Billingsley: $P_n \Rightarrow P$ and $P D_h = 0 \Rightarrow P_n h^{-1} \Rightarrow P h^{-1}$.

will apply this to distributions $P_\varepsilon \equiv \mathbb{X}_\varepsilon$, $P \equiv \bar{\mathbb{X}}_0$
$= \mathbb{P}\bar{X}_\varepsilon^{-1}$

want to show $h(\bar{x}_\varepsilon) \xrightarrow{d} h(\bar{x}_0)$, so write.

$$\int_{\omega \in C} (h \circ \bar{X}_\varepsilon)(\omega)\, d\mathbb{P}(\omega) = \int_{\omega' \in D} h(\omega')\, d\mathbb{P}\bar{X}_\varepsilon^{-1}(\omega')$$

$$\xrightarrow{*} \int_{\omega' \in D} h(\omega')\, d\mathbb{P}\bar{X}_0^{-1}(\omega)$$

$$= \int_{\omega \in C} (h \circ \bar{X}_0)(\omega)\, d\mathbb{P}(\omega).$$

so $h(\bar{X}_\varepsilon) \xrightarrow{d} h(\bar{X}_0)$, where at $*$ we use billingsley,

i.e. $\mathbb{P}\bar{X}_0^{-1} D_h = 0$.

$\mathbb{X}_0 D_h = 0$.