Dujiao Sieve

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instance

L3-3 可怜的简单题

式子推导不难，处理无穷求和是转化去一个等比数列。总之需要求的结果就是

$$\sum_{t=1}^{n}\mu(t)+n\sum_{t=2}^{n}\frac{1}{\lfloor\frac{n}{t}\rfloor-n}\mu(t)$$

inference

Note that factor with $\lfloor\rfloor$ just has $O(\sqrt{n})$ different values, divided into continous segments. So basically the difficulty lies in the prefix sum of μ.

Practice

Prove that

$$\sum_{i=1}^{n}\mu(i)=1-\sum_{t=2}^{n}\sum_{i=1}^{\lfloor n/t\rfloor}\mu(i)$$

Step1

Actually the technique here is re-combine the μ s by different direction, in $\sum_{k=1}^{n}[k=1]$. List $[k=1]$ s' μ s, row-ly. Column-ly combine them, to get

$$1=\sum_{d=1}^{n}\mu(d)\lfloor\frac{n}{d}\rfloor$$

Step2

proceed to transform the sum into prefix-like form, based on the index in the μ.

By observing that $\{\lfloor\frac{n}{d}\rfloor:d=1,\cdots,n\}$ is a ladder-shaped stuff, which was just mentioned in above content. We can exhaustively to make them into prefix-like form.

Technically, the process can also be visualized as row-ly to column-ly. Just imagine $\mu(k)$ corresponds a row with length $\lfloor\frac{n}{k}\rfloor$. The details and result will be clarified in the following step.

Step3

Enumerate $t = 1, 2, \cdots, n$, refers the position on the row of $\mu(1)$ (obviously the length is n), right now we're considering. The limit we can column-ly stretch, is the max $\boxed{X}$ which satisfies

$$\lfloor \frac{n}{\boxed{X}} \rfloor \geq t$$

The tricky technique here is that we can remove the $\lfloor\rfloor$ equivalently.

Then the thing is easy

$$\frac{n}{t} \geq \boxed{X}$$

So

$$\boxed{X} = \lfloor \frac{n}{t} \rfloor$$

Division-Segmentation

The principle of Division-Segmentation can be proved in the similar way in step3

So how to efficiently take advantage of the recursion-like formula?

algorithm

Denote that

$$S_n := \sum_{i=1}^{n} \mu(i)$$

Calculate $n = 1, \cdots, M$ in advance, do the recursion, then the time complexity should be

$$O(M + \frac{N}{\sqrt{M}})$$

So let $M = O(N^{2/3})$ to get $O(N^{2/3})$.

The details of the analysis can refer [杜教筛 - OI Wiki](). The usage of memorization lies in the fact

$$\lfloor \frac{\lfloor \frac{n}{x} \rfloor}{y} \rfloor = \lfloor \frac{n}{xy} \rfloor$$

swap x, y vice versa. So the possible value we may visit is really limited, corresponding to the very divisor.