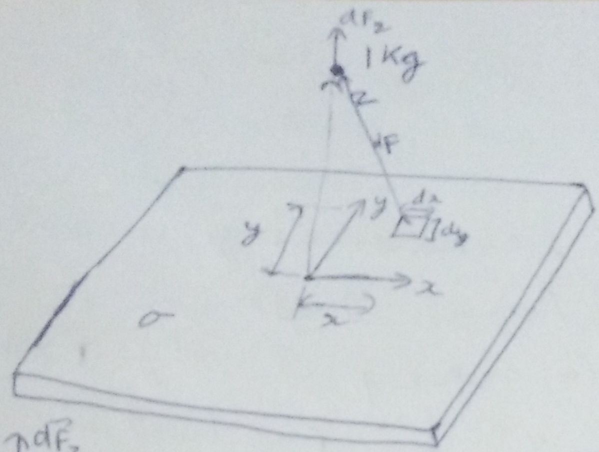
6

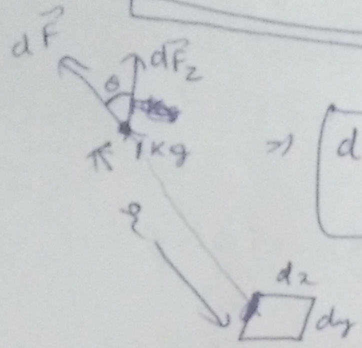


Pick a small (rectangular) element of area $(dx \cdot dy)$

Its mass is $\sigma \cdot dx \cdot dy$

$$\boxed{dm = \sigma \, dx \, dy}$$

Let it exert force dF.



$$\Rightarrow \boxed{d\vec{F} = \hat{r}\frac{(dm)G}{r^2}} \; ;$$

r is distance between point mass & small element

Coordinate of point mass is $(0,0,z)$ & element is $(x,y,0)$

$$\cancel{\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}}$$

$$\Rightarrow \boxed{r = \sqrt{x^2+y+z^2}}$$

$$\cancel{\Rightarrow dF_z = dF\cos\theta}$$

$$\Rightarrow \boxed{d\vec{F}_z = d\vec{F}\cdot\hat{z} = G\frac{dm}{r^2}\,\hat{r}\cdot\hat{z} = \frac{G(dm)}{r^3}\,\vec{r}\cdot\hat{z}} \quad \text{where } \hat{r} = \frac{\vec{r}}{r}$$

$\vec{r}$ is vector ~~between~~ pointing towards point mass from element

$$\Rightarrow \vec{r} = z\hat{z} - x\hat{x} - y\hat{y} \quad \Rightarrow \boxed{\vec{r}\cdot\hat{z} = z}$$

$$\Rightarrow d\vec{F}_z = \frac{(dm)zG}{r^3} \Rightarrow \boxed{dF_z = \frac{\sigma z G}{r^3}\,dx\,dy}$$

$$\cancel{\Rightarrow F_z = \int_{-L/2}^{L/2}\int_{-L/2}^{L/2} \frac{G\sigma z}{(x^2+y^2+z^2)}}$$

$$\Rightarrow dF_z = \frac{G\sigma z \; dx\,dy}{\left(\sqrt{x^2+y^2+z^2}\right)^3}$$

Integrating over plate, $F_z = \displaystyle\int_{-L/2}^{L/2}\int_{-L/2}^{L/2} \frac{G\sigma z \; dx\,dy}{(x^2+y^2+z^2)^{3/2}}$

$$\Rightarrow \boxed{F_z = G\sigma z \int_{-L/2}^{L/2}\int_{-L/2}^{L/2} \frac{dx\,dy}{(x^2+y^2+z^2)^{3/2}}}$$