Tephigram Method for Bounding Intensity of Tropical Cyclones

ambient: $p_{amb}(z)$, $T_{amb}(z)$, $RH(z)$ from $T(p)$, $RH(p)$ data for a convectively unstable ambient

moist adiabat:

(1) lifting condensation level for sea-level-ambient air (expanding to saturation on a dry adiabat, Y held const)

$$Y = \frac{\rho_v}{\rho} = \frac{\sigma\, RH_{ref}\, P(T_{ref})}{p_{ref}} = \frac{\sigma P(T_{lcl})}{p_{lcl}}$$

ref state is sea level at the periphery $(r = r_0,\ z = 0)$

$$p \sim \rho^{\gamma} \Rightarrow \frac{p_{lcl}}{p_{ref}} = \left(\frac{T_{lcl}}{T_{ref}}\right)^{(\gamma-1)/\gamma}$$

$$\therefore \frac{(RH)_{ref}\, P(T_{ref})}{(T_{ref})^{(\gamma-1)/\gamma}} = \frac{P(T_{lcl})}{(T_{lcl})^{(\gamma-1)/\gamma}} \Rightarrow \frac{p_{lcl}}{p_{ref}} = \left(\frac{T_{lcl}}{T_{ref}}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\text{also, } \rho_{lcl} = \frac{p_{lcl}}{R\, T_{lcl}}$$

(2) locus of moist adiabat

$$c_p\, dT + L\sigma\, d[P(T)/p] - dp/\rho \simeq 0$$

$$\left[c_p + \frac{L\sigma\, dP(T)/dT}{p}\right] dT = +\left[\frac{RT}{p} + \frac{L\sigma P(T)}{p^2}\right] dp$$

- integrate $\dfrac{dT}{dp} = \dfrac{RT/p + L\sigma P(T)/p^2}{c_p + \dfrac{L\sigma}{p}\dfrac{dP(T)}{dT}}$ from LCL

call this $T_{moist}(p)$

- find p_t: $T_{moist}(p_t) = T_{amb}(p_t)$; assign p_t, T_t, z_t to this state, where $p_{amb}(z_t) = p_t$

assigns z_t; $T_{moist}(p_t) = T_{amb}(p_t) = T_t$

subscript t refers to the lid (or tropopause)

- $$\frac{dp_{moist}}{dz} = -\rho_{moist}\, g; \quad \frac{dT_{moist}/dz}{dp_{moist}/dz} = \frac{RT_{moist}/p_{moist} + \cdots}{c_p + \dfrac{L\sigma}{p_{moist}}\dfrac{dP(T_{moist})}{dT_{moist}}}$$

$$p_{moist}(z_t) = p_t, \quad T_{moist}(z_t) = T_t$$

find $p_{moist}(0)$, $T_{moist}(0)$ -- be sure $p_{moist} < p_{lcl}$

- iterate up/down sequence until $p_{moist}(0)$, $T_{moist}(0)$ is recovered eye : $c_p T_t + L\sigma(P(T_t))/p_t + g z_t \equiv H_t$

at the tropopause, the moist content is negligible

$$c_p T_{eye}(z) + L\sigma (RH)_{eye} \frac{P(T_{eye}(z))}{p_{eye}(z)} + gz = H_t$$

can differentiate with respect to z to get ordinary differential equation

$(RH)_{eye}$ taken as const

assign $0 \leq (RH)_{eye} \leq 1$ (eye is unsaturated but not necessarily totally dry)

$$\frac{d p_{eye}}{dz} = -\rho_{eye}\, g \,,\quad p_{eye} = \rho_{eye} R T_{eye}$$

integrate from $z = z_t$ to $z = 0$

$p_{eye}(0)$ is to be compared to $p_{moist}(0)$,

$p_{amb}(0)$

- now suppose at some altitude $z_{underrun}$ (assigned), moist-adiabatic (saturated) air lies under unsaturated eye air:

$$(RH) = \begin{cases} (RH)_{eye}, & z_{underrun} < z < z_t \\ 1, & 0 \leq z < z_{underrun} \end{cases}$$

There is a contact surface at $z = z_{underrun}$, but no shock: ρ, T, Y may be discontinuous but p is continuous. From $z_{underrun} > z \geq 0$, there is no change in energy?

$$c_p T_{underrun} + L\sigma \frac{P(T_{underrun})}{p_{underrun}} + g z_{underrun} = H_t$$

$$\frac{d p_{underrun}}{dz} = -\rho_{underrun}\, g,\quad p_{underrun} = \rho_{underrun} R T_{underrun}$$

now $p_{underrun}(0)$ is to be compared to $p_{moist}(0)$,

$p_{amb}(0)$

onversion to an intensity (peak swirl):

$$\rho \frac{v^2}{r} \approx \frac{\partial p}{\partial r} \Rightarrow v_{max}^2 = \frac{\int_{p(0)}^{p_{ref}} dp}{\int_0^{r_0 \to \infty} \rho \frac{v^2(r)}{v_{max}^2\, r}\, dr}$$

$$v_{max} \cong \frac{1}{\rho_{ave}^{1/2}} \frac{\left[p_{ref} - p(0)\right]^{1/2}}{\left[\int_0^\infty \frac{v^2(x)}{v_{max}^2\, x}\, dx\right]^{1/2}}, \quad x \equiv \frac{r}{r_{max}}$$

$$\frac{v(x)}{v_{max}} = \begin{cases} x, & 0 < x \leq 1 \\ \frac{1}{x}, & 1 \leq x \leq \infty \end{cases}$$

Rankine vortex
rigidly rotating core
patched to potential vortex

$$\therefore \quad v_{max} = \frac{\left[p_{ref} - p(0)\right]^{1/2}}{\rho_{ave}^{1/2}} \quad \text{since} \int_0^\infty \frac{v^2(x)}{v_{max}^2} \frac{1}{x}\, dx = 1$$

for a Rankine vortex

- Conclusion from calculation: $\left[p_{ref} - p(0)\right]^{1/2}$ is much larger because $p(0)$ is much smaller if there is an eye, for then $p(0) \equiv p_{eye}(0)$. However: $\longrightarrow$ too moist an eye, and/or too much rainderruen even at the vertical central axis, is problematic.

Then, $p(0) \approx p_{moist}^{(0)} \Rightarrow$
and $\left[p_{ref} - p(0)\right]^{1/2}$ is much smaller.