Probability and Statistics Experiments with Python

> by Thom Ives, Ph.D.

Find this on DagsHub too. On DagsHub, I am ThomIves, and this repo is "Probability_and_Statistics_with_Python".

What is the motivation for such an approach? The approach of coding math from scratch without libraries or modules? I like the way my dear friend and brother Manjunatha Gummaraju says it best.

> "Hand crafting (without libraries & automation) helps to get a firm grip on the subject, nuances & its applications. It also helps probably to author new innovative techniques from the ground up."

Calculating (Sample) Mean and Standard Deviation

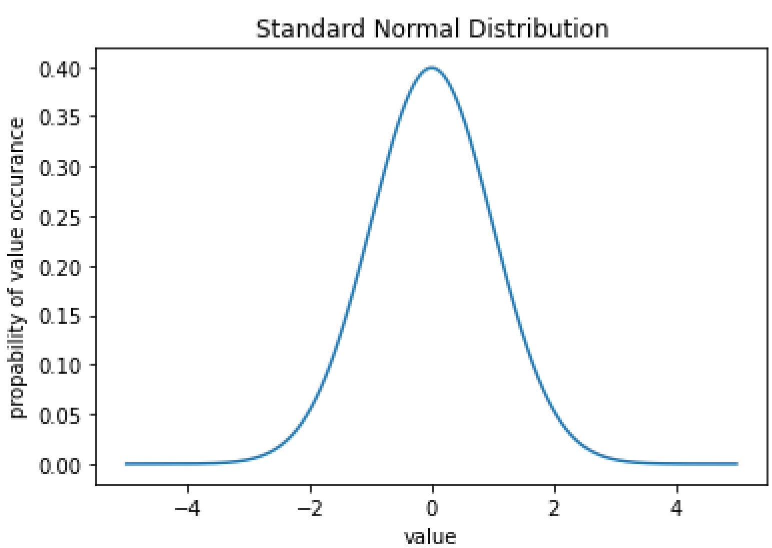
$$\mu = \frac{\sum x_i}{n} \quad \sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{n}}$$

```
def mean(X):
    mu = sum(X) / len(X)

    return mu


def standard_deviation(X, mu=None):
    if not mu:
        mu = mean(X)
    sigma = (sum([(x - mu)**2 for x in X])/len(X))**0.5

    return sigma
```

Normal Probability Distribution Function

$$p = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

```
import matplotlib.pyplot as plt


def PDF(x, mean=0, std_dev=1):
    # define e and pi explicitly
    e = 2.718281828
    pi = 3.1415927
    # calculate in two steps
    p = 1.0 / (std_dev * ((2 * pi) ** 0.5))
    p *= e ** (-0.5 * ((x - mean)/std_dev)**2)

    return p


X = [(x - 1000)/200 for x in list(range(2001))]
P = [PDF(x) for x in X]
plt.plot(X, P)
plt.title(label="Standard Normal Distribution")
plt.xlabel(xlabel="value")
plt.ylabel(ylabel="propability of value occurance")
plt.show()
```

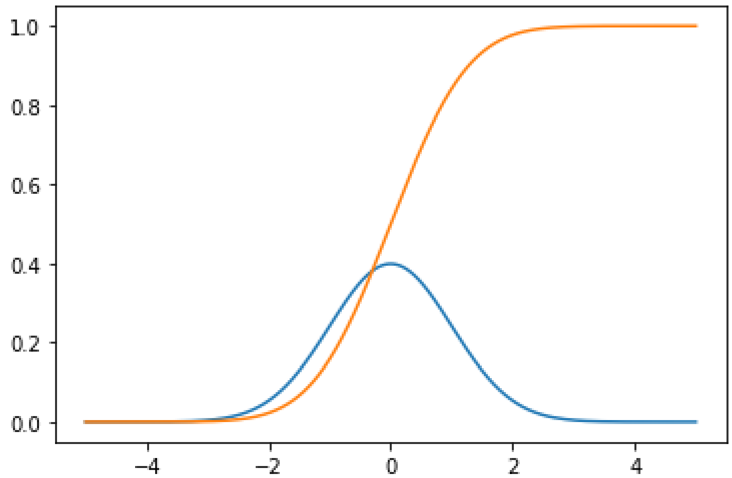


Cummulative Normal Distribution Function

$$cdf = \int_{x_{left}}^{x_{right}} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

```python
import matplotlib.pyplot as plt


def PDF(x, mean=0, std_dev=1):
    # define e and pi explicitly
    e = 2.718281828
    pi = 3.1415927
    # calculate in two steps
    p = 1.0 / (std_dev * ((2 * pi) ** 0.5))
    p *= e ** (-0.5 * ((x - mean)/std_dev)**2)

    return p


def CDF(mean=0, std_dev=1, x_left=-5, x_right=5, width=0.0001):
    CDF = 0
    X = []  # for plotting only
    CDF_y = []  # for plotting only

    x = x_left + width / 2
    while x < x_right:
        X.append(x)  # for plotting only
        panel = PDF(x, mean, std_dev) * width  # panel under PDF
        CDF += panel  # running sum of panels = integration
        CDF_y.append(CDF)  # for plotting only
        x += width  # current x value

    return CDF, X, CDF_y


total_integral, X, CDF_y = CDF()
P = [PDF(x) for x in X]
total_integral = round(total_integral, 5)
msg = f'Total integral of PDF = {total_integral}'
print(msg)
plt.plot(X, P)
plt.plot(X, CDF_y)
plt.show()
```

```
Total integral of PDF = 1.0
```



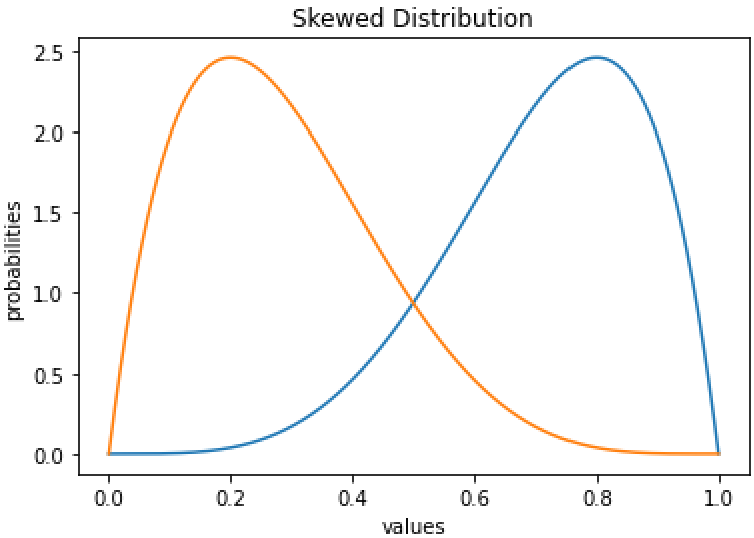
The Beta Distribution

$$f(x, \alpha, \beta) = \frac{1}{B(\alpha, \beta)}\, x^{\alpha-1}\, (1-x)^{\beta-1}$$

$$B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1} dt$$

```python
import matplotlib.pyplot as plt

class Beta_Distribution:
    def __init__(self, alpha, beta, panels=10000):
        self.alpha = alpha
        self.beta = beta
        self.panels = panels
        self.__Beta_Function__()

    def __Beta_Function__(self):
        width = 1 / self.panels
        X = [x/self.panels for x in range(self.panels)]
        # makes total integral of beta_PDF sum to 1
        self.B = sum(
            [(x**(self.alpha - 1) * \
             (1 - x)**(self.beta - 1)) * width
             for x in X])

    def beta_PDF(self, x):
        return x**(self.alpha - 1) * \
               (1 - x)**(self.beta - 1) / self.B


X = [x/1000 for x in range(1000+1)]
bd = Beta_Distribution(5, 2)
Y1 = [bd.beta_PDF(x) for x in X]
Y_integral = round(sum([y*0.001 for y in Y1]), 3)
bd = Beta_Distribution(2, 5)
Y2 = [bd.beta_PDF(x) for x in X]

print(f"The total integral of beta_PDF is {Y_integral}")
plt.plot(X, Y1)
plt.plot(X, Y2)
plt.title(label="Skewed Distribution")
plt.xlabel(xlabel="values")
plt.ylabel(ylabel="probabilities")
plt.show()
```

```
The total integral of beta_PDF is 1.0
```



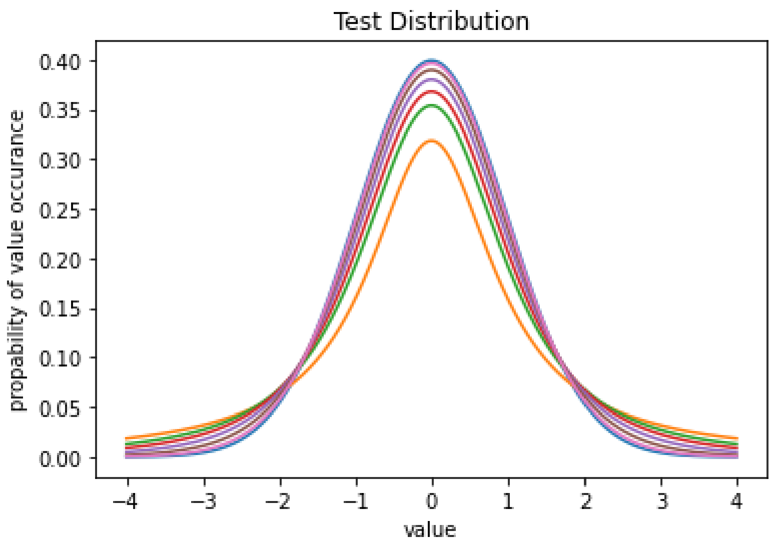
Student's T-Distribution

$$B(\alpha, \beta) = \int_0^1 t^{\alpha-1}(1-t)^{\beta-1}$$

$$PDF_t(t) = \frac{1}{\sqrt{\nu}B(\frac{1}{2}, \frac{\nu}{2})}\left(1 + \frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}}$$

```python
class T_Distribution:
    def __init__(self, dof=9):
        self.beta = self.beta_function(0.5, dof/2)

        self.front = 1 / (dof ** 0.5 * self.beta)
        self.dof = dof
        self.power = -(dof + 1)/2

    def beta_function(self, x, y):
      pw = 1 / 1000000
      beta = 0
      t = pw / 2
      while t < 1.0:
        beta += t ** (x - 1) * (1 - t) ** (y - 1) * pw
        t += pw

      return beta

    def PDFt(self, t):
        # The t probability distribution method
        f_of_t = self.front * (1 + t**2/self.dof) ** self.power

        return f_of_t

    def CDFt(self, t_left, t_right):
        # The t cummulative distribution method
        # We simply numerically integrate under the PDFt curve
        panels = self.dof * 100
        width = (t_right - t_left) / panels
        cdf = 0
        t = t_left
        prob = self.PDFt(t)
        # print(panels, width, prob)
        for i in range(panels):
            t += i * width
            prob = self.PDFt(t)
            cdf += prob * width

        return cdf
```

```python
import matplotlib.pyplot as plt


def PDF(x, mean=0, std_dev=1):
    # define e and pi explicitly
    e = 2.718281828
    pi = 3.1415927
    # calculate in two steps
    p = 1.0 / (std_dev * ((2 * pi) ** 0.5))
    p *= e ** (-0.5 * ((x - mean)/std_dev)**2)

    return p


X = [(x - 1000)/250 for x in list(range(2001))]
P = [PDF(x) for x in X]
plt.plot(X, P)
for dof in [1, 2, 3, 5, 10, 30]:
    t_dist = T_Distribution(dof=dof)
    TP = [t_dist.PDFt(x) for x in X]
    plt.plot(X, TP)

plt.title(label="Test Distribution")
plt.xlabel(xlabel="value")
plt.ylabel(ylabel="propability of value occurance")
plt.show()
```



Basic Determination Of Significance Value

A Khan Academy Problem

```
X = [80]*5 + [82.5]*24 + [85]*72 + [87.5]*181 + [90]*281 + \
    [92.5]*272 + [95]*136 + [97.5]*27 + [100]*2

mu = mean(X)
std = standard_deviation(X, mu=mu)
print(mu, std)
```

```
90.54 3.362796455332963
```

```
the_85_and_less = [x for x in X if x <= 85]
percentage_LE_85 = len(the_85_and_less)/len(X)
print(percentage_LE_85)
```

```
0.101
```

Basic Centerpoint Integration

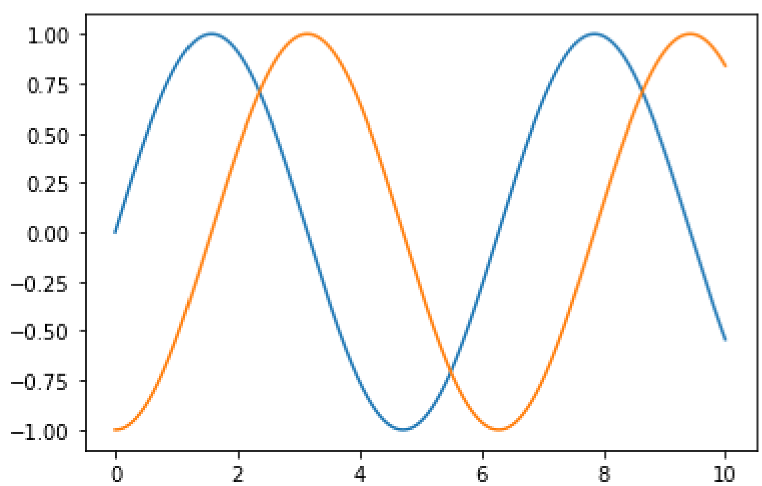
We start t at $\frac{w}{2}$ to use centerpoints for each panel.

There are other methods of numerical integration.

Centerpoint is pretty good at balancing areas above

and below the function being integrated.

```python
import matplotlib.pyplot as plt
import math

w = 1/1000
f_of_t = math.sin

T = [w/2]
S = [0]
C = [-1]
for t in range(10000):
    T.append(T[-1] + w)  # Our time step
    S.append(f_of_t(t*w))  # Our Function
    C.append(f_of_t(t*w)*w + C[-1])  # Integrating

plt.plot(T, S)
plt.plot(T, C)
plt.show()
```



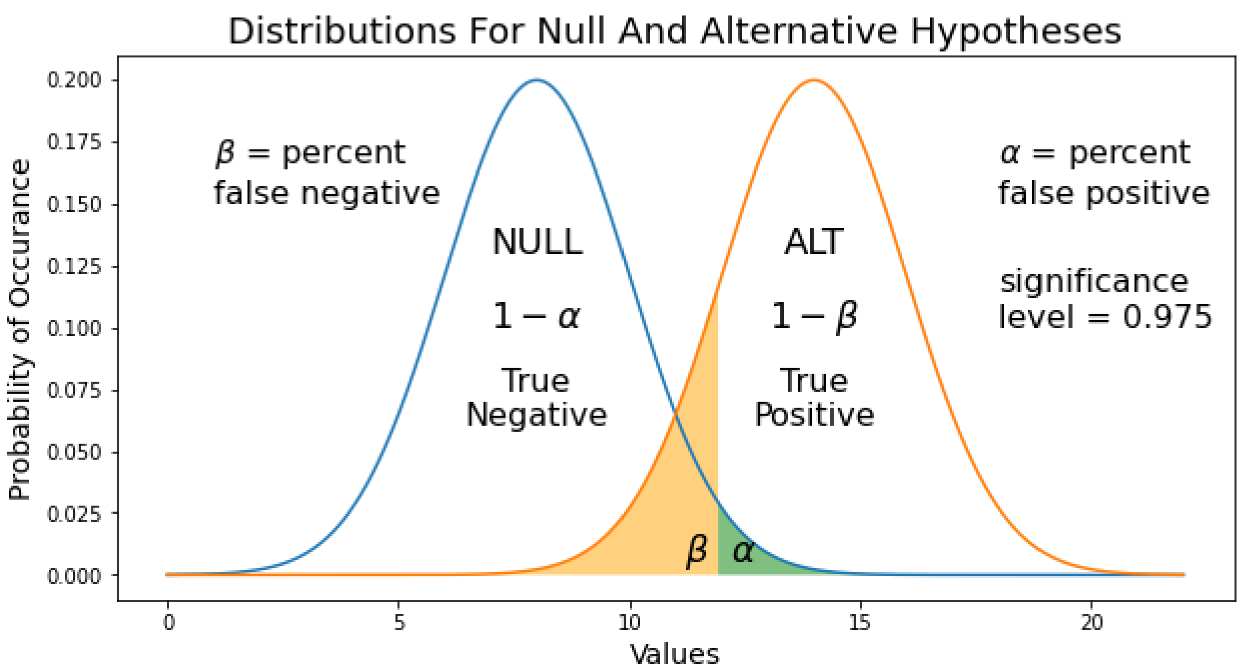
Null And Alternate Hypotheses Distributions With Dynamic Significance Level

For the LaTeX in MatPlotLib Inside Colab, See:
https://stackoverflow.com/a/62075348/996205

```python
import matplotlib
from matplotlib import rc
import matplotlib.pyplot as plt
%matplotlib inline

rc('text', usetex=True)
matplotlib.rcParams['text.latex.preamble'] = [r'\usepackage{amsmath}']
!apt install texlive-fonts-recommended texlive-fonts-extra cm-super dvipng
```

```python
import matplotlib.pyplot as plt
import math
import time

def PDF(x, mean=0, std_dev=1):
    p = 1.0 / (std_dev * ((2 * math.pi) ** 0.5))
    p *= math.e ** (-0.5 * ((x - mean)/std_dev)**2)
    return p

pw = 1/1000
X = [(x + 0.5)*pw for x in range(22000)]
P1 = [PDF(x, 8, 2) for x in X]
P2 = [PDF(x, 14, 2) for x in X]

C1 = []  # C2 = []
sum1 = 0  # sum2 = 0
for i in range(len(X)):
    sum1 += P1[i]*pw  # sum2 += P2[i]*pw
    C1.append(sum1)  # C2.append(sum2)

SigLevels = [(v/2 + 90)/100 for v in range(17)]
SigLevels = [0.975]

for sl in SigLevels:
    for i in range(len(X)):
        if C1[i] > sl:
            sig_i = i
            break

    plt.figure(figsize = (10,5))
    plt.plot(X, P1)
    plt.plot(X, P2)

    plt.title(
        label="Distributions For Null And Alternative Hypotheses", fontsize=18)
    plt.xlabel(xlabel="Values", fontsize=14)
    plt.ylabel(ylabel="Probability of Occurance", fontsize=14)

    plt.fill_between(X[sig_i:], 0, P1[sig_i:], facecolor='green', alpha=0.5)
    plt.fill_between(X[:sig_i], 0, P2[:sig_i], facecolor='orange', alpha=0.5)

    plt.text(8, 0.13, 'NULL', fontsize=18, ha='center')
    plt.text(14, 0.13, 'ALT', fontsize=18, ha='center')

    plt.text(8, 0.06, 'True\nNegative', fontsize=16, ha='center')
    plt.text(14, 0.06, 'True\nPositive', fontsize=16, ha='center')

    plt.text(1, 0.15, r'$\beta$ = percent'+'\nfalse negative',
            fontsize=16, ha='left')
    plt.text(18, 0.15, r'$\alpha$ = percent'+'\nfalse positive',
```

```
            fontsize=16, ha='left')
    this_text = f'significance\nlevel = {round(sl, 3)}'
    plt.text(18, 0.10, this_text,
            fontsize=16, ha='left')

    plt.text(8, 0.1, r'$1 - \alpha$', fontsize=18, ha='center')
    plt.text(14, 0.1, r'$1 - \beta$', fontsize=18, ha='center')

    plt.text(X[sig_i] + 0.25, 0.005, r'$\alpha$', fontsize=18)
    plt.text(X[sig_i] - 0.75, 0.005, r'$\beta$', fontsize=18)

    plt.savefig(f'hypo_{round(sl, 3)}.png')
    plt.show()
    time.sleep(1)
    plt.figure().clear()
```




```
<Figure size 432x288 with 0 Axes>
```

Null And Alternate Hypotheses Distributions With Dynamic Alternative Mean

```python
import matplotlib.pyplot as plt
import math
import time

def PDF(x, mean_=0, std_dev_=1):
    p = 1.0 / (std_dev_ * ((2 * math.pi) ** 0.5))
    p *= math.e ** (-0.5 * ((x - mean_)/std_dev_)**2)
    return p

pw = 1/1000
X = [(x + 0.5)*pw for x in range(22000)]
P1 = [PDF(x, 8, 2) for x in X]

C1 = []  # C2 = []
sum1 = 0
for i in range(len(X)):
    sum1 += P1[i]*pw
    C1.append(sum1)

sig_level = 0.975
for i in range(len(X)):
    if C1[i] > sig_level:
        sig_i = i
        break

for i in range(19):
    mu_alt = 12.4 + i * 0.2
    mu_alt = round(mu_alt, 1)

    P2 = [PDF(x, mu_alt, 2) for x in X]
    plt.figure(figsize = (10,5))
    plt.plot(X, P1)
    plt.plot(X, P2)

    plt.title(
        label="Distributions For Null And Alternative Hypotheses", fontsize=18)
    plt.xlabel(xlabel="Values", fontsize=14)
    plt.ylabel(ylabel="Probability of Occurance", fontsize=14)

    plt.fill_between(X[sig_i:], 0, P1[sig_i:], facecolor='green', alpha=0.5)
    plt.fill_between(X[:sig_i], 0, P2[:sig_i], facecolor='orange', alpha=0.5)

    plt.text(8, 0.13, 'NULL', fontsize=16, ha='center')
    plt.text(mu_alt, 0.13, 'ALT', fontsize=16, ha='center')

    plt.text(8, 0.06, 'True\nNegative', fontsize=14, ha='center')
    plt.text(mu_alt, 0.06, 'True\nPositive', fontsize=14, ha='center')

    plt.text(1, 0.15, r'$\beta$ = percent'+'\nfalse negative',
             fontsize=14, ha='left')
```

```
    plt.text(20, 0.15, r'$\alpha$ = percent'+'\nfalse positive',
             fontsize=14, ha='left')
    this_text = f'significance\nlevel = {round(sig_level, 3)}'
    plt.text(20, 0.10, this_text,
             fontsize=14, ha='left')
    plt.text(20, 0.06, r'$\mu_{alt}$' + f' = {mu_alt}',
             fontsize=16, ha='left')

    plt.text(8, 0.1, r'$1 - \alpha$', fontsize=16, ha='center')
    plt.text(mu_alt, 0.1, r'$1 - \beta$', fontsize=16, ha='center')


    plt.text(X[sig_i] + 0.25, 0.005, r'$\alpha$', fontsize=18)
    plt.text(X[sig_i] - 0.75, 0.005, r'$\beta$', fontsize=18)

    plt.xlim([0, 26])
    plt.savefig(f'hypos_{round(mu_alt, 1)}.png')
    plt.show()
    time.sleep(1)
    plt.figure().clear()
```

ROC Curve From Dynamic Significance Level

ROC = True Positive Rate vs. False Positive Rate

```python
import matplotlib.pyplot as plt
import math
import time

def PDF(x, mean=0, std_dev=1):
    p = 1.0 / (std_dev * ((2 * math.pi) ** 0.5))
    p *= math.e ** (-0.5 * ((x - mean)/std_dev)**2)
    return p

pw = 1/1000
X = [(x + 0.5)*pw for x in range(22000)]
P1 = [PDF(x, 8, 2) for x in X]
P2 = [PDF(x, 11, 2) for x in X]

C1 = []
C2 = []
sum1 = 0
sum2 = 0
for i in range(len(X)):
    sum1 += P1[i]*pw
    sum2 += P2[i]*pw
    C1.append(sum1)
    C2.append(sum2)

ROC_X = []
ROC_Y = []
pts = 41
Sig_Levels = [(v * 100/(pts-1))/100 for v in range(0, pts)]

for sig_lev in Sig_Levels:
    if sig_lev == 1:
        sig_lev = 0.999
    for i in range(len(X)):
        if C1[i] > sig_lev:
            sig_i = i
            break

    if sig_lev == 0.999:
        sig_lev = 1

    TP_Rate = 1 - C2[sig_i]
    FP_Rate = 1 - C1[sig_i]

    ROC_X.append(FP_Rate)
    ROC_Y.append(TP_Rate)

    fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(15, 5))
    ax1.plot(X, P1)
    ax1.plot(X, P2)
```

```python
msg = "Receiver Operator Curve (ROC) From Sweeping Significance Level"
fig.suptitle(msg, fontsize=18)

msg = "Significance Level On Null & Alt Distributions"
ax1.set_title(label=msg)
ax1.set_xlabel(xlabel="Values", fontsize=14)
ax1.set_ylabel(ylabel="Probability of Occurance", fontsize=14)

ax1.fill_between(X[sig_i:], 0, P1[sig_i:], facecolor='green', alpha=0.5)
ax1.fill_between(X[:sig_i], 0, P2[:sig_i], facecolor='orange', alpha=0.5)

ax1.text(1, 0.15, r'$\beta$ = percent'+'\nfalse neg',
        fontsize=16, ha='left')
ax1.text(14, 0.15, r'$\alpha$ = percent'+'\nfalse pos',
        fontsize=16, ha='left')
this_text = f'significance\nlevel = {round(sig_lev, 3)}'
ax1.text(14, 0.10, this_text,
        fontsize=16, ha='left')

ax1.text(X[sig_i] + 0.25, 0.005, r'$\alpha$', fontsize=18)
ax1.text(X[sig_i] - 0.75, 0.005, r'$\beta$', fontsize=18)

ax1.set_xlim([0, 21])

ax2.plot([0, 1], [0, 1])
ax2.plot(ROC_X, ROC_Y)
ax2.set_title(label="Receiver Operator Curve (ROC)")
ax2.set_xlabel(xlabel="False Positive Rate", fontsize=14)
ax2.set_ylabel(ylabel="True Positive Rate", fontsize=14)
ax2.set_xlim(0, 1)
ax2.set_ylim(0, 1)
plt.show()

fig.savefig(f'hypo_{round(sig_lev, 3)}.png')
time.sleep(0.2)
```

ROC Curve Changes Due To Separation Of NULL And ALT Distributions' Means

ROC = True Positive Rate vs. False Positive Rate

```python
import matplotlib.pyplot as plt
import math
import time

def PDF(x, mean=0, std_dev=1):
    p = 1.0 / (std_dev * ((2 * math.pi) ** 0.5))
    p *= math.e ** (-0.5 * ((x - mean)/std_dev)**2)
    return p

pw = 1/1000
X = [(x + 0.5)*pw for x in range(22000)]
P1 = [PDF(x, 8, 2) for x in X]

C1 = []
sum1 = 0
for i in range(len(X)):
    sum1 += P1[i]*pw
    C1.append(sum1)

pts = 101
Sig_Levels = [(v * 100/(pts-1))/100 for v in range(0, pts)]

for i in range(71):
    mu_alt = 8 + 0.1 * i
    mu_alt = round(mu_alt, 1)

    P2 = [PDF(x, mu_alt, 2) for x in X]
    C2 = []
    sum2 = 0
    for i in range(len(X)):
        sum2 += P2[i]*pw
        C2.append(sum2)

    ROC_X = []
    ROC_Y = []
    for sig_lev in Sig_Levels:
        if sig_lev == 1:
            sig_lev = 0.999
        for i in range(len(X)):
            if C1[i] > sig_lev:
                sig_i = i
                break

        if sig_lev == 0.999:
            sig_lev = 1

        TP_Rate = 1 - C2[sig_i]
        FP_Rate = 1 - C1[sig_i]

        ROC_X.append(FP_Rate)
```

```
        ROC_Y.append(TP_Rate)

    fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(15, 5))
    ax1.plot(X, P1)
    ax1.plot(X, P2)

    msg = "Receiver Operator Curve (ROC) From Sweeping Significance Level"
    fig.suptitle(msg, fontsize=18)

    msg = "Significance Level On Null & Alt Distributions"
    ax1.set_title(label=msg)
    ax1.set_xlabel(xlabel="Values", fontsize=14)
    ax1.set_ylabel(ylabel="Probability of Occurance", fontsize=14)

    ax1.text(8, 0.09, 'NULL\nHypothesis',
            fontsize=14, ha='center')
    ax1.text(mu_alt, 0.04, 'ALT\nHypothesis',
            fontsize=14, ha='center')

    ax1.set_xlim([0, 21])

    ax2.plot([0, 1], [0, 1])
    ax2.plot(ROC_X, ROC_Y)
    ax2.set_title(label="Receiver Operator Curve (ROC)")
    ax2.set_xlabel(xlabel="False Positive Rate", fontsize=14)
    ax2.set_ylabel(ylabel="True Positive Rate", fontsize=14)
    ax2.set_xlim(0, 1)
    ax2.set_ylim(0, 1)
    plt.show()

    fig.savefig(f'hypo_{round(mu_alt, 3)}.png')
    # time.sleep(0.05)
```

ROC Curve Changes Due To Changes In NULL And ALT Distributions' Standard Deviations

ROC = True Positive Rate vs. False Positive Rate

```python
import matplotlib.pyplot as plt
import math
import time

def PDF(x, mean=0, std_dev=1):
    p = 1.0 / (std_dev * ((2 * math.pi) ** 0.5))
    p *= math.e ** (-0.5 * ((x - mean)/std_dev)**2)
    return p

pw = 1/1000
X = [(x + 0.5)*pw for x in range(22000)]

pts = 101
Sig_Levels = [(v * 100/(pts-1))/100 for v in range(0, pts)]

for i in range(26):
    std_alt = 3.5 - 0.1 * i
    std_alt = round(std_alt, 1)

    P1 = [PDF(x, 8, std_alt) for x in X]
    C1 = []
    sum1 = 0
    for i in range(len(X)):
        sum1 += P1[i]*pw
        C1.append(sum1)

    P2 = [PDF(x, 12, std_alt) for x in X]
    C2 = []
    sum2 = 0
    for i in range(len(X)):
        sum2 += P2[i]*pw
        C2.append(sum2)

    ROC_X = []
    ROC_Y = []
    for sig_lev in Sig_Levels:
        if sig_lev == 1:
            sig_lev = 0.999
        for i in range(len(X)):
            if C1[i] > sig_lev:
                sig_i = i
                break

        if sig_lev == 0.999:
            sig_lev = 1

        TP_Rate = 1 - C2[sig_i]
        FP_Rate = 1 - C1[sig_i]

        ROC_X.append(FP_Rate)
```

```
        ROC_Y.append(TP_Rate)

    fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(15, 5))
    ax1.plot(X, P1)
    ax1.plot(X, P2)

    msg = "Receiver Operator Curve (ROC) From Sweeping Significance Level"
    fig.suptitle(msg, fontsize=18)

    msg = "Significance Level On Null & Alt Distributions"
    ax1.set_title(label=msg)
    ax1.set_xlabel(xlabel="Values", fontsize=14)
    ax1.set_ylabel(ylabel="Probability of Occurance", fontsize=14)

    ax1.text(8, 0.09, 'NULL\nHypothesis',
            fontsize=14, ha='center')
    ax1.text(12, 0.04, 'ALT\nHypothesis',
            fontsize=14, ha='center')

    ax1.set_xlim([0, 21])

    ax2.plot([0, 1], [0, 1])
    ax2.plot(ROC_X, ROC_Y)
    ax2.set_title(label="Receiver Operator Curve (ROC)")
    ax2.set_xlabel(xlabel="False Positive Rate", fontsize=14)
    ax2.set_ylabel(ylabel="True Positive Rate", fontsize=14)
    ax2.set_xlim(0, 1)
    ax2.set_ylim(0, 1)
    plt.show()

    fig.savefig(f'hypo_{round(sig_lev, 3)}.png')
    # time.sleep(0.05)
```

F1 Score

$$F_1 = \frac {2}{\frac {1}{Recall} + \frac {1}{Precision}}$$

$F_1 = $ Harmonic Mean Of Recall and Precision

Accuracy, Recall, Precision

$$Accuracy = \frac{TP + TN}{TP + FP + TN + FN} = \frac {What\;We\;Got\; Right}{All\;Cases}$$

$$Recall = \frac{TP}{TP + FN} = \frac {What\;Positives\;We\;Got\;Right}{All\;Actual\;Positives}$$

$$Precision = \frac{TP}{TP + FP} = \frac {What\;Positives\;We\;Got\;Right}{All\;Positive\;Predictions}$$

LASSO In Logistic Regression to Compare with Statistics Version

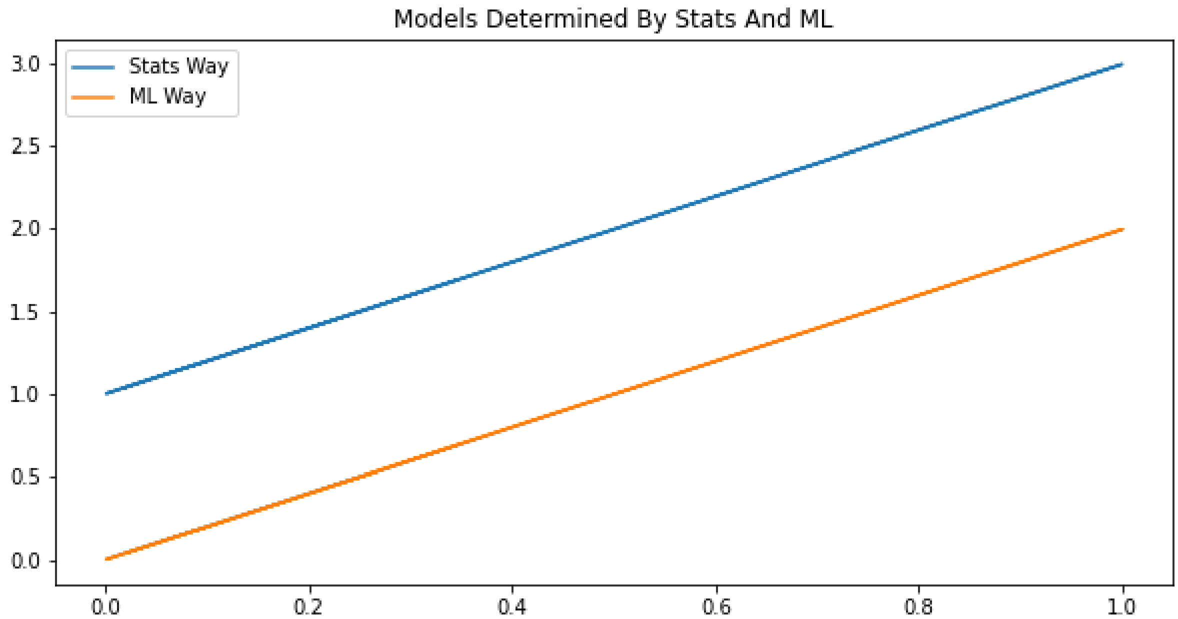
```
from sklearn.linear_model import LogisticRegression as LR
from sklearn.datasets import load_iris

X, y = X, y = load_iris(return_X_y=True)
lr_mod = LR(penalty='l1', solver='liblinear')
lr_mod.fit(X, y)
print(lr_mod.coef_)
```

```
[[ 0.          2.52235623 -2.83220134  0.        ]
 [ 0.32846823 -1.79370624  0.66582088 -1.57267348]
 [-2.62263278 -2.50833176  3.26131365  4.61826807]]
```

Regressor Coefficient From Statistics Vs. Machine Learning Methods

```python
import matplotlib.pyplot as plt
import numpy as np

# Synthesize some data (i.e. create fake data)
X = np.random.uniform(0, 1, 1000)
Y = 2.0 * X
Y_noise = np.max(Y) * 0.073
Y += np.random.normal(0, 0.073, 1000)

# Statistics Way To Create Model
X_std = np.std(X)
Y_std = np.std(Y)
r = np.corrcoef(X, Y)
Cs = Y_std / X_std * r[0, 1]
print(Cs)

# Machine Learning
mod_LR = LinearRegression(fit_intercept=False, copy_X=True)
mod_LR.fit(X.reshape(-1, 1), Y.reshape(-1, 1))
Cml = mod_LR.coef_[0, 0]
print(Cml)

# Visualize
plt.figure(figsize=(10, 5))
plt.plot(X, Cs*X+1) # + 1 separates the two exact plots
plt.plot(X, Cml*X)
plt.title('Models Determined By Stats And ML')
plt.legend(('Stats Way', 'ML Way'))
plt.show()
```

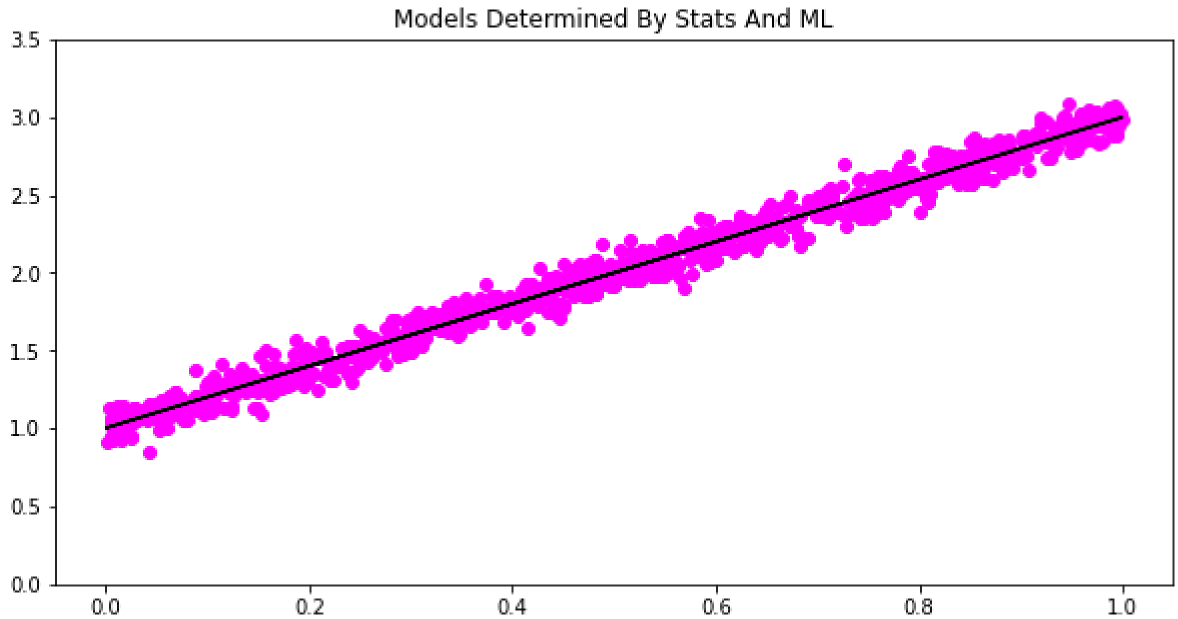
```
1.9918506427390517
1.9957689952415045
```



Regressor Coefficient AND "Intercept" FROM Statistics

```
import matplotlib.pyplot as plt
import numpy as np

# Synthesize some data (i.e. create fake data)
X = np.random.uniform(0, 1, 1000)
Y = 2.0 * X + 1
Y_noise = np.max(Y) * 0.073
Y += np.random.normal(0, 0.073, 1000)

# Statistics Way To Fit Model Coefficient
X_std = np.std(X)
Y_std = np.std(Y)
r = np.corrcoef(X, Y)
Cs = Y_std / X_std * r[0, 1]
print(Cs)

# Statistics Way To Calculate Intercept
X_mean = np.mean(X)
Y_mean = np.mean(Y)
b = Y_mean - Cs*X_mean
print(b)
```

```
1.9992493711567196
1.001206029449972
```

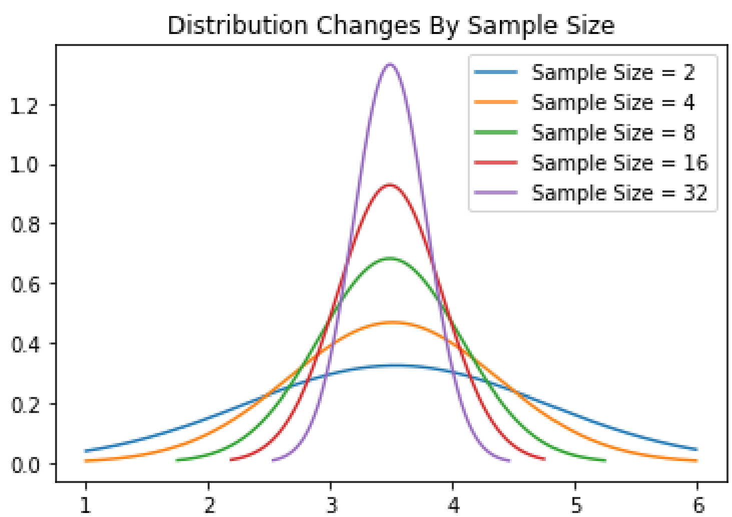



```
# Visualize
plt.figure(figsize=(10, 5))
plt.scatter(X, Y, color='magenta')
plt.plot(X, Cs*X+b, color='black')
plt.ylim((0, 3.5))
plt.title('Models Determined By Stats And ML')
plt.show()
```

Central Limit Theorem Principles

Sample Means Distribtion For Increasing Sample Sizes

```python
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm

die_values = [1, 2, 3, 4, 5, 6]
sample_sizes = [2, 4, 8, 16, 32]

for experiment in range(1):
    for sample_size in sample_sizes:
        sample_means = []
        for num_samples in range(1000):
            die_cast = np.random.choice(
            die_values, size=sample_size)
            sample_mean = np.mean(die_cast)
            sample_means.append(sample_mean)

        experiment_mean = np.mean(sample_means)
        experiment_std = np.std(sample_means)
        x_min = min(sample_means)
        x_max = max(sample_means)
        x = np.arange(x_min, x_max, 0.01)
        y = norm.pdf(x, experiment_mean, experiment_std)
        plt.plot(x, y)

    legend_texts = [f'Sample Size = {v}' for v in sample_sizes]
    plt.legend(legend_texts)
    plt.title("Distribution Changes By Sample Size")
    plt.show()
```



Approaching The Central Limit Theorem

```python
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm
import os

cwd = os.getcwd()
if not os.path.isdir(f"{cwd}/images"):
    os.mkdir(f"{cwd}/images")

no_images = True
image_num = 0
if no_images:
    die_values = [1, 2, 3, 4, 5, 6]
    sample_sizes = [2, 4, 8, 16, 32]
    num_add_samples_list = [2] + [1]*8 + [2]*5 + [10]*8 + [100]*9
    sample_means_D = {k: [] for k in sample_sizes}
    total_samples = 0

    for num_samples in num_add_samples_list:
        total_samples += num_samples
        plt.figure(figsize=(12, 6))
        for sample_size in sample_sizes:
            for sample_num in range(num_samples):
                die_cast = np.random.choice(
                die_values, size=sample_size)
                sample_mean = np.mean(die_cast)
                sample_means_D[sample_size].append(sample_mean)

            experiment_mean = np.mean(sample_means_D[sample_size])
            experiment_std = np.std(sample_means_D[sample_size])
            x_min = min(sample_means_D[sample_size])
            x_max = max(sample_means_D[sample_size])
            x = np.arange(x_min, x_max, 0.001)
            y = norm.pdf(x, experiment_mean, experiment_std)
            plt.plot(x, y)

        legend_texts = [f'Sample Size = {v}' for v in sample_sizes]
        plt.legend(legend_texts)
        title = f"Distribution Of Means For {total_samples} "
        title += "Samples For Various Sample Sizes"
        plt.title(title)
        plt.xlim([1, 6])
        if total_samples == 2:
            for i in range(5):
                plt.savefig(f"{cwd}/images/{image_num:02d}.png")
                image_num += 1
        elif total_samples == 1000:
            for i in range(5):
                plt.savefig(f"{cwd}/images/{image_num:02d}.png")
                image_num += 1
```

```
        else:
            plt.savefig(f"{cwd}/images/{image_num:02d}.png")
            image_num += 1

        plt.close()

# Run below on command line to create movie - needs ffmpeg
# ffmpeg -framerate 4 -pattern_type glob -i "*.png" output.avi
```

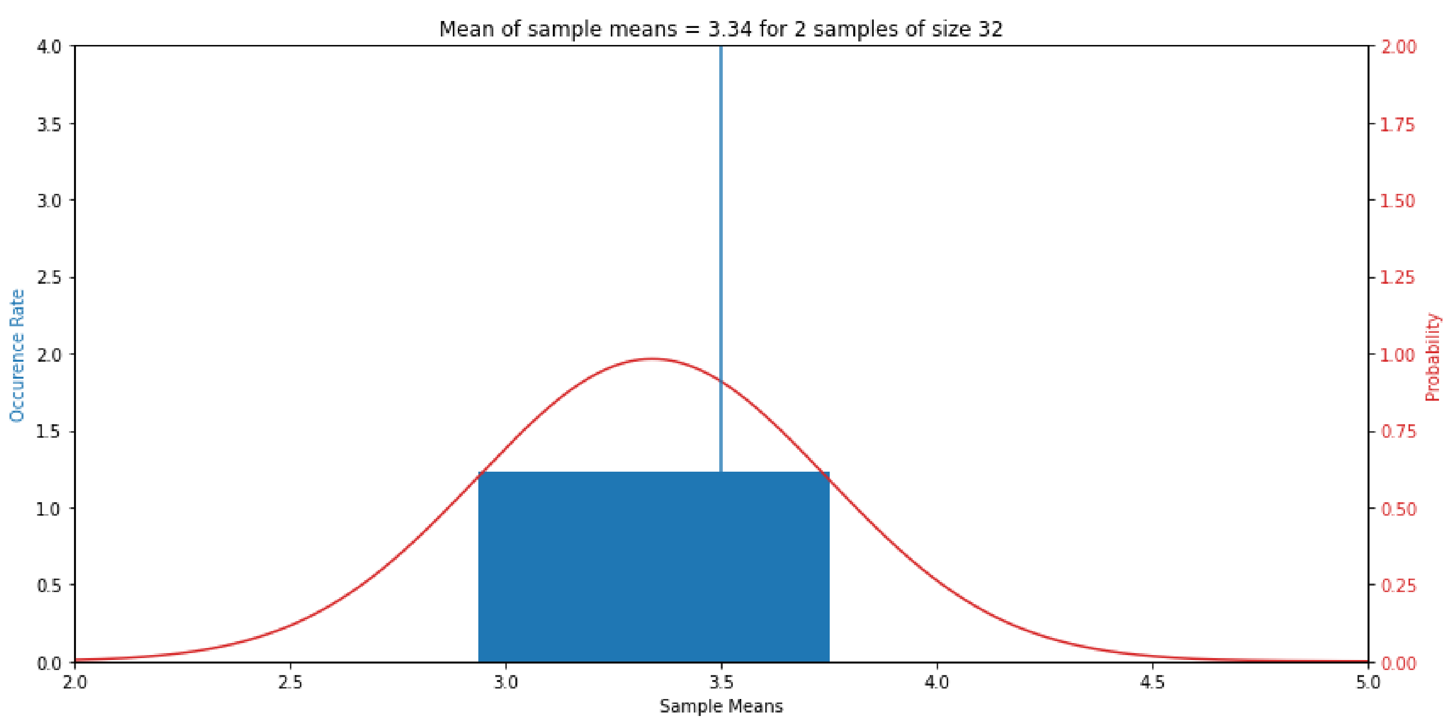
Sample From Huge Population To See When Central Limit Theorem Is Reached

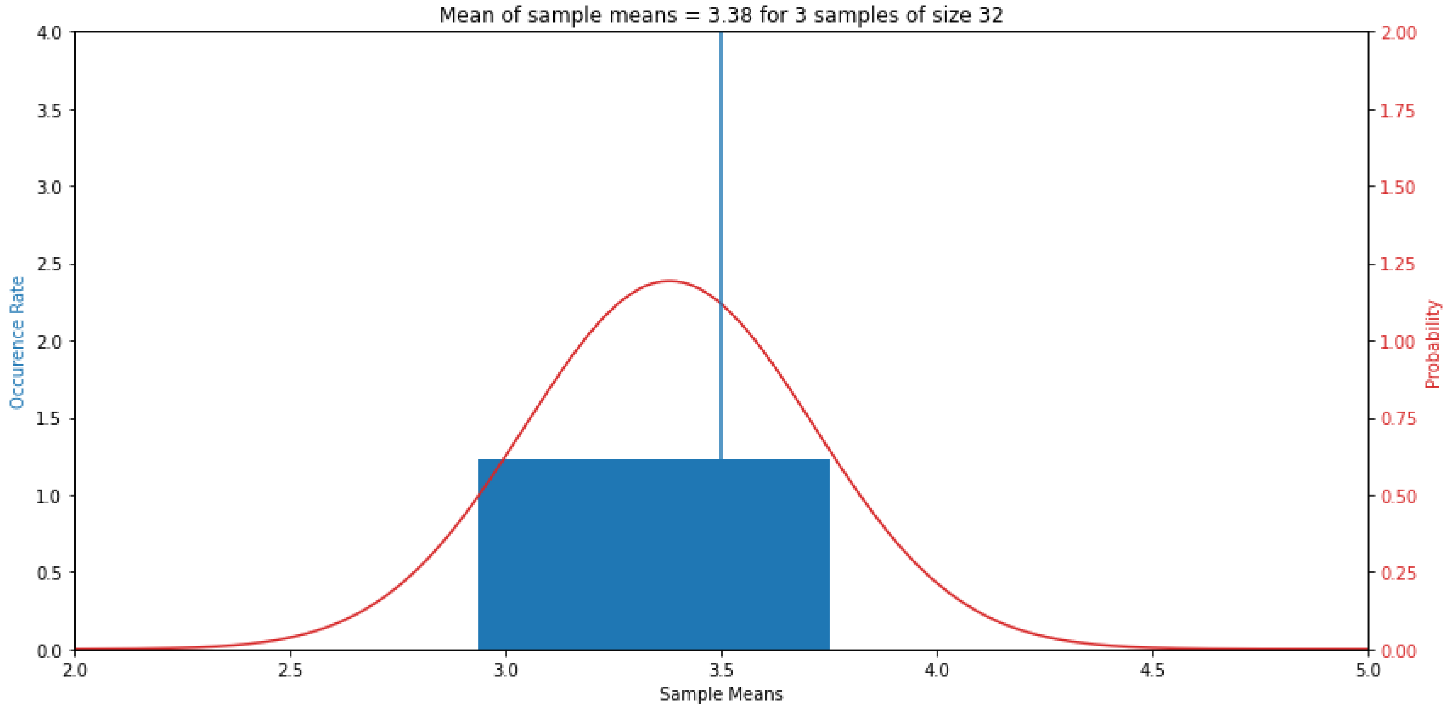
```
import numpy as np
import matplotlib.pyplot as plt

die_values = [1, 2, 3, 4, 5, 6]
die_roles = [np.random.choice(die_values, size=1)[0] for _ in range(int(1e6))]

# plt.hist(die_roles, bins=6, width=0.73)
# plt.show();

mean = round(np.mean(die_roles), 1)
print(f'Population mean is {mean}')

for sample_size in [2, 4, 8, 16, 32, 64]:
    sample_means = []
    samples = 0
    while True:
        samples += 1
        roles = np.random.choice(die_roles, size=sample_size)
        sample_mean = np.mean(roles)
        sample_means.append(sample_mean)

        running_mean = round(np.mean(sample_means), 2)
        if running_mean == 3.50:
            break

    title = f'Mean sample means = {running_mean} for {samples} samples of {sample_size}'
    running_std = np.std(sample_means)
    x = np.arange(1, 6, 0.001)
    y = norm.pdf(x, running_mean, running_std)
    plt.xlim([1, 6])
    plt.plot(x, y)
    plt.title(title)
    plt.axvline(3.5)
    plt.show();
```

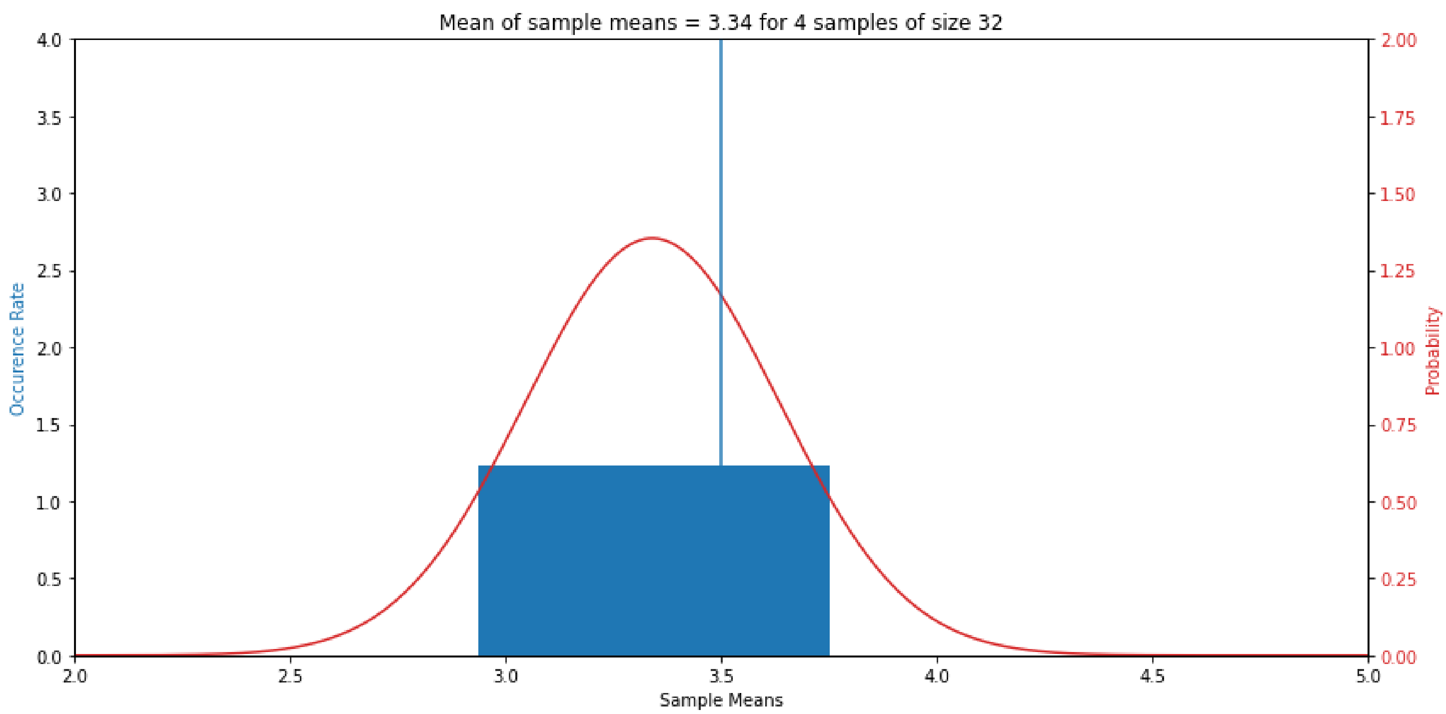
The Ugly Approach To The Central Limit Theorem

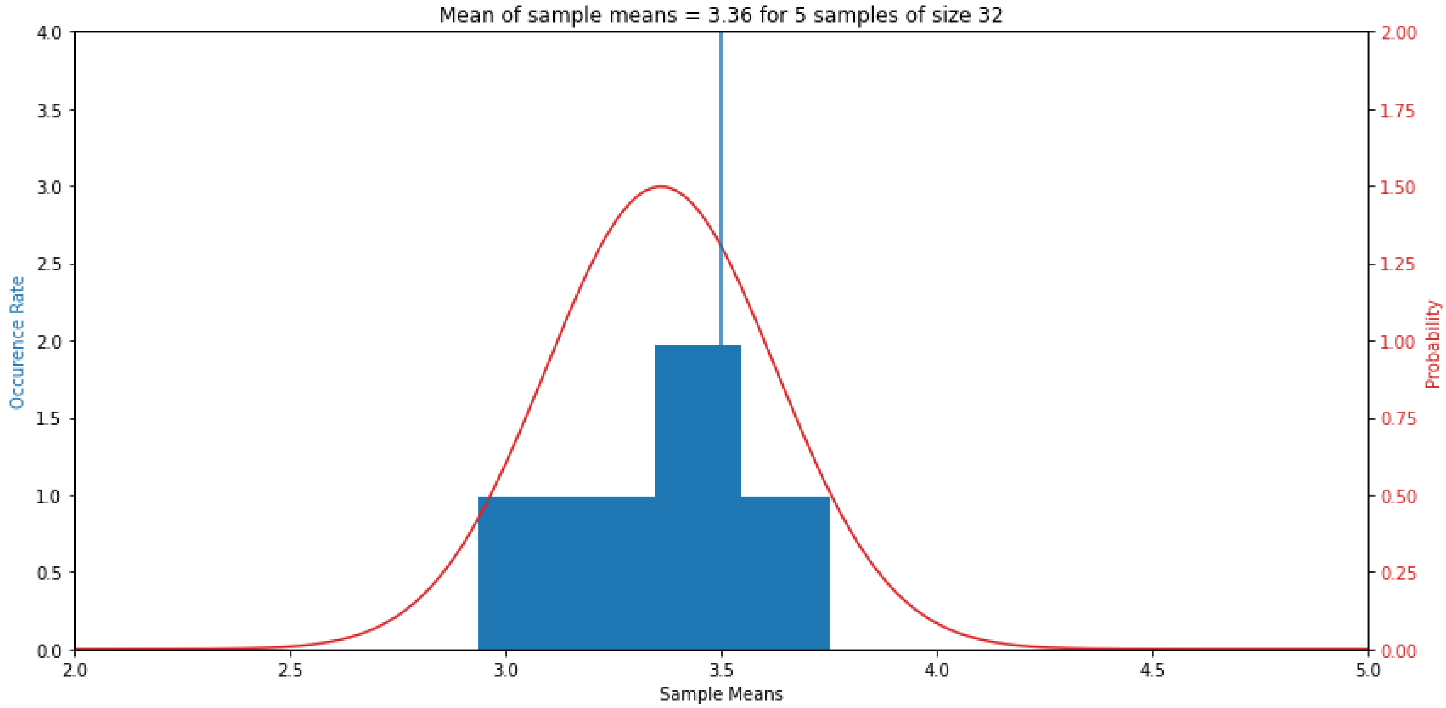
```python
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm
import os

cwd = os.getcwd()
if not os.path.isdir(f"{cwd}/images"):
    os.mkdir(f"{cwd}/images")

no_images = True
image_num = 0
if no_images:
    die_values = [1, 2, 3, 4, 5, 6]
    num_add_samples_list = [2] + [1]*8 + [2]*5 + [10]*8 + [100]*9
    num_add_samples_list += [9000] + [90000] + [900000] + [1000000]
    sample_means = []
    total_samples = 0

    for num_samples in num_add_samples_list:
        total_samples += num_samples
        for sample_num in range(num_samples):
            die_casts = np.random.choice(
                die_values, size=32)
            sample_mean = np.mean(die_casts)
            sample_means.append(sample_mean)

        fig, ax1 = plt.subplots()
        fig.set_figheight(6)
        fig.set_figwidth(12)

        color = 'tab:blue'
        ax1.set_xlabel('Sample Means')
        ax1.set_ylabel('Occurence Rate', color=color)
        bins = len(set(sample_means))
        ax1.hist(sample_means, bins=bins, density=True, stacked=True)
        ax1.set_xlim([2, 5])
        ax1.set_ylim([0, 4])

        ax2 = ax1.twinx()

        color = 'tab:red'
        ax2.set_ylabel('Probability', color=color)
        ax2.tick_params(axis='y', labelcolor=color)
        running_mean = round(np.mean(sample_means), 2)
        title = f'Mean of sample means = {running_mean} '
        title += f'for {total_samples} samples of size 32'
        running_std = np.std(sample_means)
        x = np.arange(2, 5, 0.001)
        y = norm.pdf(x, running_mean, running_std)
        ax2.plot(x, y, color=color)
```

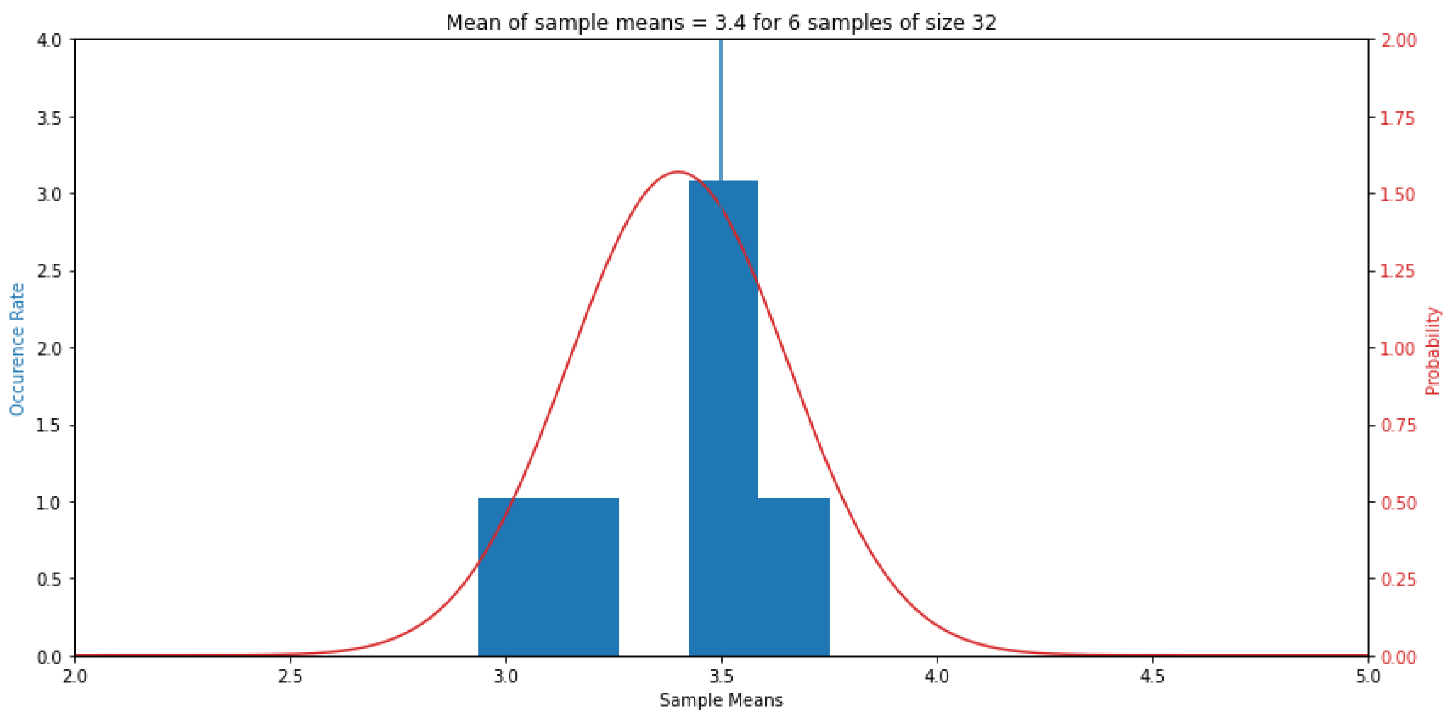
```
        ax2.set_ylim([0, 2])

        plt.title(title)
        plt.axvline(3.5)

        fig.tight_layout()

        # if total_samples == 2:
        #     for i in range(5):
        #         plt.savefig(f"{cwd}/images/{image_num:02d}.png")
        #         image_num += 1
        # elif total_samples > 1000000:
        #     for i in range(5):
        #         plt.savefig(f"{cwd}/images/{image_num:02d}.png")
        #         image_num += 1
        # else:
        #     plt.savefig(f"{cwd}/images/{image_num:02d}.png")
        #     image_num += 1

        plt.show();

        # plt.close()

# Run below on command line to create movie - needs ffmpeg
# ffmpeg -framerate 4 -pattern_type glob -i "*.png" output.avi
```

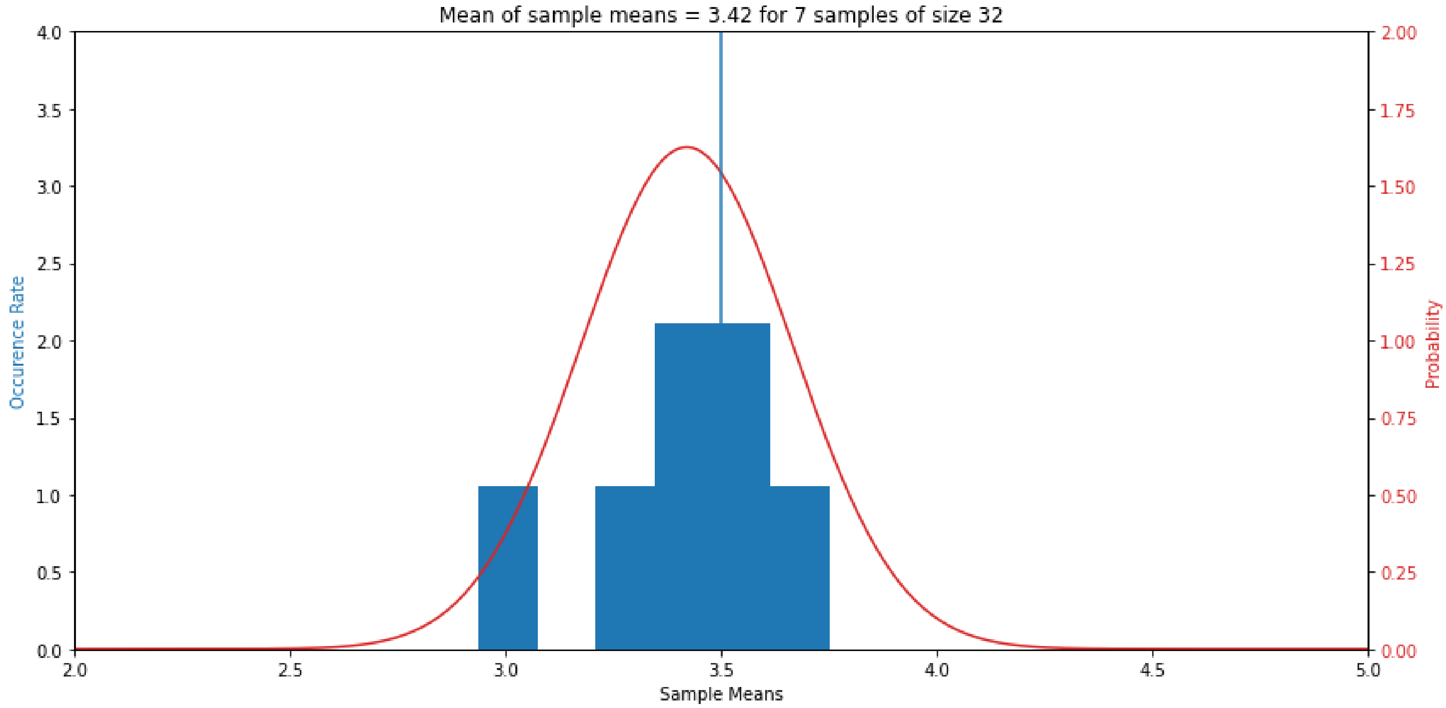


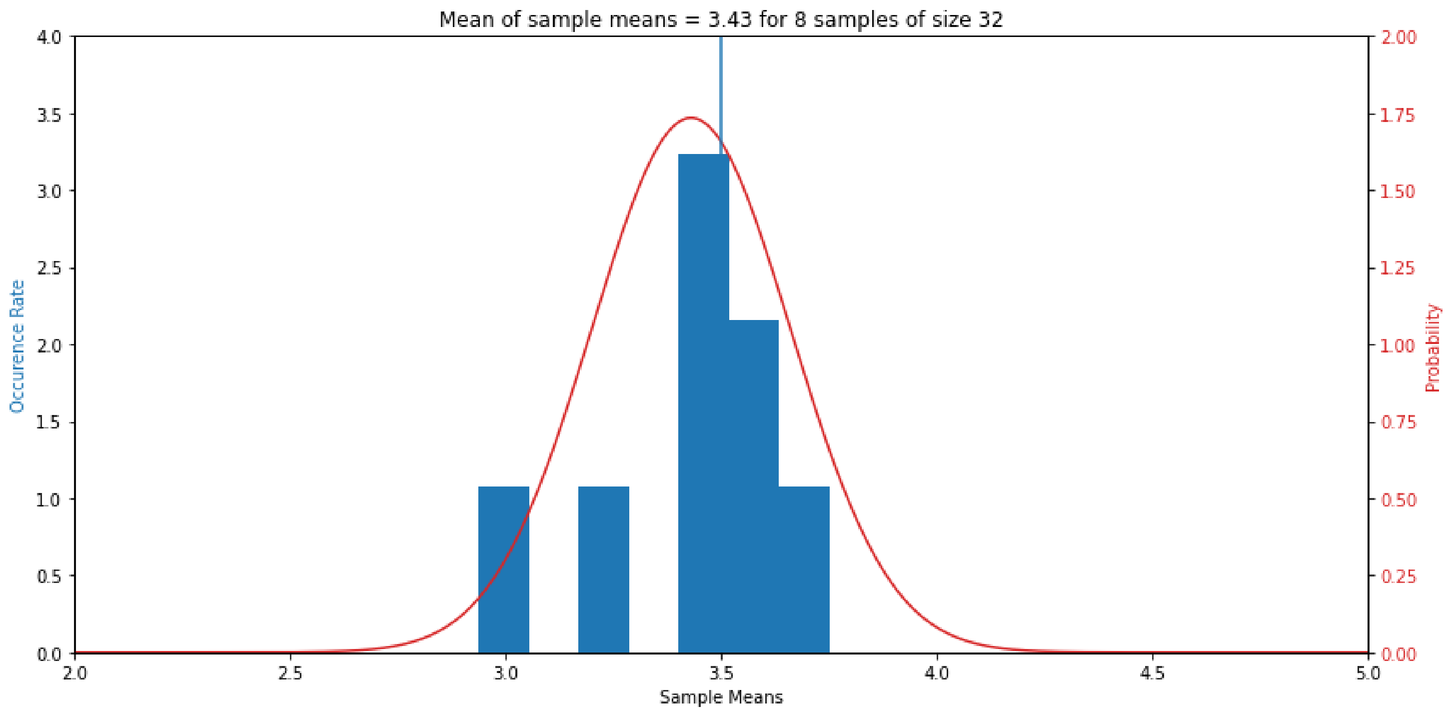

Mean of sample means = 3.38 for 3 samples of size 32
Occurence Rate
Probability
Sample Means
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.0
2.5
3.0
3.5
4.0
4.5
5.0

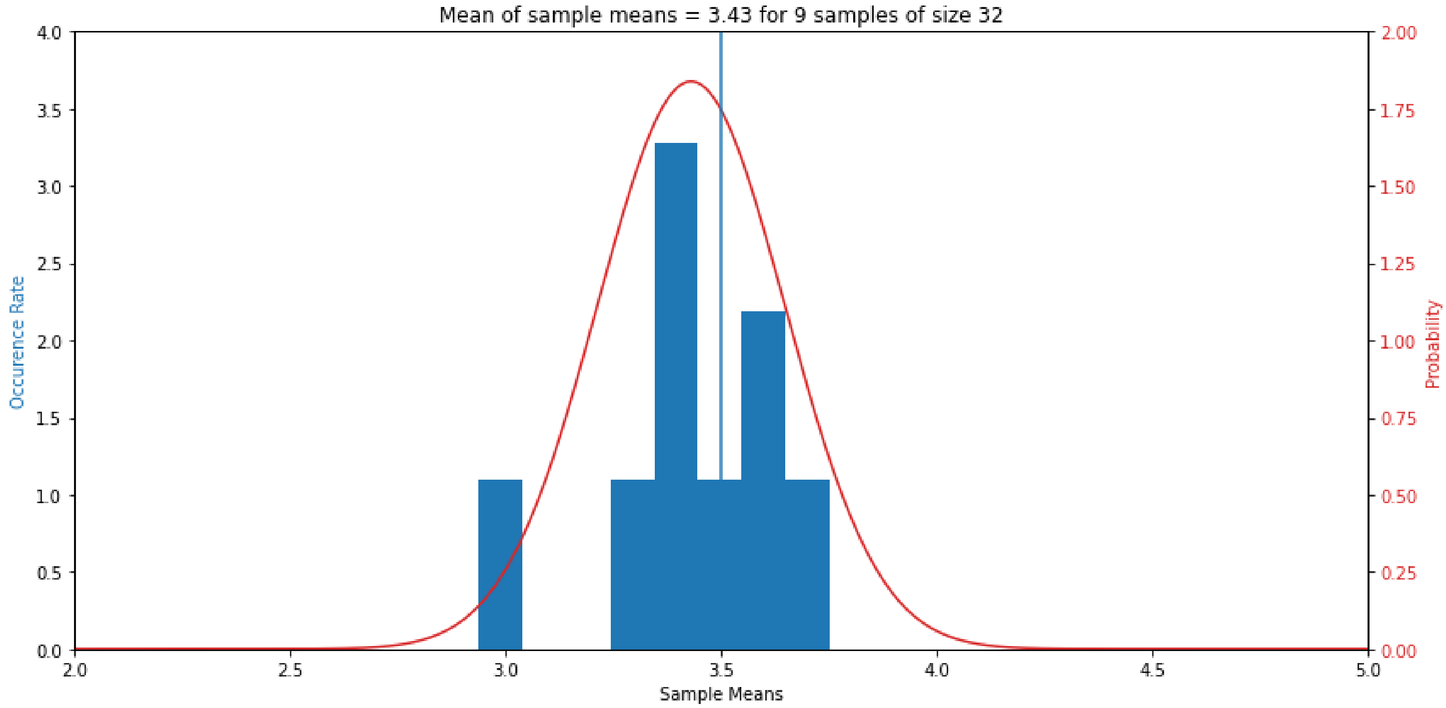

Mean of sample means = 3.34 for 4 samples of size 32
Occurence Rate
Probability
Sample Means
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.0
2.5
3.0
3.5
4.0
4.5
5.0


Mean of sample means = 3.36 for 5 samples of size 32
Occurence Rate
Probability
Sample Means
2.0
2.5
3.0
3.5
4.0
4.5
5.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

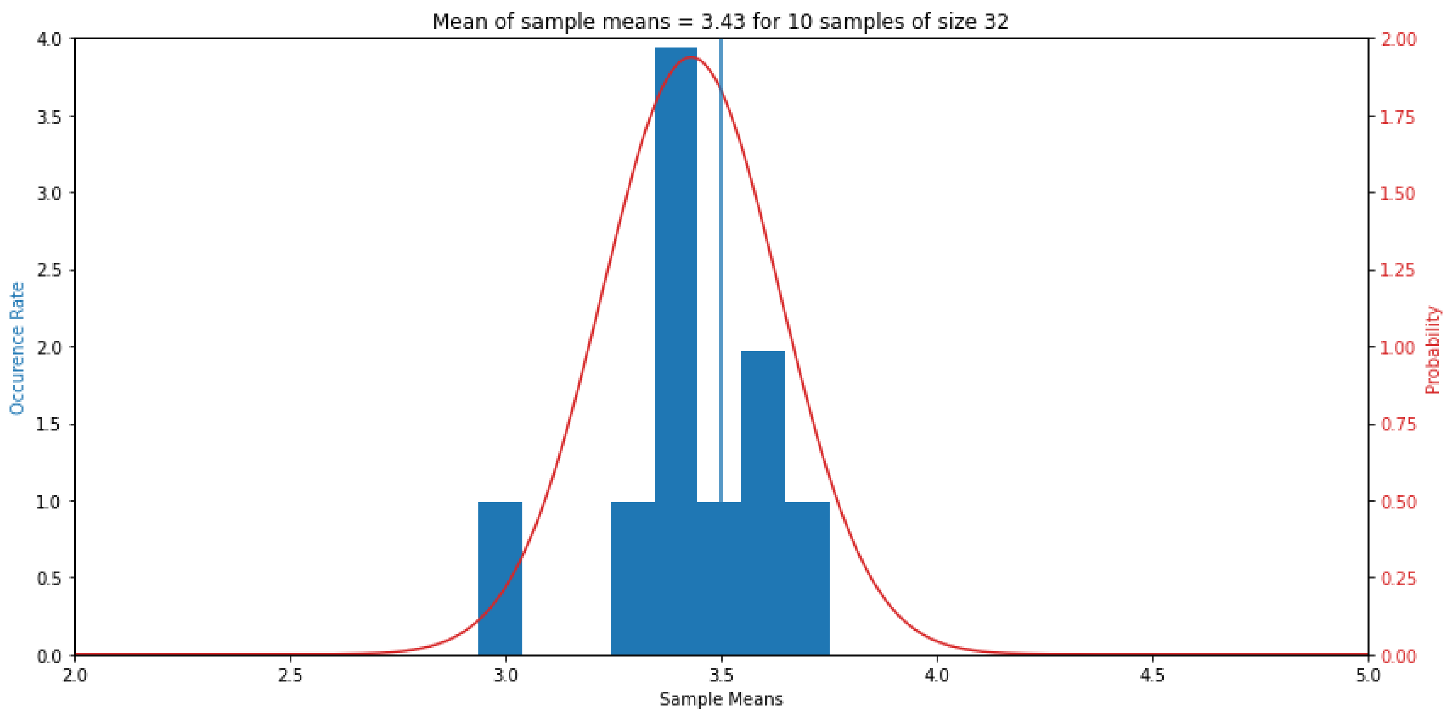

Mean of sample means = 3.4 for 6 samples of size 32
Occurence Rate
Probability
Sample Means
2.0
2.5
3.0
3.5
4.0
4.5
5.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00


Mean of sample means = 3.42 for 7 samples of size 32
Occurence Rate
Probability
Sample Means

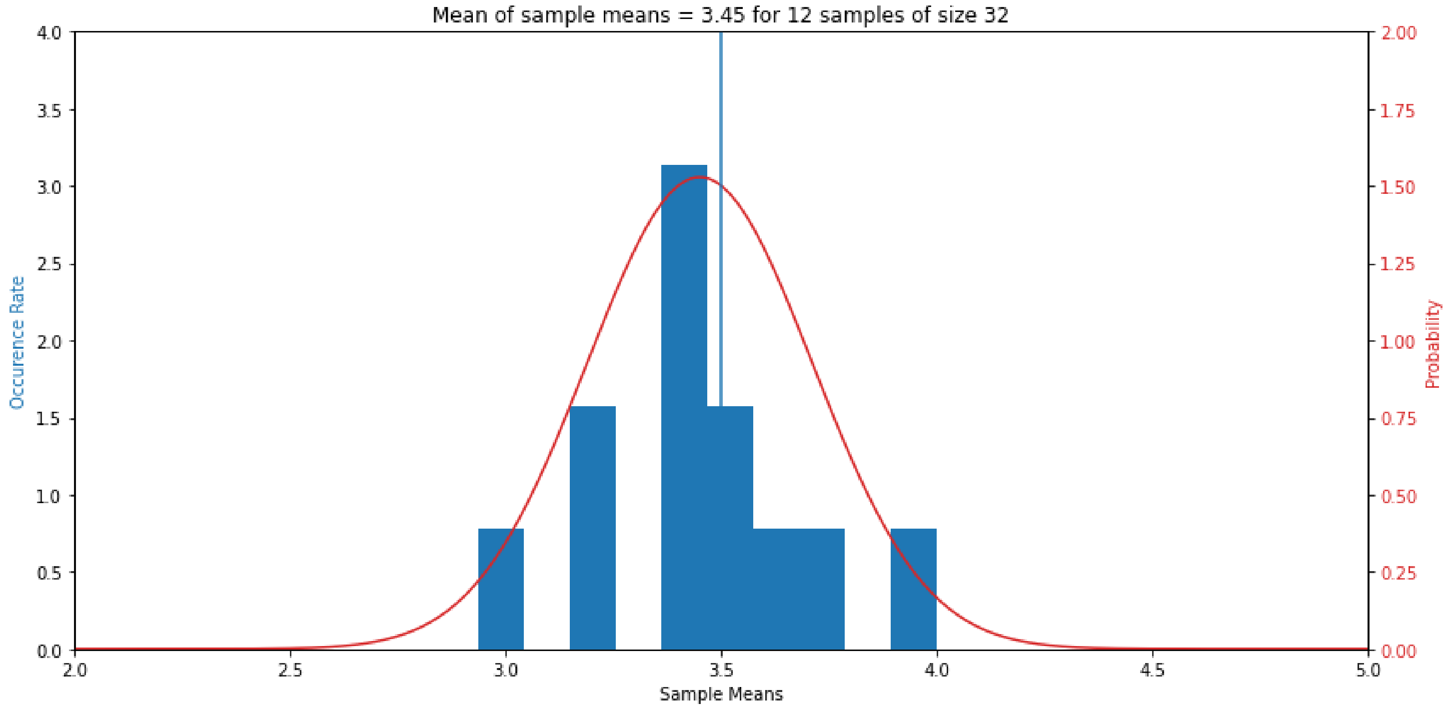

Mean of sample means = 3.43 for 8 samples of size 32
Occurence Rate
Probability
Sample Means

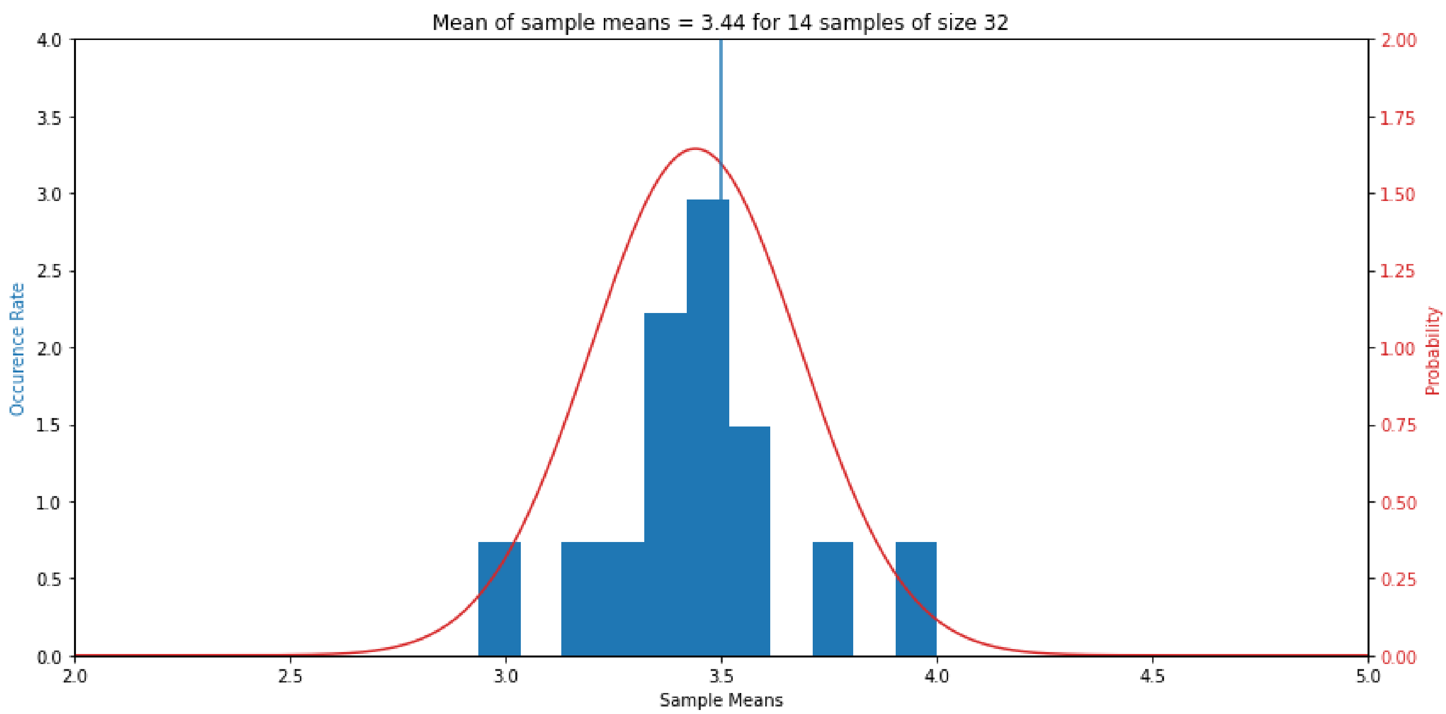


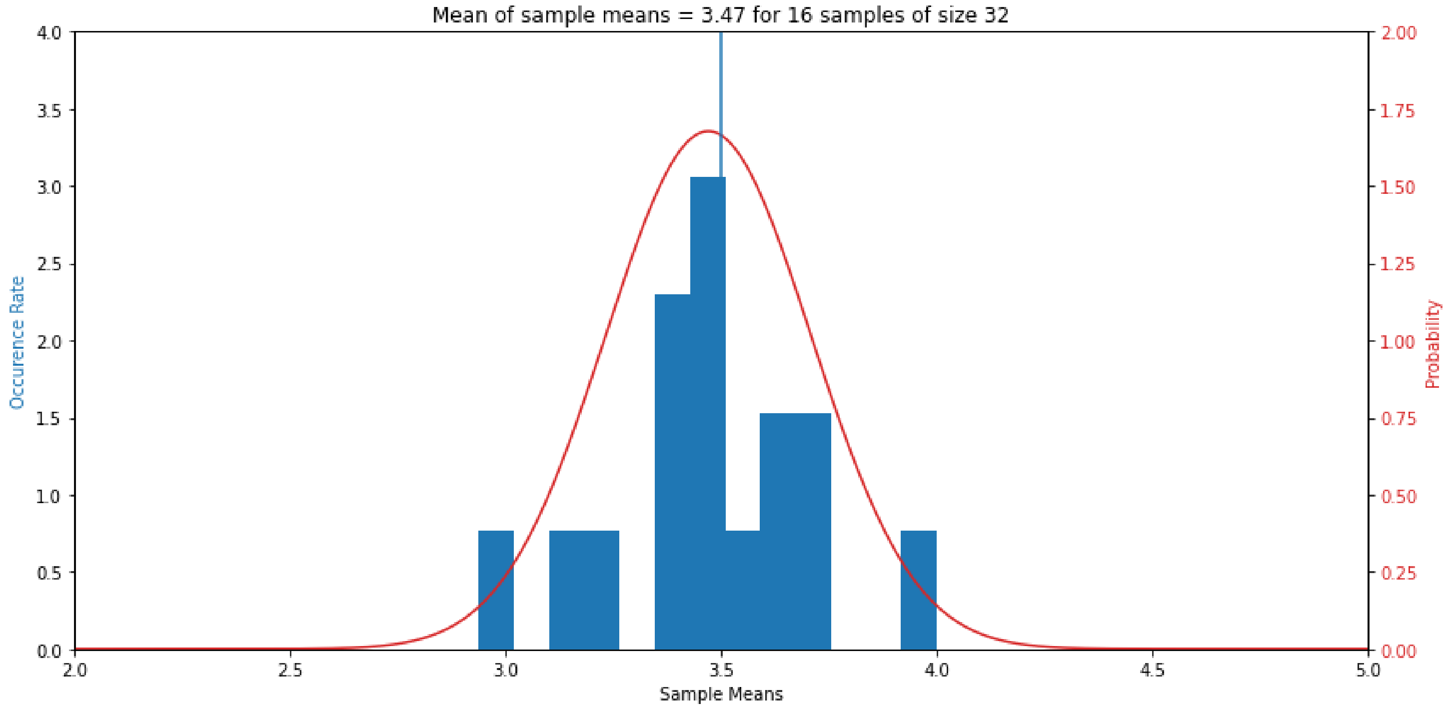
Mean of sample means = 3.43 for 9 samples of size 32
Occurence Rate
Probability
Sample Means
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.0
2.5
3.0
3.5
4.0
4.5
5.0



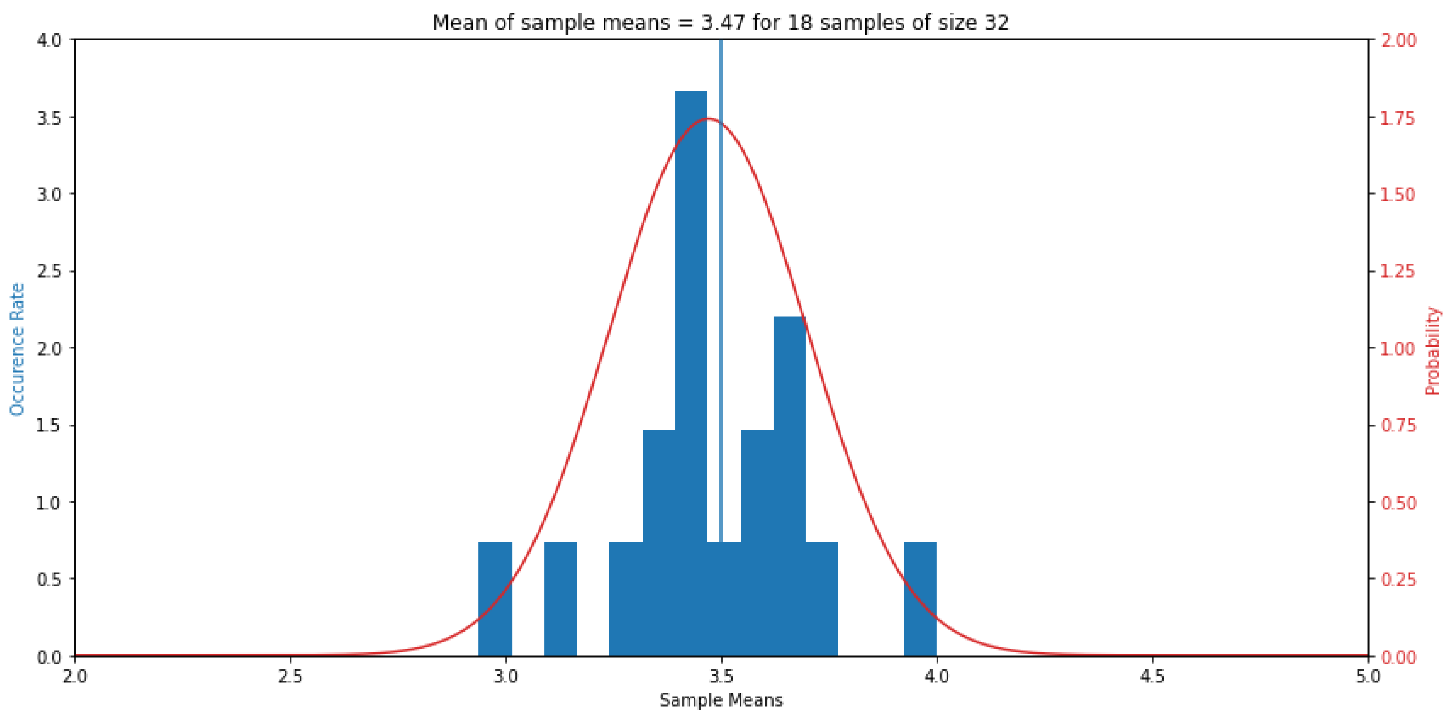
Mean of sample means = 3.43 for 10 samples of size 32
Occurence Rate
Probability
Sample Means
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.0
2.5
3.0
3.5
4.0
4.5
5.0


Mean of sample means = 3.45 for 12 samples of size 32
Occurence Rate
Probability
Sample Means
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
2.0
2.5
3.0
3.5
4.0
4.5
5.0

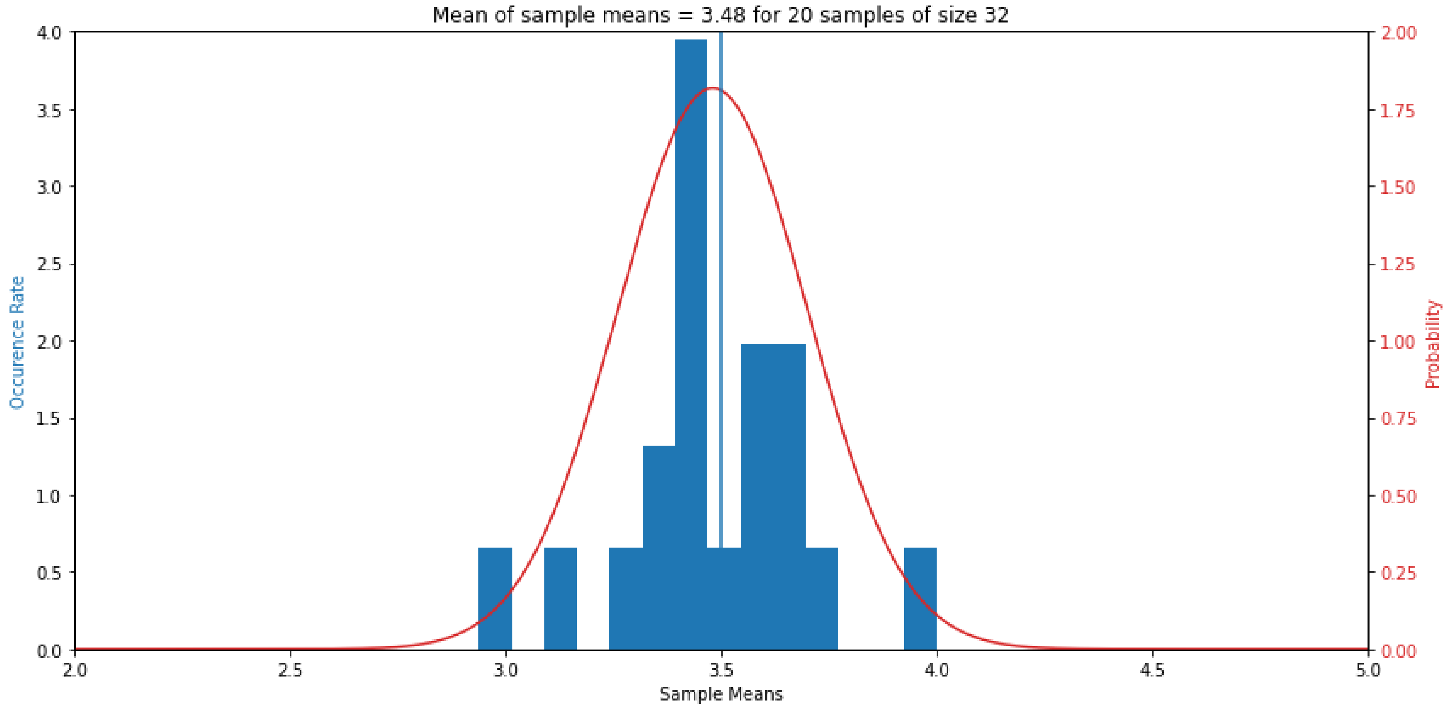

Mean of sample means = 3.44 for 14 samples of size 32
Occurence Rate
Probability
Sample Means
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
2.0
2.5
3.0
3.5
4.0
4.5
5.0



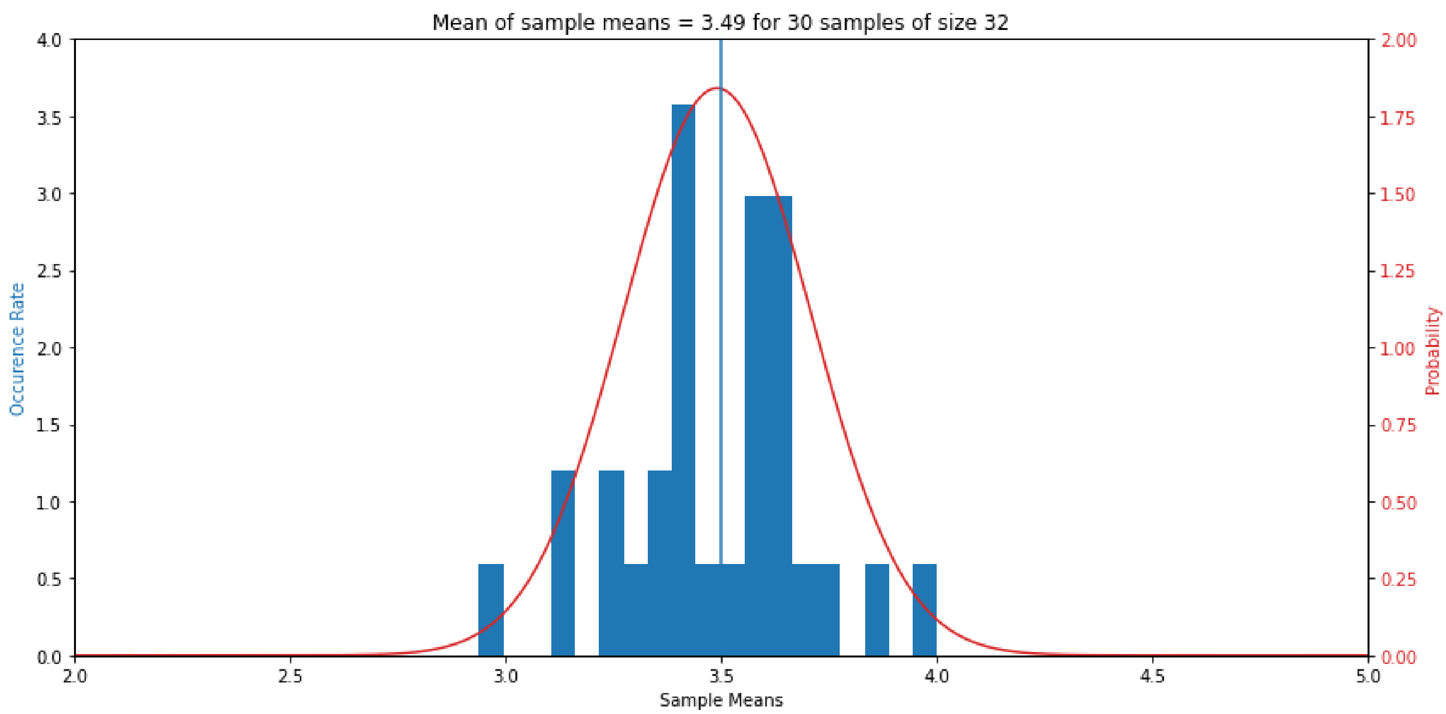
Mean of sample means = 3.47 for 16 samples of size 32
Occurence Rate
Probability
Sample Means



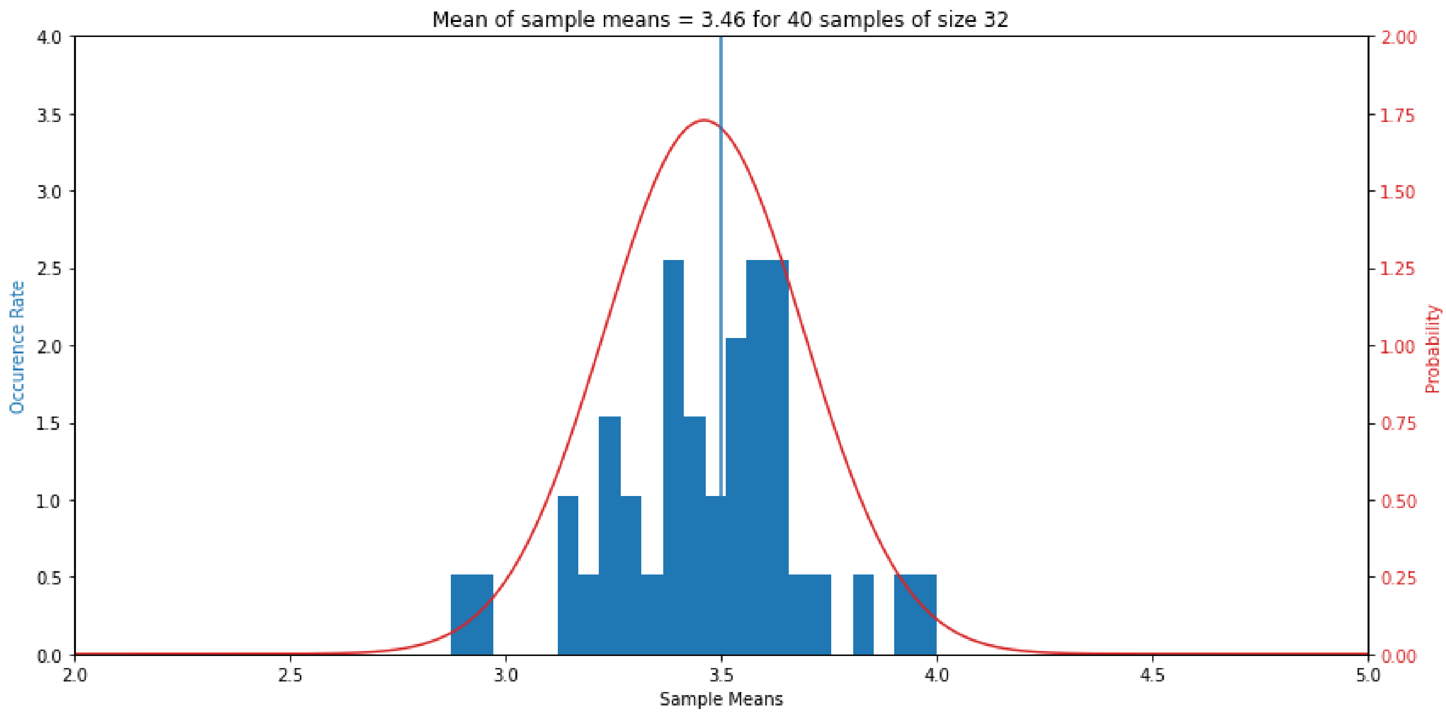
Mean of sample means = 3.47 for 18 samples of size 32
Occurence Rate
Probability
Sample Means

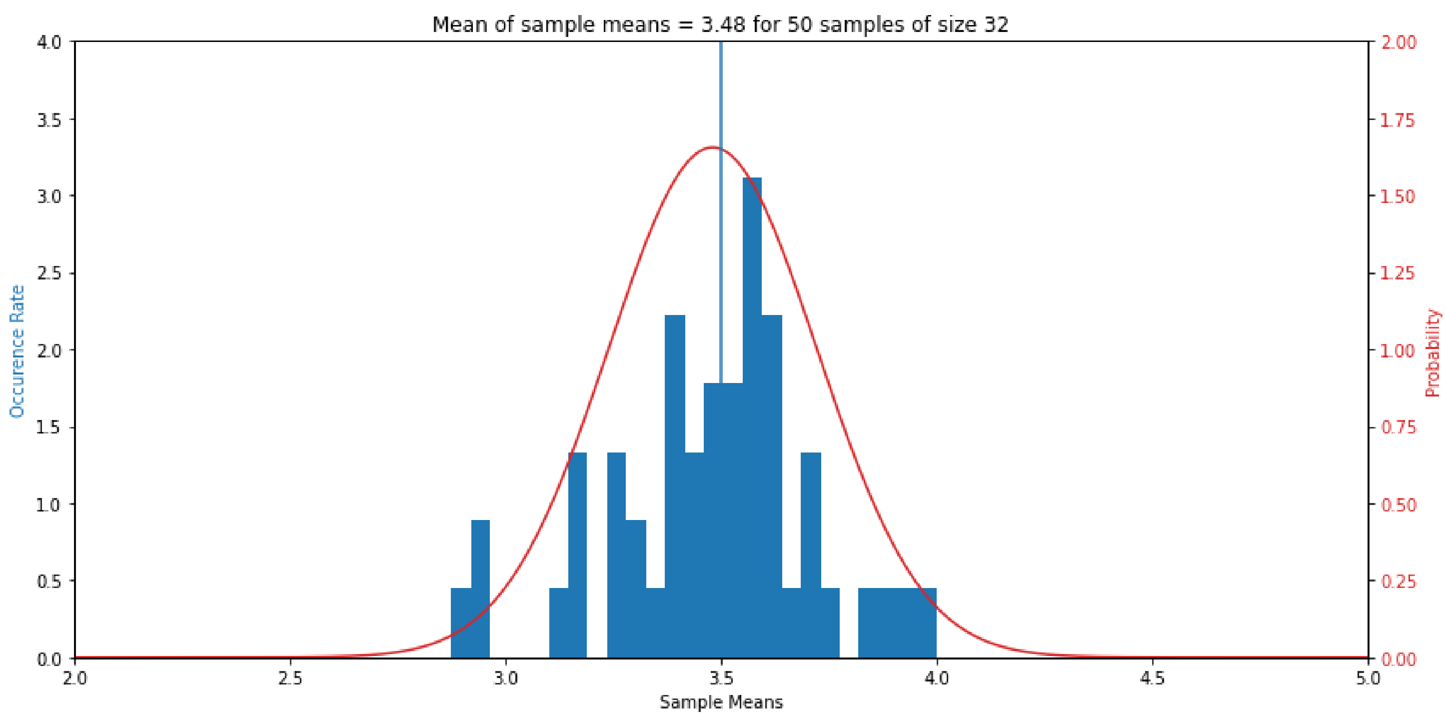


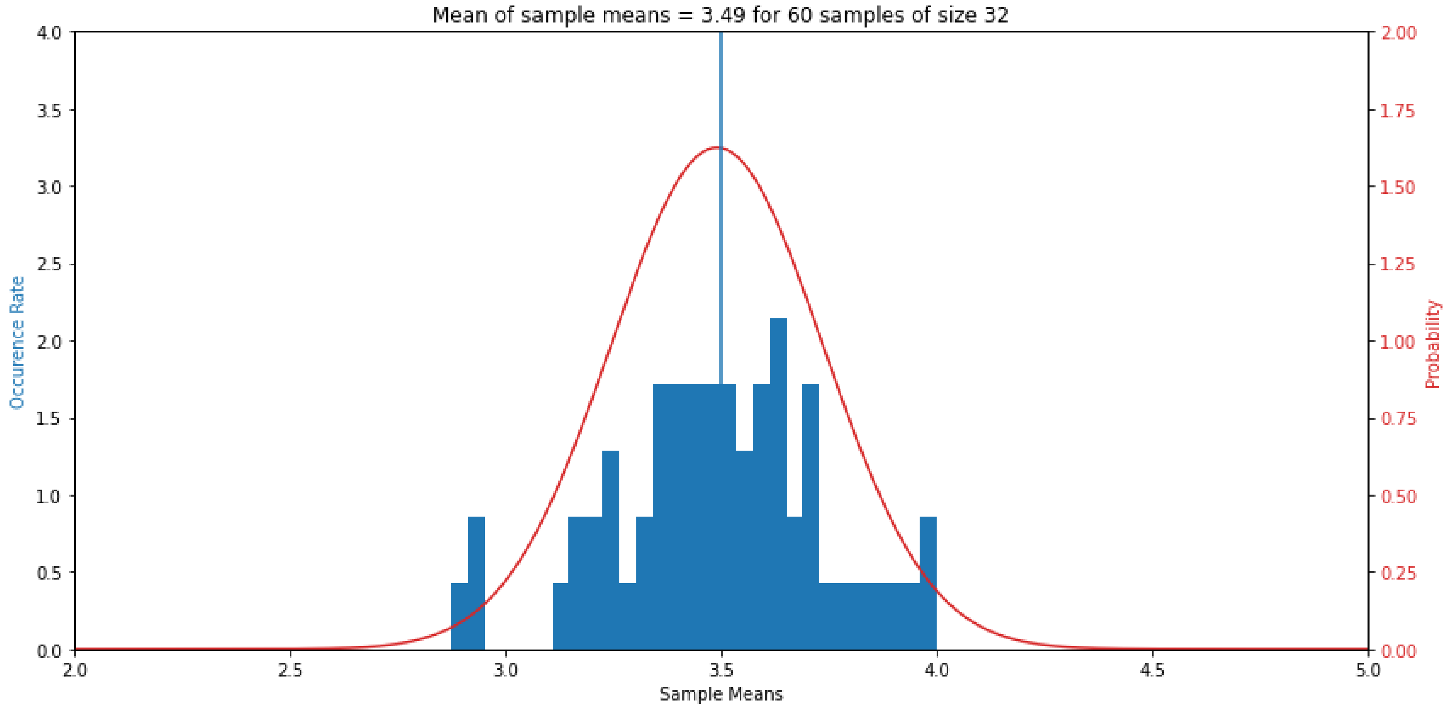
Mean of sample means = 3.48 for 20 samples of size 32
Occurence Rate
Probability
Sample Means

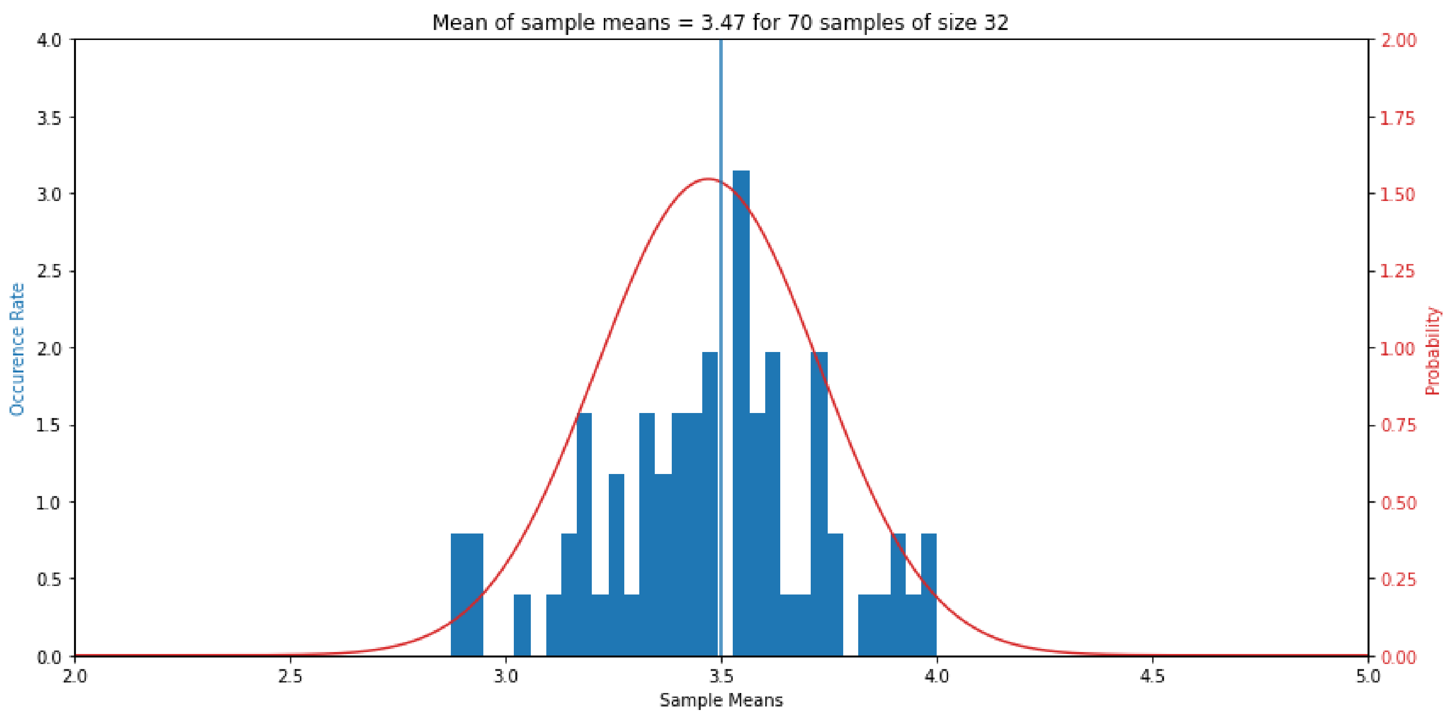


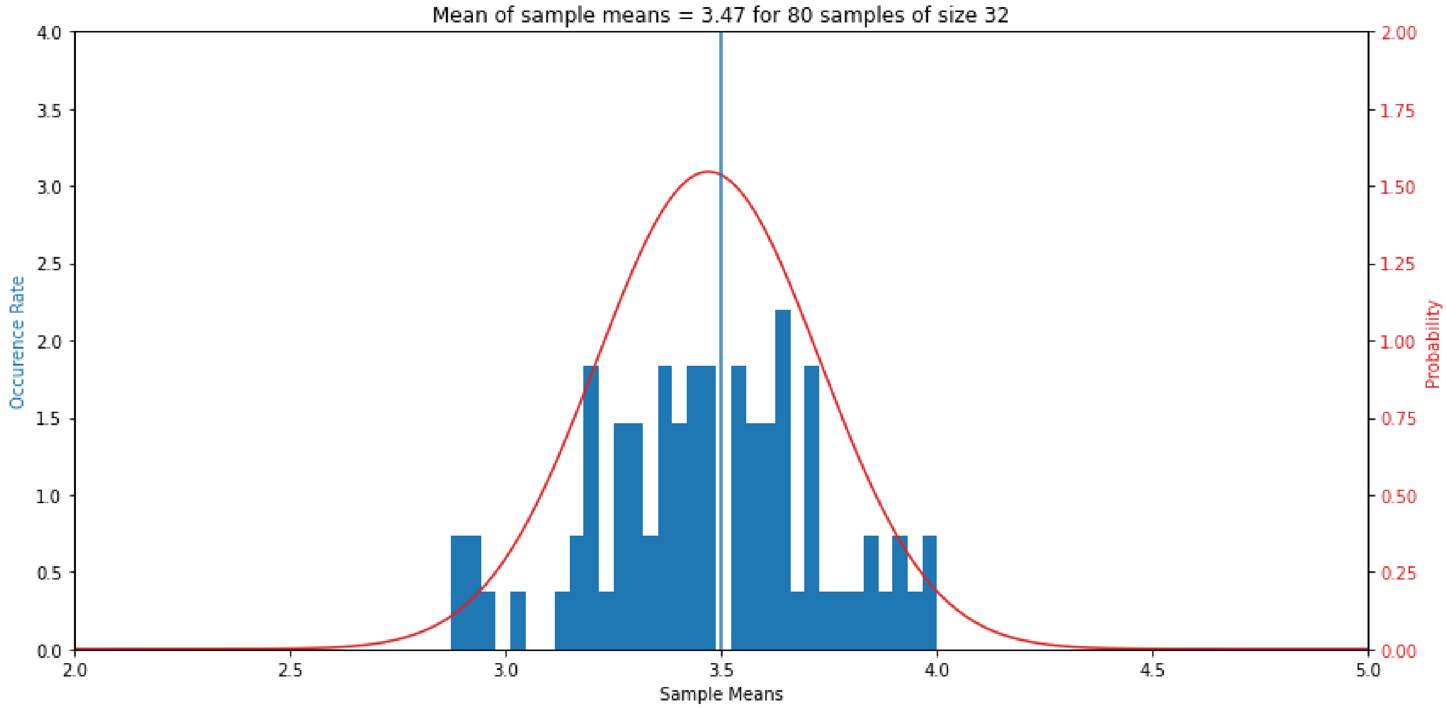
Mean of sample means = 3.49 for 30 samples of size 32
Occurence Rate
Probability
Sample Means

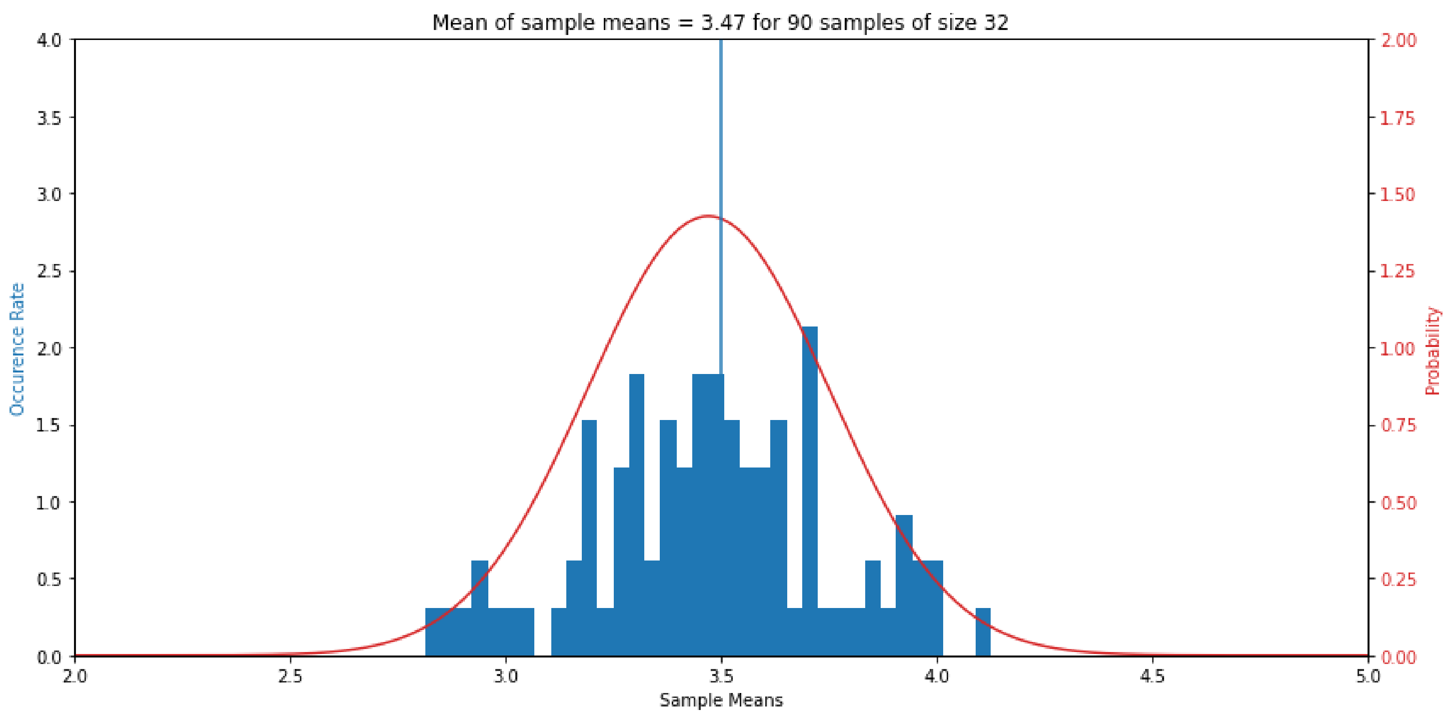

Mean of sample means = 3.46 for 40 samples of size 32
Occurence Rate
Probability
Sample Means

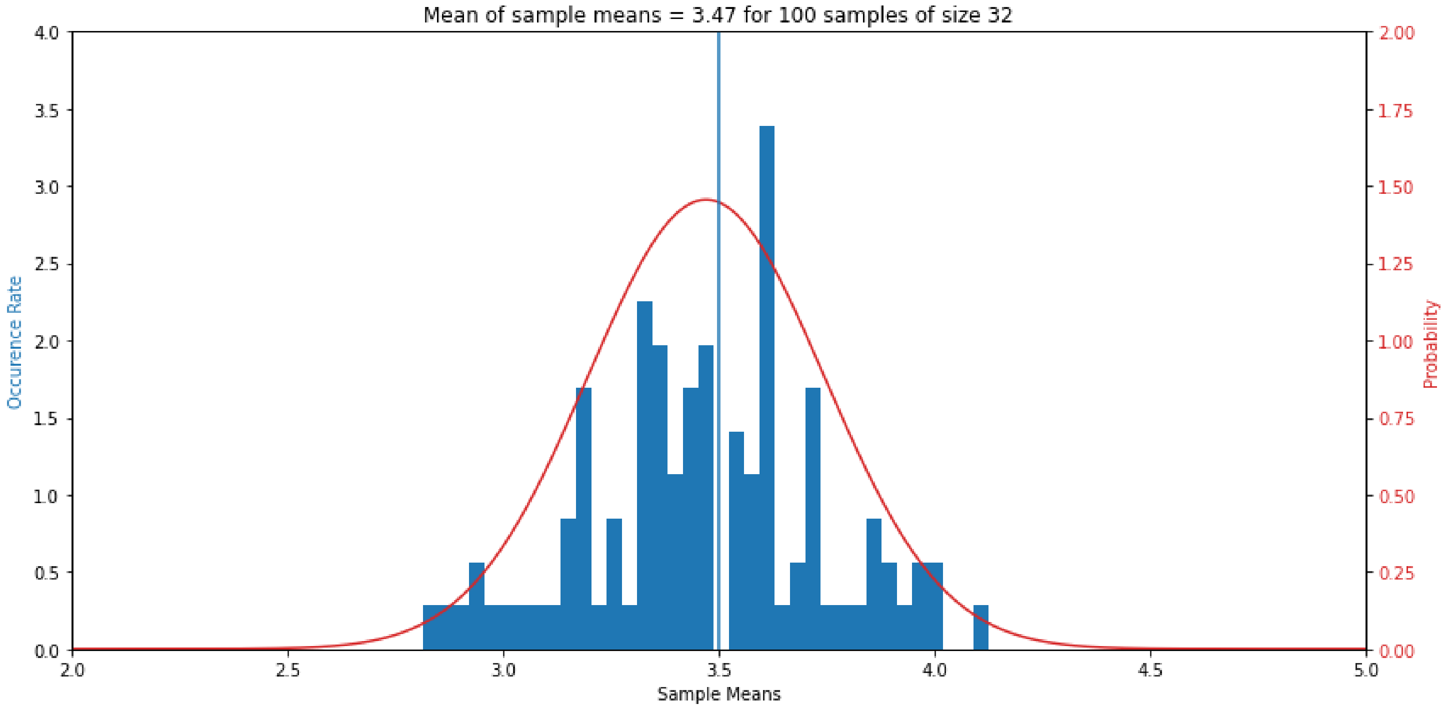

Mean of sample means = 3.48 for 50 samples of size 32
Occurence Rate
Probability
Sample Means

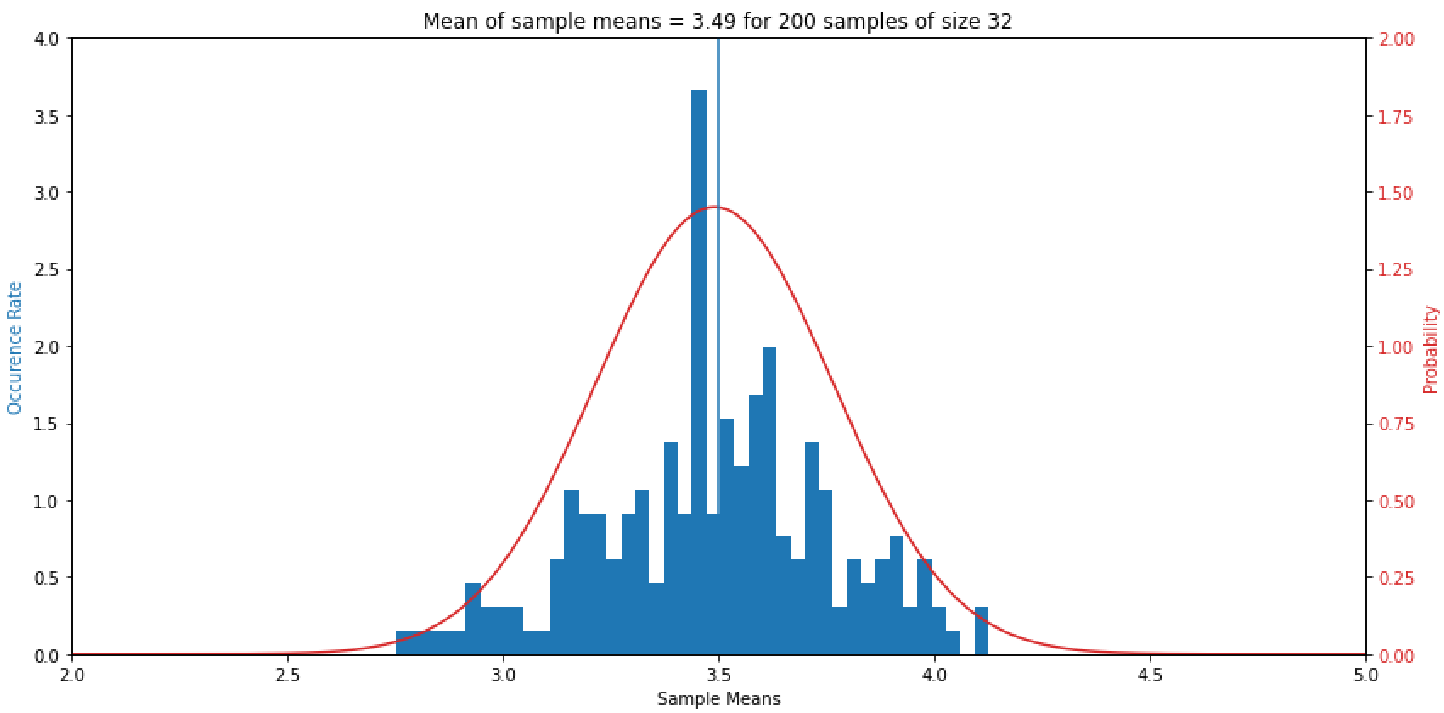

Mean of sample means = 3.49 for 60 samples of size 32
Occurence Rate
Probability
Sample Means
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

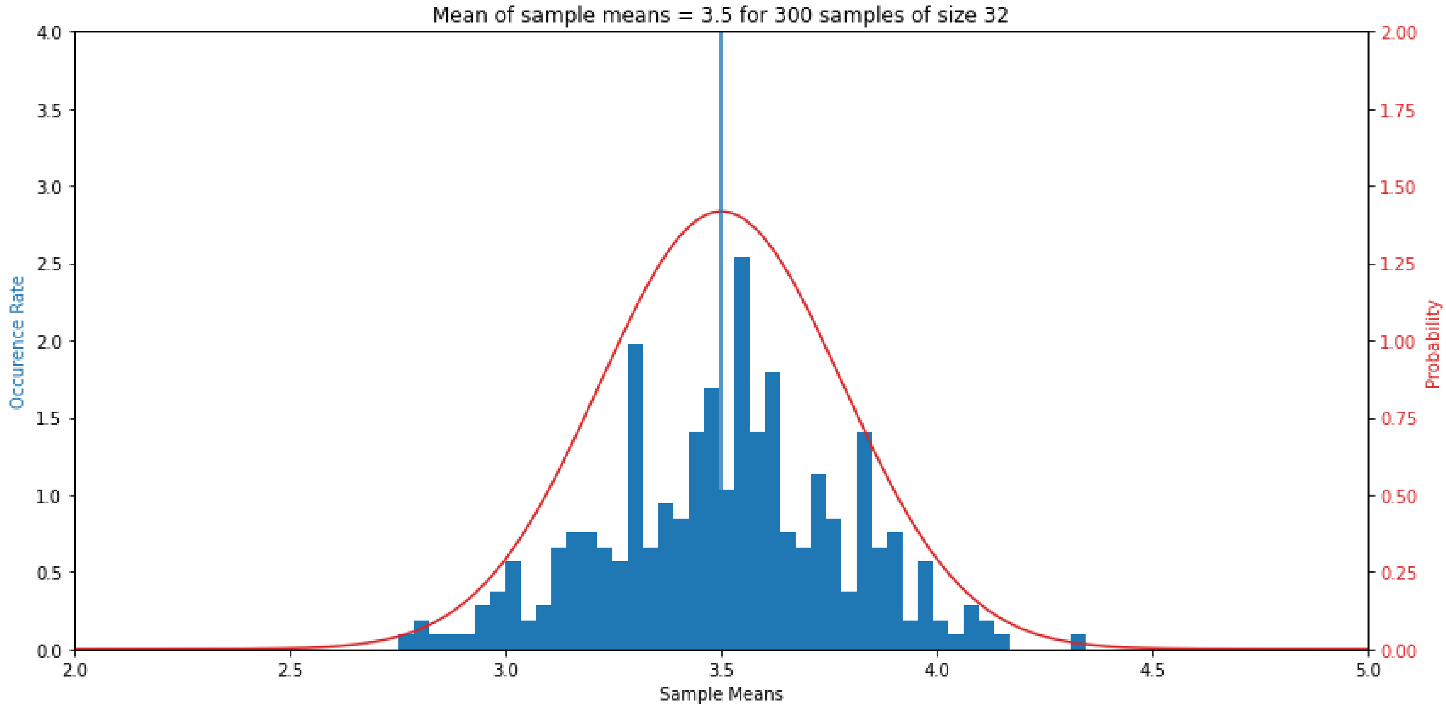

Mean of sample means = 3.47 for 70 samples of size 32
Occurence Rate
Probability
Sample Means
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

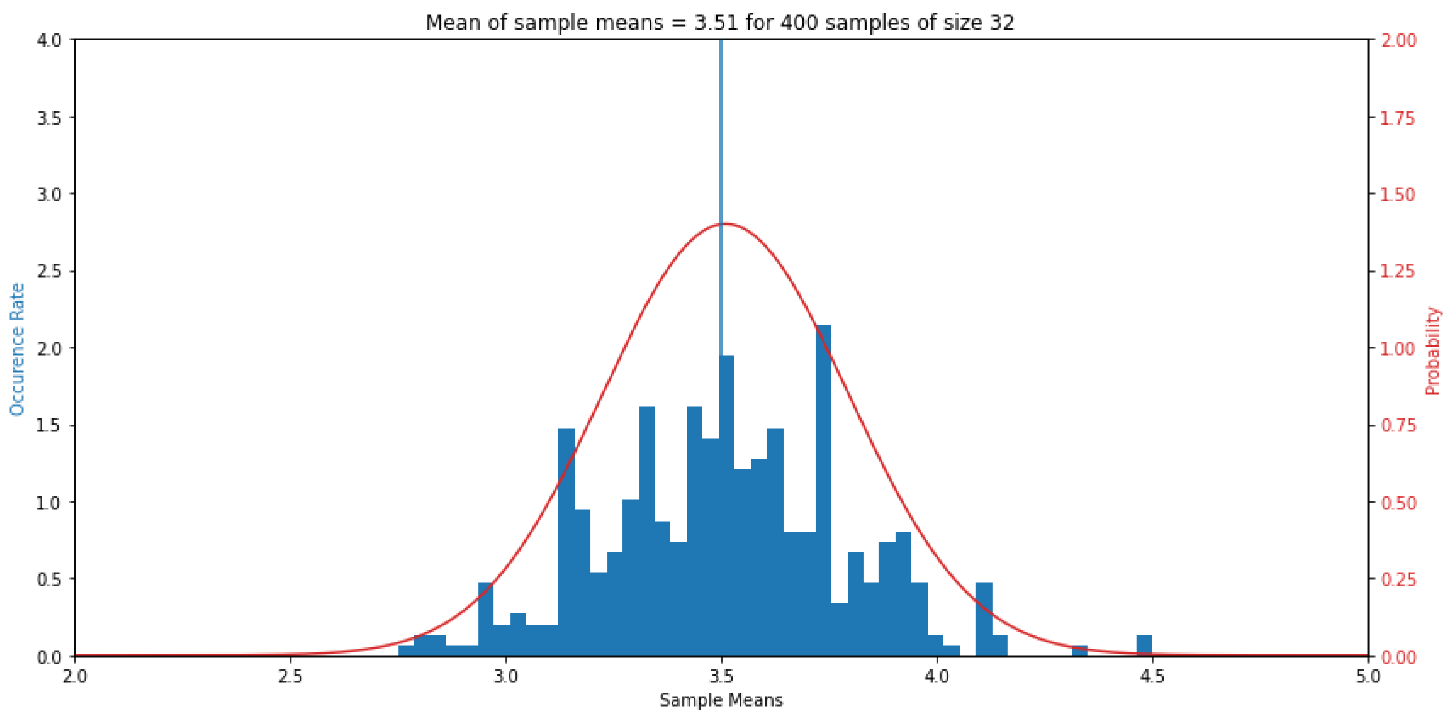

Mean of sample means = 3.47 for 80 samples of size 32
Occurence Rate
Probability
Sample Means
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.0
2.5
3.0
3.5
4.0
4.5
5.0

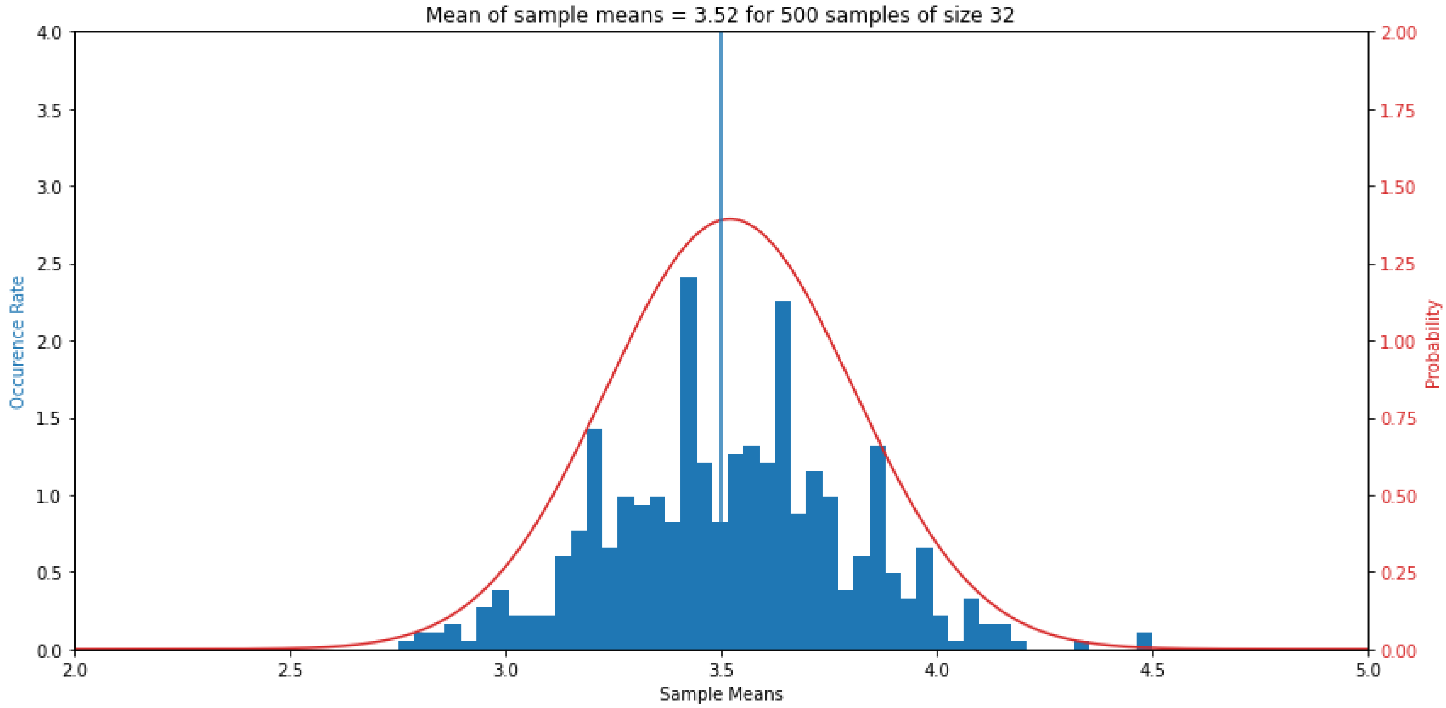

Mean of sample means = 3.47 for 90 samples of size 32
Occurence Rate
Probability
Sample Means
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.0
2.5
3.0
3.5
4.0
4.5
5.0

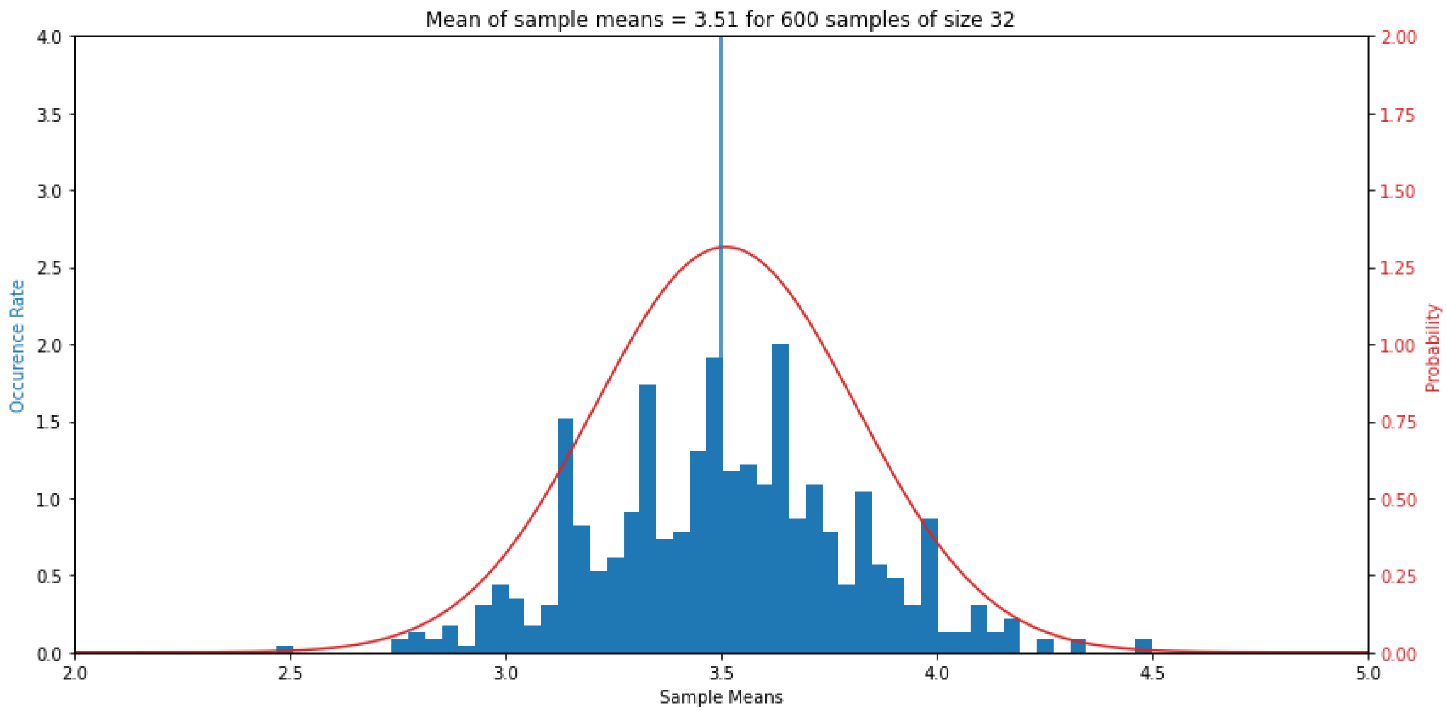

Mean of sample means = 3.47 for 100 samples of size 32
Occurence Rate
Probability
Sample Means

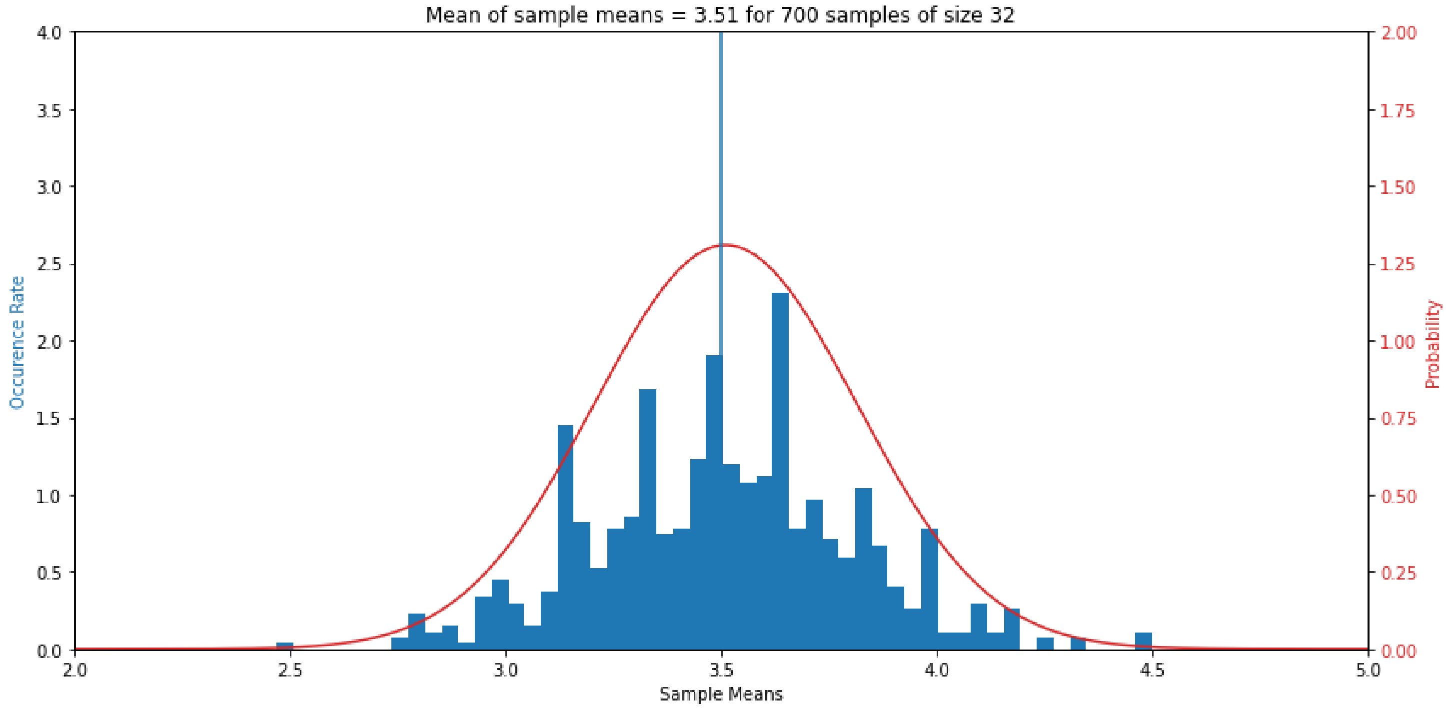

Mean of sample means = 3.49 for 200 samples of size 32
Occurence Rate
Probability
Sample Means

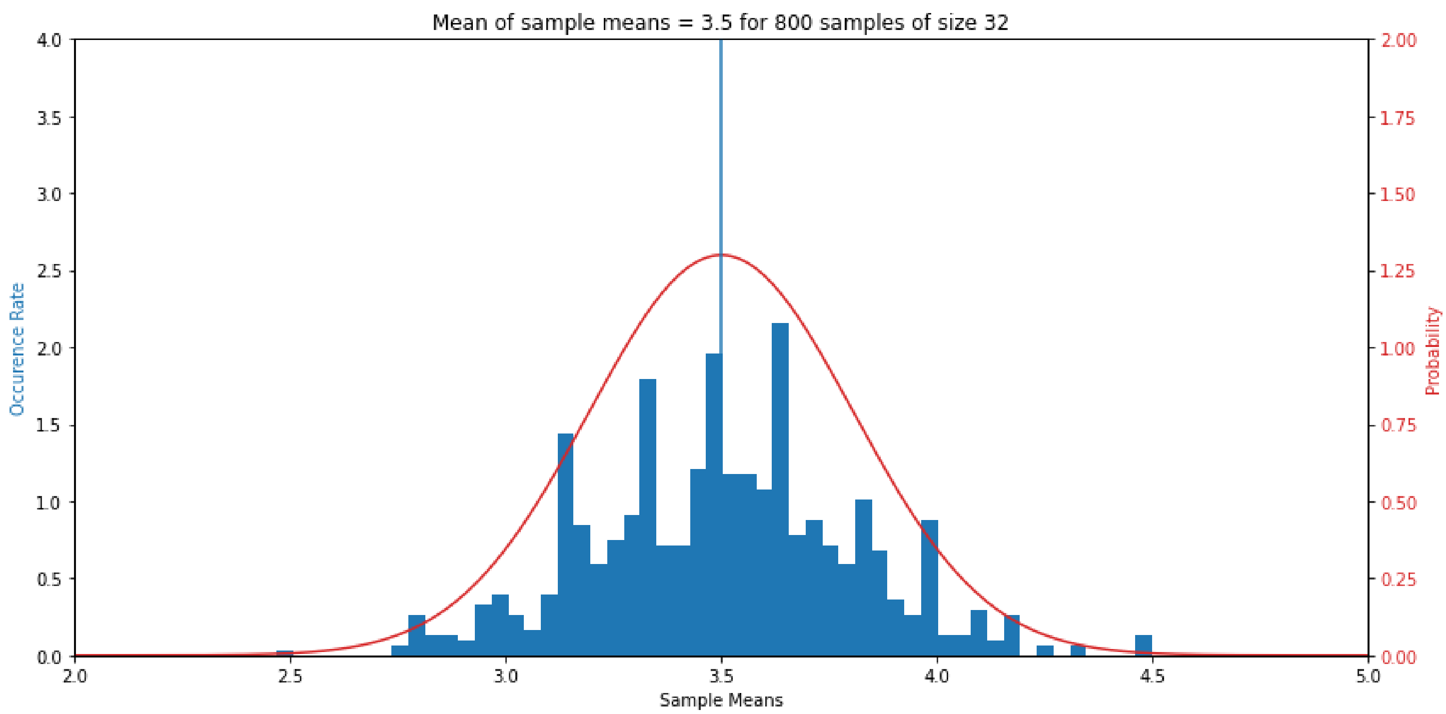

Mean of sample means = 3.5 for 300 samples of size 32
Occurence Rate
Probability
Sample Means
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.0
2.5
3.0
3.5
4.0
4.5
5.0

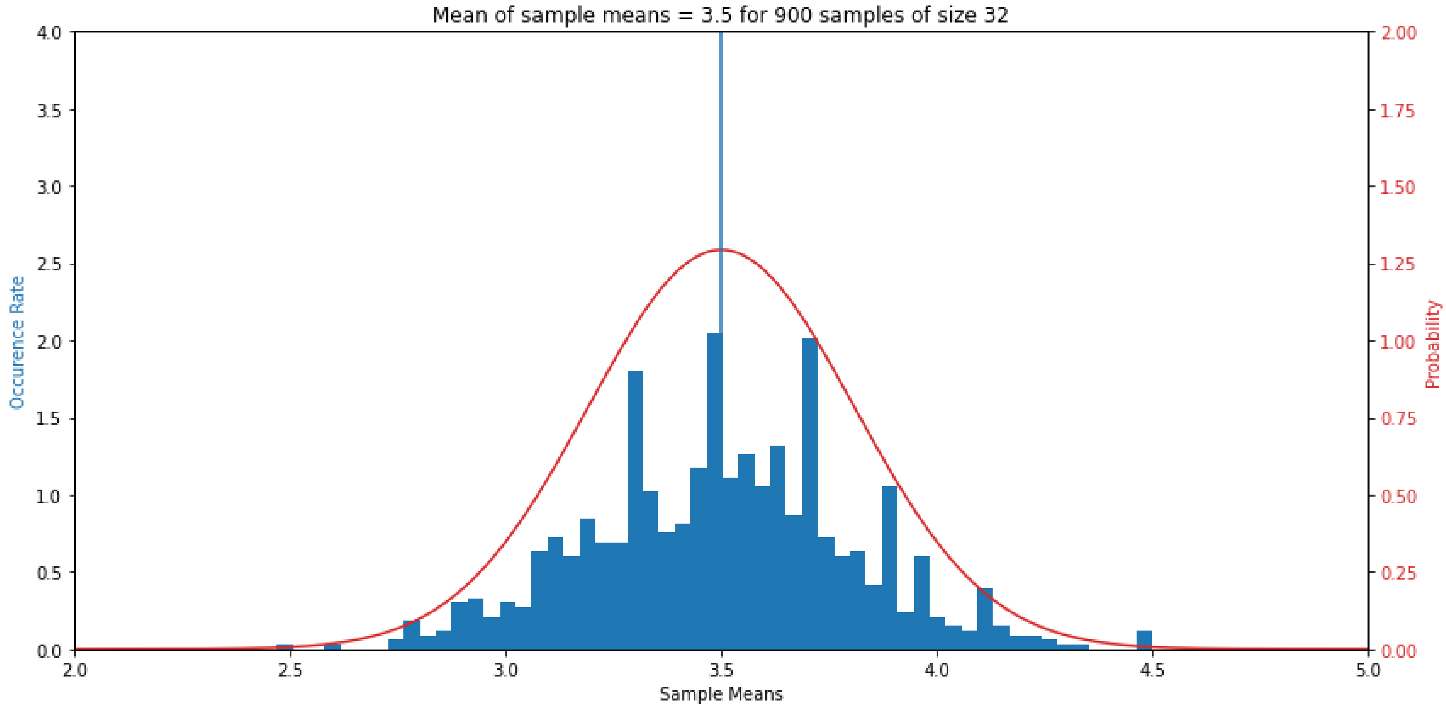

Mean of sample means = 3.51 for 400 samples of size 32
Occurence Rate
Probability
Sample Means
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.0
2.5
3.0
3.5
4.0
4.5
5.0

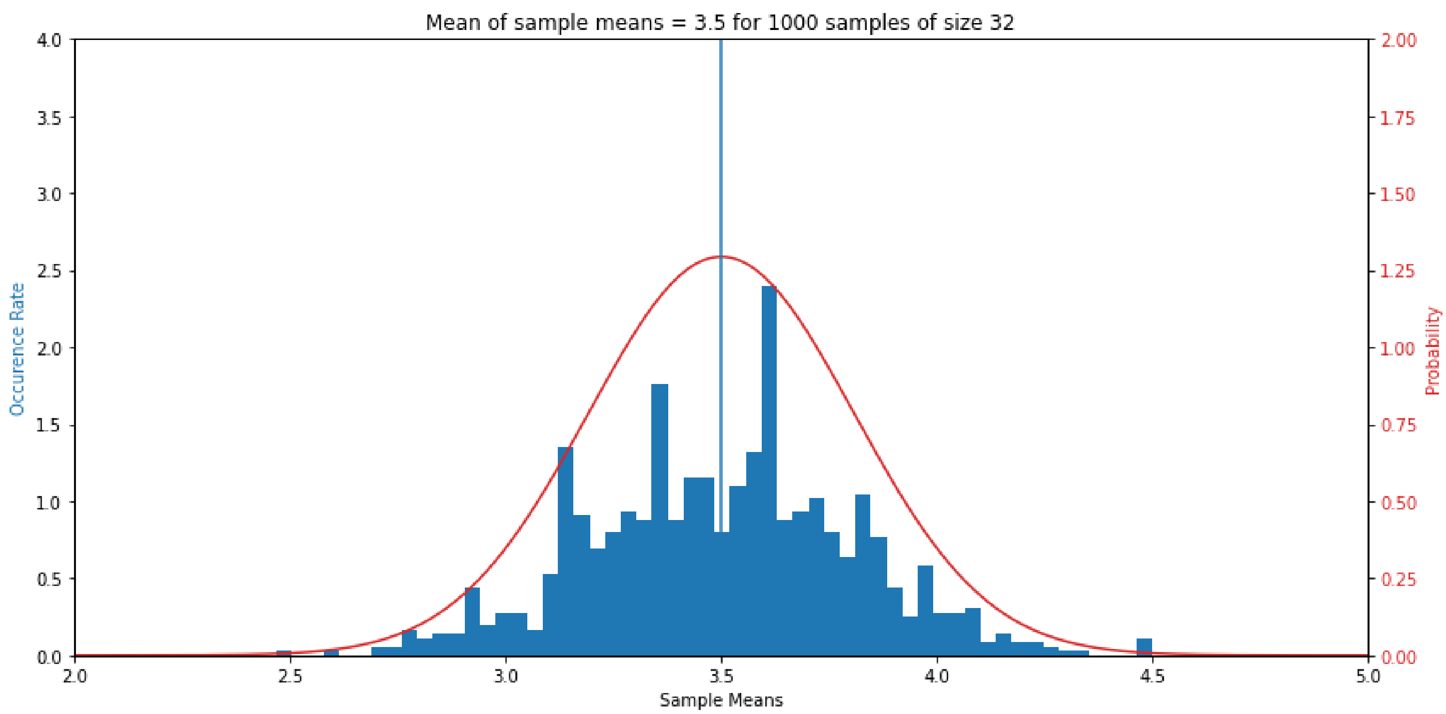

Mean of sample means = 3.52 for 500 samples of size 32
Occurence Rate
Probability
Sample Means
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.0
2.5
3.0
3.5
4.0
4.5
5.0

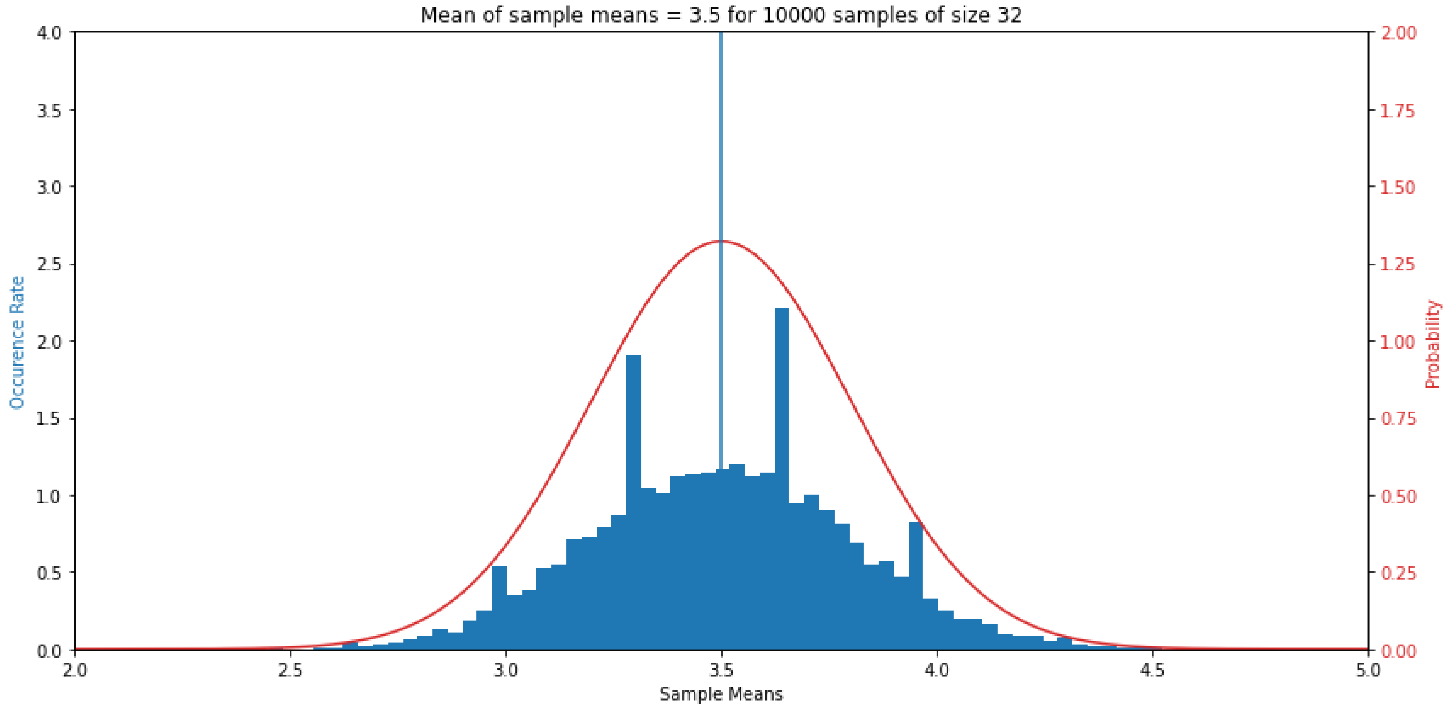

Mean of sample means = 3.51 for 600 samples of size 32
Occurence Rate
Probability
Sample Means
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.0
2.5
3.0
3.5
4.0
4.5
5.0

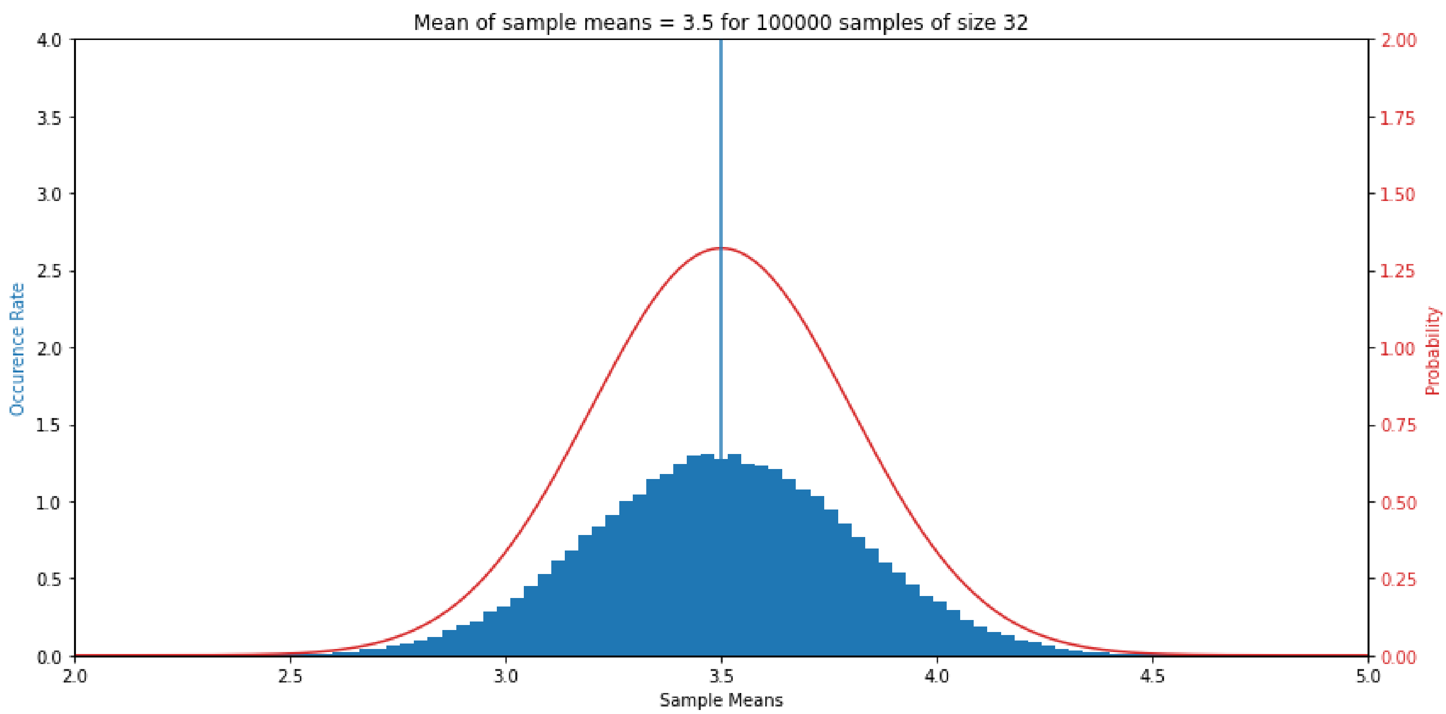

Mean of sample means = 3.51 for 700 samples of size 32
Occurence Rate
Probability
Sample Means
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.0
2.5
3.0
3.5
4.0
4.5
5.0

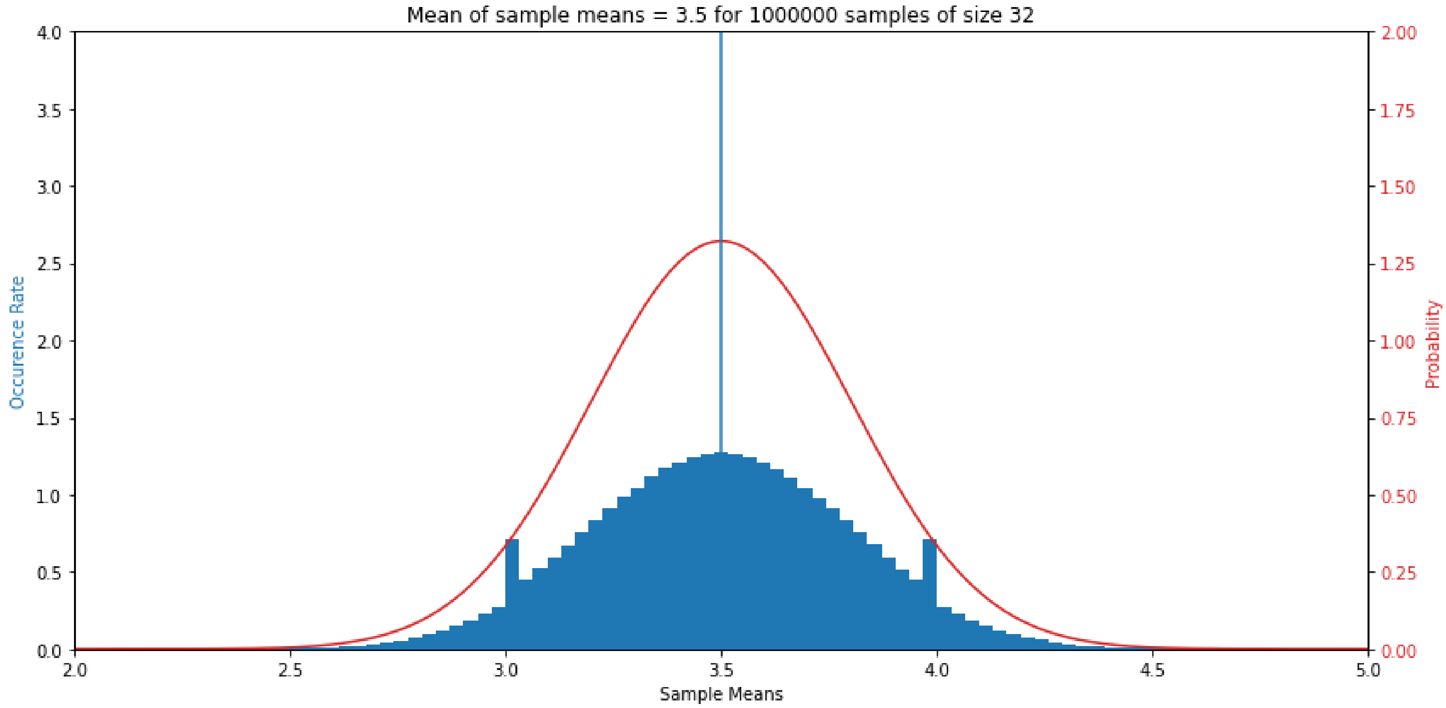

Mean of sample means = 3.5 for 800 samples of size 32
Occurence Rate
Probability
Sample Means
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.0
2.5
3.0
3.5
4.0
4.5
5.0


Mean of sample means = 3.5 for 900 samples of size 32
Occurence Rate
Probability
Sample Means
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.0
2.5
3.0
3.5
4.0
4.5
5.0

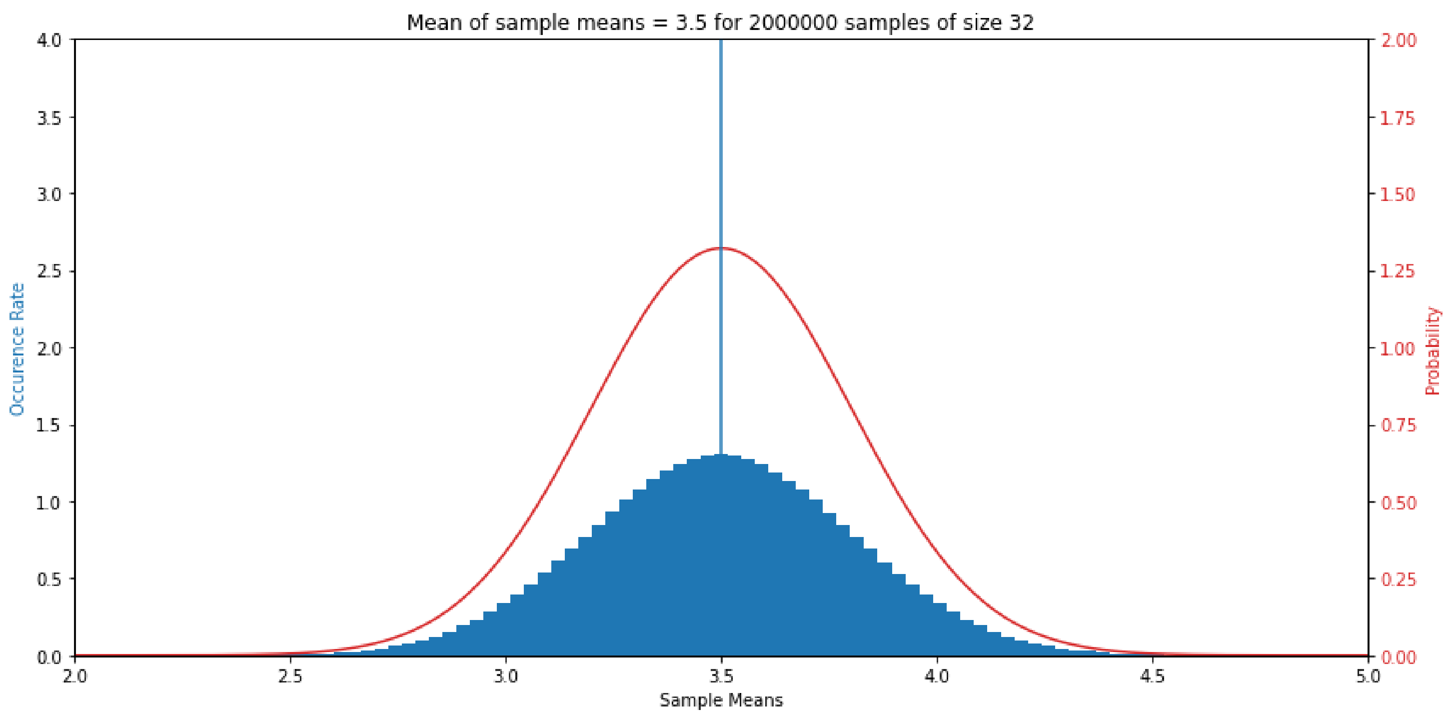

Mean of sample means = 3.5 for 1000 samples of size 32
Occurence Rate
Probability
Sample Means
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.0
2.5
3.0
3.5
4.0
4.5
5.0


Mean of sample means = 3.5 for 10000 samples of size 32
Occurence Rate
Probability
Sample Means
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.0
2.5
3.0
3.5
4.0
4.5
5.0


Mean of sample means = 3.5 for 100000 samples of size 32
Occurence Rate
Probability
Sample Means
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.0
2.5
3.0
3.5
4.0
4.5
5.0



Mean of sample means = 3.5 for 1000000 samples of size 32
Occurence Rate
Probability
Sample Means



Mean of sample means = 3.5 for 2000000 samples of size 32
Occurence Rate
Probability
Sample Means

Two Y Axes

```python
import numpy as np
import matplotlib.pyplot as plt

# Create some mock data
t = np.arange(0.01, 10.0, 0.01)
data1 = np.exp(t)
data2 = np.sin(2 * np.pi * t)

fig, ax1 = plt.subplots()

color = 'tab:red'
ax1.set_xlabel('time (s)')
ax1.set_ylabel('exp', color=color)
ax1.plot(t, data1, color=color)
ax1.tick_params(axis='y', labelcolor=color)

ax2 = ax1.twinx()  # instantiate a second axes that shares the same x-axis

color = 'tab:blue'
ax2.set_ylabel('sin', color=color)  # we already handled the x-label with ax1
ax2.plot(t, data2, color=color)
ax2.tick_params(axis='y', labelcolor=color)

fig.tight_layout()  # otherwise the right y-label is slightly clipped
plt.show()
```

Binomial Variables

Rules Binomial Variables

1. Comprised Of Independent Trials
2. Each Trial Can Be Regarded As A Success Or Failure
3. There Are A FIXED Number Of Trials
4. Probability Of Success On Each Trial Is Constant (i.e. Rule 1 Repeats)

NOTE: Use The 10% Rule For Approximate Independence Of Trials When Resampling Is Not Possible

Binomial Combinametrics

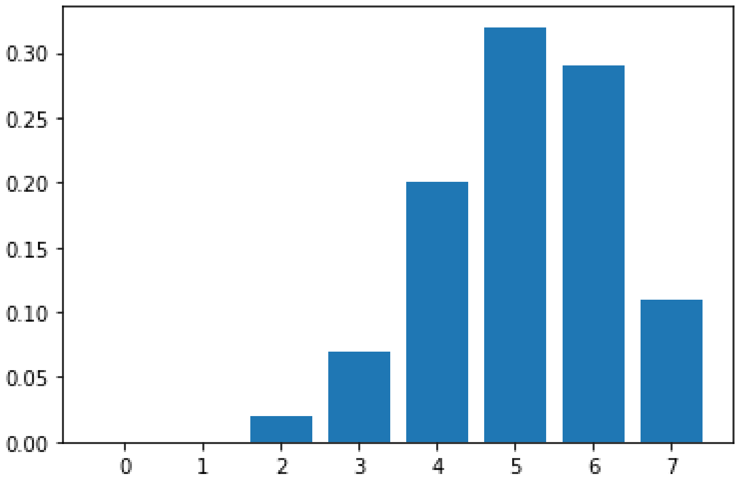
$$p(k\ of\ n) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$\left(\!\!\binom{n}{k}\!\!\right) = \binom{n+k-1}{k} = \frac{(n+k-1)!}{k!\,(n-1)!} = \frac{n(n+1)(n+2)\cdots(n+k-1)}{k!},$$

$$p(k\ of\ n) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

```
# Independent Trials Success Probability = 0.73
import math
import matplotlib.pyplot as plt

fact = math.factorial

Ps = 0.73
p_D = {}
n = 7
for k in range(n+1):
    nCk = fact(n) / (fact(k) * fact(n-k))
    p = nCk * Ps**k * (1 - Ps)**(n - k)
    p_D[k] = round(p, 2)

plt.bar(p_D.keys(), p_D.values())
plt.show()
```


0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
1
2
3
4
5
6
7