```python
import numpy as np
import pandas as pd

X_df  = pd.read_csv("./data/lab_10_10_X.csv",  header=None, sep=';', decimal=',')
Xp_df = pd.read_csv("./data/lab10_10_Xprime.csv", header=None, sep=';', decimal=','

X_full  = X_df.to_numpy()
Xp_full = Xp_df.to_numpy()

X  = X_full[:, :-1]
Xp = Xp_full[:, :-1]

print("X  shape :", X_df.shape)
print("X' shape :", Xp_df.shape)
print("Czy X' == X przesunięte o kolumnę?", np.allclose(Xp, X_full[:, 1:]))
print("X  :", X.shape)
print("X' :", Xp.shape)

def dmd_compute_A(X, Xp, r):
    U, s, VT = np.linalg.svd(X, full_matrices=False)
    Ur = U[:, :r]
    Sr = np.diag(s[:r])
    VTr = VT[:r, :]

    Atilde = np.linalg.solve(Sr.T, (Ur.T @ Xp @ VTr.T).T).T

    evals, W = np.linalg.eig(Atilde)

    Phi = Xp @ np.linalg.solve(Sr.T, VTr).T @ W

    A_dmd = Xp @ VTr.T @ np.linalg.solve(Sr, Ur.T)

    return A_dmd, Atilde, Phi, evals

r = np.linalg.matrix_rank(X)
print("rank(X) =", r)

A_dmd, Atilde, Phi, evals = dmd_compute_A(X, Xp, r)

print("A_dmd shape:", A_dmd.shape)
print("Atilde shape:", Atilde.shape)
print("Pierwsze wartości własne:", evals[:min(5, len(evals))])

Xp_pred = A_dmd @ X
rel_err = np.linalg.norm(Xp - Xp_pred, ord='fro') / np.linalg.norm(Xp, ord='fro')
print("Błąd względny", rel_err)
```

```
X  shape : (23, 36)
X' shape : (23, 36)
Czy X' == X przesunięte o kolumnę? True
X  : (23, 35)
X' : (23, 35)
rank(X) = 8
A_dmd shape: (23, 23)
Atilde shape: (8, 8)
Pierwsze wartości własne: [12.15287157 +0.j          -9.64854973+19.52264575j
 -9.64854973-19.52264575j  8.01045842 +6.81309823j
  8.01045842 -6.81309823j]
Błąd względny 4.315340024792404e-15
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In [ ]: