TUTORIAL 03

7CCMCS04
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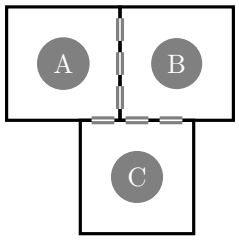
Problem 3.1 Consider a graph of N nodes, and adjacency matrix $\boldsymbol{A}$, so that $A_{ij} = 1$ if there is a link between i and j and $A_{ij} = 0$ otherwise. We denote the local degrees $k_i = \sum_j c_{ij}$. A random walk on this graph is a Markov chain with transition matrix $\boldsymbol{Q}$ such that

$$Q_{ij} = \begin{cases} \frac{1}{d_j} & \text{if } A_{ij} = 1 \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

In the equation above $d_j = \sum_i A_{ij}$ is the degree of the node j

- **a.** Justify Eq. (1).
- **b.** Show that random walks on graph are reversible Markov chains, with stationary distribution $|\boldsymbol{\Pi}\rangle$ such that $\Pi_i = \frac{d_i}{\sum_i d_i}$.

Problem 3.2 A mouse lives in a house Ω of three rooms, A, B, C, so that $\Omega = \{\mathrm{A}, \mathrm{B}, \mathrm{C}\}$. There are three doors between room A and B, two doors between room B and C, and one door between room A and C, as shown below



At regular time intervals, a door, in the room occupied by the mouse, is opened at random and the mouse is trained to change room each time. After the mouse changes room the door is closed.

- **a.** Find the transition matrix $\boldsymbol{Q}$, i.e., the probabilities for the mouse to move from one room to another.
- **b.** Approximately what fraction of its time will the mouse spend in each room?
- **c.** We now modify the dynamical rules of our process in the following way: at each time step the mouse is offered to change room, but the mouse sometimes rejects the proposed move and stay in the room it is occupying when the door is opened. In other words, we introduce a probability $a_{x'x}$ that the mouse accept the proposed move from room x to room x' and

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changes room and a probability $1 - a_{x'x}$ that the mouse rejects the move and stays in x. Find the acceptance rates $a_{x'x}$ $\forall$ x, x' for which the process will converge to a situation where the mouse spends the same time in each room, and specify the transition matrix $\boldsymbol{Q}'$ for the new dynamical process.