Exercice 1 (Vérification de primitives) Prouver dans les cas suivants que la fonction F est une primitive de la fonction f.

1.
$$f(x) = \frac{2(x^4 - 1)}{x^3}$$
, $F(x) = x^2 + \frac{1}{x^2}$

2.
$$f(x) = \frac{2x + 3x^2 - 3x^4}{1 - x^2}$$
, $F(x) = x^3 - \ln(1 - x^2)$

3.
$$f(x) = \frac{3(x^4+1)}{x^2}$$
, $F(x) = x^3 - \frac{3}{x}$

4.
$$f(x) = \frac{2x^4 + 5x^2 + 1}{x + x^3}$$
, $F(x) = x^2 + \ln(x + x^3)$

5.
$$f(x) = \frac{1}{1 + e^x}$$
, $F(x) = x - \ln(1 + e^x)$

6.
$$f(x) = \frac{1}{x \ln x}$$
, $F(x) = \ln(\ln x)$

7.
$$f(x) = \cos x - x \sin x$$
, $F(x) = x \cos x$

Exercice 2 (Primitives polynômes) Donner les primitives des fonctions suivantes:

1.
$$f(x) = x^4 - 4x^3 + x^2 - 4x + 3$$

2.
$$f(x) = -\frac{1}{x^3} + \frac{4}{x^2} - 1$$

3.
$$f(x) = x^5 + 3x^4 - x^2 + 3x - 2$$

4.
$$f(x) = \frac{1}{x^3} + \frac{3}{x^2} + 5$$
 D'autres:

$$5. \ f(x) = \frac{x^2 - 2x + 1}{3}$$

6.
$$f(x) = 1 - \frac{1}{x^3}$$

7.
$$f(x) = \frac{4}{x} + 2e^x$$

Exercice 3 (Primitives exponentielles) Donner une primitive des fonctions suivantes:

1.
$$f(x) = e^{2x-4}$$

2.
$$f(x) = 3e^{5x+1} - x$$

3.
$$f(x) = xe^{x^2+1}$$

4.
$$f(x) = e^{3x+2}$$

5.
$$f(x) = 2e^{-3x+4} + x^2$$

6.
$$f(x) = x^2 e^{x^3 - 2}$$

7.
$$f(x) = -6e^{5x}$$

8.
$$f(x) = 3e^{-0.2x} + 2x$$

9.
$$f(x) = 2e^{4x} + xe^{x^2}$$

Exercice 4 (Primitives produits et quotients)

Donner une primitive des fonctions suivantes:

1.
$$f(x) = (3x-1)(3x^2-2x+3)^3$$

2.
$$f(x) = (2x - 5)^3$$

3.
$$f(x) = \frac{x^2 - 1}{(x^2 - 3x + 1)^3}$$

4.
$$f(x) = \frac{4x+2}{x^2+x+1}$$

5.
$$f(x) = (x^2 + 1)(x^3 + 3x - 2)^5$$

6.
$$f(x) = (4x+3)^2$$

7.
$$f(x) = \frac{x+2}{(x^2+4x+3)^2}$$

$$8. \ f(x) = \frac{3x^5 - x}{x^6 - x^2 + 1}$$

9.
$$f(x) = \sin x \cos^2 x$$

10.
$$f(x) = 2x(3x^2 - 1)^3$$

11.
$$f(x) = (x^2 - 1)^2$$

12.
$$f(x) = (e^x + 1)^2$$

Exercice 5 (Primitives avec conditions) Trouver la primitive F de la fonction f qui vérifie la condition donnée:

1.
$$f(x) = x^4 + 3x^2 - 4x + 1$$
, $F(2) = 0$

2.
$$f(x) = \frac{4}{3x+2}, F(3) = 1$$

3.
$$f(x) = e^{3x+1}$$
, $F(-1) = 0$

4.
$$f(x) = x^2 + 3$$
, $F(1) = 0$

5.
$$f(x) = \frac{2}{5x-1}$$
, $F(2) = 3$

6.
$$f(x) = e^{5x+2}$$
, $F(-2) = 0$

7.
$$f(x) = \frac{2}{x^2} + x$$
, $F(1) = 0$

8.
$$f(x) = \frac{1}{(2x+1)^2}$$
, $F(0) = 0$

9.
$$f(x) = -\frac{1}{3-x}$$
, $F(1) = 1$

10.
$$f(x) = \frac{x}{(x^2 - 1)^2}$$
, $F(0) = 0$