

# *Electrical circuit project*

## *Introduction to MNA and LU decomposition*

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**Abstract**—In this project, we have tried to first understand the concept of MNA, then use the matrix solving methods to find the answer to the problem.

**Keywords**—matrix; MNA; LU; LU decomposition ;sparse; capacitor; resistor; inductor;

### I. INTRODUCTION

Questions 1 and 2 have been asked to write the MNA matrix. In question 3, we need to construct these matrices using stamps of circuit's component written in MATLAB language. In question 4, asked from us to simulate two sample circuits using question 3 styles. And in question 5, there is a comparison between matrix solving methods.

### II.PROBLEMS

#### 1. Problem 1 :

In this question, the MNA matrices are requested to be drawn for each of the given circuits, as follows.

$$\begin{bmatrix} 1/R & 0 & 1 \\ gm & sC & -1 \\ 1 & -1 & -sL \end{bmatrix} \begin{bmatrix} V1 \\ V2 \\ IL \end{bmatrix} = \begin{bmatrix} |J|e^j \\ 0 \\ 0 \end{bmatrix}$$

And for time it's domain we have :

$$\begin{bmatrix} 1/R & 0 & 1 \\ gm & 0 & -1 \\ 1 & -1 & -sL \end{bmatrix} \begin{bmatrix} V1 \\ V2 \\ IL \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & -L \end{bmatrix} \begin{bmatrix} V1' \\ V2' \\ IL' \end{bmatrix} = \begin{bmatrix} |J|\cos(\omega t + \Theta) \\ 0 \\ 0 \end{bmatrix}$$

#### 2. Problem 2:

similar to the first question. we have :

$$\begin{bmatrix} g_1+sC & 0 & -sC & 0 & 0 \\ 0 & g_2 & -g_2 & 1 & 0 \\ -sC & -g_2 & g_2+sC & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ A & -A & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V1 \\ V2 \\ V3 \\ IE \\ IOA \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 3j^{(\pi/3)} \\ IOA \end{bmatrix}$$

And for idle OP-Amp:

$$\begin{bmatrix} g_1+sC+g_2 & -g_2-sC & -1 \\ -g_2-sC & g_2+sC & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V1 \\ V2 \\ IE \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 3e^{(j\pi/3)} \end{bmatrix}$$

#### 3. Problem 3:

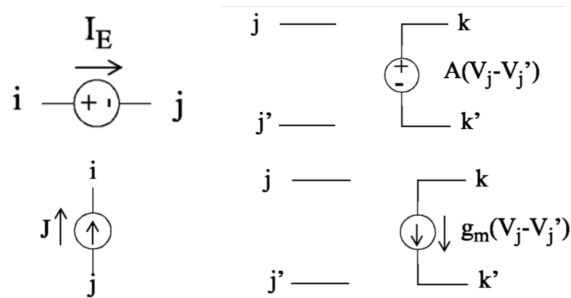
In this question, each stamp is implemented in MATLAB, and the user is asked to enter all circuit components in a specific format. Then, using the function of each of the strings, MNA matrices are constructed (here G, b are located in the equation  $GX = b$  and the matrix X is unavailable).

|                |                       |
|----------------|-----------------------|
| VSStamp        | Voltage Source        |
| VCVSStamp      | Voltage Controlled VS |
| CStamp         | Current Source        |
| VCCStamp       | Voltage Controlled CS |
| resistorStamp  | Resistor              |
| capacitorStamp | Capacitor             |
| inductorStamp  | Inductor              |
| OpAmp          | Op Amp                |

The user must also enter the circuit components in one of the following formats:

- R node1 node2 r
- C node1 node2 c
- L node1 node2 l
- VCC k k' j j' gm
- VCVS k k' j j' A
- VS i j E
- CS i j C
- OP node+ node- outNode gain

For VCC, VCVS, VS, CS, we perform the following steps.



#### 4. Problem 4:

In this question, we are asked to simulate two different circuits by using the stamps that were designed in question three, and in the end, to show the voltage-frequency in terms of frequency.

The main difference between this question and the third question is that at the end of this question, the matrix  $X$  must be computed.

This is done by the "LU Decomposition" method.

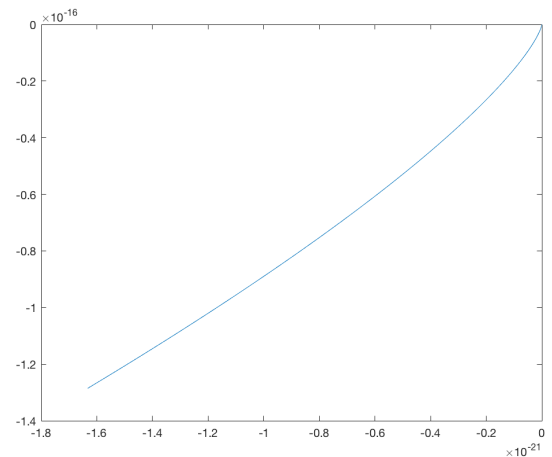
```
gSparse = sparse(G);
[L,U,P,Q] = lu(gSparse);
z = L \ (P*b);
y = U \ z;
x = Q*y;
```

As we know, the matrix  $L$  is a lower triangular matrix and  $U$  is an upper triangular matrix.

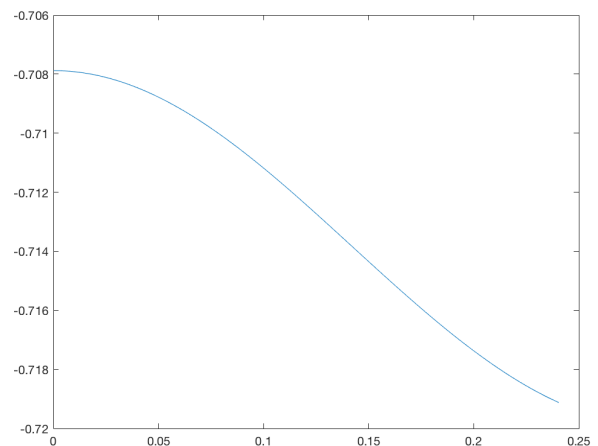
For ease, all commands are stored separately for displaying the circuit in a variable. Just uncomment one part of the Part that is in the code and see the results for the circuit.

(In fact, a variable named Part is set twice, one for the first circuit and one for the second circuit, and we have to comment on each other's results)

At the end, for the first circuit, the following diagram is displayed:



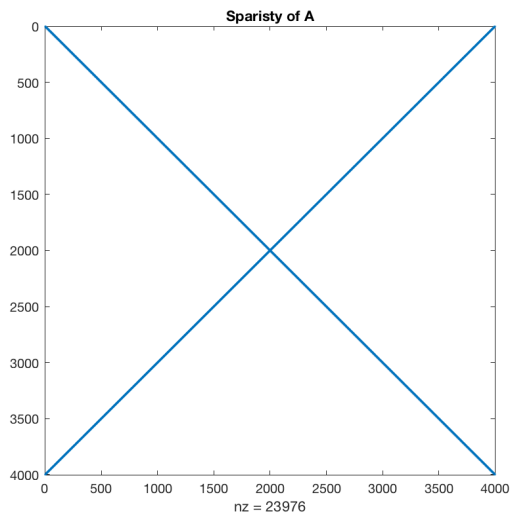
And for the second circuit, we also see the following diagram:



#### 5. Problem 5:

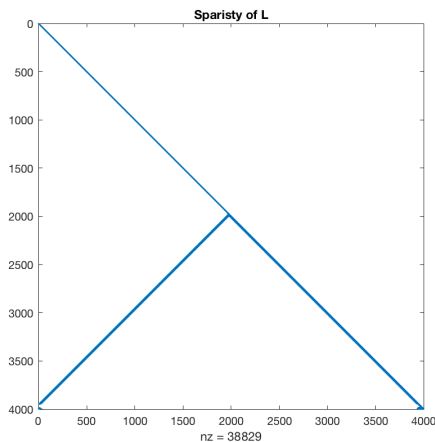
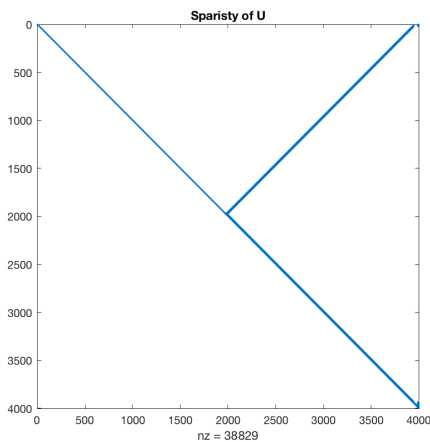
Section a:

The sparsity pattern of  $A$ :



#### Section b:

The sparsity pattern of the LU factors of A:



#### Section c:

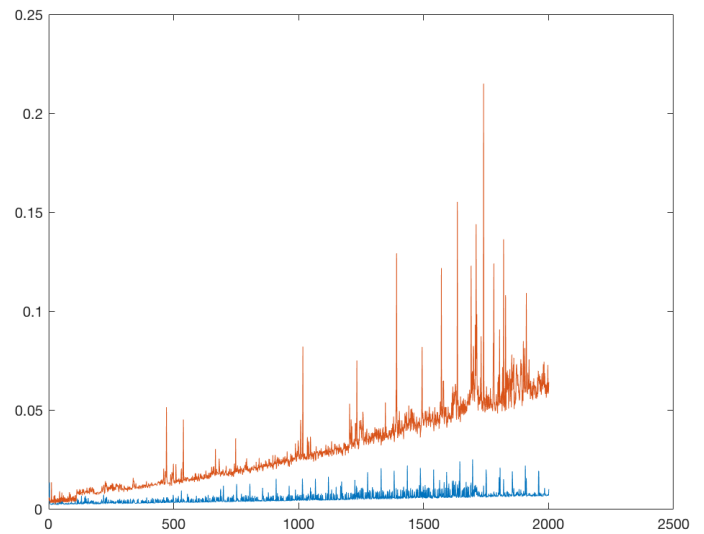
Both methods received the correct and correct response.

#### Section d:

For this matrix, with  $n = 4000$ , the calculation time is "0.0134" by the LU method and equal to "0.1038" for normal calculation.

#### Section e:

The time graph is plotted in  $n$  for  $1000 < n < 3000$ . The blue color represents the calculation by the LU method and the red color indicator is the calculation in the normal way.



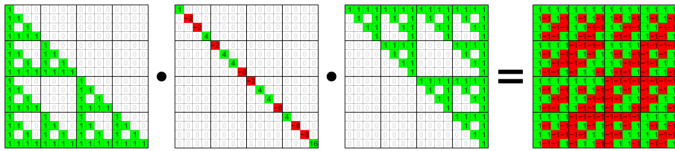
As you can see, the more  $n$ , the more time the matrix is calculated, and the growth of the calculation by the LU method is much greater than the calculated growth through the normal and exponential.

### III. CONCLUSION:

For circuit analysis, the MNA computer methodology is very practical, which can perfectly meet the demands of the problem. But calculating the large matrices that this method generates may be time consuming that can be saved in time by using the LU decomposition method and sparse matrices instead of complete matrices.

Sparse, in that it reduces the amount of additional computing by removing empty houses.

And the LU Decomposition, inasmuch as it employs a large amount of factor and places it in diameter, results in the loss of additional quantities.



So, it's possible to use MNA to respond to any large sized circuit in a small amount of time.