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Units and notation

SI units (m-kg-s, K)

pressure	p	$Pa \equiv kg/(m\ s^2)$
density	ρ	kg/m^3
temperature	T	K
altitude	z	m
water vapor mass fraction	q	(dimensionless)
relative humidity	rh	(dimensionless)

Constants

gravity	g	$9.8\ m/s^2$
gas constant for air	R	$287.1\ m^2/(s^2\ K)$
ratio, molecular wts	σ	0.622 (dimensionless)
specific heat capacity, const. p	c_p	$10^4\ m^2/(s^2\ K)$
specific latent heat, phase transition	L	$2.5 \times 10^6\ m^2/s^2$

Subscripts

ambient	amb
moist adiabat	moist
eye	eye
tropopause	trop
saturated	sat
water vapor	v
saturated water vapor	vsat
lifting condensation level	switch
ocean-surface value	s
underrunning moist air	underrun

(B)

Inputs and Functions (given)

ambient relative humidity rhamb[p], tabular
ambient temperature tamb[p], tabular

saturated vapor pressure pvsat[T]

$$pvsat[T] = 610.78 \exp\{[a(T)][T-273.16]/[T-b(T)]\}$$

$$a[T] = \begin{cases} T > 273.16, & 17.2693882 \\ T \le 273.16, & 21.8745584 \end{cases}$$

$$b[T] = \begin{cases} T > 273.16, & 35.86 \\ T \le 273.16, & 7.66 \end{cases}$$

(curve fit to data)

Equations

- Ambient Profiles with Altitude [calculated from $T_{amb}[p]$, $rh_{amb}[p]$]

$$p \doteq \rho R T \;,\; \sigma p_v = \rho_v R T \quad \text{(equations of state)}$$

$$y = \rho_v / \rho \quad \text{(definition of water vapor mass fraction)}$$

$$rh = p_v / P(T) \quad \text{(definition of relative humidity)}$$

[vapor mass fraction $y \doteq \sigma (rh) P(T)/p$]

$$\frac{\partial p}{\partial z} = -\rho g \quad \text{(hydrostatics)}$$

- Moist Adiabat Profiles

$$\left(\frac{T}{T_{ref}}\right)^{\gamma/(\gamma-1)} = \frac{p}{p_{ref}} = \frac{p_v}{(p_v)_{ref}} \quad \text{on a dry adiabat (since } y \text{ const)};$$

$$[\gamma/(\gamma-1)] = R/c_p$$

Let T be the lifting-condensation-level state, and T_{ref} be the sea-level-ambient state; $\gamma = 1.4$

$$\left[\frac{(T_{sat})_{onset}}{(T_{amb})_A}\right]^{3.5} = \frac{p_v[(T_{sat})_{onset}]}{[(rh)_{amb}]_A \, p_v[(T_{amb})_A]}$$

This gives the temp. at which surface air would saturate if lifted dry adiabatically. The temp. implies a pressure on the adiabat.

Once saturated, the moist adiabat follows the following locus of the thermodynamic states, to rough approximation (the condensate falls out):

$$c_p \, dT + L\sigma \, d\{P[T]/p\} - \frac{dp}{\rho} = 0$$

where $T(p_{onset}) = (T_{sat})_{onset}$

Where this $T(p)$ curve crosses $T_{amb}(p)$ is identified as the tropopause. The height of the tropopause is from $z_{amb}(p)$, inverse of $p_{amb}(z)$.

Note: The moist-adiabat locus is based on the total energy $c_pT + Lq + gz + q^2/2 = \text{const}$, where $q^2/2$ (the kinetic energy) is about a 1.5% contribution, and discarded.

Integrate from the tropopause seaward to find the thermodynamic state holding at the base $z = 0$:

$$\frac{dp}{dz} = -\rho g$$

$$c_p \frac{dT}{dz} + \sigma L \frac{d}{dz}\left[P(T)/p\right] - \frac{1}{\rho}\frac{dp}{dz} = 0$$

$$p(z_{tropopause}) = p_{tropopause}$$

$$T(z_{tropopause}) = T_{tropopause}$$

Remember to drop the terms with L thenceforth if T increases in value to $(T_{sat})_{onset}$ before $z = 0$ is reached; i.e., switch over to the dry adiabat below the lifting condensation level. Note $p_{moist}(z=0)$.

There is a self-consistency issue that may call for iteration of the moist-adiabatic result. The ambient-sea-level state is adopted as the reference state for the adiabat, but is really not pertinent to a vertical column in the core. The final state computed at $z = 0$ should be used as the reference state for a second calculation of temperature vs. pressure, identification of the tropopause, and assignment of altitudes. Presumably the process quickly converges, so the initial sea-level state is recovered as the sea-level state, to excellent approximation. This iteration is not part of the current notebook.

- Eye Adiabat Profiles

Integrate seaward from the tropopause for an unsaturated eye. Thus:

$$\frac{dp}{dz} = -\rho z,$$

$$c_p \frac{dT}{dz} - \sigma L (RH) \frac{d\left[P(T)/p\right]}{dz} - \frac{1}{\rho}\frac{dp}{dz} = 0$$

$$T(z_{tropopause}) = T_{tropopause},\ p(z_{tropopause}) = p_{tropopause}$$

The key new parameter here is RH, which denotes the relative humidity in the eye. If RH = 0, the eye is totally dry, and, for this extreme idealization, the pressure at sea level is quite decremented from pambo, the sea-level ambient value (given).

We envision some evaporative cooling because condensate (ice crystals and droplets) fall into the eye and are evaporated. The value of RH could vary with altitude, but the simplest procedure is to hold it constant with height, at some value between 0 and 1. At RH = 1, all the heating in the eyewall owing to condensation is reversed in the eye owing to evaporation.

In the current notebook, there is an attempt to deal with a moist inflow underrunning an eye that descends only part of the distance from the tropopause to the sea surface. So for $z_{underrun} < z < z_{tropopause}$, with $z_{underrun}$ specified (if $z_{underrun} = 0$, there is no underrun), or for $p_{tropopause} < p < p_{underrun}$, with $p_{underrun}$ specified (if $p_{underrun} > (p_{moist})_{surface}$, there is no underrun), use the above equations in the eye. In the underrun, use the same equations except $RH = 1$. However, at $z = z_{underrun}$ (or $p = p_{underrun}$ --- a value for one implies a value for the other), there is a contact surface: pressure p is continuous, but density ρ and temperature T are continuous. Also, the total energy is continuous (within our approximation that the kinetic energy is negligible). So

$$c_p T_+ + GL(RH)\,P(T_+)/p_{underrun}$$
$$= c_p T_- + GL\,P(T_-)/p_{underrun},$$

where

$$T_+ = T(z_{underrun}{}^+), \text{ known},$$
$$T_- = T(z_{underrun}{}^-), \text{ to be found},$$

and, for completeness, if $\rho_- = \rho(z_{underrun}{}^-)$,

$$\rho_- = p_{underrun}/RT_- .$$

Then one integrates moist-adiabat equations

$$\frac{dp}{dz} = -\rho g$$
$$c_p \frac{dT}{dz} + GL\,\frac{d\,[P(T)/p]}{dz} - \frac{1}{\rho}\frac{dp}{dz} = 0$$

to $z=0$ to find the pressure at the surface beneath a partially inserted edge.

- Translation of Sea-Level Pressure Anomalies to Peak Swirl

Let swirl $v(r) = \begin{cases} V_{max}\,(r/r_{max}), & 0 < r < r_{max} \\ V_{max}\,(r_{max}/r), & r_{max} < r < \infty \end{cases}$

This patching of a rigidly rotating core to a potential vortex (Rankine vortex) lets V_{max} be estimated from a cyclostrophic balance by substitution and integration

$$\frac{v^2}{r} = \frac{1}{\rho}\frac{\partial p}{\partial r}, \quad 0 \leq r < \infty,$$

provided we estimate the density ρ (e.g., ascribe ρ some average value).

For a tropical storm (no eye, so the moist adiabat holds near $r = 0$)

$$V_{max} = \left\{(P_{ambt} - P_{moist}) / [(\rho_{ambt} + \rho_{moist})/2]\right\}^{1/2}$$

For a hurricane with a partially inverted eye,

$$V_{max} = \left\{(P_{ambt} - P_{eye}) / [(\rho_{ambt} + \rho_{eye})/2]\right\}^{1/2}$$

For a hurricane with a fully inverted, nonrotating eye, so there is no rigidly rotating core,

$$V_{max} = \left\{2(P_{ambt} - P_{eye}) / [(\rho_{ambt} + \rho_{eye})/2]\right\}^{1/2}$$

There is more pressure deficit for fully inverted eye, and half need not be expended maintaining a rigidly rotating core. In any case, the absence of r_{max} in the expressions for V_{max} is noteworthy (and convenient).

Incidentally, plots of total energy (ignoring kinetic energy)

$$H(z) = c_p T(z) + L \frac{\epsilon (RH)\, P(T(z))}{p(z)} + gz)$$

for the various columns is informative.