

FX consistency

Let $X^{10} = \text{GBPUSD}$, $X^{20} = \text{EURUSD}$ so $X^{12} := \frac{X^{10}}{X^{20}} = \text{GBPEUR}$. If we specify a model for X^{10} and X^{20} under the USD measure \mathbb{Q}^0 , then we can price call options on X^{12} (in the unnatural currency USD) using

$$\mathbb{E}^0[X_T^{20}(X_T^{12} - K)_+].$$

To avoid arbitrage this must be equivalent to

$$X_0^{20} \mathbb{E}^2[(X_T^{12} - K)_+]$$

where here X^{12} dynamics are under \mathbb{Q}^2 , verifying $\frac{d\mathbb{Q}^2}{d\mathbb{Q}^0} = X^{20}$.