



introducing

**easyGSA** available from  MENDELEY DATA  GitHub

A global sensitivity analysis framework using  
advanced machine learning algorithms

- > Gaussian process regression
- > Artificial neural networks
- + Highly Increased computational efficiency
- + Reliably rank important parameters
- + Quickly identify key design parameters in a design space
- + Tap into the power of machine learning libraries

Download is available from

<https://github.com/resulal/easyGSA>

# Outline

introducing

**easyGSA**

Global sensitivity  
analysis framework  
using advanced  
machine learning  
algorithms

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 **GitHub**

Introducing syntax and features

Benchmark problems

1. Ishigami function
2. gSobol function
3. Cantilever Beam functions

Engineering applications

1. Wastewater treatment plant design space exploration

## What it can do for you

# easyGSA at a glance

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## easyGSA

Global sensitivity analysis framework using advanced machine learning algorithms

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- Easy-to-work syntax to perform GSA using Sobol method
- Sampling schemes: Sobol sequences, Latin hypercube sampling (LHS)
- Automatic hyperparameter optimization for Gaussian process models
- Gridsearch optimization algorithm for finding best neural networks configuration
- Allowing user provided data to fit surrogates and perform Sobol GSA.
- Automatic data cleaning.
- Efficient use of available parallelization architecture.

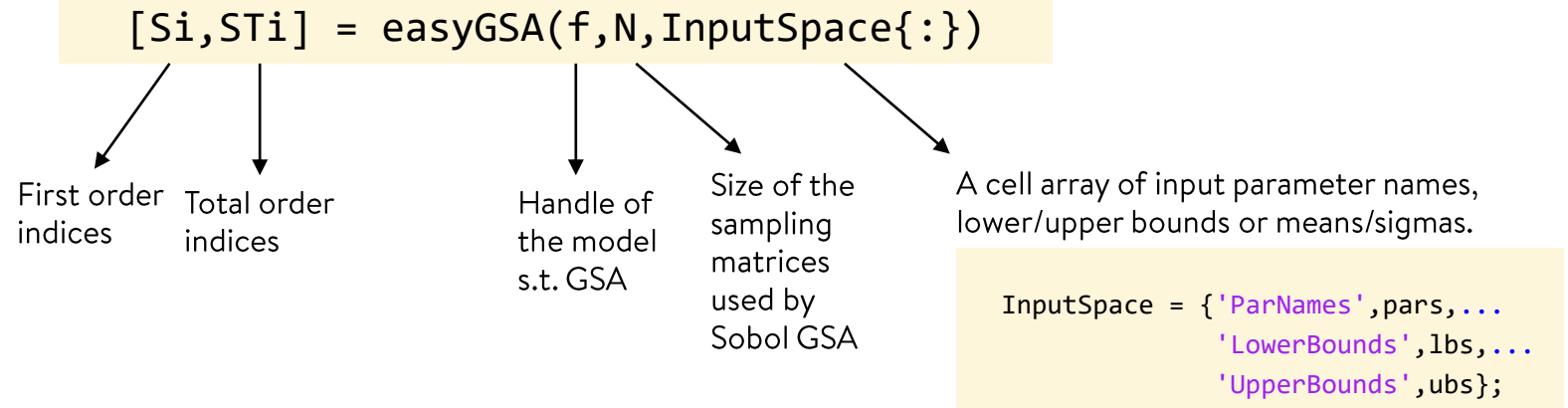
# How to use easyGSA: basic syntax

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## How to use easyGSA: detailed syntax

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```
[Si,STi,results] = easyGSA(f,N,InputSpace{:},...
```

```
'SamplingMethod','LHS',...
```

```
'Estimator','Saltelli',...
```

```
'UseSurrogate','GPR',...
```

```
'UseParallel',true,...
```

```
'Verbose',false)
```

Use Latin hypercube sampling  
to sample the input space

Use 'Saltelli' estimator for  
Sobol indices calculation

Use Gaussian  
process models  
as a surrogate

More detailed results of the  
analysis containing all the  
models and simulation results.

Suppress command  
line messages

Activate parallel  
computing

How to use

# easyGSA: Input arguments overview

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<b>'Model'</b> @ishigami @mymodel.m	<b>'N'</b> 2e3 ..	<b>'InputSpace'</b> 'LowerBounds' 'UpperBounds'
<b>'SamplingMethod'</b> 'Sobol' 'LHS'	<b>'Estimator'</b> 'Jansen' 'Saltelli'	<b>'UseSurrogate'</b> 'GPR' 'ANN'
<b>'UserData'</b> Data.X Data.Y	<b>'UseParallel'</b> true false	<b>'Verbose'</b> true false

Required argument

Optional argument

Available options

Default setting in bold

## Benchmark problems

# Ishigami function

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Easy syntax to perform GSA using Sobol method

```
% Test on Ishigami function: Analytical sensitivities are known
f = @(x) sin(x(:,1)) + 7.*sin(x(:,2)).^2 + 0.1.*x(:,3).^4.*sin(x(:,1));
N = 1e3; % Number of MC samples

% define input parameter space
pars = strseq('x',1:3); % input parameter names
lbs = -pi.*ones(1,3); % lower bounds of input parameters
ubs = pi.*ones(1,3); % upper bounds of input parameters
InputSpace = {'ParNames',pars,'LowerBounds',lbs,'UpperBounds',ubs};

% call easyGSA to perform Sobol sensitivity analysis with MC approach
[Si,STi] = easyGSA(f,N,InputSpace{:})
```

## Benchmark problems

# Ishigami function

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## Easy syntax to perform GSA using Sobol method

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lbs = -pi.*ones(1,3); % lower bounds of input parameters
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InputSpace = {'ParNames',pars,'LowerBounds',lbs,'UpperBounds',ubs};

% call easyGSA to perform Sobol sensitivity analysis with MC approach
[Si,STi] = easyGSA(f,N,InputSpace{:})
```

## Change sampling method and MC estimator

```
[Si,STi] = easyGSA(f,N,InputSpace{:}, ...
    'SamplingMethod','LHS',... % also: 'Sobol'
    'Estimator','Jansen')      % also: 'Saltelli'
```



## Benchmark problems

# Ishigami function

Use a Gaussian Process regression model to do the GSA

```
% use a GPR model instead
[gprSi, gprSTi] = easyGSA(f,N,parameters{:}, ...
    'UseSurrogate', 'GPR')
```

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## Benchmark problems

# Ishigami function

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Use a Gaussian Process regression model to do the GSA

```
% use a GPR model instead
[gprSi, gprSTi] = easyGSA(f,N,parameters{:}, ...
                        'UseSurrogate', 'GPR')
```

Use an artificial neural network model to do the GSA

```
% use an ANN model instead
[annSi, annSTi] = easyGSA(f,N,parameters{:}, ...
                        'UseSurrogate', 'ANN')
```

## Benchmark problems

## Ishigami function

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Use a Gaussian Process regression model to do the GSA

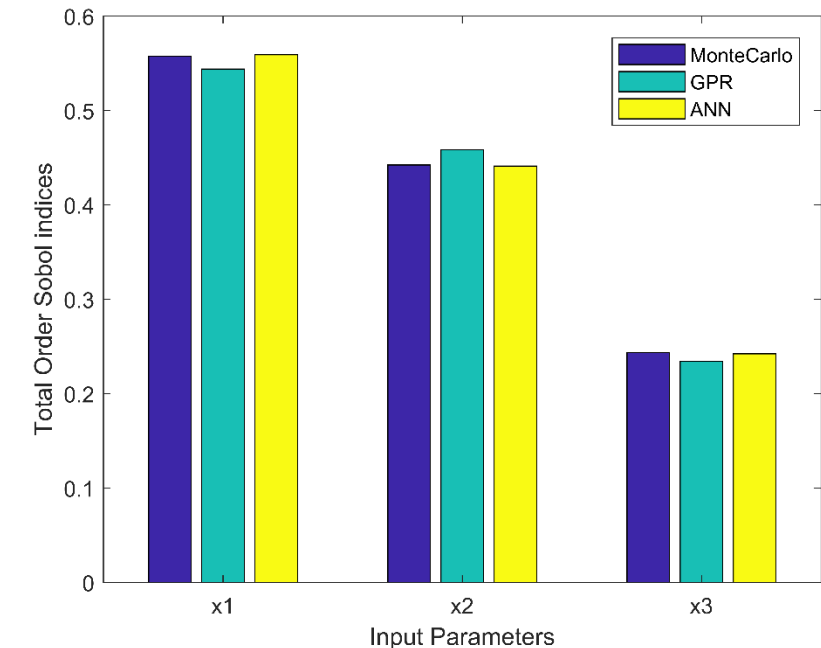
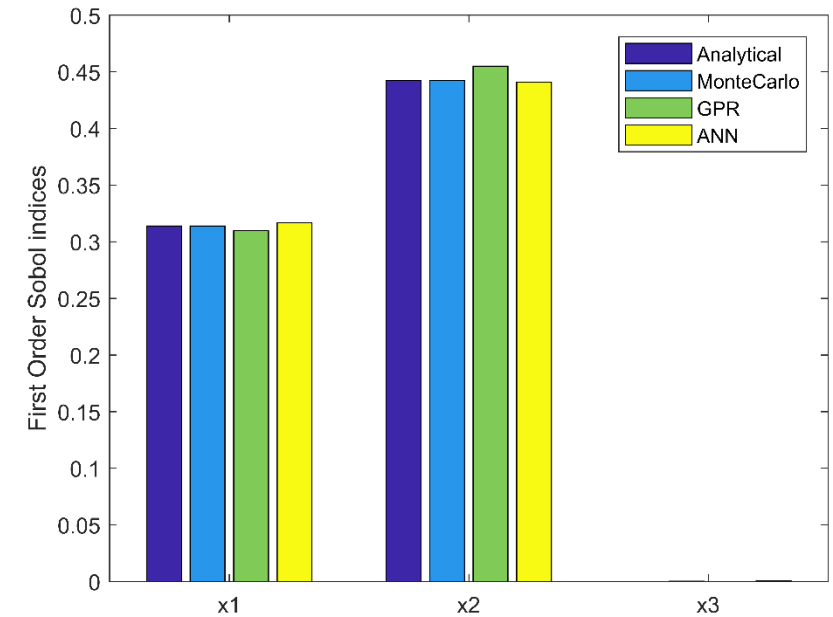
```
% use a GPR model instead
[gprSi, gprSTi] = easyGSA(f,N,parameters{:}, ...
    'UseSurrogate', 'GPR')
```

Use an artificial neural network model to do the GSA

```
% use an ANN model instead
[annSi, annSTi] = easyGSA(f,N,parameters{:}, ...
    'UseSurrogate', 'ANN')
```

Use parallel computing to speed up

```
% use an ANN model instead
[annSi, annSTi] = easyGSA(f,N,parameters{:}, ...
    'UseSurrogate', 'ANN', ...
    'UseParallel', 'true')
```



# g-function of Sobol

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```
f = @(x) gSobol(x);
N = 1e6; % Number of MC samples

pars = strseq('x',1:5); % input parameter names
lbs = zeros(1,5);      % lower bounds of input parameters
ubs = ones(1,5);       % upper bounds of input parameters
parameters = {'ParNames',pars,'LowerBounds',lbs,'UpperBounds',ubs};

% Monte Carlo indices from the original model
[mcSi,mcSTi] = easyGSA(f,N,parameters{:});

% GPR indices
[gprSi, gprSTi] = easyGSA(f,N,parameters{:},'UseSurrogate','GPR')

% ANN indices
[annSi, annSTi] = easyGSA(f,N,parameters{:},'UseSurrogate','ANN')
```

# g-function of Sobol

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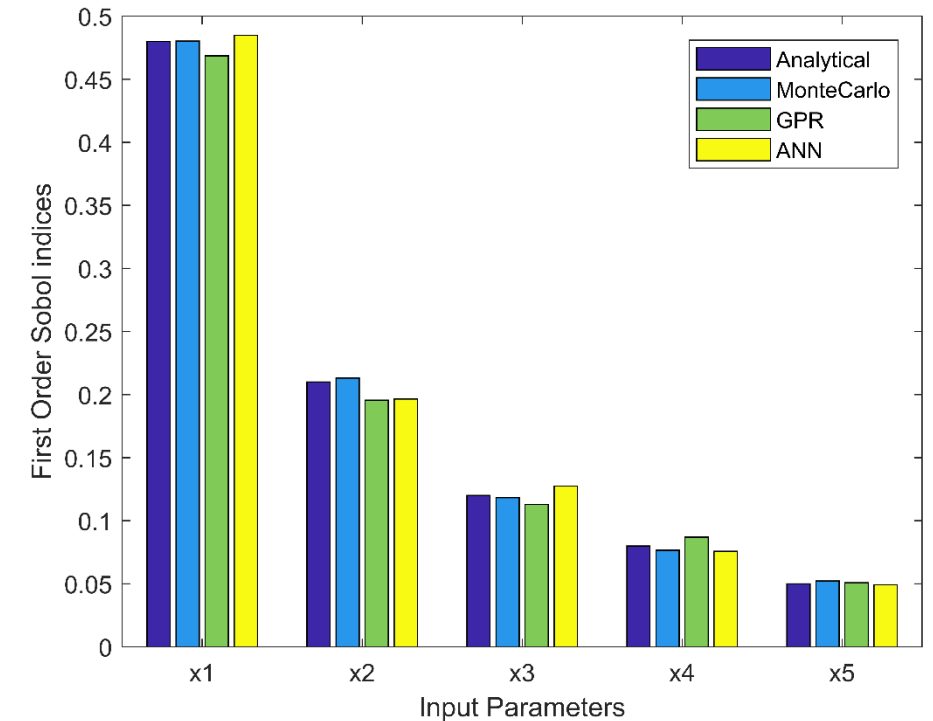
```
f = @(x) gSobol(x);
N = 1e6; % Number of MC samples

pars = strseq('x',1:5); % input parameter names
lbs = zeros(1,5);      % lower bounds of input parameters
ubs = ones(1,5);       % upper bounds of input parameters
parameters = {'ParNames',pars,'LowerBounds',lbs,'UpperBounds',ubs};

% Monte Carlo indices from the original model
[mcSi,mcSTi] = easyGSA(f,N,parameters{:});

% GPR indices
[gprSi, gprSTi] = easyGSA(f,N,parameters{:},'UseSurrogate','GPR')

% ANN indices
[annSi, annSTi] = easyGSA(f,N,parameters{:},'UseSurrogate','ANN')
```



# The Cantilever Beam functions

## Multiple outputs

$$D(x) = \frac{4L^3}{E\omega t} \sqrt{\left(\frac{Y}{t^2}\right)^2 + \left(\frac{X}{\omega^2}\right)^2}$$

$$S(x) = \frac{600Y}{\omega t^2} + \frac{600X}{\omega t^2}$$

The Cantilever Beam functions, used for uncertainty quantification, model a simple uniform cantilever beam with horizontal and vertical loads. The beam length  $L$  and displacement tolerance  $D0$  at the free end of the beam are problem constants, with values  $L = 100$  inches, and  $D0 = 2.2535$  inches. The parameters  $w$  and  $t$  are width and thickness of the cross-section.

The responses are displacement ( $D$ ) and stress ( $S$ ).

## Normally distributed input space

$R \sim N(\mu=40000, \sigma=2000)$	yield stress
$E \sim N(\mu=2.9E7, \sigma=1.45E6)$	Young's modulus of beam material
$X \sim N(\mu=500, \sigma=100)$	horizontal load
$Y \sim N(\mu=1000, \sigma=100)$	vertical load

Model implementation from  
<https://www.sfu.ca/~ssurjano/canti.html>

## Related tutorial

demo\_canti.m

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Normally distributed input space with defined  $\mu$  and  $\sigma$

```
f = @cantibeam; % handle to the cantibeam.m model file
N = 2e3; % Number of Monte Carlo samples

% Input Space definition
pars = {'R','E','X','Y'}; % input parameter names
means = [4e4 2.9e7 5e2 1e3]; % mean values of input parameters
stds = [2e3 1.45e6 1e2 1e2]; % std deviations of input parameters
InputSpace = {'ParNames',pars,'Means',means,'Sigmas',stds};

% call easyGSA tool to perform Sobol GSA with MC approach
[Si,STi,results] = easyGSA(f,N,InputSpace{:},'UseParallel',true)
```

## Benchmark problems

## The Cantilever Beam functions

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Normally distributed input space with defined  $\mu$  and  $\sigma$ 

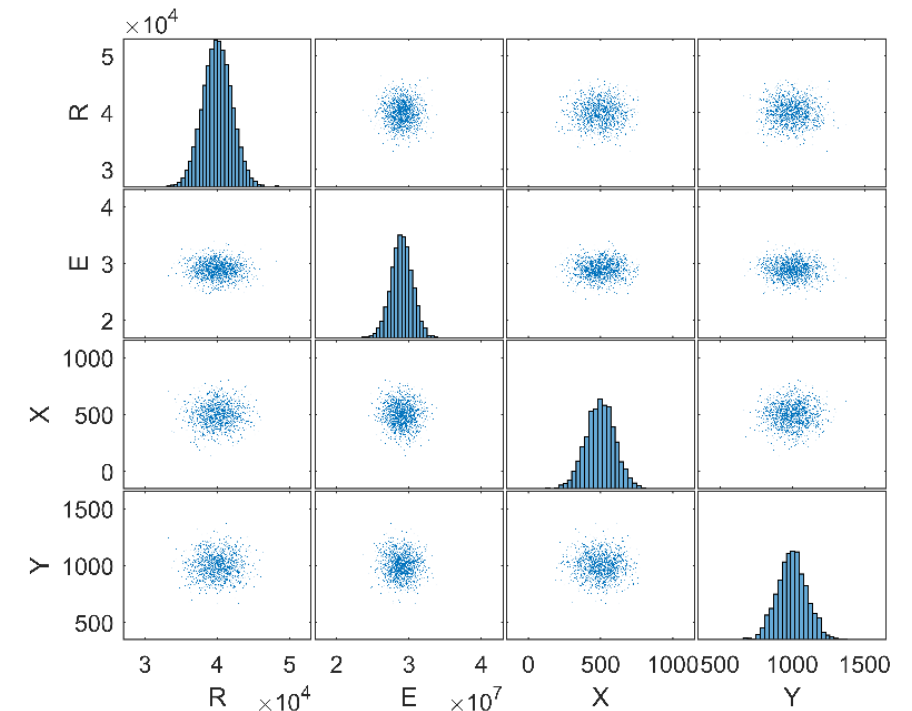
```
f = @cantibeam; % handle to the cantibeam.m model file
N = 2e3; % Number of Monte Carlo samples

% Input Space definition
pars = {'R','E','X','Y'}; % input parameter names
means = [4e4 2.9e7 5e2 1e3]; % mean values of input parameters
stds = [2e3 1.45e6 1e2 1e2]; % std deviations of input parameters
InputSpace = {'ParNames',pars,'Means',means,'Sigmas',stds};

% call easyGSA tool to perform Sobol GSA with MC approach
[Si,STi,results] = easyGSA(f,N,InputSpace{:},'UseParallel',true)
```

Plot input sampling matrices

```
% visualize input sampling matrices
figure; [~,ax]=plotmatrix(results.A); np=numel(pars);
for i=1:np
    ylabel(ax(i,1),pars(i)); xlabel(ax(np,i),pars(i));
end
```





# The Cantilever Beam functions

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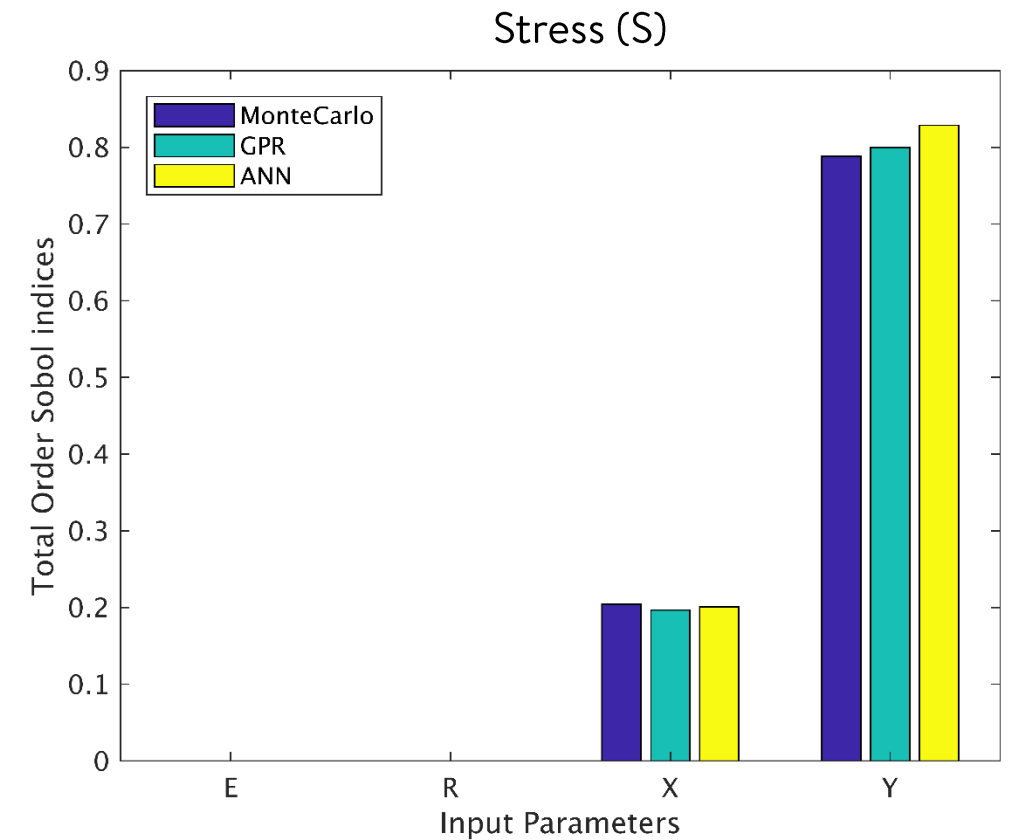
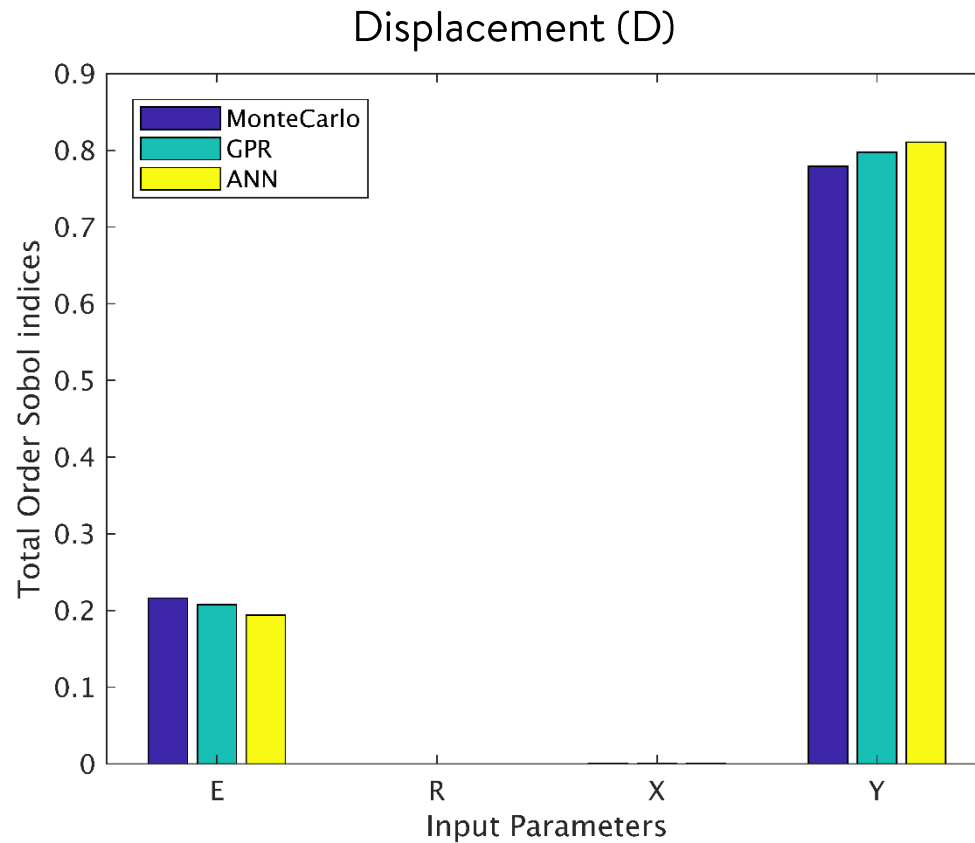
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## Using your own data

# User data support

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Use your own Monte Carlo Simulation data to quickly fit GPR and ANN models and perform the GSA

```
% Inputting your own dataset to perform GPR and ANN-based GSA

% Step 1: Load your own data, eg. simulation results, etc.
[X,Y] = chemical_dataset; X=X'; Y=Y'; % a standard MATLAB dataset

% Step 2: Put your data into a struct. Only X and Y fields are expected.
Data.X = X; % inputs
Data.Y = Y; % outputs

% Step 3: pass your data into easyGSA
[Si,STi,results] = easyGSA('UserData',Data) % uses GPR models by default.

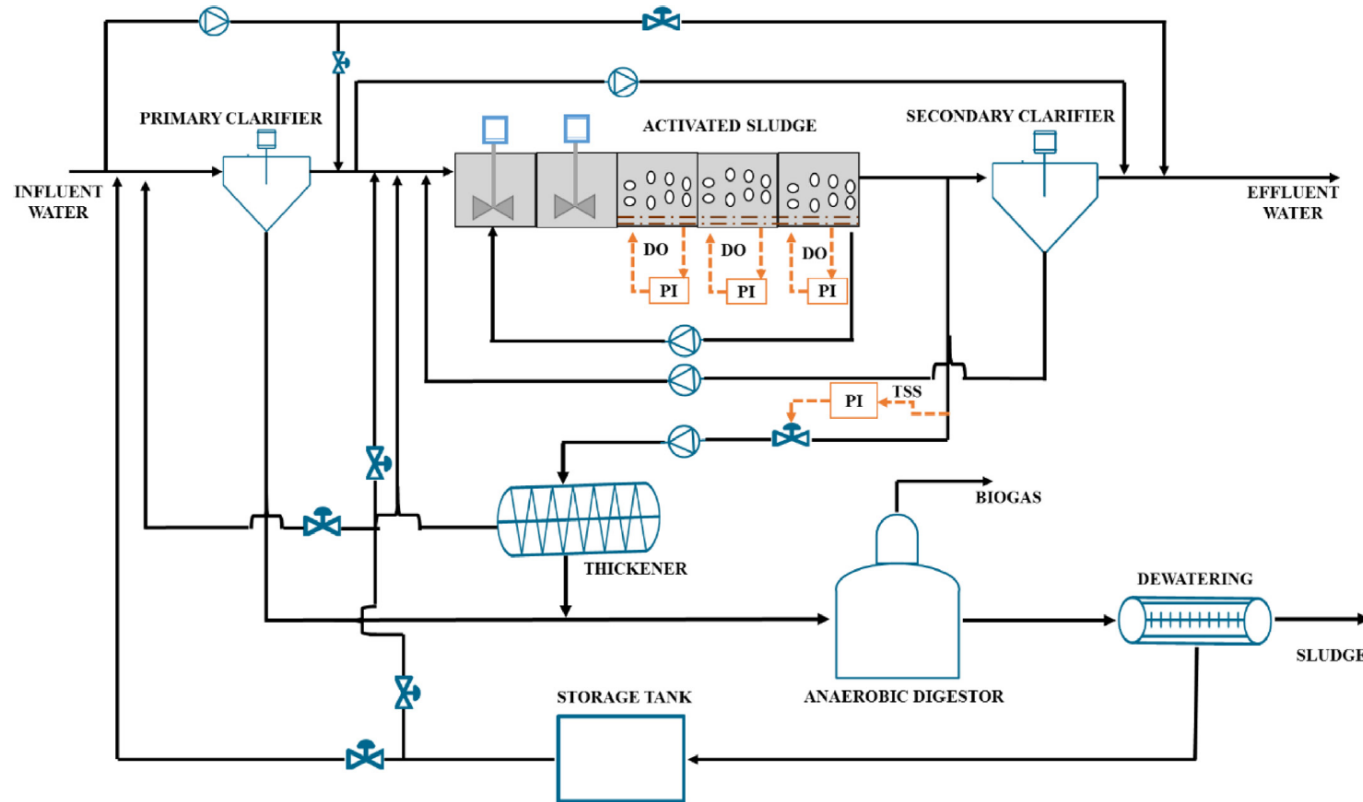
% Step 4: Fit ANN models and perform a Sobol GSA
[Si,STi,results] = easyGSA('UserData',Data,...
    'UseSurrogate','ANN')
```

Related tutorial

[demo\\_UserData.m](#)

# Wastewater treatment plant application

Find the design decisions that are most influencing the key plant performance indicators (KPIs).

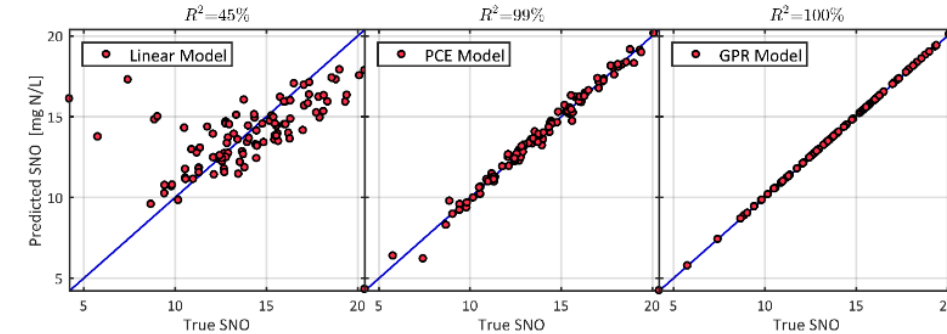
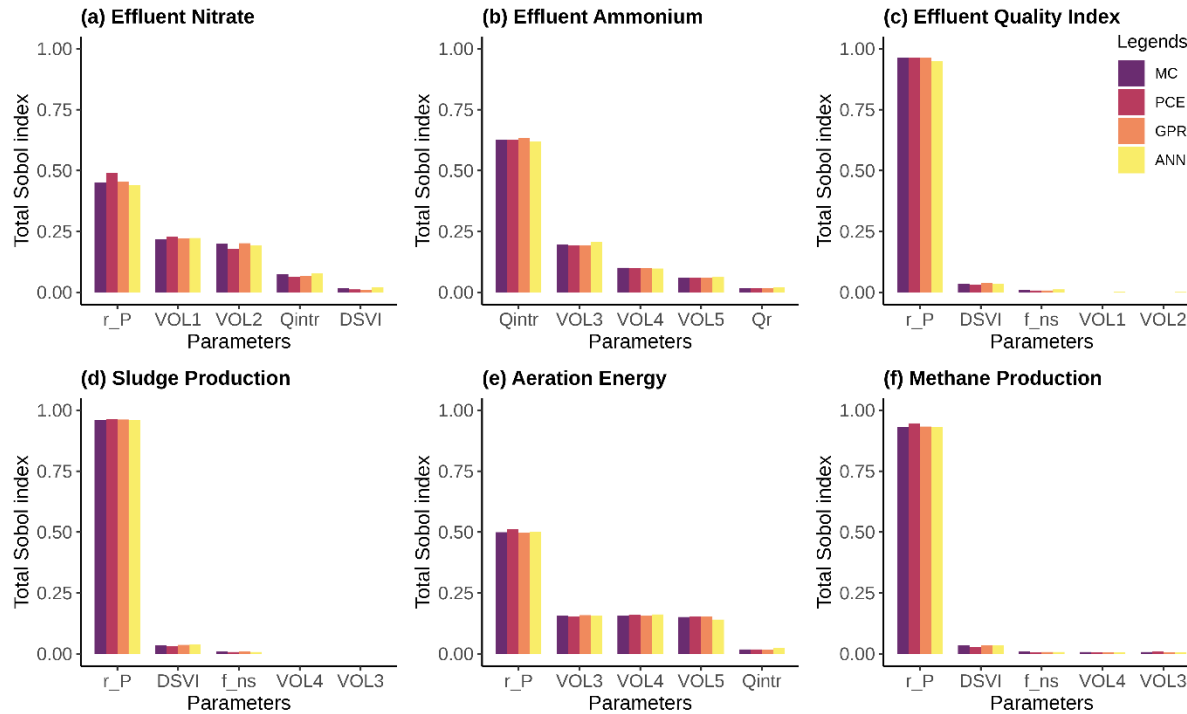


Benefits of surrogate-based methodology becomes the most evident when you have expensive-to-evaluate simulation models.

e.g. Benchmark Simulation Model 2

## Wastewater treatment plant application

Refine engineering design spaces with Global sensitivity analysis



Al et al., 2019. *Comput Chem Eng* 127

*Computers and Chemical Engineering* 127 (2019) 233–246



Contents lists available at [ScienceDirect](https://www.sciencedirect.com)

**Computers and Chemical Engineering**

journal homepage: [www.elsevier.com/locate/comchemeng](https://www.elsevier.com/locate/comchemeng)



Meta-modeling based efficient global sensitivity analysis for wastewater treatment plants – An application to the BSM2 model

Resul Al, Chitta Ranjan Behera, Alexandr Zubov, Krist V. Gernaey, Gürkan Sin\*

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## Computational gain

## Wastewater treatment plant application

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## How much is the computational gain?

Comparison of computational costs of different approaches for global sensitivity analysis.

Approach	# of plant-wide simulations used				Total computational cost
	Scenario 1 ( $d = 7$ )	Scenario 2 ( $d = 20$ )	Scenario 3 ( $d = 10$ )	Scenario 4 ( $d = 37$ )	
SRC with MCS	1000	1000	1000	1000	$4000 \times t_{BSM2}$
Sobol indices with MCS using BSM2	18,000	44,000	24,000	78,000	$164000 \times t_{BSM2}$
Sobol indices with MCS using GPR	150	100	100	250	$600 \times t_{BSM2}$
Sobol indices with MCS using ANN	150	100	100	450	$800 \times t_{BSM2}$
Sobol indices with PCE	250	150	100	250	$750 \times t_{BSM2}$

Al et al., 2019. *Comput Chem Eng* 127

On average, surrogates provide 200 times faster results.

## For questions

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*PROSYS Research Centre*

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