Teaching Portfolio

Greg Knapp

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Contents

1 Teaching Experience

1.1 University of Oregon

Position title: Graduate Employee
Time at organization: August 2017–Present
Courses for which I have been instructor of record:

- Math 105—University Mathematics I
- Math 111—College Algebra
- Math 112—Elementary Functions
- Math 242—Calculus for Business and Social Sciences II
- Math 251—Calculus I: Differentiation
- Math 252—Calculus II: Integration
- Math 253—Calculus II: Sequences and Series
- Math 347—Fundamentals of Number Theory I
- Math 348—Fundamentals of Number Theory II

Courses for which I have been a teaching assistant:

- Math 111—College Algebra
- Math 241—Calculus for Business and Social Sciences I
- Math 242—Calculus for Business and Social Sciences II

Mentoring opportunities:

- Directed Reading Program: I mentored an undergraduate reading project in formal logic, ranging from the Gödel's Incompleteness Theorem to subsystems of Peano Arithmetic.
- Logic research: the previously mentioned reading project became a research project where my mentee and I are currently working on improving the results of my master's thesis. We have made meaningful progress and are currently writing up the results.

Other teaching opportunities:

- "Preschool" Instructor: I co-ran a two week long problem solving session with incoming first-year PhD students to help them decide which classes to take during their first year.
- AWM K–12 Outreach Committee: I designed and presented educational programs for K–12 students in museum and festival settings.

Teaching awards:

- Anderson Graduate Teaching Award—this award is given to two mathematics graduate students completing their fifth year of graduate studies for excellent teaching practices.

1.2 Case Western Reserve University

Position title: Peer tutor
Time at organization: January 2015–May 2017
Courses for which I was a peer tutor:

- Calculus I, II, III
- Differential Equations
- Linear Algebra
- Abstract Algebra
- Intro to Logic
- Math Logic and Model Theory

1.3 Boonshoft Museum of Discovery

Position title: Summer Educator
Time at organization: Summer 2012–2017
Summer camps I created and taught:

- Project Mindbender: A math/logic/computer science camp focusing on teaching basic reasoning skills through game playing, scavenger hunts, and magic tricks
- eMagination: A computer science camp in which students learned to create their own Minecraft modifications
- Minecamp: A Minecraft-themed camp in which students learned the basics of materials science, computer programming, fencing, and cryptography

1.4 Cleveland School of the Arts

Position title: Tutor
Time at organization: August 2013–December 2013
Courses for which I tutored: Math, Physics, Chemistry, English

2 Teaching Statement

My teaching philosophy is fundamentally humanist. Teaching is done by people, for people, and it comes with all of the challenges and joys associated with human interaction. Good education engages students, gives them choice, and celebrates their individuality. A successful math teacher does this by teaching students the "whys" more than the "whats" so that students can see the structure of mathematics in their own lives. Moreover, this hypothetical educator gives students the tools to communicate mathematics with others, and shows students that mathematics is not a solitary activity, but a project collectively undertaken by humanity. I implement this holistic picture of education by enabling my students to construct knowledge themselves, teaching my students the language of mathematics, and giving my students and peers a voice in the classroom.

One of the most important lessons I have learned as an educator—whether as a summer camp teacher at a children's science museum, a tutor at my undergraduate university, or an instructor in graduate school—is that students learn best by constructing knowledge for themselves. I use this principle when designing worksheets and assignments where mere scaffolding[1] may not be enough to help students acquire a deep understanding. In the Sequences and Series course that I taught in the spring term of 2021, my students were struggling a great deal with understanding the distinction between the numeric objects in the course (the sequences and series) and the functional objects in the course (the standard calculus functions and their Taylor series). Rather than give a lecture to address their concerns, I designed a worksheet titled "Putting the Pieces Together" which gives students the opportunity to encounter the necessary cognitive dissonance to learn the relationships between these concepts[2]. For example, I wanted my students to understand the statement "the fourth Taylor polynomial of $f(x)$ is the fourth partial sum in the Taylor series for $f(x)$," so I designed a section of this worksheet to juxtapose questions about partial sums of series with questions on Taylor polynomials. Several students told me that putting the pieces together for themselves on this worksheet helped them clarify concepts that had previously been confusing.

I also enable students to construct knowledge for themselves through in-class activities. For example, when teaching trigonometry, I gave each student a compass and asked them to draw a circle of any radius they chose. Next, students cut a piece of Twizzler candy whose length was equal to the radius of their circle and then measured how many times that piece of candy wrapped around the circle. We collected the class data, averaged it out, and found that there are about 2π radians in a circle, no matter what the radius is. After this activity, I received very few questions about what a radian actually was—after all, the students had eaten one! With these kinds of activities, students are able to see through imposing mathematical jargon to the deeper structures underlying the natural world.

While constructing knowledge is incredibly important for internalizing mathematics, that knowledge is hardly usable without the ability to communicate it. Moreover, communication and language learning are some of the most transferable skills students can acquire in any class, so I actively help my students learn the linguistic component of mathematics. To help students acquire these skills, I provide students with resources to help them internalize the new vocabulary they are expected to learn: I clearly mark definitions on their lecture guides, I provide Frayer model diagrams to help them learn new terms, I include a vocabulary section on each exam, and before each exam I assign a self-evaluation for the vocabulary terms. In the most recent courses I taught, I also assigned oral presentations so that students could practice speaking the language of math. For example, in Integral Calculus, students chose among the following topics: a modeling problem about a backyard

[1] Scaffolding is an assignment-design technique which helps students build up to complex problems from basic ones.

[2] This worksheet is found in section 4.2.

marathon runner, a problem comparing the differential calculus method of linear approximation with a Riemann sum approximation method, a modeling problem calculating the volume of a red blood cell, and a problem revealing how integration by parts plays a role in understanding the prime number theorem. In Fundamentals of Number Theory, I assigned a group expository paper and presentation on a topic of the group's choosing. Each of these projects allowed my students to practice writing and speaking mathematics, and I often hear from students that these projects are their favorite part of the course.

When students have built their own intuitive understanding of math and possess the skills to communicate their knowledge, they are ready to participate in the mathematical community. I model my belief in the power of the mathematical community by incorporating my students' and my peers' ideas in the classroom. With my students, I focus on listening to their input, advocating for their future, and mentoring them during their mathematical journey. Every term, I give my students extra credit for completing a midterm feedback survey on which I ask for their input. Often, students give me helpful suggestions, and their input has led to restructured due dates, a revamped peer review system, and more time for in-class worksheets.

After listening to my students and getting to know them better, I am then able to effectively advocate for them. I wrote recommendations and served as references for my students at all levels, from University Mathematics I to Fundamentals of Number Theory. I also served as a mentor in the Directed Reading Program (DRP) at UO. In 2020, I organized a reading project on Gödel's Incompleteness Theorem with an undergraduate student, and in 2021 we continued that project by reading together about subsystems of first-order arithmetic in the DRP. This spun into a research project which we continue today by investigating geometry in Elementary Function Arithmetic. Our research helped him decide that he wanted to do math research for a career, and he is now a PhD student at Montana State University.

Moreover, I build community with my students by incorporating inclusive and equitable policies and practices. I accomplish this in part by minimizing the impact of my personal bias in student assessments and being transparent about my grading process. Towards this goal, I use UO's learning management system to implement blind grading and to publish the rubric I am using for the assessment. I also try to help my students see themselves in the math community. For example, when creating word problems, I will write something like "You are trying to send your friend a message" instead of "Alice is trying to send Bob a message."

I could improve upon my classroom's inclusivity by leading more effective classroom discussions on race. In my Fundamentals of Number Theory course in the winter term of 2022, I tried to lead a class discussion on the differences between the names "Sun-Tsu's Theorem" and "the Chinese Remainder Theorem." This conversation was intended to help my students think about the impact of their word choices; however, it was largely unsuccessful. In the future, I would like to improve the authenticity of the discussion, which I could work toward in three ways. First, I would ask my students to think about the theorem names before class so that they could collect some preliminary thoughts. Next, I would begin the discussion in a think-pair-share format so that every student has a chance to grapple with these issues. During the full class discussion, I would then ask my students what role (if any) the historical difficulty of ascertaining exactly who first stated/proved the theorem should play in the discussion of its name.

My students are an important part of the math education community, but so too are my peers. I build an education community with my peers through vertical integration: I seek out mentors who can help me improve my teaching practices, and I try to help others who are newer to teaching than I am. Three years ago, I co-founded and have since co-organized the Graduate Student Teaching Seminar at UO—a weekly seminar where graduate students meet to discuss best practices and pedagogy. Much of the specific implementation of my teaching philosophy arose from conversations in this seminar: our readings about metacognition yielded my self-assessment vocabulary assignments, our meeting on project-based learning inspired me to assign presentations, and our discussion of APOS theory influenced my worksheet designs.

I have come a long way in developing a teaching philosophy since I began my educational journey as a camp counselor in 2012. Many of my foundational ideas about teaching—like students constructing knowledge for themselves and learning math as a language—came from my own influential teachers. Other ideas—like learning from student feedback and talking with my peers about best teaching practices—have arisen naturally from the experience of teaching. And while my teaching practices have changed a great deal over time, there are many good ideas I have yet to experiment with in my teaching, from assigning reading and study habit reflections to mastery grading. I'm excited to continue learning, growing, and helping students discover the joy of math!

3 Teaching Evaluations

3.1 Professional Reviews

Each year in graduate school, my teaching has been evaluated by a UO faculty member. The faculty member will typically review my syllabus (if relevant) and attend a class on which they will provide some comments. I believe I was scheduled to be evaluated during the spring term of 2020, but this was canceled due to remote teaching during the COVID-19 pandemic.

3.1.1 Calculus for Business and Social Science I, Winter 2018 (TA)

In this course, I was a TA and Mike Price sat in on one of my discussion sections. The lead instructor for the course had provided me with a rough outline for how to structure the discussion section, so I only had control over the implementation of that structure.

Price's evaluation was separated into two sections: a list of pre-written teaching objectives on which I was scored as "needs improvement," "satisfactory," or "nearly perfect" and a free-form list of comments on successes and suggestions for improvement.

In the first section, I received scores of "satisfactory" or "nearly perfect" in every category. In the second section, Price's comments are listed below. As an early instructor, I was successful with my explanations of course material (and my time as a tutor gave me practice with explanations), but I struggled with integrating student input into the lecture component of the discussion.

<u>Comments on Successes</u>

- Good introduction to weekly focus items; can a brief summary be written on board as well?
- Interesting conceptual quiz questions, are these ones you wrote on your own or as provided by lead instructor?
- Good check in about whether students wanted additional problems done before they start on their own work
- The analogy about d/dx as an action to take is a good distinction. although a hard one for them to grasp (you could relate it to an operation they're familiar with, e.g. "multiplying by d/dx is like multiplying by a plus sign, it just doesn't make sense")—often the subtlety between the term $\frac{dy}{dx}$ and the operator $\frac{d}{dx}$ is what trips them up

<u>Suggestions for Improvement</u>

- Especially for a quiz that students have already worked on, ask for student contributions in solving individual exercises
- Similar comment for working on new exercises, at least a couple of students were eager to contribute to its solution,use them and invite others to answer little components (e.g. "what's the derivative of this constant?"), they should know the answer, but it paces it out so they are with you
- Structure of discussion is largely determined by lead instructor, so we omit a conversation about use of class time

3.1.2 Differential Calculus, Winter 2019 (Instructor of Record)

The department lost the records of this evaluation, but the evaluator still had some notes from the session and sent the following to me.

- You started and finished on time
- Your handwriting was clear (you used markers and a projector to work on a handout), nicely organized and neat and your presentation style facilitated note taking (you provided students with handout with extra space to write on)
- You knew names of some students and encouraged student thought and participation
- You answered student questions
- You demonstrated content-competence
- You included graphical illustrations (even using different colors of markers)
- You made connections between things that students have already learned (Chain Rule) and a graph of $e^{-x)}$ having no horizontal intercept (meaning $e^{-x} = 0$ has no real solution)
- You explained some concepts in more than one way (first derivative sign analysis done via graph and a sign change on a real number line)

My few suggestions were mostly regarding how to set up technology-related components, namely the projector was not doing very well with all lights on (that was in Deady 106) as there was not enough contrast and so dimmed lights would be advised, if possible. You also used different colors to emphasize different things (the sign change of f'), yet they were again almost indistinguishable due to lack of contrast while using the projector. My other remark was to allow students to help you to do some things (like factoring out $e^{-x} - xe^{-x} = 0$).

3.1.3 Integral Calculus, Winter 2021 (Instructor of Record)

This was the third course which I taught over Zoom and I was evaluated by Peter Ralph. Ralph examined my course syllabus and webpage (on Canvas, the UO's learning management system) and sat in on my class. He then provided comments. Ralph noted that I was successful with my course organization, my rapport with my students, and my willingness to improve my pedagogy. He suggested that I spend more time in class talking about the big picture rather than the details. Here are his comments:

> I am writing this teaching review on the basis of observing Greg Knapp's "Calculus II" class in Winter 2021, after attending a class (remotely) and reviewing teaching materials (on Canvas). In summary, I found Greg's class to be well-organized, and Greg's teaching and explanations to be very clear and well-illustrated. I was impressed with his teaching philosophy and his motivation to learn and work on good teaching practice.
> **Professional teaching:** I was favorably impressed by Greg's course organization on Canvas, including comprehensive syllabus information, including learning outcomes, expectations, outlines of course assignments, tips for success, inclusivity, and accessibility. The Canvas site makes it easy to see what's due and what's expected in the class, and instructions are clear and reasonable. The activities are well-designed to encourage

student learning, with questions that reflect the topics covered in class that week of various levels of difficulty. Homework solutions are provided. Greg has clearly put a lot of thought and work into structuring the class to be easy and clear to interface with.
Inclusive teaching: Expectations are outlines in the syllabus that provide flexibility, for instance, by allowing participation credit either through answering clicker questions or by engaging in discussion on Canvas. I especially liked the lists of "things I expect from you", "things you should expect from me", and "tips for success" on the syllabus, provided to make explicit the things students are expected to know (but might not). The statement on inclusivity in the syllabus is clearly sincere and supportive. In class (over Zoom), students asked questions either by video or chat, and Greg often explained things in more than one way (e.g., algebraically and visually). Greg has extremely clear diction and excellent "boardwork" (drawing electronically on a pdf).
Engaged teaching: I enjoyed discussing pedagogy with Greg after class - he's clearly thought a lot about it, and has many good (and research-informed) ideas. For instance, two innovative strategies are: a verbal assignment, where students explain how to do a problem to him, and giving students lists of vocabulary word topics as a way of checking understanding. Greg is also very engaged, enthusiastic, and accessible in class – students did not seem to have a problem asking questions or engaging with him (which is particular impressive in a zoom classroom). My main feedback for Greg was to spend a bit more time on the big picture – the class that I attended spent a lot of time in the details of particular examples (although at an appropriate pace and with a break) – however, this was a minor suggestion. **Research-informed teaching:** As I mentioned above, Greg has actively worked on teaching pedagogy, joining discussion groups with other math department instructors. In particular, he's working on teaching math as a language, including awareness of vocabulary and creative thinking. Furthermore, he explicitly discussed teaching pedagogy during class, which can help students better learn how to learn. In class, when explaining new tools (when I observed, the topic was finding areas between two curves) he built up from simple tools to higher-level thinking, and dealt with mistakes well.
In summary, I found Greg to be very organized, engaged, and thoughtful teacher, actively working on his pedagogical practice, and running a good calculus class.

3.1.4 Fundamentals of Number Theory I, Winter 2022 (Instructor of Record)

This was the first proof-based math course I taught and I was observed by Ellen Eischen. Eischen provided comments on the class in which she sat in. She observed that I had a clear and well-organized lecturing format and successful student interaction during class, and she suggested that I spend more time engaging with students during class. Her full comments are here:

> I observed Graduate Teaching Fellow Greg Knapp's Math 347 (Inroduction to Number Theory I) class on Monday, February 28, 2022. This is a 50-minute-long number theory course aimed at undergraduates who are seeing the topics for the first time. The class was taught in a hybrid mode, with 10 students attending in-person and at least 4 students attending remotely. The day I visited, the lecture focused primarily on Wilson's Theorem. Overall, this was a well-taught class. Strengths include:
>
> - The entire lecture, which included interaction with students, progressed at an appropriate pace and was well-organized.

- Knapp appeared to be a calm and confident instructor, like someone who has a lot of experience teaching undergraduates and has reflected on best practices.
- Knapp projected well and had a clear voice. He also checked to make sure that people in the back of the classroom could see okay.
- Students appear to be comfortable asking and answering questions. Several students contributed questions or answers during the lecture I observed.
- Knapp did an excellent job motivating Wilson's Theorem. He said that his class's goal would be to motivate their study of quadratic reciprocity, and in turn, he linked quadratic reciprocity to some exercises students had already encountered on the homework.
- Knapp did an engaging interactive example in which students explored cases of Wilson's Theorem. Every student in the class, even two near the back who had originally been scrolling on the phones, got engaged in this example and discussed it with a partner. It was impressive to see all the students in the class work actively with a partner on the exercise. One student did not have a partner, so Knapp discussed the problem with him.
- In interactions with his students, Knapp responded with a calm but enthusiastic demeanor. He is clearly skilled at interacting with his students.

Area for improvement:

- I would strongly recommend interacting with your students earlier in the class period and more frequently. I was only there for one class, so I don't know for sure if the class I visited was typical. That said, the day I was there, the first 8 minutes were spent going over LaTex commands on the screen, and the students seemed completely unengaged for that period. At least two were doing unrelated things on their phones in the back of the room. As soon as you introduced an interactive example, though, everyone became actively engaged. This was the rare class where the energy-level and engagement-level seemed to increase throughout the lecture period. It would be great to have more of this starting at the beginning, especially since it is clear that you are highly skilled at interacting with your students.

Below are some additional details concerning the university's four new required categories:

Professional Teaching

Knapp's in-class exercise, described above, was clearly designed to maximize students' understanding of the material.

Inclusive Teaching

All the students participated in the interactive exercise. Even if they did not speak aloud to the entire class, each student got a chance to discuss the material with a partner. Greg also prompted students for questions and encouraged them to ask questions.

Engaged Teaching

Knapp clearly cares a lot about teaching and actively seeks out opportunities to improve. In fact, he took the initiative to approach me and ask for my feedback (since from being a student in my classes, he knows I also care a lot about teaching), even though he was not scheduled to be observed this term.

Research-Informed Teaching

Knapp did a great job linking together different topics (e.g. linking the upcoming study of quadratic reciprocity to the current focus on Wilson's theorem) and different parts of the course (e.g. introducing aspects of examples of quadratic reciprocity in earlier homework assignments).

Concluding remarks

Knapp is a skilled and thoughtful instructor. Students clearly respond well to his prompts to engage and interact with the material and with him. My only suggestion for improvement is to schedule some interaction earlier in the lecture and maybe even more often.

3.2 Student Evaluations

The University of Oregon changed evaluation systems partway through my time in graduate school and rather than attempt to describe and give data from both systems, I will merely report selected student comments from different courses.

3.2.1 College Algebra, Fall 2017 (Instructor of Record)

- "Greg is an exceptional teacher and does an amazing job at making difficult concepts clear and digestible. Only possible improvement would be spending less time on some of the more simple concepts, which is beneficial to students who are struggling. Though tends to slow the overall pace of the course."
- "Explained problems with much detail. Needs to face classmates more when he speaks and writes on the board."
- "The instructor is clearly new to teaching and has some areas to improve on. The areas for improvement were that we would use metaphors a lot when teaching and not show an example of a problem that wasn't a metaphor. Seeing a real problem worked out helps a student break down and learn how to do a problem on their own. He also wouldn't go over problems enough times or slow enough to the point where the class as a whole would miss chunks of topics."
- "Greg Knapp explained the course material in an exceptional manner and was always available to help when there were questions about material."
- "There was a disconnect between the homework and the material taught in class and the exams. The written work and webwork covered material that was very difficult and in class we usually covered the basics and a very easy example problem. It would have been useful to go over harder problems in class. However, I've felt prepared taking all the exams in this class because what is on the exams is never something we hadn't covered with the coursework."

3.2.2 Calculus for Business and Social Sciences I, Winter 2018 (Teaching Assistant)

- "He was very enthusiastic and open to feedback. He was also very good at adapting to what was needed of the discussion. He could manage time a little better as class often felt rushed."
- "Greg was very good at communicating with the class about what would be discussed during the discussion time. Also he did a nice job of going through examples at a speed in which

everyone could understand. Although he did a good job of fixing this halfway through the term, he could definitely do more examples on the board and focusing on material that would be on the quiz. I give him credit for asking us for feedback and implementing this change."

- "Greg was very friendly and helpful. He made an adjustment in the format of the discussion which was much more fluid. I went from getting a 1 on the quiz, to a 9, and 10 on the following after the change."

3.2.3 Elementary Functions, Spring 2018 (IR)

- "At first he was rushing through material and expecting us to learn it quickly, but after the midterm he slowed down and made sure we understood what we were learning."
- "good teaching and always willing to work with schedule for office hours, however, consistency between homework problems and test problems and review sheets felt a little off. even with all the materials, I found it hard to study because the tests were so much different than the homework."
- "He made much more of an effort to communicate with us and make sure we really understood the material than most professors do (and he's not even an actual professor), which I really appreciated because sometimes the material felt hard to grasp."

3.2.4 Calculus for Business and Social Sciences II, Fall 2018 (IR)

- "The class had hard material that was well explained. The examples helped, and the exams weren't shockingly harder that what we have been taught. If you went to class and did homework the class was manageable. Extra reviews sessions before exams would've been helpful as well."
- "could tell he really cared about us learning and wanted to help us learn if we had any problems. Was also flexible and realistic with expectations helping us as students."
- "I think the instructor did a good job of breaking down the steps to get to the answer and also explaining all the necessary things needed to pass the class. I think one thing he could improve on is to assign less homework because for weeks where students have a lot on their mind already, it can become really tedious at times just to do the homework. I would recommend one written homework and one webwork that covers all that is learned throughout that week that is due Sunday or something."
- "It was helpful that quizzes were a check for the classes understanding, not graded for correct answers."

3.2.5 University Mathematics I, Summer 2019 (IR)

- "Greg was hands down the best graduate student teacher that I have had for math at UO. He took the time to explain things and genuinely cared and took the time to make sure that everyone understood what was going on. He balanced out class time very well which is hard to do in such a short 8 [sic] week course. He was able to relay the information in an easy to understand way."
- "Greg is incredibly passionate and knowledgeable about Math. He actively accepts and reflects upon class comments and feedback, and has a very dynamic teaching style as such."

3.2.6 Calculus I, Winter 2020 (IR)

- "I loved this course. Although the material got pretty challenging towards the end, he made sure to help us as much as he possibly could. He encouraged us to always ask questions and also for feedback which was great!"
- "Greg is always there to help if you need him to, very accessible and open to meet one on one. he explains concepts thoroughly and with a cheerful attitude, which is much appreciated."
- "I personally like the course textbook, the fact that it is free, and online versus a physical copy and or pricey. also LOVE the lecture guides, they really make a difference in not only understanding the class material, but just overall make taking notes in class much easier than trying to copy things off the board and into a notebook. I feel like reading the definition and having the problems/questions in Greg's words is much easier and more beneficial than if we were to copy things from the board."
- In response to the question "What specific changes in the organization of the course would help your learning?" a student said "The only thing I would like is for the material to be in order instead of skipping through and going back."

3.2.7 Calculus II, Spring 2020 (IR, Remote)

- Note: there were no university course evaluations this term due to the COVID-19 pandemic. However, a student sent me the following unsolicited note: "I just wanted to say that you have been a phenomenal teacher. You somewhat seamlessly transitioned to the online format, you've been incredibly kind the whole term, and you've done a exceptional job articulating complex concepts that otherwise would have been rather overwhelming. I remember at the start of the term you saying that your end goal was to find a job teaching at a small liberal arts college. While plans may change you are a cut above any other math teacher I've had. I am certain you'll thrive if you continue Pursuit of a career in education."

3.2.8 Calculus I, Fall 2020 (IR, Remote)

- "The teacher had lots of office hours, and was always available through email. This really helps in a virtual learning environment."
- "There was no active learning involved whatsoever in this course. I think there needs to be a lot more discussion and questions because every class was just lecture and note taking."
- "I'm not sure where this falls, but not a fan of break rooms; especially when they take up a lot of time"
- "I feel like this course is very conducive to my learning. Although it does go very fast sometimes and I feel lost, I've learned a lot and the exams are definitely a good test of our knowledge."

3.2.9 Calculus II, Winter 2021 (IR, Remote)

- "Greg's expectations were clear and he genuinely wants his students to succeed, he is a great instructor and any university would be lucky to hire him!!"

- "I liked the teacher's feedback because I always had a good idea of how to improve the quality of my work on the written assignments."
- "I like how the lectures are organized and don't go too fast. I also like that you have lecture guides because when teachers don't it's hard to know what is important."
- "I don't really like the oral presentation because I am not a great presenter and it makes me nervous to speak in front of someone, especially about math."

3.2.10 Calculus III, Spring 2021 (IR, Remote)

- "The support from the teacher was 100/10. Office hours are the best thing ever and super super helpful"
- "There was one time in this class that everyone agreed we could as a whole have done better and we spent the day after this happening just talking about what happened and how we could all do better. I felt this was really good and a really good way of keeping everyone engaged in the course."
- In response to the question "What specific change in the use of active learning would help your learning?" a student said "This is really only an issue I feel like because things were online. Even with things being online Greg but an obvious effort in to make things 'active'."

3.2.11 Calculus II, Fall 2021 (IR, Hybrid)

- "Greg was super supportive in making sure that the whole class understood the subjects we were learning. When someone asked a question he would answer it multiple ways until they understood it, not just give it one answer. He took time to ask the students how we wanted to make changes to the course structure and almost immediately put those changes into the course, quickly reorganizing everything for us."
- "Greg was very accommodating with me and my ADHD."
- "I really enjoyed the guided notes. That always makes learning a lot easier when you can easily follow and organize your own notes."
- "Participation through Socrative felt a bit like a chore rather than an opportunity to apply. The groupwork format was a good change."

3.2.12 Fundamentals of Number Theory I, Winter 2022 (IR, Hybrid)

- "The instructor has done a great job at making office hours available for everyone. I feel like I learned the most over all during office hours and outside communication from the instructor."
- "With the very specific and detailed feedback from homeworks, I've learned a lot what I did good and what I did not good. It really helps me to answer a math problem in a professional and mathematical way."
- In response to the question "What specifically about the use of active learning helped your learning?" a student said "Greg does a great job encouraging students to ask questions and providing weekly opportunities to apply what we've learned during class time"

- "Every Friday we were given worksheets to work as a group with other students and this helped me. How? Well I am the kind of person that does not do much for class other than the homework and assignments. Within a group of students I was able to reinforce my understanding of class material and work through different applications that we might not cover on the homework."
- "I liked the Draft/Review/Final model for homework assignments since I got to see other ways to solve problems and it forced me to start thinking about each assignment early."
- "Greg offers both in person as well as a synchronous zoom option, which is very accommodating and allows everyone to be able to attend class and receive the same course material."
- "The course organization is beneficial to my learning because I am able to do a draft of a homework get it peer reviews and then revise to submit the final. But this is like 3 assignment s for one HW so it might be beneficial to my overall learning but it can be overwhelming during midterms weeks with other classes."
- "Assignments usually have little connection with the textbook or with lectures. The lectures tend to be primarily running through proofs with little examples or explanation of their uses, leaving students ill-prepared for any use or knowledge to solve problems. The peer review for the class is more harmful than helpful, leaving students with incorrect comments that could have easily been remedied if the comments that the instructor left on everyone's peer reviews were instead focused on the person's work who was being reviewed. The instructor seems to have many too many assignments as he many times would fall weeks behind in grading leaving students with little feedback so they continue to be marked down for making the same mistakes over and over again from lack of timely feedback."
- "I think Greg has been doing a great job as an instructor. He cares about every student and I feel supported thought the term. The format of this class was designed in a good way so that we have time to practice in class and have enough time to finish homework and really learn stuff from revising homework. Especially the peer review session and using portfolio to replace 'regular in class paper exam'. The peer review session gives me a chance to learn how other student approach to the answer of same problem. The portfolio part provides a really good way to summarize and conclude what I have learned so far, it's kind of like a review but let you dive further into the knowledge."

3.2.13 Fundamentals of Number Theory II, Spring 2022 (IR, Hybrid)

- "He helps us a lot with projects and assignments. He always uses different ways to follow and answer our questions."
- "I had a very positive experience with the final project which included some independent study."
- "Having group work was helpful to discuss and think through the concepts that we learned each week."
- "Although challenging, I really like the course portfolio and final presentation/paper. I felt like doing these instead of exams helped me learn more useful, longterm, mathematics skills."

- “I would recommend taking another look at the section of the curriculum on partitions. I think that week’s homework was much harder than the other ones. Going from the usual course material to more combinatorial thinking was jarring.”
- “Course organization needs improvement, specifically in regards to assignments, group work, and peer review. I suggest reading assignments to help prepare for each week, more time to discuss problems in class and work with peers, and better rubrics and structure for peer reviews. (Perhaps doing them during class so students can receive feedback from instructor so students can receive beneficial feedback.)”
- “I think I’ve warmed up to the peer review idea. This second term I’ve had some good reviews that pointed out when I’ve cut too many corners in my proofs. I’ve also had some reviews of lower quality but I think this is unavoidable.”

4 Teaching Materials

4.1 Sample Syllabus

Below is the syllabus for my Fundamentals of Number Theory II course that I taught in the spring term of 2022. I always try to provide a course syllabus that can function as a complete course guide which can answer any student question about the course (or at least point them to where they can get their questions answered). The syllabus covers policies about the classroom environment, the who/what/when/where on course meetings and office hours, a course schedule, learning outcomes, the grading scheme, weekly assignment schedule, generic assignment rubrics, list of my expectations for my students, list of things my students can expect from me, and tips for success in the course.

Instructor: Greg Knapp (he/him)
Office: Fenton 312
Email: gknapp4@uoregon.edu

Office Hours:
Mo/We: 11:00 am–12:00 pm
Thu: 9:00–9:50 am
Fri: 2:00–3:10pm

1 Classroom Environment

1.1 Inclusivity

Historically, mathematics has been an exclusive discipline and has shut out people for their race, gender, sexual orientation, political views, etc. While terrible, this cannot be undone. However, we can work to create a more inclusive environment for current mathematics students. Racism, sexism, ableism, other discrimination or harrassment, or general behavior that creates an unwelcoming environment will not be tolerated in this classroom. Furthermore, even if you do not see people like you represented in the mathematical community, you still belong here. Know that I will support each of you in your mathematical journey because math is for *everyone*.

What can you do if someone is creating an unwelcoming environment in this classroom?

1. Talk to me. I will do my best to work with the individual who is creating an issue and resolve it. If I can't do this, I will talk to someone at the university who can resolve this. If I'm creating the problem, I will listen to you and do my best to correct my behavior.

2. If you don't feel comfortable talking to me, my supervisor is Mike Price (mprice@uoregon.edu).

3. Talk to a university official. You can find more information about your options at `https://respect.uoregon.edu`, `https://safe.uoregon.edu`, and `https://investigations.uoregon.edu`. You can also contact the non-confidential Title IX office/Office of Civil Rights Compliance (541-346-3123) or Dean of Students offices (541-346-3216) or call the 24-7 hotline 541-346-SAFE for help.

1.2 My Reporting Obligations

I am a student-directed employee. In short, this means that I listen to your request when deciding whether or not to report something that you disclose to me to the university. For detailed information about my reporting obligations as an employee, please see my Employee Reporting Obligations at `http://titleix.uoregon.edu/employee-reporting-obligations`. I am also a mandatory reporter of child abuse. You can find more information about the Mandatory Reporting of Child Abuse and Neglect at
`http://hr.uoregon.edu/policies-leaves/general-information/mandatory-reporting-child-abuse-and-neglect`

1.3 Mental Health and Well Being

College can be overwhelming in a number of ways. If you are struggling with your mental health and need some deadline flexibility, talk to me. You can also find support through the Duck Nest (`https://health.uoregon.edu/ducknest`) and through University Counseling Services (`https://counseling.uoregon.edu` or 541-346-3227). I've been to UCS and I had a really positive experience with them.

1.4 Academic Integrity

You are free to work with others when studying or doing homework. Unless explicitly instructed otherwise, however, you must submit your own work. For a full description of academic misconduct, see the Student Conduct Code at `https://policies.uoregon.edu/vol-3-administration-student-affairs/ch-1-conduct/student-conduct-code`. Academic misconduct will be reported to the university and it will result in a zero on the assignment on which academic misconduct occurred. Multiple or egregious instances of cheating will result in an 'F' in the course. *This policy is non-negotiable and I am not willing to discuss alternate consequences.*

You are now at the point in your career where it is critical to acknowledge your collaborators and sources. You are encouraged to work with others, you are allowed to look up homework problems online, and you are ***required*** to acknowledge your collaborators and sources. For each problem you submit: if you work with another student or if you find a fact online or in another book, you must write something like "This problem completed in collaboration with [student here]" or "By theorem 1.5 of Milne's *Algebraic Number Theory*,..." or "This solution based off the Stack Exchange post here:..."

1.5 Accessibility

For those of you who are currently registered with Accessible Education Center for any kind of accommodations, please communicate with me about this during the first week of the term so that we can design a plan for you. If you need learning accommodations but are not registered with the AEC, talk to them *as soon as possible*. It is much more likely that measures can be taken to provide adequate accommodation if the organization is done through AEC. I have attempted to provide documents that are accessible. Please let me know if you need additional accommodations. You can find the AEC at `https://aec.uoregon.edu/`.

2 Class Meetings

10:00–10:50 AM in University 301 every Monday, Wednesday, and Friday. You can also attend lecture over Zoom or find the recordings on the course Canvas page. The Zoom meeting ID for class is 922 2305 6596.

3 Office Hours

Office hours are your chance to ask me questions! I will be in my office (Fenton 312) or in the atrium on the top floor of Fenton at the times listed at the top of the syllabus and if you show up, I'll do my best to answer your questions and help you learn some math. You don't need to make an appointment to attend office hours. If you can't attend any of my office hours and my schedule permits, I'm also happy to meet with you individually. Just send me an email asking to meet with me and I'll let you know if/when I'm available!

If necessary, some office hours may be moved to Zoom during the term. In that case, I will send out a Canvas announcement with the appropriate details.

Since a member of my household is immune-compromised, I kindly ask that you wear a mask whenever spending a prolonged period of time with me indoors.

4 Materials

4.1 Textbook

We will use *Elementary Number Theory & its Applications*, sixth edition, by Kenneth Rosen. You can find this textbook in the usual places you would expect to find textbooks. Email me if you have trouble finding it.

4.2 A "Scanner"

You will be peer-reviewing each other's homework, typically by handwriting your comments and then scanning and uploading to Canvas. You can use a phone app (the Microsoft OneDrive app will do this for free and we have unlimited storage through UO) or an actual scanner (which the library probably has). If you have a way of annotating .pdf files on a tablet, however, you are also welcome to use that option.

4.3 A Computer

You will be required to type your homework using the typesetting program LaTeX (more on this later). Typing in this program will require access to a computer with a physical keyboard.

5 Tentative Schedule

Here is my goal for the material we will cover each week along with the corresponding assignments due that week (see assignment details later in the syllabus):

Week	**Section(s)**	**Assignments Due**
1	6.3	HW 1 draft
2	7.1–7.2	HW 1 peer review, final copy; HW 2 draft
3	7.5	HW 2 peer review, final copy; HW 3 draft
4	7.5	HW 3 peer review, final copy; HW 4 draft
5	9.1	HW 4 peer review, final copy
6	9.2–9.3	**Portfolio**; HW 5 draft
7	9.3–9.4	HW 5 peer review, final copy; HW 6 draft
8	9.4, 13.1	HW 6 peer review, final copy; HW 7 draft
9	13.1, 13.3	HW 7 peer review, final copy; HW 8 draft
10	**Presentations**	HW 8 peer review, final copy

6 Learning Outcomes

By the end of the course, a successful student should be able to:

- Write clear, concise proofs using the techniques of
  - mathematical induction
  - contradiction
  - cases
  - disproof by counterexample

  when relevant
- Provide constructive and thoughtful feedback to other students
- Exhibit understanding of how to use basic algorithms to do certain computations with integers, like computing values of multiplicative functions, primitive roots, and discrete logarithms.
- Precisely state important theorems like Euler's Theorem, the Euler Parity Theorem, Euler's Partition Formula, the classification of Pythagorean Triples, and the Four Square Theorem
- Use important theorems to give proofs about multiplicative functions, partitions, primitive roots, and sums of squares.
- Write clear, long-form exposition explaining a topic in number theory to other students
- Clearly present new mathematical information to your peers

7 Grading

Grading will be determined according to the following scheme:

Participation	5%
Written Homework	30%
Written Feedback	15%
Midterm Portfolio	15%
Final Paper	20%
Final Presentation	15%

I will make a strong effort to make the standard grading system applicable to this course (e.g. grades in the 80% to 89% range will be Bs, those in the 70% to 79% range are Cs, etc., with plus and minus grades being awarded to the upper and lower 2% of a bracket). If grades are too low, I will curve them up. If grades are too high, I will *not* curve down.

7.1 Participation

The purpose of the participation credit is to encourage you to engage in mathematical conversation each week. Mathematical conversation will expose you to new ideas and perspectives, help you learn to critique your own thinking, teach you to constructively push back on others' ideas, and help you learn the course content in more depth.

Participation will occur in the form of weekly question-asking and question-answering. Each week, you will have two different options for how you can earn this credit.

1. Attend class and participate in a *group* on worksheet days.
2. Ask or answer a math question. This can occur in office hours or on the weekly Canvas discussion. If you ask a particularly good question or provide a particularly insightful answer, you may receive extra credit.

7.2 Written Homework and Feedback

Most weeks, you will have to complete some or all of the following three tasks:

1. Draft solutions to a new homework assignment.
2. Peer-review another student's homework assignment.
3. Finalize your solutions to the previous week's homework.

Here is the schedule for *weeks 1 and 2* (and you can infer the general schedule for most other weeks from this sample):

Week 1

Monday	Homework 1 posted
Thursday	Draft of homework 1 due
Friday	Peer review for homework 1 assigned

Week 3

Monday	Homework 2 posted
Tuesday	Peer review of homework 1 due
Thursday	Draft of homework 2 due
Friday	Final copy of homework 1 due
Friday	Peer review for homework 2 assigned

Note: Canvas will tell you that the draft assignments are due on Thursdays at 11:59 pm and the final copies of the assignments are due on Fridays at 11:59 pm. However, since there's no chance I'll be assigning the peer review assignment or grading your homework at midnight, there's no reason you can't turn it in up until the next morning. The assignment will remain available and you can submit your homework without penalty until 10:00 am on the day after the stated due date.

7.2.1 Format

In this class, all of your homework must be submitted as a .pdf file which you create using the typesetting program LaTeX. This is industry-standard for all mathematical documents (and publishing in other fields as well). More instructions for how to create documents using LaTeX can be found on Canvas (and we will spend some time in class on this).

Important: Please do NOT put your name on any of your submissions. This will reduce the role that implicit biases play when peers are commenting on your work and when I am grading your work.

7.2.2 Peer Review

Before submitting a final copy for me to grade, you will submit a draft for a peer to review and then you will be assigned a peer's homework to review.

The purpose of this assigment is to encourage you to examine your own thinking by putting you in the perspective of the audience rather than the writer. Often, when writing, my own perspective and thoughts are crystal clear to me—because those are my thoughts!—but it can be hard to tell when I've been sufficiently clear in my writing so that another person can understand my thinking. By reviewing your peers' assignments, you will learn how to make sure you are conveying enough information so that people other than you can read your work. Additionally, you may learn other ways of doing the problems or other formatting or typesetting tricks and techniques.

Important: Your peers are relying on your feedback. If you are unable to review a peer's assignment in any given week—whether for an "excusable" reason or not—please let me know so that I can remove you from that week's rotation. As of now, I'm not sure how this will work with Canvas, so also keep an eye on your email throughout the term in case I need to email you individual instructions.

Logistically, when you are assigned a peer's homework to review, you will either...

1. Print your peer's assignment, hand write your comments in the margins, then scan the annotated copy and upload it to Canvas.
2. Download your peer's assignment, hand write your comments on a tablet (using a program like Drawboard PDF, e.g.), then upload the annotated copy to Canvas.

When you are peer reviewing, you will want to make sure that your comments...

1. Are thorough: Comment on every relevant aspect of their homework.
2. Are constructive: When giving criticism, focus on what your peer can do better rather than what they did poorly. When giving positive feedback, give *encouragement* rather than *praise*. Encouragement focuses on the deed where praise focuses on the doer. (Consider the difference between the comments "Wow, you study math? You must have worked really hard" and "Wow, you study math? You must be really smart.") It's better to focus on the choices they made (which they can change in the future) rather than their state of being (which can't be changed).
3. Focus on the logic: Do your peer's claims follow from their previous claims and the assumptions of the problem statements? Are their proofs complete?
    - Important: while you can give your peers hints how to do the problem, do *not* tell them how to do the problem.
    - Important: make sure that you are giving *correct* feedback to your peers. If you are not sure that something you are saying is correct, do not say it.
4. Focus on the writing: Does your peer state when they are using named theorems? Do they state when they are doing a proof by contradiction? Are there shorter or clearer ways of saying the same thing? Did it take you a long time to read and understand a particular sentence?
5. Focus on the format/organization: Is their LaTeX easy to read? Could their notation be improved? Could they rearrange the parts of their proof to improve the clarity?

The latter three categories are the components which I will be using to grade your completed homework. You are welcome to use my rubric (see below) to assist your reviewing.

7.2.3 Grading Rubric

Your peer review assignments will be graded for thoroughness, accuracy, and constructivity. It is critically important that you catch the author's serious errors and that you give correct feedback to them.

Your final homework assignments should have the following important qualities:

- Format
    - Problems are easy to read
    - Math uses math font and text uses text font
    - Inline equations and newline equations are used appropriately
    - Problem statements are included and your solutions are clearly separated from the problem statements
- Writing
    - Student uses complete sentences
    - Student only includes necessary information
    - Student states assumptions and named theorems
    - Student uses definitions and theorems appropriately (without quoting them word-for-word)
    - Student uses variables appropriately
        - Student does not use the same variable for multiple purposes
        - Student treats uppercase and lowercase variables as distinct
    - Student uses precise language
    - Student is honest about logical gaps/imprecision
    - Student does not include examples unless asked for them

  - Student's equations flow left-to-right
  - Student discriminates between assumptions and implications

- Reasoning
  - Student avoids logical errors
  - Student justifies logical steps/implications
  - Student uses appropriately sized logical steps
  - Student puts logical steps in a linear sequence
  - Student does not include extraneous reasoning
  - Student avoids arithmetic errors
  - Student correctly uses contrapositive or contradiction
  - Student includes all cases
  - Student uses precise language
  - Student checks all necessary details
  - Student completes the argument
  - Important: Student does not assume what needs proved
  - Student uses definitions correctly
  - Student avoids unwarranted assumptions
  - Student avoids conflating an implication with its converse.

Note: Qualities marked "important" above count for two qualities in the below rubric. Also note that the format category will apply to an entire assignment where the writing and reasoning category apply to each graded problem. Each category will be worth 5 points. Here is what each grade means:

0. None of the below grades applies (e.g. more than four of the previous qualities needs improvement).
1. Four of the previous qualities could use improvement OR two qualities could use improvement and one quality needs lots of attention OR two qualities need lots of attention.
2. Three of the previous qualities could use improvement OR one area needs lots of attention and one area needs improvement.
3. Two of the previous qualites could use improvement OR one area needs lots of attention.
4. One of the previous qualities could use improvement.
5. None of the previously listed qualities needs improvement.

Note: solutions which are missing a substantial amount of work (possibly because they are incomplete or because of a logical error) will be capped at three points in both the writing and reasoning categories.

7.3 Course Portfolio

In place of a midterm exam, you will instead submit a midterm portfolio. This will be due on Friday, May 6 (week 6). It will formatted (and graded) like homework assignments, but you will not be allowed to discuss the problems with others, you will not be allowed to use the internet for help with the problems, and there will be no peer review component. The portfolio will be cumulative in the sense that it will require you to know and use material from everything you've learned in the class up until that point.

7.4 Final Paper and Presentation

At some point early during the term, you will be asked to choose a group with whom you will write a paper and give a presentation on a number theory topic which will not be covered in lecture. Groups will include two or three students, all of whom must contribute to both the presentation and the paper. Topics, a rubric, a schedule, and additional details will be provided in a separate document during week 1. Presentations will occur in class during week 10. Final papers will be due on **Friday, June 3** (week 10).

7.5 Absences

IF YOU ARE SICK, STAY HOME. This includes illnesses other than COVID. No one wants your germs.

During the term, you may miss class for any reason at all without telling me. If you want to keep a good grade, however, it would behoove you to...

- ...find a way to make up your participation credit for that week. See 7.1 for more details.
- ...get any announcements for the day from me or one of your peers.
- ...get any notes for the day from Canvas or one of your peers.
- ...contact me if you are going to be absent for an extended period of time.

7.6 Late Work/Make Up Work

No late work will be accepted *if* you wait until after a deadline has passed to contact me. If you contact me ahead of time, I *may* be able to make arrangements for you to submit some work late. Generally, you may submit homework assignments up until solutions are posted, though you may not submit peer review assignments late. Since life happens sometimes and you miss deadlines sometimes, I will drop your lowest homework grade and your lowest peer review grade.

8 Things I Expect From You

- Communicate with me. Tell me what problems you're having and how I can help. This is why there's a "Feedback" discussion on the Canvas page—so you can give me suggestions about how I can improve the class! But this is only effective if you actually give me feedback.
- Read the textbook! It is incredibly helpful to have an idea of what's going to be talked about in class before you show up to class. Even just skimming the textbook ahead of time to know what terms you should expect to hear can turn difficult lectures into easy lectures.
- Spend the appropriate amount of time on this class. This is a 4 credit-hour course, which means that you should expect to spend 12 hours each week, including class time, for this class. If you find that you are not spending this much time on this class just by attending class, doing the homework, and reading the book, find other ways of spending time on this class: do extra problems out of the textbook, study with a group, or attend office hours.

9 Things You Should Expect From Me

- You should expect me to want you to learn. A trait that I find to be unfortunately too common, especially at large research institutions, is that instructors aren't invested in their students' success. You can expect me to care about your education and your learning of the material.
- You should expect me to communicate clearly with you about what I expect from you. If you have questions about how I grade or how I expect you to write your answers, please ask!
- You should expect me to have good reasons for setting the course policies to be what they are. I've been tinkering with course policies for over four years now and I'm still working on creating good course policies that help you learn. That said, none of my policies are perfect, so we may need to adjust some things as the term goes along. If you have questions about why course policies are the way they are, please ask!
- You should expect me to post solutions to assigned problems. I forget to post solutions more frequently than I'd like to admit. Please email me when I forget to post solutions to something.
- You should not expect me to be perfect, but you should expect me to make amends when I make mistakes. If you think I've fallen short in any way, please let me know and I'll make up for that mistake in a reasonable way.

10 Tips For Success

- Spend your time efficiently. One of the worst ways to spend your time is to go through your notes searching for relevant theorems and definitions for each homework problem you have. That will result in you spending lots of time searching and not much time learning. Instead...
  - Skim the textbook before class. This will help you absorb the lecture material in more depth and will save you time going through your notes later.
  - Review the lecture notes after class. This will also help you absorb the notes better as you put together the "big idea" of a given lecture.
    - Reviewing the lecture notes means making sure that you can precisely state each definition and important theorem without referencing notes.
    - Also take this time to make sure you understand what each definition and theorem *means*. Precisely quoting theorems is important for using them, but you also need to have some intuitive sense of what each theorem says and why it is significant.
  - Review the week's lecture notes before looking at the homework problems.
  - If you have prepared properly, upon reading a homework problem, you should immediately know which (if any) theorems and definitions are immediately applicable. If something is applicable, apply it. If none are applicable, you'll know that you need to fiddle a bit and maybe "have an idea" before you can apply something. This will save you a substantial amount of time on homework.
  - These are not the only things you should be spending your time on; practice problems, reading proofs, proof-reading your own proofs, and so on should also be a part of your weekly study routine.
- Know the vocabulary. Math is a language—in order to properly do math, you need to know how to read it, write it, and speak it.
  - On Canvas, you can find something called the "Frayer model" to help you learn vocabulary words with which you are struggling. The best way to learn vocabulary is to internalize each vocab term and understand why the definition is the way it is rather than merely memorizing the sequence of words which comprises its definition.
- If you are having a hard time completing a problem, you can make additional "simplifying assumptions" to demonstrate that you mostly know how to do the problem. For example if you need a number to be even, but are having a hard time showing it, you can say something like "I would like to show that n is even, but I don't know how. Assuming now that n is even, we proceed to show..." This is better than making up a reason (which you know to be false) to claim that n is even since the former approach demonstrates honesty and self-awareness (for which you will be rewarded).
- Work with other students in the class periodically. If you are struggling, you may find that other students have a better idea of what you're struggling with than I do, since they've learned the material more recently than I have. If you (think you) are doing well, you will find that explaining the concepts to other students will solidify your understanding of the material and identify gaps in your knowledge that you didn't know were there.
- Put in the appropriate time and quality of work. If the time that you are spending on this class is broken up by distractions like roommates, TV shows, or computer or cell phone use, you will not get the same benefit from that time as you would have gotten without the distractions.
- Make use of office hours. You don't have to have an appointment to attend office hours—you can just show up! I think there's a perception among undergraduates that instructors don't like holding office hours and whether or not this is true for other instructors, it is not true for me! I find office hours to be enjoyable since I can have more of a conversation with you, rather than lecturing at you as I do in class.
- Free tutoring may be available. There are two primary math tutoring resources on campus, the Math Library and the Teaching and Academic Engagement Center. You can find information about your tutoring options at the TAEC at `https://engage.uoregon.edu/tutoring`. You can find information about your tutoring options at the math library at `https://library.uoregon.edu/scilib/mathlib`.

4.2 Sample Worksheet (Putting the Pieces Together)

Below is the worksheet that I describe in my teaching statement in section 2. This worksheet was created and administered at the end of my Calculus III: Sequences and Series course in the spring term of 2021. The goal of this worksheet is to enable students to provide students with the necessary cognitive dissonance to connect course concepts that had not been covered near one another during the course. The first section covers the components of series and gives students the opportunity to connect finite sums, partial sums of infinite series, and Taylor polynomials/terms/coefficients. The second section covers series convergence and gives students the chance to connect the familiar series convergence tests with questions about the radius/interval of convergence for power series. The third section allows students to see familiar calculus functions and practice finding series for those functions and finding simple function descriptions of series. The final section covers remainder estimates and gives students to compare those remainder estimates for numeric series and power series.

1 Components of Series

Ex 1 Write out every term of $\displaystyle\sum_{n=1}^{5} \frac{1}{n^2+1}$

Ex 2 Write out every term of $\displaystyle\sum_{\ell=-1}^{4} \sqrt{\ell+1}$

Ex 3 Write out every term of $\displaystyle\sum_{k=0}^{3} \frac{1}{k!}x^k$

Ex 4 Write out every term of $\displaystyle\sum_{r=0}^{3} \frac{2^r}{3^r}x^{2r+1}$

Ex 5 Give the second term of $\displaystyle\sum_{n=1}^{9} \frac{2}{3n}$

Ex 6 Give the third partial sum of $\displaystyle\sum_{s=1}^{\infty} \frac{2^s}{s+1}$

Ex 7 Give the fifth partial sum of $\displaystyle\sum_{t=0}^{\infty} \frac{t^2-1}{t!}$

Ex 8 Give the fourth partial sum of $\displaystyle\sum_{u=1}^{\infty} \frac{1}{u^2 \cdot 2^u} \cdot x^u$

Ex 9 The series $\displaystyle\sum_{u=1}^{\infty} \frac{1}{u^2 \cdot 2^u} \cdot x^u$ is the Maclaurin series for some function $f(x)$. Give the fourth Taylor polynomial of $f(x)$ centered at $a = 0$.

Ex 10 The series $\displaystyle\sum_{u=1}^{\infty} \frac{1}{u^2 \cdot 2^u} \cdot x^u$ is the Maclaurin series for some function $f(x)$. Give the fourth term in the Maclaurin series of $f(x)$.

Ex 11 The series $\displaystyle\sum_{u=1}^{\infty} \frac{1}{u^2 \cdot 2^u} \cdot x^u$ is the Maclaurin series for some function $f(x)$. Give the fourth Taylor coefficient for $f(x)$ centered at $a = 0$.

Ex 12 Consider the series $\displaystyle\sum_{k=0}^{\infty} a_k$ where $a_k = \frac{1}{2^k}$

(a) Fill out the following table where a_k is defined above and S_k denotes the kth partial sum of the above series.

k	0	1	2	3	4	5	6	7	8
a_k									
S_k									

(b) What is $\lim_{k\to\infty} a_k$?

(c) What is $\lim_{k\to\infty} S_k$?

2 Convergence of Series

Ex 1 Does $\displaystyle\sum_{n=0}^{\infty} \frac{n}{n^3+1}$ converge or diverge?

Ex 2 Does $\displaystyle\sum_{k=0}^{\infty} \frac{k^3}{5^k}$ converge or diverge?

Ex 3 Does $\displaystyle\sum_{\ell=2}^{\infty} \frac{1}{\ell\sqrt{\ln(\ell)}}$ converge or diverge?

Ex 4 Does $\displaystyle\sum_{j=0}^{\infty} \frac{(-1)^j}{\sqrt{j+1}}$ converge or diverge?

Ex 5 Does $\displaystyle\sum_{a=1}^{\infty} \frac{\cos(3a)}{1+(1.2)^a}$ converge or diverge?

Ex 6 Does $\displaystyle\sum_{b=2}^{\infty} \ln\left(\frac{2b+1}{2b+3}\right)$ converge or diverge?

Ex 7 Does $\displaystyle\sum_{n=0}^{\infty} \frac{(x+2)^n}{3^n}$ converge or diverge...

(a) ...when $x = -1$?
(b) ...when $x = 2$?
(c) ...when $x = -3$?
(d) ...when $x = -6$?
(e) ...when $x = 1$?
(f) ...when $x = -5$?
(g) ...when $x = 0.99$?
(h) ...when $x = -4.99$?

Ex 8 Use your answers to the previous question to answer this question. What is the interval of convergence for $\displaystyle\sum_{n=0}^{\infty} \frac{(x+2)^n}{3^n}$? How about the radius of convervence?

Ex 9 How does the interval of convergence for $\displaystyle\sum_{n=0}^{\infty} \frac{(x+2)^n}{(n+1)\cdot 3^n}$ compare to the interval of convergence in the previous problem? How about the radius of convergence?

Ex 10 How does the interval of convergence for $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^n}{(n+1)^2\cdot 3^n}$ compare to the interval of convergence in the previous problem? How about the radius of convergence?

3 Functions and Series

Ex 1 Let $f(x) = \frac{1}{1-x}$.

(a) Find a power series which converges to $f(x)$. (Yes, this is as easy as it looks.)
(b) Using the definition of the Maclaurin series, find the Maclaurin series for $f(x)$.

Ex 2 Find the Maclaurin series for each of the following functions. Also include *an interval on which that series converges*. You may use whatever methods you like, but you should practice using the geometric series at least once and using the definition of the Maclaurin series at least once.

(a) $\frac{x^2}{1+x}$

(b) $\arctan(x^3)$

(c) xe^{2x}

(d) 10^x

(e) $\frac{21}{(10-x)(1+2x)}$

Ex 3 Find an explicit formula (i.e. give a formula for the function that doesn't use a series) for each of the following series.

(a) $\sum_{n=0}^{\infty} x^n$

(b) $\sum_{n=1}^{\infty} x^n$

(c) $\sum_{n=0}^{\infty} \frac{3 \cdot x^n}{4^n}$

(d) $\sum_{k=0}^{\infty} (-1)^k x^{2k}$

(e) $\sum_{r=0}^{\infty} e^{rx}$

4 Remainder Estimates

Ex 1 Consider the series $\sum_{s=1}^{\infty} \frac{2s}{s+7}$. Using sigma notation, write out R_3. Write down the first four terms of R_3.

Ex 2 Consider the power series $\sum_{k=1}^{\infty} -\frac{x^k}{k}$. This is the Maclaurin series for $\ln(1-x)$.

(a) Write down the second Taylor polynomial for $\ln(1-x)$.

(b) Graph $\ln(1-x)$ and the second Taylor polynomial for $\ln(1-x)$ on the same axes.

(c) Write down (without using an infinite sum) the function $R_2(x)$.

(d) Write a sentence describing what $R_2(x)$ represents.

(e) Using a calculator, compute $R_2(1/2)$, $R_2(-1/2)$, $R_2(1/10)$, $R_2(-1/10)$, and $R_2(0)$. How does the size of $R_2(x)$ change as x approaches 0?

Ex 3 Consider the series $\sum_{n=0}^{\infty} \frac{3n^2}{1+n^6}$.

(a) Find k so that the kth partial sum of $\sum_{n=0}^{\infty} \frac{3n^2}{1+n^6}$ is within 0.00001 of the actual sum.

(b) Use your favorite computational software to give the corresponding approximation of the actual sum.

Ex 4 Consider the series $\displaystyle\sum_{\ell=0}^{\infty}(-1)^{\ell}\frac{1}{\sqrt{\ell^{3/2}+1}}$.

(a) Find k so that the kth partial sum of $\displaystyle\sum_{\ell=0}^{\infty}(-1)^{\ell}\frac{1}{\sqrt{\ell^{3/2}+1}}$ is within 0.00001 of the actual sum.

(b) Use your favorite computational software to give the corresponding approximation of the actual sum.

Ex 5 Consider the power series $\displaystyle\sum_{k=1}^{\infty}-\frac{x^k}{k}$. This is the Maclaurin series for $f(x)=\ln(1-x)$.

(a) Use Taylor's estimate to find an upper bound for $|R_1(1/2)|$.

  i. What are x, a, I, and k?

  ii. What is $f''(z)$?

  iii. Find an upper bound for $|f''(z)|$ for $z \in I$.

  iv. Why is the previous part important?

(b) Use Taylor's estimate to find an upper bound for $|R_3(1/2)|$.

(c) Find a value of k so that $|R_k(1/2)| < 0.0001$.

(d) Find a value of k so that $|R_k(x)| < 0.0001$ for each $x \in [-1/2, 1/2]$.

4.3 Sample Project Instructions

Below is the set of instructions I gave for the final project in my Fundamentals of Number Theory II course in the spring term of 2022. The structure of this document was inspired by the teaching seminar discussion on project-based learning and so the document includes:

- Project objectives so that the students know what to focus on learning
- A project timeline so that students know when things are due, can plan ahead, and make incremental progress throughout the term
- List of specifications for the presentation and paper
- Rubric for each graded component of the project
- Principles for good expository writing
- List of possible topics, including notes on any additional background material that students would find helpful.
- List of resources to help students with the technological component of the project

Final Paper and Presentation

Math 348

Spring 2022

1 Objective

There are three primary objectives for this project.

First, this project serves as a way for you to explore a number theory topic in more depth than we otherwise would in class. You'll be exploring a more advanced topic and you'll have a chance to examine some research-level mathematics, up to and including some open research questions.

Second, this project will allow you to practice math as a spoken language. You already practice speaking math when doing group work in class or when working on homework with others, but each of those is a two-way conversation where imprecise language is acceptable. In the presentation portion of the project, you will be practicing explaining math to someone else in a formal setting.

Third, this project gives you the opportunity to write exposition. Exposition differs substantially from the type of writing you do on homework. Your writing on homework ought to be as concise as possible: only include required information, don't provide examples unless asked for them, cite theorems rather than quoting them, and so on. Expository writing is generally longer and the point is give your reader a good big-picture understanding of your topic. Good principles for expository writing are given below.

2 Timeline

Week	Date	Description
2	Wednesday, April 6	Topic and Group Preferences Due
5	Wednesday, April 29	Progress update due
8	Wednesday, May 18	First draft of paper due
10	Any class day	Presentation
10	Friday, June 3	Paper due

2.1 Topic and Group Preference

By Wednesday, April 6, please send me an email (gknapp4@uoregon.edu) including your topic preferences and your group preferences. Topics will be assigned on a first-come first-serve basis. If you have a group preference, each member of the group must email me indicating that preference. For example, if Harry, Ron, and Hermione want to be a group, Harry needs to email me requesting Ron and Hermione, Ron needs to email me requesting Harry and Hermione, and Hermione needs to email me requesting Harry and Ron.

2.2 Progress Update

Your progress update in week 5 should include a detailed outline of your proposed paper, including a bibliography. The outline should describe which sections will be included in your paper and the author of each section. Each section should be accompanied by a brief description written by that author.

This update will be graded for completion. I will provide feedback on your progress and suggest sources and subtopics that may be of interest.

2.3 First Draft

The first draft of your paper should be 80–90% complete. This draft will be graded for completion and I will provide feedback on the content and writing.

3 The Presentation

Your presentation may either be a (non)traditional “chalk talk” (either at the board or via the projector) or a slideshow. The technology and setup for a chalk talk is much simpler, though you’ll find that it’s much harder to get through the material you want to cover in the appropriate amount of time. The technical setup for a slideshow is much more involved, but the presentation is much smoother.

The presentation will be on either Wednesday or Friday of week 10. You will be sharing the class period with one other group, so you should plan to give a 20 minute talk and leave about 5 minutes for questions.

4 The Paper

4.1 Format and Length

The final paper should be formatted as follows:

1. Use LaTeX!
2. Use 1 inch margins
3. Use 12 point (or smaller) font. This can be accomplished by passing the “12pt” option to the document class at the beginning of the preamble, i.e. including the command “\documentclass[12pt]{article}”
4. Single space. You may choose to indent paragraphs or you may also choose to put a blank line between paragraphs (or both).
5. Follow all formatting requirements and good practices for homework
6. Make sure to number any equations you reference. Make sure any equations you do not reference do not have a number (i.e. an equation should be numbered if and only if it is referenced).
7. Your bibliography should use an industry-standard citation format. It doesn’t matter whether you use MLA, APA, IEEE, BiBTeX’s “plain” style, etc. But pick one and stick with it.
8. Each group member should write at least three pages. This may include equations and figures unless those equations and figures take up a “substantial” amount of room. If you have questions about the definition of “substantial,” ask. I’d rather deal with that on a case-by-case basis.

4.2 Writing

The final paper should be written with the following good practices:

1. Follow all writing requirements for homework outlined in the syllabus EXCEPT

    - Student only includes necessary information
    - Student does not include examples unless asked for them

    Note that it is, in fact, a good idea to include more information than necessary and to include examples for this project

2. Write in the first-person plural (e.g. "we will show that"), though you may address the reader as "you."

3. Edit your writing: avoid typos and mathematical mistakes

4.3 Content

The final paper should include the following content:

1. An introduction that gives background and motivation for the topic at hand

2. A mathematically rigorous body. You don't need to prove every assertion you make, but please define any new terms carefully. You are encouraged to use examples, data tables, illustrations, etc. to make your results easier to understand.

3. A conclusion. Remind the reader of the big ideas from your paper and suggest further questions for the reader to think about.

4. A bibliography.

5 Rubric

5.1 Progress Update

It is possible to receive grades in between the grades listed in this table.

Progress Update	5/5	2/5	0/5
Outline	Authors include a detailed description of which topics they'll be writing about and who will be writing about each topic	Authors include a list of which topics they'll be writing about and who will be writing about each topic	Authors do not include an outline
Bibliography	Authors provide a potential list of sources which includes at least two more sources than I included in my email. The list of sources is formatted consistently and includes the title, author, and type of source, along with a brief description of how the authors plan to use each source.	Authors provide a potential list of sources which may be difficult to read or may lack a description of how the authors plan to use those sources.	Authors do not include a bibliography.

Note: formatting of your bibliography is unimportant for your progress update. Just make it consistent and clear. You don't even need to pick anything like MLA or APA or whatever.

5.2 First Draft

It is possible to receive grades in between the grades listed in this table. Note that this rubric applies to a particular student rather than to an entire group (so as not to penalize entire groups if a single member doesn't complete their work on time).

First Draft	5/5	2/5	0/5
Quantity	Student has at least two pages of drafted material and a clear plan for a third page.	Student has at least one page of drafted material	Student does not have drafted material
Quality	Student uses complete sentences and good reasoning with few exceptions. Student is honest about missing details.	Student's work is rushed, consistently difficult to read, or incomplete.	Student does not have drafted material.
Bibliography	Bibliography is formatted consistently and includes complete citation information. Bibliography contains at least two more sources than I included in my email.	Bibliography may be formatted inconsistently and may lack source information.	Bibliography is incomplete and formatted inconsistently.

Note: you may choose an appropriate style for your bibliography. I recommend using BiBTeX and using the "plain" or "unsrt" BiBTeX styles. If you prefer something like MLA or APA or IEEE, you are welcome to use that.

5.3 Final Paper

The following categories will be used to grade the final paper. It is possible to receive a grade not listed in this table. Notice that some categories are individual where other categories apply to the entire group.

Category (Point Value)	100%	50%	0%
Reasoning (40)	Every claim student makes is accurate and well-reasoned	Student makes multiple inaccurate, misleading, or unjustified statements	Student's statements are generally inaccurate or unjustified.
Writing (40)	Student's writing adheres completely to the writing guidelines in section 4.2.	About 75% of student's writing adheres to writing guidelines in section 4.2.	No more than 50% of student's writing adheres to writing guidelines in section 4.2.
Formatting (12)	Group's formatting adheres completely to the formatting guidelines in section 4.1.	Group's formatting adheres to some guidelines in section 4.1.	Group's formatting does not adhere to guidelines in section 4.1
Content (16)	Group adheres to content guidelines in section 4.3.	Group mostly adheres to content guidelines in section 4.3.	Group is missing some major component of the content listed in section 4.3.
Cohesiveness (16)	Group's paper transitions effectively from topic to topic independent of author.	Group's paper and transitions may be disconnected in places.	Group's paper reads like several different papers stapled together.
Bibliography Format (8)	Bibliography is formatted correctly according to an industry standard citation format.	Bibliography may be formatted inconsistently or may be formatted without reference to an industry standard format.	Bibliography is incomplete and formatted inconsistently.
Reference Quantity (8)	Group references at least two sources beyond what I sent in my email.	Group references one source beyond what I sent in my email.	Group's references are all from my email.
Reference Quality (8)	Group's references are from reputable sources.	Group's references rely heavily on questionable sources (e.g. Wikipedia)	Group exclusively relies on questionable sources.
Reference Usage (12)	Group cites each reference in bibliography. Group regularly uses references when appropriate.	Group does not cite all sources in bibliography or group does not use references when necessary.	Group cites only some sources in bibliography and does not use references when necessary.

5.4 Final Presentation

As with the other rubrics, intermediate point values are possible. Everyone in the group will probably receive the same grade, but exceptions are possible.

Category (Point Value)	100%	50%	0%
Clarity (30)	Presented materials are easy to read. Group defines necessary terms and notation and uses terms and notation properly.	Presented materials are hard to read OR group lacks precise definitions of terms/notation OR group uses terms/notation improperly.	Presented materials are hard to read AND group lacks precise definitions of terms/notation AND group uses terms/notation improperly.
Accuracy (30)	Presented content is entirely factual.	Presentation includes minor mistakes or misleading statements.	Presentation includes major mistakes.
Motivation (15)	Presentation includes motivation for why the chosen topic is of interest.	Presentation may mention why the chosen topic is of interest.	Presentation does not indicate why the chosen topic is of interest.
Big Idea/Main Result* (15)	Presentation presents a clear big idea/main result.	Presentation may be fractured into small, yet related pieces.	Presentation has no clear direction.
Balance (10)	Everyone in the group speaks for roughly equal amounts of time	Some group members speak noticeably more often than others	Some group members clearly dominate the presentation

*Note that the "big idea/main result" does not have to be a single theorem or result; it can be a philosophy like "analysis is a useful tool for studying primes."

6 Progress Update Exemplars

Suppose Harry, Ron, and Hermione are doing their project on partitions. Here are some exemplars to indicate what different levels of a progress update might look like.

<u>A 10/10 Progress Update</u>

> The partition function is one of the most fascinating topics in mathematics. On one hand, it answers a simple question: given a positive integer n, in how many distinct ways can you sum positive integers to yield n? On the other hand, it is one of the most difficult functions in mathematics and is still the subject of much modern research. Historical greats like Euler, Hardy, and Ramanujan have made major contributions by creating or taking advantage of cutting-edge mathematics (generating functions in Euler's case and complex analytic methods in Hardy and Ramanujan's). Modern mathematicians like Ono have used the theory of modular forms with great success as well. Still, many questions about partitions (especially restricted partitions) remain open. In this project, we will explore the theory of partitions through Ferrers diagrams, generating functions, and computation.
>
> <u>Introduction</u>
>
> We will together write the introduction to this project in which we motivate why people might be interested in the partition function. We will rigorously define the partition function and we will describe why the partition function is defined the way it is. We will describe restricted partition functions and give the corresponding notation. We will summarize some of the history of partition functions and state important results.

Ferrers Diagrams

Harry will write the portion of this project about Ferrers diagrams. He will define a Ferrers diagram, give examples, and give a proof demonstrating the utility of the Ferrers diagram. Harry will also write about generalizations of the Ferrers diagram like the Young tableaux and plane partitions and will list some techniques, results, and open questions associated to Ferrers diagrams.

Generating Functions

Ron will write the portion of this project about generating functions. He will define generating functions, give examples, and give a proof demonstrating the utility of generating functions. He will focus on how generating functions have been used in partition theory, but will also mention applications of generating functions to fields like differential equations. He will also write about analogues of generating functions, like Dirichlet series.

Computational Results

Hermione will write the portion of this project about computations and algorithms related to the partition function. She will describe some of the history of how people computed $p(n)$ before computers existed, including how asymptotic formulas shaped the development of some of these algorithms. She will describe how recurrence relations have impacted computations, in particular, Euler's Partition Formula. She will also provide some code she has written to compute various restricted partitions functions along with table summarizing the data she developed.

Sources

Rosen, Kenneth H. *Elementary Number Theory & its Applications.* Textbook.

We will use Rosen's textbook to formulate an outline of our project and we will give more detailed versions of several proofs found in Rosen; namely, the proof of the factorization of the generating function for $p(n)$ and the proof of Euler's pentagonal number theorem.

[More sources here]

A 4/10 Progress Update

The partition function, $p(n)$ counts the number of ways to add up positive integers to get n. In this project, we will examine the partition function through Ferrers diagrams, generating functions, and computation. Harry will write the segment of the paper about Ferrers diagrams, Ron will write the segment of the paper about generating functions, and Hermione will write the segment of the paper about computation.

Here are the sources we will use:
Rosen, Kenneth H. *Elementary Number Theory & its Applications.* Textbook.

[More sources here]

7 Writing Principles

When doing math writing in general, you want to have the answers to two question in mind:

1. What is the purpose of this writing?
2. Who is the audience of this writing?

The answers to these questions impact every decision you make about the writing itself. In general, you are probably most familiar with writing homework assignments. On homework assignments, the audience is your grader/instructor and the purpose of the writing is for you to practice working with and communicating about new concepts to which you were exposed in class. As a result, the best type of homework writing is information-dense in the sense that you want to prove exactly what you were asked to prove and no more. After all, it is a mark of mastery to say exactly what you want in as few words as possible.

You're probably also quite familiar with the type of math writing that you see in textbooks. The audience of a textbook is often "a student who has met all of the prerequisites to learn about this topic in detail" and the purpose of the textbook is to teach the reader in detail about a topic so that the reader leaves the book with a certain new skill set that they didn't already have before.

With this project, however, the answers to the two fundamental questions changes. You want to give a lot more room for discussion than you otherwise would on a homework assignment. You don't want to give the same level of detail and depth that a textbook would, but you do want to give a lot more breadth and context. To see why, we'll talk about choosing a purpose and audience. The purpose and audience are somewhat tied together here, so let's start by discussing audience.

For your audience, you may assume that your reader is one of

1. A general undergraduate math student who has taken an intro proof course, an intro number theory course, and an intro computer programming course
2. A general math graduate student who has taken all of the above and courses in abstract algebra, real and complex analysis, and point-set topology

I recommend choosing "general undergraduate" as your audience, but some of you may have topics that are difficult to introduce without assuming that your audience has some familiarity with groups, fields, absolute convergence, or some other such concept that you don't have space to introduce in your paper.

Once you have your audience in mind, you'll want think about the purpose of your writing.

- Why should your audience care about your topic? Is it seemingly simple but deceptively complicated? Does it have really important applications? Is it analogous to an object that everyone is familiar with but different in some crucial way?
- What are the big ideas that you want your audience to walk away with?
- What are some of the clever techniques that are used when doing math in this area?
- What are the major accomplishments in this area?

It's impossible to focus on all of these questions, but you will at least want to touch on all of them in a meaningful way. So while you won't be writing with just one purpose in mind, each paragraph should probably correspond to just one purpose. Once you've chosen a purpose for a particular paragraph, however, you can begin to see that your writing will differ substantially from homework writing.

When writing homework problems, you don't need to give clear definitions because the grader already knows the terms you're using. Here, the audience (including me, the grader) might not know new terms and so you'll want to make sure to define those terms carefully. This probably includes some exposition about why the definition includes the components that it does. E.g. if you're writing about the ElGamal cryptosystem and you want to define a discrete log, you might say something like

> The idea behind defining a discrete log is the same as the idea behind defining the usual logarithm $\mathbb{R}_{>0} \to \mathbb{R}$. The standard logarithm is defined so that $\log(a)$ is the unique x so that $e^x = a$. When defining the discrete logarithm mod m, we want to define $\log_b(a)$ to be the unique x so that $b^x \equiv a \mod m$, but we require some conditions.
>
> First, we need to guarantee that such an x exists. Such an x cannot be guaranteed if b and a are not relatively prime to m, for instance. There is no solution to $3^x \equiv 2 \mod 6$ after all. Hence, we only define $\log_b(a)$ for $b, a \in (\mathbb{Z}/m\mathbb{Z})^\times$. Even so, such an exponent might not be guaranteed to exist as we can see by the fact that there are no solutions to $7^x \equiv 5 \mod 9$. Hence, we further require b to be a primitive root modulo m.
>
> Under these conditions (that b is a primitive root mod m and that a is relatively prime to m), there is guaranteed to be an x so that $b^x \equiv a \mod m$ (see Theorem Blah that we proved earlier). However, such an x is not unique. Note that $x = 2, 8, 14, 20, \ldots$ are all solutions to $2^x \equiv 4 \mod 9$. However, since the order of 2 mod 9 is 6, any exponent x satisfying $2^x \equiv 4 \mod 9$ must be unique modulo $6 = \varphi(9)$. More generally, with a primitive root whose order is $\varphi(m)$, any x satisfying $b^x \equiv a \mod m$ must be unique modulo $\varphi(m)$. Now that we understand some of the obstacles to making this definition, we proceed to give the definition:
>
> **Definition 1.** *Suppose m is an integer greater than 1 and b is a primitive root modulo m. For any a in $(\mathbb{Z}/m\mathbb{Z})^\times$, define the index (or discrete logarithm) base b of a modulo m to be the residue class modulo $\varphi(m)$ of any x with $b^x \equiv a \mod m$.*

Only including the definition of "index" could leave your reader confused about a lot of the details like the "why"s and the "how"s.

Another thing that you'll want to notice is that when you want to pass on big ideas or clever techniques, it can often be helpful to give that in the context of an example rather than the general case. In fact, when passing on a big idea, it is often impossible to give the full idea without getting too deep into the weeds. In that case, it may be helpful to present an example and give some commentary on where the specific details of the example fail to generalize.

Notice also that the purpose of this assignment is to get you to practice audience-centered writing rather than self-centered writing. When you complete homework assignments, your purpose is to demonstrate your understanding of the problems and hence is self-centered. Here, your writing ought to be audience-centered where you put their understanding above your need to demonstrate mastery. This setting is somewhat contrived, of course, because ultimately you are being graded and so you want to demonstrate some mastery, but you're being graded on your ability to write audience-centered mathematics, so I hope that provides some authenticity to this assignment. Moreover, I plan to post each of your papers to Canvas so that your classmates may read them and use them as a reference should they want to do so in the future.

Of course, these are not the only things you'll want to consider when writing math. More good practices for writing mathematics can be found at this link: `https://kconrad.math.uconn.edu/blurbs/proofs/writingtips.pdf`. Note that these are Keith Conrad's opinions, but they constitute an excellent description of modern mathematical convention, they are well-reasoned, and he's honest when his opinion isn't necessarily universally agreed upon.

8 Some Details

8.1 Possible Topics

You may choose any of the following topics and you may also propose a topic yourself if there's something you're curious about! Note that "topic**" denotes a topic requiring abstract algebra, "topic++" denotes a topic requiring analysis, and ([topic]) denotes a topic about which I have fewer resources to share.

- Algebraic number fields**: factorization of ideals, class numbers
- Approximations by rational numbers: how well do rational numbers approximate irrational numbers?
- ([Bernoulli numbers]): a very mysterious pattern, 400 years old at least
- ([Bounded gaps between primes]): an exciting, recent discovery
- Continued fractions: representing real numbers in the form $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \frac{1}{a_3 + \ddots}}}$
- Elliptic curves**: ubiquitous in modern research, including the proof of Fermat's last Theorem
- ([Factoring algorithms]): highly applicable and clever
- Finite fields**: elegant results used across number theory
- Geometry of Numbers (Lattices): number theory that you can visualize
- Goldbach's conjecture: easy to state, still unproven! Relates additive and multiplicative number theory
- Lagrange's Four Square Theorem: A surprising pseudo-converse to the fact that every perfect square is nonnegative
- Modular forms++**: feels closer to algebra/analysis than number theory, but has massive number theoretic consequences
- Multiplicative functions and Möbius inversion: beyond Euler's φ function
- p-adic numbers++**: a different way of measuring distance
- Partial summation and some consequences: one of the most useful ways of computing finite sums; turn them into integrals!
- Pell's equation: a neat example of solving a polynomial equation in integers
- ([Primality testing algorithms]): these can be fast!
- Primes in arithmetic progressions++: incredible results
- The Prime Number Theorem: for any positive number t, (approximately) how many primes are $\leqslant t$?
- ([Special Cases of Fermat's Last Theorem]): for cubes, fourth powers, etc.
- Riemann zeta function++: essential for understanding primes
- ([Twin primes and Brun's sieve]): a challenging and beautiful result

8.2 Technology

For your final paper, it is probably best to collaborate on Overleaf. You can share and collectively edit a single .tex document on Overleaf with very little hassle. You can also use a more complicated solution like GitHub, but I can't provide tech support if you choose to go that route.

For the bibliography of your final paper, I recommend that you use BiBTeX. This is a great tool for formatting your bibliographies and it makes reformatting them a breeze. Here is Overleaf's BiBTeX tutorial; it's excellent: `https://www.overleaf.com/learn/latex/Bibliography_management_with_bibtex`

For your final presentation, if you choose to do a slideshow, you'll want to look at the beamer package in LaTeX. It makes very nice looking slideshows. Rather than trying to put together my own introduction to beamer, I will link you to the experts: `https://www.overleaf.com/learn/latex/Beamer_Presentations%3A_A_Tutorial_for_Beginners_(Part_1)%E2%80%94Getting_Started`

4.4 Sample Lecture Guide

Below is a sample lecture guide that I gave my students during the Differential Calculus course I taught during the winter of 2020. This section was about continuity and the Intermediate Value Theorem and I designed this lesson from the principles of backwards design. My primary objective was to helps students understand the statement of the Intermediate Value Theorem, the necessity of the hypotheses, and the reason why the definition of continuity is the way it is. As a result, I began the lesson with a true claim (which was an application of the Intermediate Value Theorem to height) and some false variants of that claim and I had my students discuss with each other why each of the variants was false. This led to a natural discussion of why continuity was important, what continuity was, which sorts of functions are continuous, and finally, the formal statement of the Intermediate Value Theorem.

Claim: If you were five feet tall last year and you are seven feet tall this year, at some point in between last year and this year, you were ~~five~~ six feet tall.

- true

Variant 1: If you were five feet tall last year and you are seven feet tall this year, at some point in between last year and this year, you were eight feet tall.

- false because the height in the conclusion is not between the heights in the hypotheses

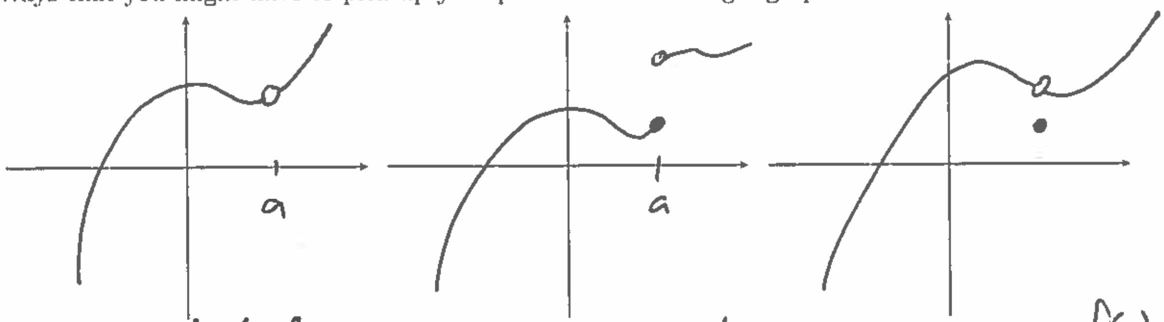
Variant 2: If you were five feet tall last year and you are seven feet tall this year, at some point next year, you will be ~~eight~~ six feet tall.

- false because the time in the conclusion is not between the times in the hypotheses

Variant 3: If you had five dollars last year and you have seven dollars this year, at some point in between last year and this year, you had six dollars.

- false because the amount of money you have can "jump"

Ways that you might have to pick up your pencil when drawing a graph:



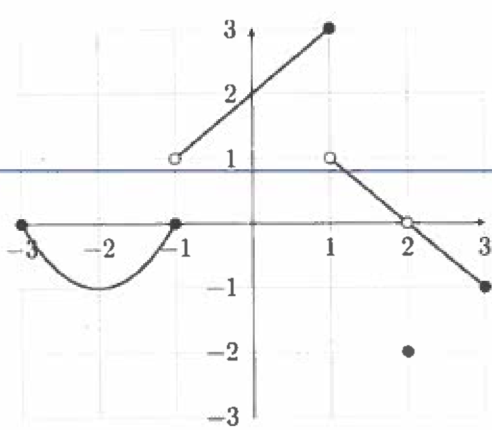
$f(a)$ needs to be defined

$\lim_{x\to a} f(x)$ needs to be defined

need $\lim_{x\to a} f(x) = f(a)$

Def: Let $a \in \mathbb{R}$ (shorthand for a is a real number) and let $f(x)$ be a function defined on an interval containing a. Then $f(x)$ is continuous at a if $\lim_{x\to a} f(x) = f(a)$

Ex 1 Consider the graph of $f(x)$ below



(a) Is $f(x)$ continuous at $x = -2$?

Yes; $\lim_{x\to -2} f(x) = -1 = f(-2)$

(b) Is $f(x)$ continuous at $x = -1$?

No; $\lim_{x\to -1} f(x)$ DNE

(c) Is $f(x)$ continuous at $x = 2$?

No; $\lim_{x\to 2} f(x) = 0 \neq f(-2)$

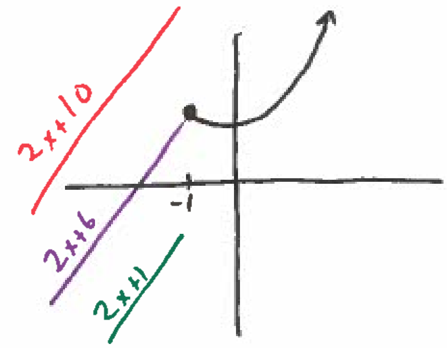
Ex 2 Let $f(x) = \begin{cases} 2x + B & x < -1 \\ x^2 + 3 & x \geq -1 \end{cases}$. Find the value of B which makes $f(x)$ continuous at $x = -1$.

$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} 2x + B = -2 + B$

$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} x^2 + 3 = 4$

Need $-2 + B = 4 \rightarrow B = 6$

Now $\lim_{x \to -1} f(x) = 4 = f(-1)$



Thm: Let $p(x)$ and $q(x)$ be polynomials. Then they are continuous at every real number (i.e. $\lim_{x \to a} p(x) = p(a)$ for all $a \in \mathbb{R}$). Furthermore, $\frac{p(x)}{q(x)}$ is continuous at every a for which $q(a) \neq 0$.

Ex 3 At which points (if any) is the function $f(x) = x^{150} - 3x^{72} + 44$ discontinuous?

$f(x)$ is a polynomial, so $f(x)$ is never discontinuous

Ex 4 At which points (if any) is the function $f(x) = \frac{(x-2)(x+1)}{x^2+1}$ discontinuous?

$f(x)$ is discontinuous at real numbers where $x^2 + 1 = 0$

$x^2 + 1 = 0 \iff x^2 = -1$ ↪ no real solutions

$f(x)$ is never discontinuous

Ex 5 At which points (if any) is the function $f(x) = \frac{x^2-2x+1}{x^2+2x-3}$ discontinuous? Does $\lim_{x\to a} f(x)$ exist at any of the points of discontinuity?

$f(x)$ is discontinuous when $x^2+2x-3=0$

$(x+3)(x-1)=0$

$x=-3,1$

For $x \neq -3,1$, $f(x) = \frac{(x-1)^2}{(x+3)(x-1)} = \frac{x-1}{x+3}$

$\lim_{x\to 1} f(x) = \frac{1-1}{1+3} = \frac{0}{4} = 0$

$\lim_{x\to -3^-} f(x) = -\infty$

$\lim_{x\to -3^+} f(x) = \infty$

$\lim_{x\to 1} f(x)$ exists, but $\lim_{x\to -3} f(x)$ does not

Ex 6 At which points (if any) is the function $f(x) = \begin{cases} \frac{x^2-2x+1}{x^2+2x-3} & x \neq -3,1 \\ 0 & x=-3,1 \end{cases}$ discontinuous?

$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{x^2-2x+1}{x^2+2x-3} = 0$ by Ex 5

$f(1) = 0$, so $f(x)$ is continuous at 1

$\lim_{x\to -3} f(x)$ DNE, so $f(x)$ is only discontinuous at -3

Thm: (Building continuous functions) If $f(x)$ and $g(x)$ are continuous at a, then

(a) $(f \pm g)(x)$ is continuous at a

(b) $(f \cdot g)(x)$ is continuous at a

(c) $\left(\frac{f}{g}\right)(x)$ is continuous at a provided $g(a) \neq 0$

(d) $f(x)^c$ is continuous at a provided x^c is defined in an open interval of $f(a)$

Aside (Function Composition)

(a) For $f(x) = x^2 - 3x + 2$, what is $f(x+h)$?

$f(x+h) = (x+h)^2 - 3(x+h) + 2$

(b) For $f(x) = \sqrt{x}$ and $g(x) = 10x - 15$, what is $(f \circ g)(x)$?

$(f \circ g)(x) = \sqrt{10x - 15}$

(c) For $f(x) = \sin(x)$ and $g(x) = x - \pi$, what is $(f \circ g)(x)$?

$(f \circ g)(x) = \sin(x - \pi)$

Def: (informal) For the composition $(f \circ g)(x) = f(g(x))$, I refer to $f(x)$ as the outside function and $g(x)$ as the inside function.

Thm: (Composite Function Theorem) Suppose that $a \in \mathbb{R}$ with functions $f(x)$ and $g(x)$ so that $g(x)$ is continuous at a and $f(x)$ is continuous at g(a). Then $(f \circ g)(x)$ is continuous at a.

Special Case: If $f(x)$ and $g(x)$ are continuous at every point then so is $(f \circ g)(x)$.

Fact: $\sin(x)$ and $\cos(x)$ are continuous at every point

Ex 7 At which points (if any) is $\tan(x)$ discontinuous?

$\tan(x) = \frac{\sin(x)}{\cos(x)}$. Because $\sin(x)$ and $\cos(x)$ are continuous everywhere, the only points of discontinuity are where $\cos(x) = 0$, ie. $\frac{\pi}{2} + 2\pi n$ and $\frac{3\pi}{2} + 2\pi n$ for integers n.

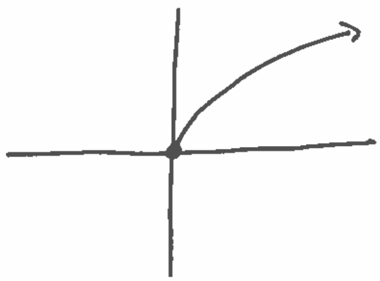
Ex 8 At which points (if any) is $\csc(\pi(x+1))$ discontinuous?

$\csc(\pi(x+1)) = \frac{1}{\sin(\pi(x+1))}$. Because $\sin(x)$ and $\pi(x+1)$ are continuous, the only points of discontinuity are where $\sin(\pi(x+1)) = 0$, ie. $\pi(x+1) = 2\pi n \rightarrow x+1 = 2n \rightarrow x = 2n-1$

$\pi(x+1) = \pi + 2\pi n \qquad x+1 = 1+2n \qquad x = 2n$ for integers n

Ex 9 Is $f(x) = \sqrt{x}$ continuous at 0?

$\lim_{x \to 0} \sqrt{x}$ DNE so $\sqrt{x}$ is not continuous at 0



Def: A function $f(x)$ defined on an interval $[a, b]$ is continuous from the right at a if $\lim_{x \to a^+} f(x) = f(a)$

Def: A function $f(x)$ defined on an interval $[b, a]$ is ____________ if

Def: Let $a, b \in \mathbb{R}$ with $a < b$. Let $f(x)$ be a function defined on (a, b). Then

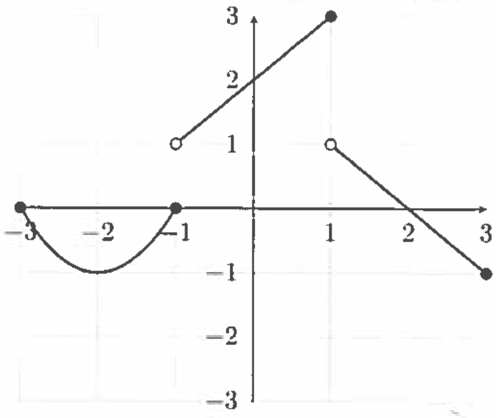
(a) $f(x)$ is continuous on (a,b) if $f(x)$ is continuous at all points in (a, b)

(b) $f(x)$ is Continuous on $[a,b]$ if $f(x)$ is continuous at all points

AND $f(x)$ is continuous from the right at a

AND $f(x)$ is continuous from the left at b

Ex 10 Consider the graph of the function $f(x)$ given below



(a) Is $f(x)$ continuous on $(-3,-1)$? What about $[-3,-1]$?

$f(x)$ is cts. on $(-3,-1)$

$\lim_{x\to -3^+} f(x) = 0 = f(-3)$ and $\lim_{x\to -1^-} f(x) = 0 = f(-1)$, so

$f(x)$ is cts on $[-3,-1]$

(b) Is $f(x)$ continuous on $(-2,0)$? What about $[-2,0]$?

$f(x)$ is discontinuous at -1, so it is cts. on neither $(-2,0)$ nor $[-2,0]$

(c) Is $f(x)$ continuous on $(-1,1)$? What about $[-1,1]$?

$f(x)$ is cts. on $(-1,1)$

However, $\lim_{x\to -1^+} f(x) = 1 \neq f(-1)$, so

$f(x)$ is not continuous from the right at -1.

So f is not continuous on $[-1,1]$

Ex 11 Let $f(x) = \begin{cases} x - 7 & x \leqslant 1 \\ x^2 - 2x + 3 & x > 1 \end{cases}$

(a) Is $f(x)$ continuous on $(-5, -1)$?

$f(x) = x - 7$ on $(-5, -1)$, so f is continuous on $(-5, -1)$

(b) Is $f(x)$ continuous on $(0, 2)$?

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x - 7 = -6$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x^2 - 2x + 3 = 1^2 - 2 \cdot 1 + 3 = 2$

$f(x)$ is discontinuous at 1, so it is not cts. on $(0,2)$

(c) Is $f(x)$ continuous on $[0, 1]$?

$f(x) = x - 7$ on $[0,1]$, so f is cts. on $[0,1]$

(d) Is $f(x)$ continuous on $[1, 2]$?

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x^2 - 2x + 3 = 1^2 - 2 \cdot 1 + 3 = 2$

$f(1) = 1 - 7 = -6$

Since $\lim_{x \to 1^+} f(x) \neq f(1)$, f is not cts. on $[1,2]$

Ex 12 Let $f(x) = \begin{cases} \frac{\sin(x)}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$ Is $f(x)$ continuous on $(-1, 1)$?

Only possible problem is at $x = 0$

$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{\sin(x)}{x} = 1 = f(0)$

So $f(x)$ is cts. on $(-1,1)$

x – time , f(x) – height

Revisit If you were five feet tall a year ago and you are seven feet tall this year, at some point in between, you were six feet tall.

f(a) x = a f(b) b c a<c<b

z

Thm: (Intermediate Value Theorem) Let f(x) be continuous on the interval (a, b). If z is any number between f(a) and f(b), then there is a c in (a, b) so that f(c) = z

Ex 13 Show that $f(x) = x - \cos(x)$ has at least one zero. Recall that if $f(x)$ is a function, then a zero of f is an x value where $f(x) = 0$.

$f(-100) = -100 - \cos(-100) < 0$

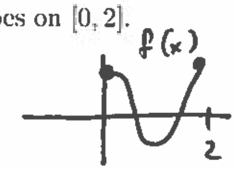
$f(100) = 100 - \cos(100) > 0$

f(x) is continuous at all real numbers

By the IVT, f(x) = 0 for some x with $-100 < x < 100$

Ex 14 Always true or sometimes false? Let $f(x)$ be a continuous function on $[0, 2]$ with $f(0) > 0$ and $f(2) > 0$. Then $f(x)$ has no zeroes on $[0, 2]$.

False



Ex 15 Let $f(x) = \frac{2}{x}$. Note that $f(-1) = -2$ and $f(1) = 2$. Does $f(x)$ have a zero on $[-1, 1]$?

f(x) is not cts on [-1, 1], so the IVT does not imply that there is a zero.

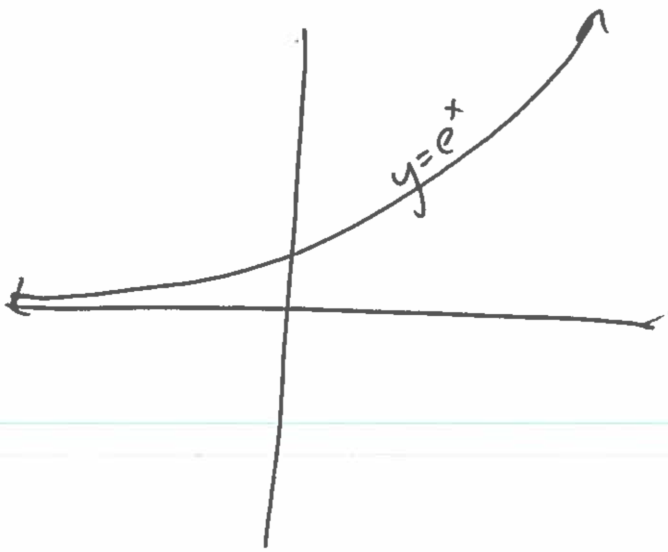
if $\frac{2}{x} = 0$, then $2 = 0$ (contradiction!)

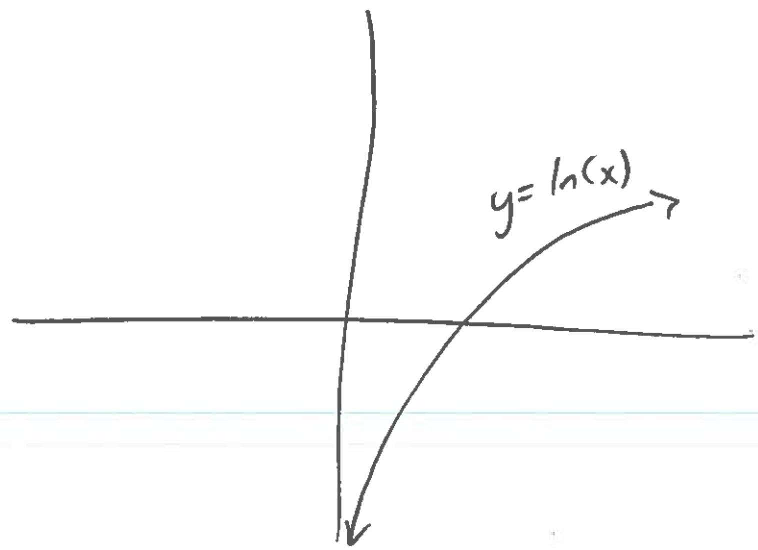
so f(x) has no zeroes at all

Facts: e^x is cts everywhere

$\ln(x)$ is cts. for all $x > 0$

Why?





4.5 Sample Exam

Below is a sample exam from my Integral Calculus course in the fall term of 2021. On this exam, I test multiple areas of Bloom's Taxonomy, from knowledge (the vocabulary section) to understanding (problem 3) to applying (problem 9) to analyzing (problem 6) to evaluating (problem 4) to creating (problem 7).

Exam 1

Name: ______________________________

Tell me what you know: If you don't know the entire answer, showing a formula or writing something illustrating that you understand any concept involved in the problem will allow me to give partial credit. I have to give you a 0 if you write nothing down. You will earn more points by writing down one true thing and no nonsense things than you will by writing down one true thing and a bunch of nonsense things.

Check your answers. Take the time before you turn in your test to make sure you have read the directions correctly and in their entirety, that all your work shown is correct, and that you have clearly stated your answer (by boxing or circling it where appropriate).

Pace yourself: If you're stuck on a problem, move on and come back to it later. Don't risk forcing yourself to give partial answers if you run out of time near the end of the test. Do the easy ones first. This exam will last 50 minutes and there are 89 points on this exam. That means you should budget about 0.6 minute(s) for each point a problem is worth in order to complete the exam in time.

Reminder: The only resources that you are allowed to use in conjunction with the exam are the two pages of notes that you bring to the exam and a scientific calculator with neither graphing nor calculus capabilities. If you use any other resources or if you communicate with other individuals about the exam before everyone has completed the exam, you will receive a zero on the exam and you will be reported to the university for academic misconduct.

Exam 1

1. **Vocabulary**

   (a) (8 points) Let $f(x)$ be a continuous function on the interval $[a, b]$. State both versions of the **Fundamental Theorem of Calculus** (it doesn't matter which order you state them in):

      i.

      ii.

   (b) (4 points) State the **integral power rule**.

Exam 1

2. (4 points each) True or false? For each statement, *circle* the word "true" if the statement is true and "false" otherwise. No work is necessary, but you can show work for some partial credit if you get the answer wrong.

   (a) **True** or **False**? If $f(x)$ is a continuous function on $[a, b]$, then the right Riemann sum with 5 rectangles for $f(x)$ on $[a, b]$ has to be greater than or equal to $\int_a^b f(x)\,dx$

   (b) **True** or **False**? If an object travels a distance of 8 meters, then its net displacement could be 5 meters.

   (c) **True** or **False**? If $g(x)$ and $h(x)$ are differentiable functions, then $\int_a^b g'(h(x))h'(x)\,dx = 0$.

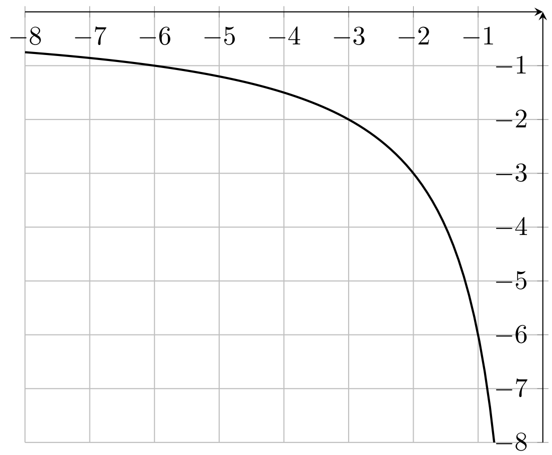
   (d) **True** or **False**? The sums $\sum_{i=10}^{100} 2^i$ and $\sum_{i=0}^{90} 2^{i+10}$ are equal.

3. (5 points) Is $\dfrac{x^2}{2}\sin(x)$ an antiderivative of $x\cos(x)$? Why or why not?

4. (5 points) Briefly explain why the expression $\displaystyle\int_{-3}^{3} \frac{1}{u^2+1}\,dx$ does not make sense.

5. (3 points) Compute $\displaystyle\frac{d}{dx}\int_0^5 \frac{t}{t^7+1}\,dt$

6. Below is the graph of the function, $f(x) = \dfrac{6}{x}$



(a) (4 points) On the graph, draw the rectangles which correspond to the left Riemann sum with four rectangles for $f(x)$ on the interval $[-3, -1]$.

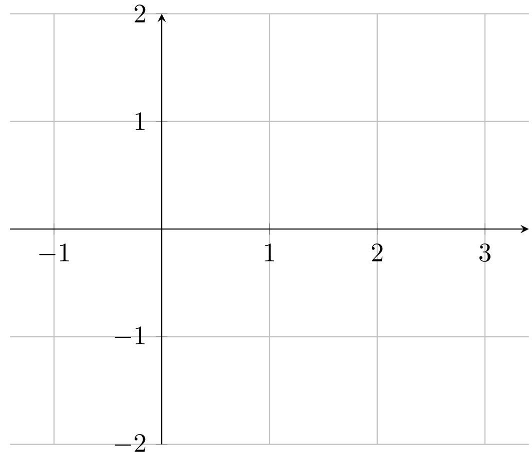
(b) (4 points) Compute the left Riemann sum with four rectangles to approximate $\int_{-3}^{-1} f(x)\,dx$. You can use your picture in part (a) to help.

7. (6 points) On the below axes, draw the graph of a function $g(t)$ with the following properties:

   (a) $g(t)$ is continuous on $[-1, 3]$

   (b) $\displaystyle\int_{-1}^{1} g(t)\,dt < 0$

   (c) $\displaystyle\int_{-1}^{3} g(t)\,dt > 0$



8. (a) (8 points) Compute $\int_1^7 \sqrt{9-(x-4)^2}\,dx$

(b) (12 points) Use your answer from part (a) to compute $\int_1^7 \sqrt{9-(x-4)^2} + \frac{x}{x^2+1}\,dx$

If you didn't get an answer in part (a), make one up and then use it here!

9. (14 points) t years after January 1, 2020, the rate of change in the number of members of a family of river otters is approximately $\dfrac{e^{1/t}}{t^2}$ otters per year. If there were 12 otters in the population on January 1, 2022, how many otters will there be in the family on January 1, 2025?

4.6 Sample Lectures

You can find many of my lectures for the Integral Calculus course I taught remotely and asynchronously in spring 2020 on my YouTube channel here: `https://www.youtube.com/channel/UC8pcvjDge4M3MMqpdci0yCQ/featured`