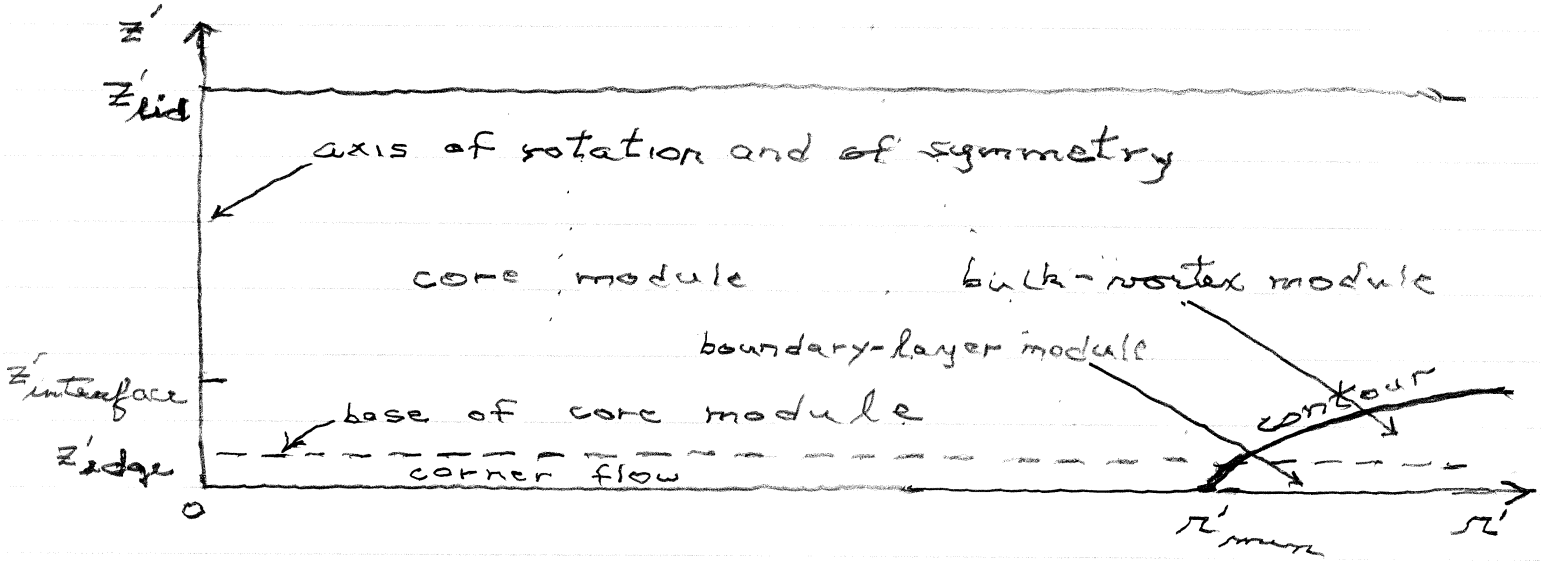
Notes on Starting Conditions for the Core Module



This sketch is a little more appropriate for the saucer-like proportions of a tropical cyclone, which has diameter readily 60 times its height, though the inner portion of the core has a diameter closer to 2.5-5 times its height: $z'_{lid} \approx 12$ km, while $r'_{min} \approx 30-60$ km and $z'_{edge} \approx 1$ km.

The Prandtl condition that $\partial p'/\partial z' \approx 0$ across the boundary-layer module (aside from hydrostatic changes) does <u>not</u> imply $\partial p'/\partial r' \approx 0$ across the lower core module. The average updraft in the lower core may average 100 times the downdrift speed into the boundary layer. The pressure fall from $r' = r'_{min}$ to $r' = 0$ may equal to pressure fall from $r' = r'_0$ to $r' = r'_{min}$ in the lower vortex. Short-lived core cbs may have 20-mph updrafts.

The case depicted is a tropical depression or tropical storm. There is no eye module, and the corner flow extends to the axis.

— We relate conditions at the exit from the corner $(0 < r' < r'_{min},\ z' = z'_{edge})$ to conditions at the entry to the corner $\{r' = r'_{min}$, or $x = x_{min}$; $0 < z' < z'_{edge}$; or $0 \le \zeta < \zeta_{edge}$, where $\zeta = z' / \{\nu'/2\Omega'\}^{1/2}$, $x = r'^2 / r_0'^2$. We conserve mass flux (total mass/time), and distribute the total stagnation energy over streamlines at the exit as at the entry, so the total-stagnation-energy flux is conserved.

— We do not conserve total angular momentum in the corner flow because this would imply that those fluid elements that approach closest to the axis of rotation may spin up the most. Instead, we suggest that the relative-swirl velocity component is distributed over streamlines at the exit as at the entry.

— In summary, whereas the dynamics is taken to be inviscid, and the energetics to be diffusive, in the bulk-vortex module, in contrast for the corner flow, the dynamics is taken to be dissipative, and the energetics inviscid. Saturation occurs in the corner flow. What holds along $0 < r' < r'_{min}$, $z' = z'_{edge}$, is significant because the core flow is taken to be inviscid (and saturated). Thus, the total angular momentum, total stagnation enthalpy, and entropy remain constant on streamlines at the value holding at entry on each streamline.

- At $z' = z'_{edge}$, $0 < r' \le r'_{min}$,

$$\frac{\partial \eta'}{\partial r'} = \rho' w' r'$$

where η' is the streamfunction. So

$$\eta'(r'_{min}, z'_{edge}) - \eta'(0, z'_{edge})$$
$$= \int_0^{r'_{min}} \rho'(r', z'_{edge})\, w'(r', z'_{edge})\, r'\, dr',$$

or, if $\rho'(r', z'_{edge})\, w'(r', z'_{edge})$ is const,

$$\eta'_{max} - 0 = \rho'(r', z'_{edge})\, w'(r', z'_{edge}) \frac{r'^2_{min}}{2};$$

$$\rho'(r', z'_{edge})\, w'(r', z'_{edge}) = 2\eta'_{max} / r'^2_{min}$$

Hence,

$$\eta'(r', z'_{edge}) = \frac{2\eta'_{max}}{r'^2_{min}} \left(\frac{r'^2}{2}\right) \Rightarrow \frac{\eta'(r', z'_{edge})}{\eta'_{max}} = \frac{r'^2}{r'^2_{min}}.$$

The magnitude of η'_{max} is $\left[\rho'_{ref} \equiv \rho'(r'_0, 0) = \rho'_{amb}(0)\right]$

$$\rho'_{ref}\, \Omega' r'^3_0 (E/2)^{1/2} \int_{x_{min}}^{1} \rho(x, \zeta_{edge}) \left[-W(x)\right] dx,$$

Ekman number $E \equiv \nu'/(\Omega' r'^2_0)$. $\eta'(r', z'_{edge}) \equiv \eta'_{edge}(r')$

- We take

$$\Gamma'(r', z'_{edge}) = r' v'(r', z'_{edge}) + \Omega r'^2$$

— We relate $v'(r', z'_{edge}) = v'_{edge}\left[\eta'_{edge}(r')\right]$, $\eta'_{edge}(r') = \eta'_{max}\left(\frac{r'}{r'_{min}}\right)^2$

$$v'_{edge}(\eta'_{edge}(r')) = v'_{bl}\left(\eta'_{max} \frac{\eta^*}{\eta^*_{edge}}\right) = v'_{bl}(\eta^*) = \Omega' r'_0 \frac{\psi\left[x_{min}, \zeta(\eta^*)\right]}{x^{1/2}_{min}},$$

where $\eta^*(x_{min}, \zeta) = -\int_0^{\zeta} \phi(x_{min}, \underline{\zeta})\, d\underline{\zeta}$, $\psi = \frac{r'}{r'_0} \frac{v'_{bl}}{r'_0 \Omega'} = r\, v_{bl}$

$$\psi(x_{min}, \zeta) = r_{min}\, v(r_{min}, \zeta) = x_{min}^{1/2}\, v(x_{min}, \zeta) = x_{min}^{1/2}\, v_{bl}(\zeta) \quad 4$$

where

$$0 \le \eta^* \le \eta^*_{max}, \quad 0 \le \zeta \le \zeta_{edge}, \quad 0 \le v'_{edge} \le V'(r'_{min})$$

. We take

$$E'(r', z'_{edge}) = c_p' T'(r', z'_{edge}) + L' \sigma P'[T'(r', z'_{edge})]/p'(r', z'_{edge}) + g' z'_{edge} + \frac{V'^2}{2}, \quad V' = \Omega r_0' \frac{(1+\epsilon - x_{min})}{x_{min}^{1/2}}$$

$$E'(r', z'_{edge}) = E'_{edge}[\eta'_{edge}(r')]$$

$$\doteq E'_{bl}\left(\eta'_{max} \frac{\eta^*}{\eta^*_{edge}}\right) = E'_{bl}(\eta^*)$$

$$= c_p' T_{ref} E[x_{min}, \zeta(\eta^*)]$$

. We take

$$\frac{\partial p'(r', z'_{edge})}{\partial r'} \doteq \frac{p'(r', z'_{edge})}{R' T'(r', z'_{edge})} v'^2(r', z'_{edge})$$

$$p'(r'_{min}, z'_{edge}) = p'_{min}$$

— $\rho'(r', z'_{edge}) = p'(r', z'_{edge})/R'T'(r', z'_{edge}) \Rightarrow w'(r', z'_{edge}) = \ldots$

This is integrated along with the equation for $E(r', z'_{edge})$.

. We take $S'(r', z'_{edge}) = S'_{edge}(\eta'_{edge}(r'))$

$$\frac{S'(r', z'_{edge})}{R'} = \ln\left[T'(r', z'_{edge})/T_{ref}\right]^{\gamma/(\gamma-1)}$$

$$- \ln\left[p'(r', z'_{edge})/p'_{ref}\right]$$

$$+ \frac{L' \sigma P'[T'(r', z'_{edge})]}{R' T'(r', z'_{edge})\, p'(r', z'_{edge})}$$