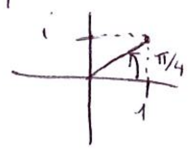
Éléments de correction du DS Blanc

1) $\rho = \left|\frac{-3}{1+i}\right| = \frac{|-3|}{|1+i|} = \frac{3}{\sqrt{1^2+1^2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$ et $\theta = \text{Arg}\left(\frac{-3}{1+i}\right) = \text{Arg}(-3) - \text{Arg}(1+i)$

$= \pi - \frac{\pi}{4}$ car

$= \frac{3\pi}{4}$

donc la F.E. est $\boxed{\frac{3\sqrt{2}}{2} e^{i3\pi/4}}$



2) $\rho = |\sqrt{3}+i| = \sqrt{\sqrt{3}^2+1^2} = \sqrt{4} = 2$

$\left.\begin{array}{l}\cos\theta = \frac{a}{\rho} = \frac{\sqrt{3}}{2} \\ \sin\theta = \frac{b}{\rho} = \frac{1}{2}\end{array}\right\} \theta = \pi/6$

donc $(\sqrt{3}+i)^6 = \left(2e^{i\pi/6}\right)^6 = 2^6 e^{i6\pi/6}$
$= 64e^{i\pi} = 64(\cos\pi + i\sin\pi) = 64(-1+i0)$
$= -64$

4) • f est définie si $x^2-2x-4 \neq 0$ or $\Delta_{x^2-2x-4} = (-2)^2-4\times1\times(-4) = 20 > 0$

donc il y a 2 V.I. $x = \frac{-(-2)\pm\sqrt{20}}{2} = \frac{2\pm2\sqrt{5}}{2} = 1\pm\sqrt{5}$

• $f' = \frac{(2x+1)(x^2-2x-4)-(x^2+x-1)(2x-2)}{(x^2-2x-4)^2} = \frac{-3x^2-6x-6}{(x^2-2x-4)^2}$ or $\Delta_{-3x^2-6x-6} = 36-4\times3\times6 < 0$

or $-3x^2-6x-6$ est toujours du signe de $a = -3 < 0$:

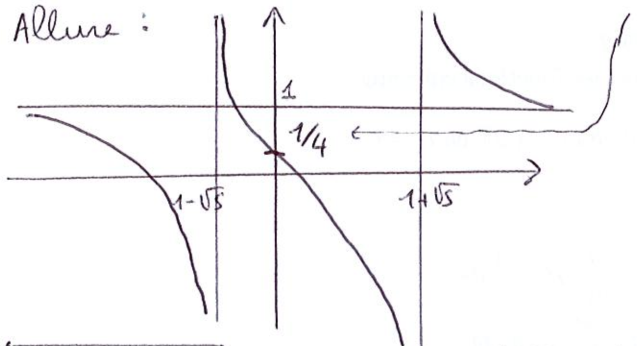
x		$1-\sqrt{5}$		$1+\sqrt{5}$	
f'	−	‖	−	‖	−
f	↘	‖	↘	‖	↘

• $\lim_{+\infty} f = \lim_{+\infty} \frac{x^2}{x^2} = \lim_{+\infty} 1 = 1$ et idem $\lim_{-\infty} f = 1$

termes de + haut degré

car $f(0) = \frac{0^2+0-1}{0^2-2\times0-4} = \frac{1}{4}$

• Allure :



3)a) $\sqrt{(3x-1)(5x+10)}$ définie si $(3x-1)(5x+10) \geq 0$:

x		-2		$1/3$	
$3x-1$	−		−	0	+
$5x+10$	−	0	+		+
$(3x-1)(5x+10)$	+	0	−	0	+

donc $D_f =]-\infty;-2] \cup [1/3;+\infty[$

3b) $\ln\left(\frac{4x-5}{3-3x}\right)$ déf si $\frac{4x-5}{3-3x} > 0$

x		1		$5/4$	
$4x-5$	−		−	0	+
$3-3x$	+	0	−		−
$\frac{4x-5}{3-3x}$	−	‖	+	0	−

donc $D_f =]1;5/4[$

5) $D_f(g) =]0;+\infty[$ à cause du $\ln(x)$

$g'(x) = \frac{1}{x} - 2x + 3 = \frac{1-2x^2+3x}{x}$
$= \frac{-2x^2+3x+1}{x}$

ici $x > 0$ d'après le $D_f(g)$ donc tout dépend du signe de $-2x^2+3x+1$

or $\Delta_{-2x^2+3x+1} = 9+4\times2\times1 = 17 > 0$ donc $x = \frac{-3\pm\sqrt{17}}{-4} = \frac{3\pm\sqrt{17}}{4}$

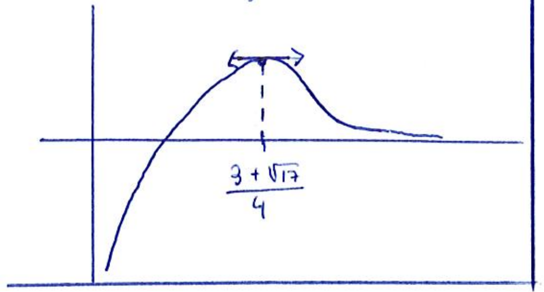
x	0		$(3+\sqrt{17})/4$		$+\infty$
g'		+	0	−	
g	$-\infty$	↗	$2,7$	↘	0

$\lim_{+\infty} \ln(x) - x^2 + 3x = \lim_{+\infty} -x^2 = -\infty$

$\lim_{0} \ln(x) - x^2 + 3x = \ln(0) - 0^2 + 3\times0 = -\infty$

$g\left(\frac{3+\sqrt{17}}{4}\right) \approx 2,7$

(Suite de la 5)



6a) $f = \ln(2x-5) = \ln u$

$f' = \frac{u'}{u} = \frac{(2x-5)'}{2x-5} = \frac{2}{2x-5}$

donc $f(3) = \ln(2\times3-5) = \ln(1) = 0$

$f'(3) = \frac{2}{2\times3-5} = 1$

donc $y = 1(x-3)+0 = x-3$

6b) $g = x\sqrt{x} = x^1x^{1/2} = x^{3/2}$

$g' = \frac{3}{2}x^{3/2-1} = \frac{3}{2}x^{1/2} = \frac{3}{2}\sqrt{x}$

$g(1) = 1\sqrt{1} = 1$

$g'(1) = \frac{3}{2}\sqrt{1} = \frac{3}{2}$

donc $y = \frac{3}{2}(x-1)+1 = \frac{3}{2}x - \frac{1}{2}$

7a) $\int_0^2 3x+1\,dx = \left[3\frac{x^2}{2}+x\right]_0^2 = \left(3\frac{4}{2}+2\right) - \left(3\frac{0^2}{0}+0\right) = 8$

7b) $\int_1^3 5e^{3t-1}dt = \left[\frac{5}{3}e^{3t-1}\right]_1^3 = \frac{5}{3}e^8 - \frac{5}{3}e^2 = \frac{5}{3}(e^8-e^2)$

7c) $\int_1^2 x^2 - \frac{3}{x}dx = \left[\frac{x^3}{3} - 3\ln x\right]_1^2 = \left(\frac{8}{3}-3\ln2\right) - \left(\frac{1^3}{3} - 3\ln1\right) = \frac{7}{3} - 3\ln2$

7d) $\int_0^\pi \sin(\theta)d\theta = [-\cos\theta]_0^\pi = (-\cos\pi) - (-\cos0) = (-(-1)) - (-1) = 2$

8a) $f' = (x^2\ln x)' = 2x\ln x + x^2\times\frac{1}{x} = 2x\ln x + x = x(2\ln x+1)$

8b) On a
$g = (x+1)e^{2x}$
$g' = 1\,e^{2x} + (x+1)2e^{2x} = (1+2(x+1))e^{2x} = (2x+3)e^{2x}$
$g'' = (2x+3)'e^{2x} + (2x+3)(e^{2x})' = 2e^{2x} + (2x+3)2e^{2x} = (4x+8)e^{2x}$

donc
$g''+4g = (4x+8)e^{2x} + 4(x+1)e^{2x} = (8x+12)e^{2x}$
$4g' = 4(2x+3)e^{2x} = (8x+12)e^{2x}$
} c'est bien égal

9a) $y'-3y = 0$
$y' = 3y$
donc $y = \alpha e^{3x}$

9b) $2y'-y = 5y'+y$
$2y'-5y' = y+y$
$-3y' = 2y$
$y' = -\frac{2}{3}y$
donc $y = \alpha e^{-3/2x}$

9c) $y' = 2y+3 \Leftrightarrow y'-2y = 3$

ESSM : $y'-2y = 0$
$y' = 2y$
donc $y_{ESSM} = \alpha e^{2x}$

SP : on cherche $y = \beta$
$\beta' - 2\beta = 3$
$-2\beta = 3$
$\beta = -\frac{3}{2}$
donc $y_P = -\frac{3}{2}$

SG :
$y = y_P + y_{ESSM}$
$= -3/2 + \alpha e^{2x}$

9d) $3y'-4y = 4$ ~~[illegible]~~

ESSM : $3y'-4y = 0$
$\Longrightarrow y' = 4/3\,y$
donc $y_{ESSM} = \alpha e^{4/3x}$

SP : on cherche $y = \beta$
$3\beta' - 4\beta = 4$ donc $\beta = -1$ (y_P)

SG :
$y = y_P + y_{ESSM}$
$= -1 + \alpha e^{4/3x}$