1017

When a 5000 lb car driven at 60 mph on a level road is suddenly put into neutral gear (i.e. allowed to coast), the velocity decreases in the following manner:

$$V = \frac{60}{1 + (\frac{t}{60})} \text{ mph} ,$$

where t is the time in sec. Find the horsepower required to drive this car at 30 mph on the same road.
Useful constants: $g = 22$ mph/sec, 1 H.P. $= 550$ ft.lb/sec, 60 mph $=$ 88 ft/sec.

(*Wisconsin*)

주석: 1.H.P.(horsepower)=550 ft.lb wt/sec
lb wt는 힘의 단위로 1lb wt= g ft.lb/s^2
lb는 질량의 단위, ft는 길이의 단위

1018

A child of mass m sits in a swing of negligible mass suspended by a rope of length l. Assume that the dimensions of the child are negligible compared with l. His father pulls him back until the rope makes an angle of one radian with the vertical, then pushes with a force $F = mg$ along the arc of a circle until the rope is vertical, and releases the swing. For what length of time did the father push the swing? You may assume that it is sufficiently accurate for this problem to write $\sin\theta \approx \theta$ for $\theta < 1$.

(*Wisconsin*)

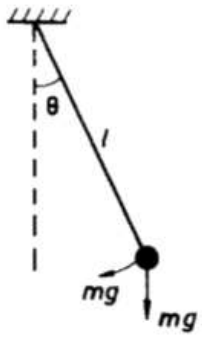

Fig. 1.10.

1019

A particle of mass m is subjected to two forces: a central force $\mathbf{f}_1$ and a frictional force $\mathbf{f}_2$, with

$$\mathbf{f}_1 = \frac{\mathbf{r}}{r} f(r),$$
$$\mathbf{f}_2 = -\lambda \mathbf{v} \qquad (\lambda > 0)\ ,$$

where $\mathbf{v}$ is the velocity of the particle. If the particle initially has angular momentum $\mathbf{J}_0$ about $r = 0$, find its angular momentum for all subsequent times.

(*Wisconsin*)

1035

A ball of mass M is suspended from the ceiling by a massless spring with spring constant k and relaxed length equal to zero. The spring will break if it is extended beyond a critical length l_c ($l_c > Mg/k$). An identical spring hangs below the ball (Fig. 1.22). If one slowly pulls on the end of the lower spring, the upper spring will break. If one pulls on the lower spring too rapidly, the lower spring will break. The object of this problem is to determine the force $F(t)$ which, when applied to the end of the lower spring, will cause both springs to break simultaneously.

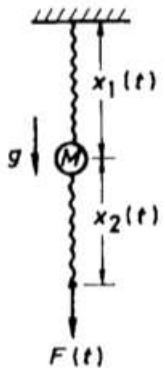

Fig. 1.22.

(a) Find an integral expression relating the length $x_1(t)$ of the upper spring to the applied force $F(t)$.

(b) Using any technique you like, find $x_1(t)$ and $x_2(t)$ for $t > 0$ when $F(t)$ has the particular form

$$F(t) = \begin{cases} 0, & t < 0 \\ \alpha t, & t > 0 \end{cases},$$

where α is a constant.

(c) Use a careful sketch of your solutions to show that if α is too small, the upper spring will break. Similarly, shown that if α is too large, then the lower spring could break first.

(d) Show that both springs break simultaneously when α is a solution of the equation

$$\sin\left(\frac{kl_c}{\alpha}\sqrt{\frac{k}{M}}\right) = \frac{Mg}{\alpha}\sqrt{\frac{k}{M}}\,.$$

(MIT)

주석:미분방정식

1036

A pendulum, made up of a ball of mass M suspended from a pivot by a light string of length L , is swinging freely in one vertical plane (see Fig. 1.25). By what factor does the amplitude of oscillations change if the string is very slowly shortened by a factor of 2?

(*Chicago*)

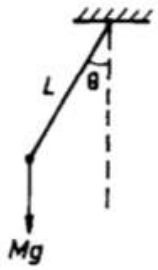

Fig. 1.25.

주석:줄의 길이가 절반으로 줄어들었다. 2가지 풀이가 가능

1039

A particle of mass m is projected with velocity v_0 toward a fixed scattering center which exerts a repulsive force $\mathbf{F} = (mv_1^2/2)\delta(r - a)\hat{\mathbf{r}}$, where $\hat{\mathbf{r}}$ is a unit vector along the radius from the force center, a is a fixed radius at which the force acts, and v_1 is a constant having the dimensions of velocity. The impact parameter is s, as shown in Fig. 1.27.

(a) Find the potential energy.

(b) Show that if $v_0 < v_1$, the particle does not penetrate the sphere $r = a$, but bounces off, and that the angle of reflection equals the angle of incidence.

(c) Sketch carefully the orbit you would expect for $v_0 > v_1$, $s = a/2$.

(*Wisconsin*)

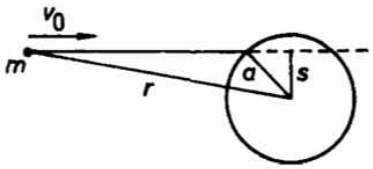

Fig. 1.27.

주석:에너지는 델타함수 형태로 주어져 있다
(c)의 조건에 대해 조금 더 생각해 보자
v0>v1이면 항상 통과 가능한가?