos\theta) = -\frac{5g}{8} .$$

The time of upward motion of the brick is then

$$t_1 = \frac{\dot{x}_0}{-\ddot{x}} = 5/(5g/8) = 0.25 \text{ s}$$

and the displacement of the brick is

$$x_1 = \dot{x}_0 t_1 + \frac{1}{2}\ddot{x}t_1^2 = \frac{5}{8} \text{ ft} .$$

For $t > t_1$, $\dot{x} < 0$ and the equation of motion becomes

$$m\ddot{x} = -mg\sin\theta + \mu mg\cos\theta$$

or

$$\ddot{x} = -g(\sin\theta - \mu\cos\theta) = -\frac{3g}{8} .$$

The displacement during the time interval $t_1 = 0.25$ s to $t_2 = 0.5$ s is

$$\Delta x = \ddot{x}\frac{t^2}{2} = -\frac{1}{2} \cdot \frac{3g}{8} \cdot \frac{1}{16} = -\frac{3}{8} \text{ ft} \ ,$$

so that the displacement of the brick at $t = 0.5$ s is

$$S = x_1 + \Delta x = 5/8 - 3/8 = 0.25 \text{ ft}.$$

Solution:

The person has mechanical energy $E_1 = mg(h+s)$ just before he lands. The work done by him during deceleration is $E_2 = fs$, where f is the total force on his legs. As $E_1 = E_2$,

$$f = \frac{mgh}{s} + mg = \left(\frac{80 \times 1}{0.01} + 80\right) g = 8080g \text{ N} \ .$$

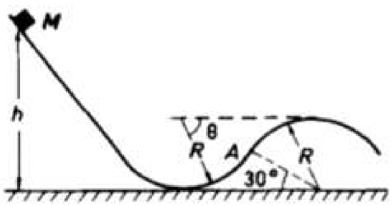

Fig. 1.4.

Solution:

Before the inflection point A of the track, the normal reaction of the track on the mass, N, is

$$N = \frac{mv^2}{R} + mg\sin\theta \ ,$$

where v is the velocity of the mass. After the inflection point,

$$N + \frac{mv^2}{R} = mg\sin\theta \ ,$$

for which $\sin\theta = \frac{R}{2R}$, or $\theta = 30°$.

The mass loses contact with the track if $N \le 0$. This can only happen for the second part of the track and only if

$$\frac{mv^2}{R} \ge mg\sin\theta \ .$$

The conservation of mechanical energy

$$mg[h - (R - R\sin\theta)] = \frac{1}{2}mv^2$$

then requires

$$h - R + R\sin\theta \ge \frac{R\sin\theta}{2} \ ,$$

or

$$h \ge R - \frac{R\sin\theta}{2} \ .$$

The earliest the mass can start to lose contact with the track is at A for which $\theta = 30°$. Hence the minimum h required is $\frac{3R}{4}$.

Solution:

Let g and g' be the gravitational accelerations at the pole and at the equator respectively and consider a body of mass m on the surface of the planet, which has a mass M. At the pole,

$$mg = \frac{GMm}{R^2} ,$$

giving

$$GM = gR^2 .$$

At the equator, we have

$$\frac{mV^2}{R} = \frac{GMm}{R^2} - mg' = mg - \frac{mg}{2} = \frac{mg}{2} .$$

Hence $g = 2V^2/R$.

If we define gravitational potential energy with respect to a point at infinity from the planet, the body will have potential energy

$$-\int_{\infty}^{R} -\frac{GMm}{r^2} dr = -\frac{GMm}{R} .$$

Note that the negative sign in front of the gravitational force takes account of its attractiveness. The body at the pole then has total energy

$$E = \frac{1}{2}mV^2 - \frac{GMm}{R} .$$

For it to escape from the planet, its total energy must be at least equal to the minimum energy of a body at infinity, i.e. zero. Hence the escape velocity v is given by

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = 0 ,$$

or

$$v^2 = \frac{2GM}{R} = 2gR = 4V^2 ,$$

i.e.

$$v = 2V .$$

Solution:

The maximum static friction between the mass and the disk is $f = \mu mg$. When the small mass slides off the disk, its horizontal velocity v is given by

$$\frac{mv^2}{R} = \mu mg \ .$$

Thus

$$v = \sqrt{\mu R g} \ .$$

The time required to descend a distance h from rest is

$$t = \sqrt{\frac{2h}{g}} \ .$$

Therefore the horizontal distance of travel before landing on the floor is equal to

$$vt = \sqrt{2\mu R h} \ .$$

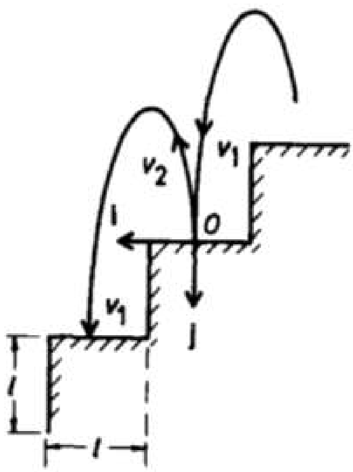

Fig. 1.5.

Solution:

Use unit vectors **i**, **j** as shown in Fig. 1.5 and let the horizontal velocity of the marble be v_h. The velocities just before and after a bounce are respectively

$$\mathbf{v}_1 = v_h \mathbf{i} + v_i \mathbf{j}$$

and

$$\mathbf{v}_2 = v_h \mathbf{i} + v_f \mathbf{j} \ .$$

As the conditions at each step remain exactly the same, v_i, v_f and v_h are all constant. The conservation of mechanical energy

$$\frac{1}{2} m v_1^2 = \frac{1}{2} m v_2^2 + mgl$$

gives

$$v_i^2 = v_f^2 + 2gl \ .$$

As by definition

$$v_f = -e v_i \ ,$$

the above gives

$$v_i^2 = \frac{2gl}{1 - e^2} \ .$$

The time required for each bounce is

$$t = \frac{v_i - v_f}{g} = \frac{l}{v_h} \ ,$$

giving

$$v_h = \frac{gl}{v_i - v_f} = \frac{gl}{(1+e) v_i} = \sqrt{\frac{gl}{2} \frac{1-e}{1+e}} \ ,$$

which is the necessary horizontal velocity. The bouncing height H is given by the conservation of mechanical energy

$$\frac{m v_f^2}{2} = mgH \ .$$

Therefore,

$$H = \frac{v_f^2}{2g} = \frac{e^2}{2g} \frac{2gl}{1 - e^2} = \frac{e^2 l}{1 - e^2} \ .$$

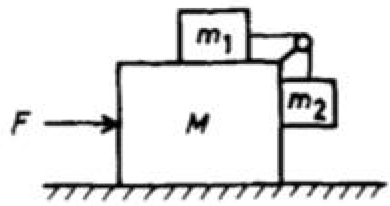

Fig. 1.6.

Solution:

The forces f_1, F and mg are shown in Fig. 1.7. The accelerations of m_1, m_2 and M are the same when there is no relative motion among them. The equations of motion along the x-axis are

$$(M + m_1 + m_2)\ddot{x} = F ,$$
$$m_1\ddot{x} = f_1 .$$

As there is no relative motion of m_2 along the y-axis,

$$f_1 = m_2 g .$$

Combining these equations, we obtain

$$F = \frac{m_2(M + m_1 + m_2)g}{m_1} .$$

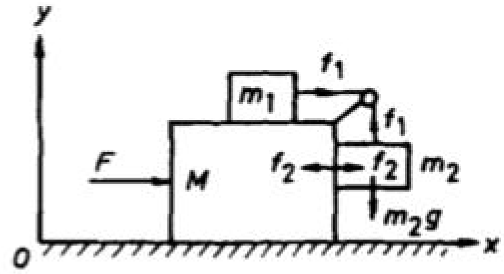

Fig. 1.7.

Solution:

For the motion of the earth around the sun,

$$\frac{mv^2}{r} = \frac{Gmm_s}{r^2} ,$$

where r is the distance from the earth to the sun, v is the velocity of the earth, m and m_s are the masses of the earth and the sun respectively.

For the motion of the sun around the center of the galaxy,

$$\frac{m_s V^2}{R} = \frac{Gm_s M}{R^2} ,$$

where R is the distance from the sun to the center of the galaxy, V is the velocity of the sun and M is the mass of the galaxy.

Hence

$$M = \frac{RV^2}{G} = \frac{R}{r}\left(\frac{V}{v}\right)^2 m_s .$$

Using $V = 2\pi R/T$, $v = 2\pi r/t$, where T and t are the periods of revolution of the sun and the earth respectively, we have

$$M = \left(\frac{R}{r}\right)^3 \left(\frac{t}{T}\right)^2 m_s .$$

With the data given, we obtain

$$M = 1.53 \times 10^{11} m_s .$$

Solution:

(a)

$$V_0 = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 10} = 14 \text{ m/s} .$$

The time elapsed from dive to impact is

$$t = \frac{V_0}{g} = \frac{14}{9.8} = 1.43 \text{ s} .$$

(b) As the gravitational force on the diver is balanced by the buoyancy, the equation of motion of the diver through the water is

$$m\ddot{x} = -b\dot{x}^2 ,$$

or, using $\ddot{x} = \dot{x}d\dot{x}/dx$,

$$\frac{d\dot{x}}{\dot{x}} = -\frac{b}{m}dx .$$

Integrating, with $\dot{x} = V_0$ at $x = 0$, we obtain

$$V \equiv \dot{x} = V_0 e^{-\frac{b}{m}x} .$$

(c) When $V = V_0/10$,

$$x = \frac{m}{b}\ln 10 = \frac{\ln 10}{0.4} = 5.76 \text{ m} .$$

(d) As $dx/dt = V_0 e^{-\frac{b}{m}x}$,

$$e^{\frac{b}{m}x}dx = V_0 dt .$$

Integrating, with $x = 0$ at $t = 0$, we obtain

$$\frac{m}{b}(e^{\frac{b}{m}x} - 1) = V_0 t ,$$

or

$$x = \frac{m}{b}\ln\left(1 + bV_0\frac{t}{m}\right) .$$

Solution:

When the maximum speed is achieved, the propulsive force is equal to the resistant force. Let F be this propulsive force, then

$$F = aV \qquad \text{and} \qquad FV = 600 \text{ W} .$$

Eliminating F, we obtain

$$V^2 = \frac{600}{a} = 150 \text{ m}^2/\text{s}^2$$

and the maximum speed on level ground with no wind

$$v = \sqrt{150} = 12.2 \text{ m/s} .$$

Solution:

Take the mass m as a point mass. At the instant when the pendulum collides with the nail, m has a velocity $v = \sqrt{2gl}$. The angular momentum of the mass with respect to the point at which the nail locates is conserved during the collision. Then the velocity of the mass is still v at the instant after the collision and the motion thereafter is such that the mass is constrained to rotate around the nail. Under the critical condition that the mass can just swing completely round in a circle, the gravitational force is equal to the centripetal force when the mass is at the top of the circle. Let the velocity of the mass at this instant be v_1, and we have

$$\frac{mv_1^2}{l-d} = mg ,$$

or

$$v_1^2 = (l-d)g .$$

The energy equation

$$\frac{mv^2}{2} = \frac{mv_1^2}{2} + 2mg(l-d) ,$$

or

$$2gl = (l-d)g + 4(l-d)g$$

then gives the minimum distance as

$$d = \frac{3l}{5} .$$

Solution:

(a) The tension in the string provides the centripetal force needed for the circular motion, hence $F = mv_0^2/R_0$.

(b) The angular momentum of the mass m is $J = mv_0R_0$.

(c) The kinetic energy of the mass m is $T = mv_0^2/2$.

(d) The radius of the circular motion of the mass m decreases when the tension in the string is increased gradually. The angular momentum of the mass m is conserved since it moves under a central force. Thus

$$mv_0R_0 = mv_1\left(\frac{R_0}{2}\right),$$

or

$$v_1 = 2v_0 .$$

The final kinetic energy is then

$$T_1 = \frac{mv_1^2}{2} = \frac{m(2v_0)^2}{2} = 2mv_0^2 .$$

(e) The reason why the pulling of the string should be gradual is that the radial velocity of the mass can be kept small so that the velocity of the mass can be considered tangential. This tangential velocity as a function of R can be calculated readily from the conservation of angular momentum.

Solution:

Let $V_0 = 60$ mph, then

$$\frac{t}{60} = \frac{V_0}{V} - 1 .$$

Hence

$$\frac{dV}{dt} = \frac{-V^2}{60V_0} ,$$

and the resistance acting on the car is $F = mV^2/(60V_0)$, where m is the mass of the car. The propulsive force must be equal to the resistance F' at the speed of $V' = 30$ mph in order to maintain this speed on the same road. It follows that the horsepower required is

$$\begin{aligned}
P' = F'V' = \frac{mV'^3}{60V_0} &= 37500 \frac{\text{mph}^2.\text{lb.}}{\text{s}} \\
&= \frac{37500}{g} \frac{\text{mph}^2.\text{lb wt}}{\text{s}} = \frac{37500}{22} \text{ mph.lb wt} \\
&= \frac{37500}{22} \cdot \frac{88}{60} \frac{\text{ft.lb wt}}{\text{s}} \\
&= 2500 \frac{\text{ft.lb wt}}{\text{s}} = 4.5 \text{ H.P.}
\end{aligned}$$

Note that pound weight (lb wt) is a unit of force and 1 lb wt $= g$ ft.lb/s^2. The horsepower is defined as 550 ft.lb wt/ s .

Solution:

According to Fig. 1.10, the equation of the motion of the child is

$$ml\ddot{\theta} = -mg - mg\sin\theta \ ,$$

or

$$\ddot{\theta} + (\frac{g}{l})\sin\theta = \frac{-g}{l} \qquad (\theta \geq 0) \ .$$

With $\omega^2 = g/l$, $\sin\theta \approx \theta$, the above becomes

$$\ddot{\theta} + \omega^2\theta = -\omega^2 \ .$$

The solution of this equation is $\theta = A\cos(\omega t) + B\sin(\omega t) - 1$, where the constants A and B are found from the initial conditions $\theta = 1$, $\dot{\theta} = 0$ at $t = 0$ to be $A = 2$, $B = 0$. Hence

$$\theta = 2\cos(\omega t) - 1 \ .$$

When $\theta = 0$,

$$\cos(\omega t_1) = \frac{1}{2} \ ,$$

giving

$$\omega t_1 = \frac{\pi}{3} \ ,$$

i.e.

$$t_1 = \frac{1}{\omega} \cdot \frac{\pi}{3} = \frac{\pi}{3}\sqrt{\frac{l}{g}} \ .$$

This is the length of time the father pushed the swing.

Solution:

Write out the equations of motion of the particle in polar coordinates:

$$m(\ddot{r} - r\dot{\theta}^2) = f(r) - \lambda\dot{r} \ ,$$
$$m(2\dot{r}\dot{\theta} + r\ddot{\theta}) = -\lambda r\dot{\theta} \ ,$$

or

$$\frac{1}{r}\frac{d(mr^2\dot{\theta})}{dt} = -\lambda r\dot{\theta} \ .$$

Letting $J = mr^2\dot{\theta}$, we rewrite the last equation as follows:

$$\frac{dJ}{dt} = \frac{-\lambda J}{m} \ .$$

Integrating and making use of the initial angular momentum $\mathbf{J}_0$, we obtain

$$\mathbf{J} = \mathbf{J}_0 e^{-\frac{\lambda}{m}t} \ .$$

Solution:

(a) Consider the limiting case that the Crab pulsar is just about to disintegrate. Then the centripetal force on a test body at the equator of the Crab pulsar is just smaller than the gravitational force:

$$\frac{mv^2}{R} = mR\omega^2 \leq \frac{GmM}{R^2} ,$$

or

$$\frac{M}{R^3} \geq \frac{\omega^2}{G} ,$$

where m and M are the masses of the test body and the Crab pulsar respectively, R is the radius of the pulsar, v is the speed of the test body, and G is the gravitational constant. Hence the minimum density of the pulsar is

$$\rho = \frac{M}{\frac{4}{3}\pi R^3} \geq \frac{3\omega^2}{4\pi G} = \frac{3(2\pi \times 30)^2}{4\pi \times 6.7 \times 10^{-11}} \sim 1.3 \times 10^{14} \text{ kg/m}^3 .$$

(b) As $\frac{3M}{4\pi R^3} \geq \rho_{\min}$,

$$R \leq \left(\frac{3M}{4\pi\rho_{\min}}\right)^{\frac{1}{3}} = \left(\frac{6 \times 10^{30}}{4\pi \times 1.3 \times 10^{14}}\right)^{\frac{1}{3}} = 1.5 \times 10^5 \text{ m} = 150 \text{ km} .$$

(c) The nuclear density is given by

$$\rho_{\text{nuclear}} \approx \frac{m_p}{4\pi R_0^3/3} ,$$

where m_{p} is the mass of a proton and is approximately equal to the mass m_{H} of a hydrogen atom. This can be estimated as follows:

$$m_{\text{p}} \approx m_{\text{H}} = \frac{2 \times 10^{-3}}{2 \times 6.02 \times 10^{23}} = 1.7 \times 10^{-27} \text{ kg} .$$

With

$$R_0 \approx 1.5 \times 10^{-15} \text{ m} ,$$

we obtain

$$\rho_{\text{nuclear}} \approx 1.2 \times 10^{17} \text{ kg/m}^3 .$$

If $\rho = \rho_{\text{nuclear}}$, the pulsar would have a radius

$$R \approx \left(\frac{6 \times 10^{30}}{4\pi \times 1.2 \times 10^{17}}\right)^{\frac{1}{3}} \approx 17 \text{ km} .$$