

$$\underline{12} \quad \vec{F}_1 = \frac{G m_1 m_2}{|\vec{r}_{12}|^2} \hat{r}_{12} + \frac{G m_1 m_3}{|\vec{r}_{13}|^2} \hat{r}_{13} \quad \left\{ \begin{array}{l} \text{each star} \\ \text{experiences} \\ \text{gravitational} \\ \text{force due to} \\ \text{other two stars.} \end{array} \right.$$

$$\vec{F}_2 = \frac{G m_3 m_2}{|\vec{r}_{23}|^2} \hat{r}_{23} + \frac{G m_2 m_1}{|\vec{r}_{21}|^2} \hat{r}_{21}$$

$$\vec{F}_3 = \frac{G m_1 m_3}{|\vec{r}_{31}|^2} \hat{r}_{31} + \frac{G m_2 m_3}{|\vec{r}_{32}|^2} \hat{r}_{32} \quad \left\{ \vec{F}_i = \sum_{j=1, j \neq i}^3 \frac{\hat{r}_{ij} G m_i m_j}{|\vec{r}_{ij}|^2} \right.$$

where \vec{F}_1, \vec{F}_2 & \vec{F}_3 are forces acting on star 1, 2 & 3 respectively and \vec{r}_{ij} is vector pointing from star i to star j and m_i is mass of star i .

Let \vec{r}_i be position vector of star i .

$$\Rightarrow \vec{F}_i = m_i \frac{d^2 \vec{r}_i}{dt^2} \Rightarrow \boxed{\frac{d^2 \vec{r}_i}{dt^2} = \frac{\vec{F}_i}{m_i}}$$

Since \vec{r}_{ij} is vector from i to j ,

$$\boxed{\vec{r}_{ij} = \vec{r}_j - \vec{r}_i} \Rightarrow \boxed{\hat{r}_{ij} = \frac{\vec{r}_{ij}}{|\vec{r}_{ij}|} = \frac{\vec{r}_j - \vec{r}_i}{|\vec{r}_j - \vec{r}_i|}}$$

$$\Rightarrow \vec{F}_i = \sum_{j=1, j \neq i}^3 \frac{G m_i m_j}{|\vec{r}_j - \vec{r}_i|^3} (\vec{r}_j - \vec{r}_i) \Rightarrow \frac{\vec{F}_i}{m_i} = \frac{1}{m_i} \cdot m_i \sum_{j=1, j \neq i}^3 \frac{G m_j}{|\vec{r}_j - \vec{r}_i|^3} (\vec{r}_j - \vec{r}_i)$$

$$\Rightarrow \boxed{\frac{d^2 \vec{r}_i}{dt^2} = \sum_{j=1, j \neq i}^3 \frac{G m_j}{|\vec{r}_j - \vec{r}_i|^3} (\vec{r}_j - \vec{r}_i)}$$

$$\Rightarrow \frac{d^2 \vec{r}_1}{dt^2} = \frac{G m_2}{|\vec{r}_2 - \vec{r}_1|^3} (\vec{r}_2 - \vec{r}_1) + \frac{G m_3}{|\vec{r}_3 - \vec{r}_1|^3} (\vec{r}_3 - \vec{r}_1)$$

$$\frac{d^2 \vec{r}_2}{dt^2} = \frac{G m_1}{|\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2) + \frac{G m_3}{|\vec{r}_3 - \vec{r}_2|^3} (\vec{r}_3 - \vec{r}_2)$$

$$\frac{d^2 \vec{r}_3}{dt^2} =$$

$$\frac{G m_2}{|\vec{r}_2 - \vec{r}_3|^3} (\vec{r}_2 - \vec{r}_3) + \frac{G m_1}{|\vec{r}_1 - \vec{r}_3|^3} (\vec{r}_1 - \vec{r}_3)$$