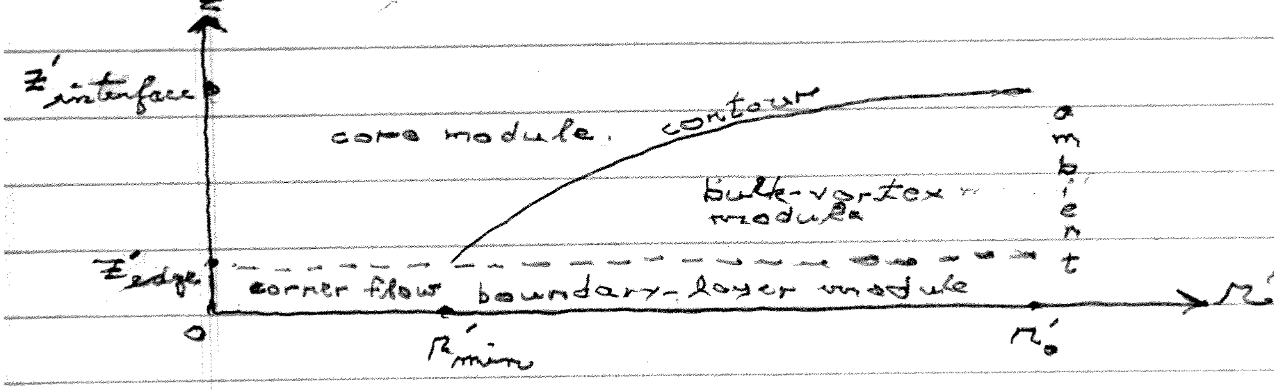
Specifying the Position of, and Properties Holding on, the Contour between the Bulk-Vortex Module and Core Module



- We seek to define the curve ("contour") between (r'_{min}, z'_{edge}) and $(r'_0, z'_{interface})$, given end points

— The contour position is defined by continuity of pressure $p'(r', z')$ because the dynamics on each side is inviscid. The swirl $v'(r', z')$ is continuous across the contour, but the radial velocity component $u'(r', z')$ and axial velocity component $w'(r', z')$ are discontinuous, so contour is a vortex sheet for the "secondary flow". The pressures $p'(r'_{min}, z'_{edge})$ and $p'(r'_0, z'_{interface})$ are known from the bulk-vortex analysis, with $p'(r'_{min}, z'_{edge}) > p'(r'_0, z'_{interface})$ in cases of interest.

— The temperature and density are discontinuous across the contour, which is a contact surface. The air is unsaturated in the bulk-vortex module, and saturated in the core module.

— The contour is a streamline: there is no flow across it.

— The entropy, angular momentum, and total stagnation energy are constant on streamlines on the core side, which is inviscid. The value of each variable varies from streamlines to streamline.

— Only the angular momentum is constant on streamlines on the bulk-vortex side, and for tractability we examine only the special case for which the constant is invariant from streamline to streamline. Thus the model for the swirl in the bulk-vortex module is a potential vortex, generalized to the noninertial coordinate system.

— We anticipate that the pressure $p'(r', z')$ on the contour decreases monotonically from $p'(r'_{min}, z'_{edge})$ to $p'(r'_o, z'_{interface})$, and that the curve $z'_{contour}(r')$ has monotonically decreasing but nonnegative slope as one goes from (r'_{min}, z'_{edge}) to $(r'_o, z'_{interface})$. (The inverse ~~reprosentation~~ representation $R'_{contour}(z')$ has a monotonically increasing and nonnegative slope.) According to our model, r'_{min} decreases, but z'_{edge}, r'_o, and $z'_{interface}$ do not change, as the vortex intensifies, so the contour shape is not anticipated to be a sensitive indicator of intensity for a fixed ambient. The position r'_{min} is a sensitive indicator of intensity.

- Bulk-Vortex Module
  - conservation of angular momentum

$$r'v'(r') + \Omega' r'^2 = r_o' v_o' + \Omega' r_o'^2 = \Omega' r_o'^2 (1+\epsilon) = \Gamma_o'$$

  - pressure field

$$\rightarrow p'(r', z') - p'(r_o', z'_{edge}) \doteq -g' \int_{z'_{edge}}^{z'} \rho'_{amb}(\underline{z}')\, d\underline{z}'$$

$$- \int_{r'}^{r_o'} \rho'(\underline{r}', z'_{edge}) \left\{ 2\Omega' v'(\underline{r}) + \frac{v'^2(\underline{r}')}{\underline{r}'} \right\} d\underline{r}'$$

The two integrals can be tabulated ahead of time. Recall that $\rho'(r', z'_{edge})$ is obtained in the bulk-vortex-module equations. We much prefer this algebraic approximation for $p'(r', z')$ than to deal with a differential-equation/algebraic-equation mixture in defining the contour.

- Core Side of Contour Streamline

$$r'v'(r') + \Omega' r'^2 = \Gamma_o'$$

  - 4 eqs. for 4 unknowns: $T'_{core}(r'_{min}, z'_{edge})$, $T'_{core}(r_o', z'_{interface})$, E'_{core}, S'_{core}

$$E'_{core}(r'_{min}, z'_{edge}) \doteq c_p' T'_{core}(r'_{min}, z'_{edge}) + \frac{L' \sigma P'[T_{core}(r'_{min}, z'_{edge})]}{p'(r'_{min}, z'_{edge})}$$

$$+ g' z'_{edge} + \frac{v'^2(r'_{min})}{2}$$

$$E'_{core}(r'_{min}, z'_{edge}) = E'_{core}(r_o', z'_{interface}) = c_p' T'_{core}(r_o', z'_{interface})$$

$$+ \frac{L' \sigma P'[T'_{core}(r_o', z'_{interface})]}{p'(r_o', z'_{interface})} + g' z'_{interface} + \frac{v'^2(r_o')}{2}$$

$$\frac{S'_{core}(r'_{min}, z'_{edge})}{R'} = \ln\left(\frac{T'_{core}(r'_{min}, z'_{edge})}{T'_{ref}}\right)^{\frac{\gamma}{\gamma-1}} - \ln\left(\frac{p'(r'_{min}, z'_{edge})}{p'_{ref}}\right)$$

$$+ \frac{L'\sigma P'[T'_{core}(r'_{min}, z'_{edge})]}{R' T'_{core}(r'_{min}, z'_{edge})\, p'(r'_{min}, z'_{edge})}$$

$$T'_{ref} = T'(r'_o, 0), \quad p'_{ref} = p'(r'_o, 0)$$

$$\frac{S'_{core}(r'_{m}, z'_{edge})}{R'} = \frac{S'_{core}(r'_o, z'_{interface})}{R'} = \ln\left(\frac{T'_{core}(r'_o, z'_{interface})}{T'_{ref}}\right)^{\gamma/(\gamma-1)}$$

$$- \ln \frac{p'(r'_o, z'_{interface})}{p'_{ref}}$$

$$+ \frac{L'\sigma P'[T'_{core}(r'_o, z'_{interface})]}{R' T'_{core}(r'_o, z'_{interface})\, p'(r'_o, z'_{interface})}$$

— Pressure field

$$\rightarrow \quad E'_{core}(r'_{min}, z'_{edge}) = c'_p T'_{core}(r', z') + \frac{L'\sigma P'[T'_{core}(r', z')]}{p'(r', z')}$$

$$+ g' z' + \frac{v'^2(r')}{2}$$

$$\rightarrow \quad \frac{p'(r', z')}{p'_{ref}} = \left(\frac{T'_{core}(r', z')}{T'_{ref}}\right)^{\gamma/(\gamma-1)}$$

$$\cdot \exp\left[-\frac{S'_{core}(r'_{min}, z'_{edge})}{R'} + \frac{L'\sigma P'[T'_{core}(r', z')]}{R' T'_{core}(r', z')\, p'(r', z')}\right]$$

- The three equations with arrows are three coupled nonlinear algebraic equations for $\{r', z', T'_{core}(r', z')\}$, where r', z' is the point on the contour where the pressure is $p'(r', z')$, an assigned value within the range $p'(r'_{min}, z'_{edge})$, $p'(r'_o, z'_{interface})$

— This is a step-by-step progression, starting from one end point on the contour, and proceeding to the other end point on the contour. The first guess at the next point in the progression is the triplet of results holding at the last converged point.

— This is a parametric solution:

$$r'[p'],\ z'[p'],\ T'_{core}[p'].$$

Of course, one of the other variables could have been selected as the parameter, at least in theory.

— In the bulk-vortex module

$$\frac{S'(r',z')}{R'} = \ln\left(T'(r',z')/T'_{ref}\right)^{\gamma/\gamma-1} - \ln\left(p'(r',z')/p'_{ref}\right)$$

The core-side streamline at the contour spends so little time in the boundary layer no angular momentum is lost, but the onset of saturation changes density, temperature, vapor mass fraction, and entropy across the contour.