TUTORIAL 04

7CCMCS04
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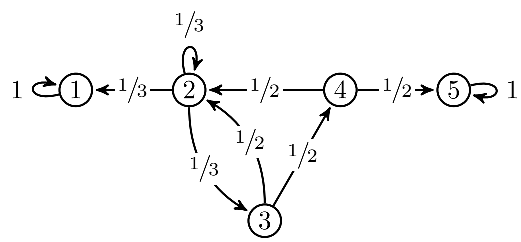
Problem 4.1 Consider the following finite Markov chain for a stochastic process X_t:



Each arrow connecting, e.g., the state i to the state j is associated to a number corresponding to the probability $\mathbb{P}_{1|1}[\mathsf{X}_{t+1} = j|\mathsf{X}_t = i]$.

a. Write the transition matrix $\boldsymbol{Q}$ for the represented chain.
b. Identify the absorbing states. Also, partition the states in *recurrent* and *transient*.
c. Starting from a transient state j, what is the number of required steps to hit one of the absorbing states?

Problem 4.2 A university degree course is two-years long and students have to resit a year if they fail their final examinations each year. If a student passes the first year final, she accesses to the second year; if she passes her second year final exam, she graduates. However, if she fails the second year final, she is sent back to first year. Students can drop this course at any point. The frequencies of all possible events are summarised below.

	Passed	Failed	Dropped
First year final	$1/3$	$1/3$	$1/3$
Second year final	$2/3$	$1/6$	$1/6$

Write a representation of a Markov chain describing this dynamics. How many years after entering the university does a student usually graduate? What is the probability of graduating?

Problem 4.3 A gambler plays a game in which, starting from $\mathsf{X}_0 = 1$ coin, then on each successive gamble either wins 1 coin or loses 1 independently from the past with probabilities p and $q = 1 - p$ respectively. Let X_t denote the total fortune after the tth round. The gambler's objective is to reach a total fortune of 3 coins, without first getting ruined, i.e., touching the 0 values. If the gambler

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succeeds, he win the game. The gambler stops playing after winning or getting ruined, whichever happens first. Draw the diagram of the chain, and give the corresponding transition matrix. For which value $p^\star$ of p the game lasts longer on average? What is the probability of winning the game?