

2)

Roots given by 1st method are

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Roots given by 2nd method are

$$x_1 = \frac{2c}{-b - \sqrt{b^2 - 4ac}} x_2 = \frac{2c}{-b + \sqrt{b^2 - 4ac}}$$

When  $|b|$  is large compared to geometric mean of  $a$  and  $c$  (i.e.  $|b| \gg \sqrt{ac}$ ),  $b^2 - 4ac$  is very close to  $b^2$ . So,  $\sqrt{b^2 - 4ac}$  is very close to  $|b|$ .

If  $b \geq 0$ , then  $|b| = b$ . Then, for large  $|b|$ ,  $-b + \sqrt{b^2 - 4ac} \approx -b + |b| = -b + b = 0$ . So,  $-b + \sqrt{b^2 - 4ac}$  is very small and numerical errors occur due to limited double precision representation of  $b$  and  $\sqrt{b^2 - 4ac}$ . So, in this case, we adopt roots which term  $-b - \sqrt{b^2 - 4ac}$ .

If  $b < 0$ , then  $|b| = -b$ . Then, for large  $|b|$ ,  $-b - \sqrt{b^2 - 4ac} \approx -b - |b| = -b + b = 0$ . So,  $-b - \sqrt{b^2 - 4ac}$  is very small and numerical errors occur due to limited double precision representation of  $b$  and  $\sqrt{b^2 - 4ac}$ . So, in this case, we adopt roots which term  $-b + \sqrt{b^2 - 4ac}$ .

So, this leads us to our composite method.

Roots given by composite method are

$$x_1 = \frac{2c}{-b - \sqrt{b^2 - 4ac}}, x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \text{ if } b \geq 0$$
$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, x_2 = \frac{2c}{-b + \sqrt{b^2 - 4ac}} \text{ if } b < 0$$