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Problem 1

```
% A car traveling along a straight road is clocked at a number of points. The
% data from the obsercations are given in the following table, where the time
% is in seconds, the distance is in feet, and the speed is in feet per second.
```

```
% Time      0   3   5   8   13
% Distance  0  225 383 623 993
% Speed     75  77  80  74  72
```

a. Use a Hermite polynomial to predict the position of the car and it's speed when $t = 10s$.

```
time      = [ 0;   3;   5;   8;  13];
distance  = [ 0; 225; 383; 623; 993];
speed     = [75;  77;  80;  74;  72];

hermite(time, distance, speed);
```

The Hermite polynomial generated by the data in the table is

```
% p(x) = 75x + 0.222222*x^2*(x - 3) - 0.0311111*x^2*(x - 3)^2
%       - 0.00644444*x^2*(x - 3)^2*(x - 5)
%       + 0.00226389*x^2*(x - 3)^2*(x - 5)^2
%       - 0.00091319*x^2*(x - 3)^2*(x - 5)^2*(x - 8)
%       + 0.00013053*x^2*(x - 3)^2*(x - 5)^2*(x - 8)^2
%       - 0.00002022*x^2*(x - 3)^2*(x - 5)^2*(x - 8)^2*(x - 13)
```

at $t = 10$, the position of the car is at ...

```
syms x
p = 75*x + 0.222222*x^2*(x - 3) - 0.0311111*x^2*(x - 3)^2 ...
    - 0.00644444*x^2*(x - 3)^2*(x - 5) ...
    + 0.00226389*x^2*(x - 3)^2*(x - 5)^2 ...
    - 0.00091319*x^2*(x - 3)^2*(x - 5)^2*(x - 8) ...
    + 0.00013053*x^2*(x - 3)^2*(x - 5)^2*(x - 8)^2 ...
    - 0.00002022*x^2*(x - 3)^2*(x - 5)^2*(x - 8)^2*(x - 13);

double(subs(p, x, 10))

% p(10) = 742.5003 approx. 743 feet,
```

```
ans =

742.5003
```

at $t = 10$, the value of the speed of the car is ...

```
p_prime = diff(p);
double(subs(p_prime, x, 10))

% p'(10) = 48.3781 approx. 48 feet/sec.
```

```
ans =

48.3781
```

b. Use the derivative of the Hermite polynomial to determine whether the car ever exceeds a 55 mph speed limit on the road. If so, what is the first time the car exceeds this speed?

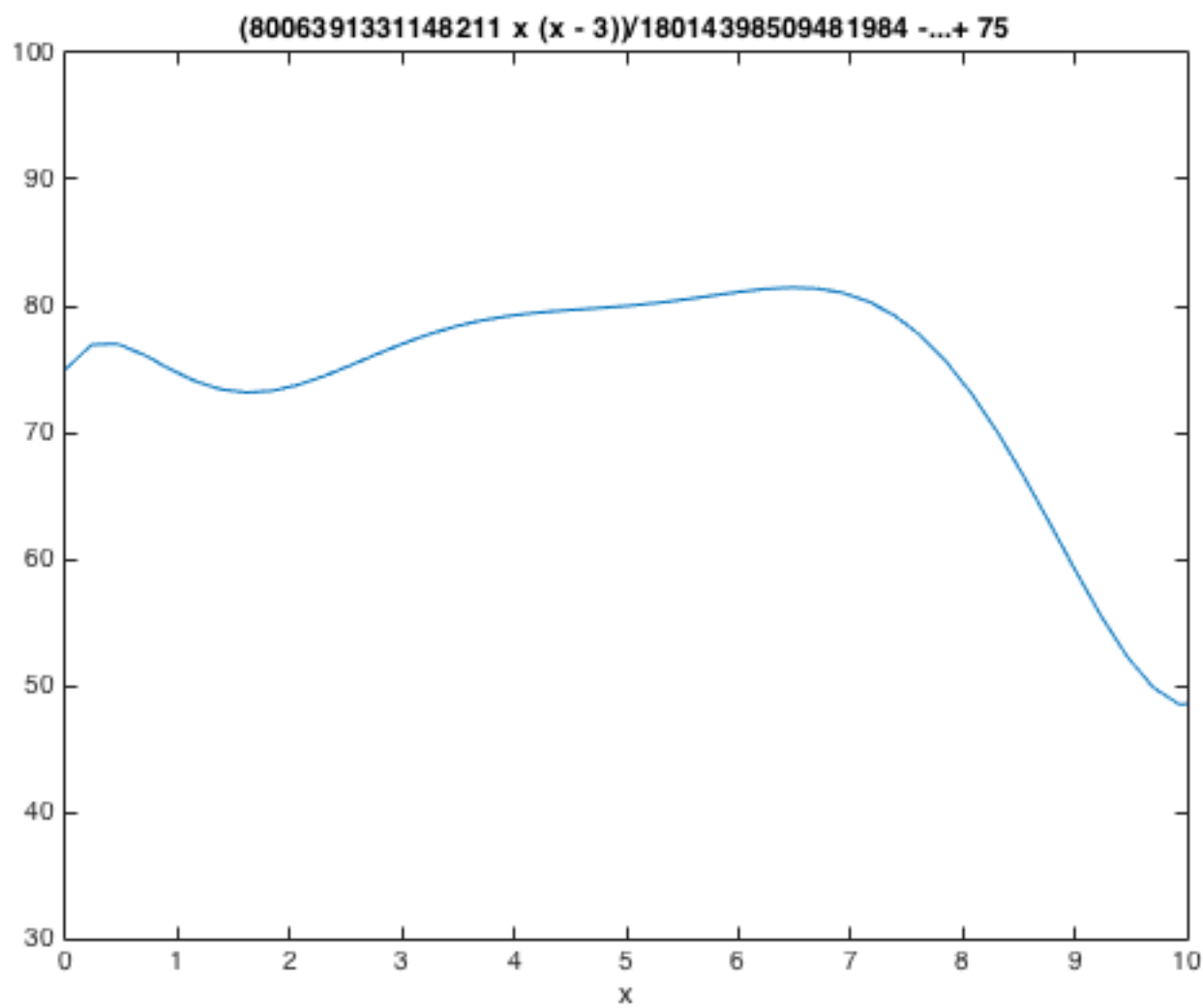
```
% 55 mi/h (80.67 ft/s)
% looking at the graph below, the value exceeds 80.67 ft/s

% approximately, by looking at the graph
% ----- 5.64880 -----

ezplot(p_prime, [0,10, 30, 100])
double(subs(p_prime, x, 5.64880))
```

```
ans =

80.6666
```



c. What is the predicted maximum speed for the car?

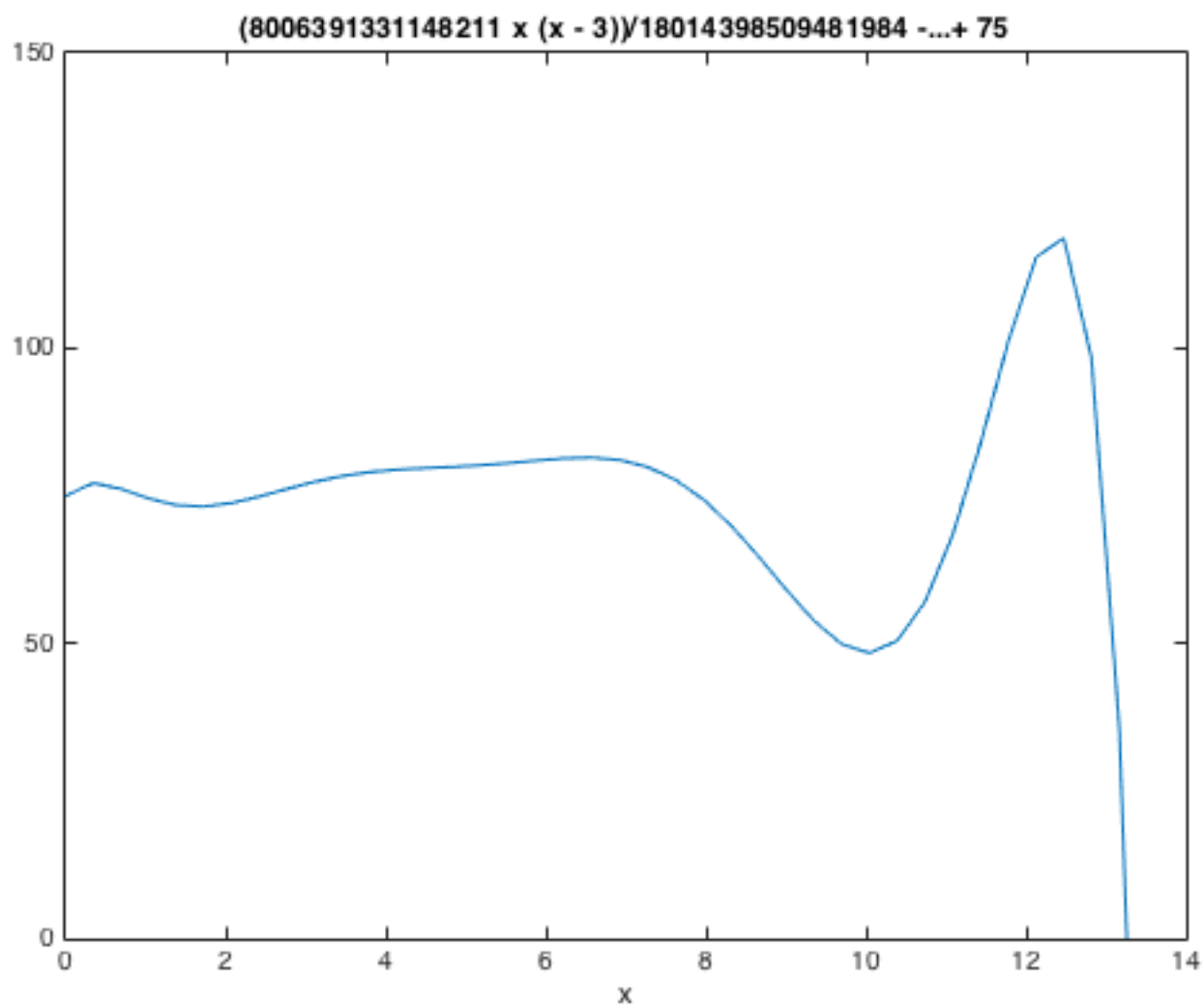
```
% looking at the graph of the derivative, the maximum speed of the car occurs somewhere between
n t = 12 and t = 13.
```

```
ezplot(p_prime, [0, 14, 0, 150])
double(subs(p_prime, x, 12.3737))
```

```
% approximately, by looking at the graph
% ----- 12.3737 -----
% and the speed is about
% ----- 119.4955 -----
```

```
ans =
```

```
119.4955
```



Problem 2

figure

```
% m = 20
% [x, y] = ginput(m)

% got the points below using the ginput
```

```
x = [0.678571428571429
0.524193548387097
0.411290322580645
0.275345622119816
0.275345622119816
0.406682027649770
0.494239631336406
0.586405529953917
0.692396313364055
0.733870967741935
0.717741935483871
0.567972350230415
0.404377880184332
0.328341013824885
0.243087557603687
0.173963133640553
0.109447004608295
0.111751152073733
0.141705069124424
0.190092165898617];
```

```

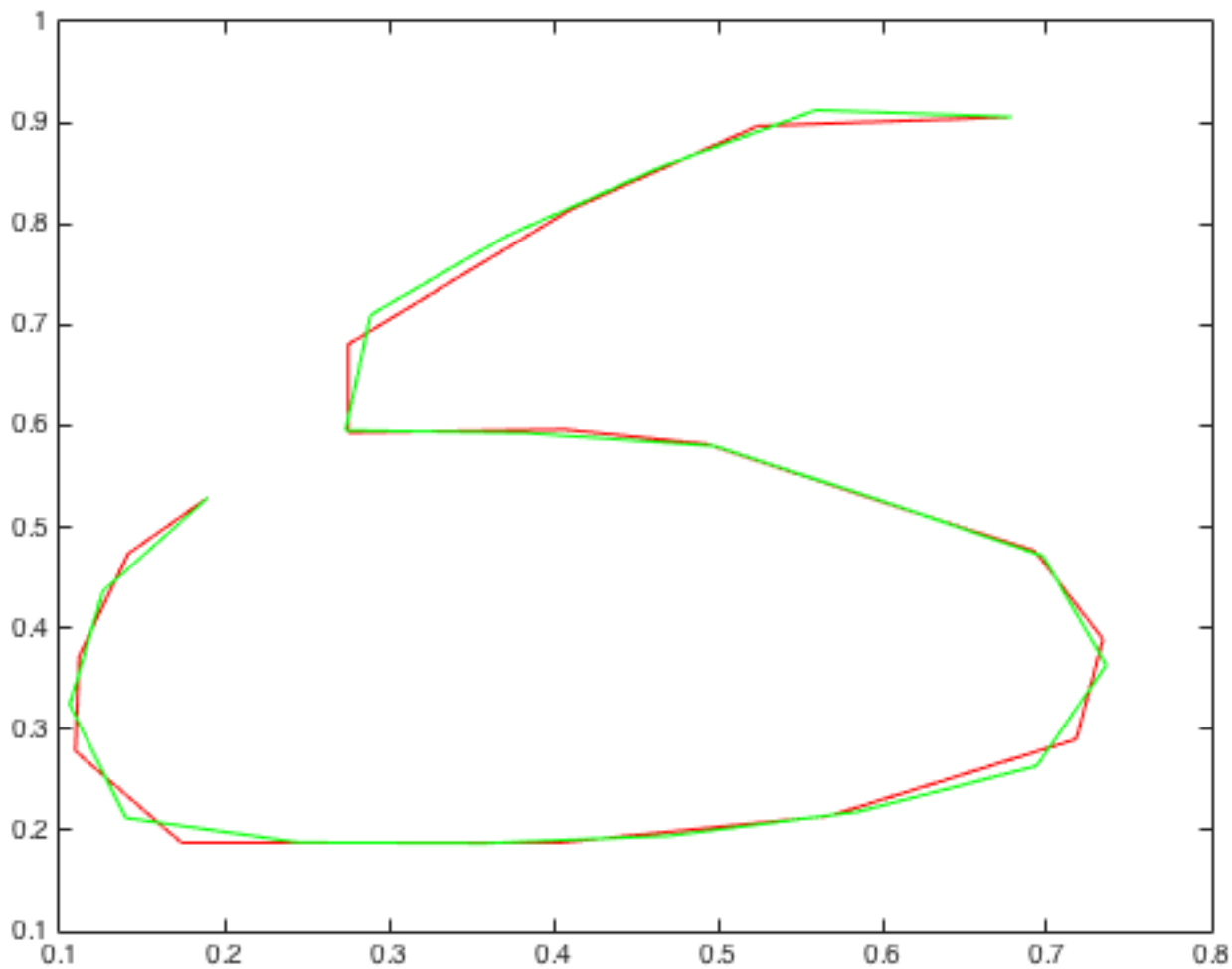
y = [0.905247813411079
      0.896501457725947
      0.814868804664723
      0.680758017492711
      0.593294460641399
      0.596209912536443
      0.581632653061224
      0.532069970845481
      0.476676384839650
      0.389212827988338
      0.290087463556851
      0.214285714285714
      0.188046647230321
      0.188046647230321
      0.188046647230321
      0.188046647230321
      0.278425655976676
      0.371720116618076
      0.473760932944606
      0.529154518950437];

```

```

plot(x, y, 'r')
hold on
[xi, yi] = SplineInPlane(x, y, m);
plot(xi, yi, 'g')

```



```
% Twelve data points (xi, yi) are given by
%     xi = cos(pi*i/6)
%     yi = sin(pi*i/6)
% These are obviously points on a unit circle with center at (0, 0). To
% close the circle, we get the 13th point whose coord. is identical
% to the 1st coord.
```

a. Use natural splines to fit x and y to parameter p for p values from 1 to 10 in increments of 0.1, and this generate a seq. of (x, y) points for plotting the circle

```
figure

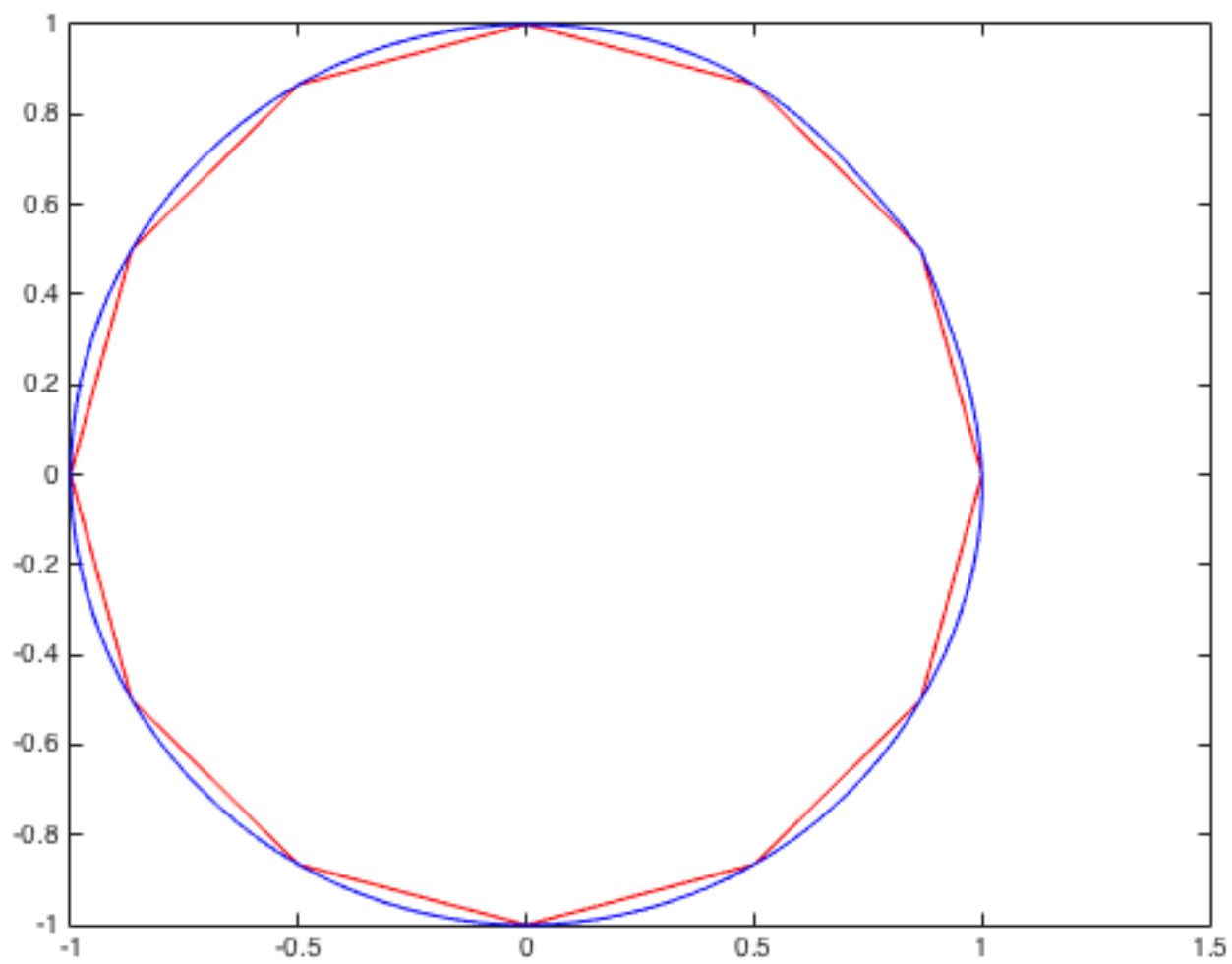
i = linspace(1, 13, 13);
x = cos((pi*i)/6);
y = sin((pi*i)/6);

[a_x, b_x, c_x, d_x] = natural_cubic_spline(i, x);
[a_y, b_y, c_y, d_y] = natural_cubic_spline(i, y);

% evenly spaced 121 intervals
intervals = linspace(1.0, 13.0, 121);

for j=1:121
    q = 1;
    for k=1:13
        if intervals(j) > i(q+1)
            q = k;
        end
    end
    t = intervals(j);
    nth_f = i(q);
    xx(j) = a_x(nth_f) + b_x(nth_f)*(t - nth_f) + c_x(nth_f)*(t - nth_f)^2 + d_x(nth_f)*(t - nth_f)^3;
    yy(j) = a_y(nth_f) + b_y(nth_f)*(t - nth_f) + c_y(nth_f)*(t - nth_f)^2 + d_y(nth_f)*(t - nth_f)^3;
end

plot(x, y, 'r', xx, yy, 'b')
```



b. Use a periodicity property of functions \cos and \sin to generate a periodic spline. compare this spline to natural spline in a) and to the unit circle.
