Homework 4

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P1

6.8.) a.)

```
tr1 <- matrix(c(6, 7, 5, 9, 8, 6, 4, 9, 7, 9), nrow = 2, ncol = 5)
tr1
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    6    5    8    4    7
## [2,]    7    9    6    9    9
```

```
tr2 <- matrix(c(3, 3, 1, 6, 2, 3), nrow = 2, ncol = 3)
tr2
```

```
##      [,1] [,2] [,3]
## [1,]    3    1    2
## [2,]    3    6    3
```

```
tr3 <- matrix(c(2, 3, 5, 1, 3, 1, 2, 3), nrow = 2, ncol = 4)
tr3
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    2    5    3    2
## [2,]    3    1    1    3
```

```
xbar1 <- rowMeans(tr1)
xbar1
```

```
## [1] 6 8
```

```
xbar2 <- rowMeans(tr2)
xbar2
```

```
## [1] 2 4
```

```
xbar3 <- rowMeans(tr3)
xbar3
```

```
## [1] 3 2
```

```
#xl1 <- rbind(tr1[1,], tr2[2,], tr3[3,])

xbar <-
  c((ncol(tr1) * xbar1[1] + ncol(tr2) * xbar2[1] + ncol(tr3) * xbar3[1]) / (ncol(tr1) + ncol(tr2) + nco
    ((ncol(tr1) * xbar1[2]) + (ncol(tr2) * xbar2[2]) + (ncol(tr3) * xbar3[2])) / (ncol(tr1) + ncol(tr2)
  )
xbar
```

```
## [1] 4 5
```

```
#cat("=", )

#xbarl1 <- matrix(4, nrow = 3, ncol = 5) +
# cat("=")
# matrix(4, nrow = 3, ncol = 5)
# print('+' )
# matrix(c(2,-2,-1), nrow = 3, ncol =5)
# print('+')
# matrix(c(2,-2,-1), nrow = 3, ncol =5)
```

$$x_{1j} = \begin{pmatrix} 4 & 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & & \\ 4 & 4 & 4 & 4 & \end{pmatrix} + \begin{pmatrix} 2 & 2 & 2 & 2 & 2 \\ -2 & -2 & -2 & & \\ -1 & -1 & -1 & -1 & \end{pmatrix} + \begin{pmatrix} 0 & -1 & 2 & -2 & 1 \\ 1 & -1 & 0 & & \\ -1 & 2 & 0 & -1 & \end{pmatrix}$$

$$x_{2j} = \begin{pmatrix} 5 & 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & & \\ 5 & 5 & 5 & 5 & \end{pmatrix} + \begin{pmatrix} 3 & 3 & 3 & 3 & 3 \\ -1 & -1 & -1 & & \\ -3 & -3 & -3 & -3 & \end{pmatrix} + \begin{pmatrix} -1 & 1 & -2 & 1 & 1 \\ -1 & 2 & -1 & & \\ -1 & 2 & 0 & -1 & \end{pmatrix}$$

b.) Variable 1

```
mat1 <- rbind(tr1[1,], c(tr2[1,], rep(0, 2)), c(tr3[1,], 0))
mat1
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    6    5    8    4    7
## [2,]    3    1    2    0    0
## [3,]    2    5    3    2    0
```

```
SSobs1 <- sum(mat1 ^ 2)
SSobs1
```

```
## [1] 246
```

```
SSmean1 <- xbar[1] ^ 2 * 12
SSmean1
```

```
## [1] 192
```

```
SStrt1 <-
  ncol(tr1) * (mean(tr1[1,] - xbar[1])) ^ 2 + ncol(tr2) * (mean(tr2[1,] - xbar[1])) ^
  2 + ncol(tr3) * (mean(tr3[1,] - xbar[1])) ^ 2
SStrt1
```

```
## [1] 36
```

```
SSres1 <-
  sum((mat1 - rbind(rep(xbar1[1], 5),
                    c(rep(
                      xbar2[1], 3
                    ), 0 , 0),
                    c(rep(
                      xbar3[1], 4
                    ), 0))) ^ 2)
SSres1
```

```
## [1] 18
```

```
SStotal1 <- SSobs1 - SSmean1
SStotal1
```

```
## [1] 54
```

Variable 2

```
mat2 <- rbind(tr1[2,], c(tr2[2,], rep(0, 2)), c(tr3[2,], 0))
SSobs2 <- sum(mat2 ^ 2)
SSobs2
```

```
## [1] 402
```

```
SSmean2 <- xbar[2] ^ 2 * 12
SSmean2
```

```
## [1] 300
```

```
SStrt2 <-
  ncol(tr1) * (mean(tr1[2,] - xbar[2])) ^ 2 + ncol(tr2) * (mean(tr2[2,] - xbar[2])) ^
  2 + ncol(tr3) * (mean(tr3[2,] - xbar[2])) ^ 2
SStrt2
```

```
## [1] 84
```

```
SSres2 <-
  sum((mat2 - rbind(rep(xbar1[2], 5),
                    c(rep(
                      xbar2[2], 3
                    ), 0 , 0),
                    c(rep(
                      xbar3[2], 4
                    ), 0))) ^ 2)
SSres2
```

```
## [1] 18
```

```
SStotal2 <- SSobs2 - SSmean2
SStotal2
```

```
## [1] 102
```

Cross Product

```
SSobs_x <- sum(mat1 * mat2)
SSobs_x
```

```
## [1] 275
```

```
SSmean_x <- xbar[1] * xbar[2] * 12
SSmean_x
```

```
## [1] 240
```

```
SStrt_x <- ncol(tr1) *  (mean(tr1[1,] - xbar[1]))* (mean(tr1[2,] - xbar[2])) + ncol(tr2) * (mean(tr2[1,]
SStrt_x
```

```
## [1] 48
```

```
SSres_x <- sum((mat2 - rbind(rep(xbar1[2], 5), c(rep( xbar2[2], 3), 0 , 0),
                    c(rep(xbar3[2], 4
                    ), 0))) * (mat1 - rbind(rep(xbar1[1], 5),
                    c(rep(
                      xbar2[1], 3
                    ), 0 , 0),
                    c(rep(
                      xbar3[1], 4
                    ), 0))))
SSres_x
```

```
## [1] -13
```

```
SStotal_x <- SSobs_x - SSmean_x
SStotal_x
```

```
## [1] 35
```

```
cat("Treatment \n")
```

```
## Treatment
```

```
B <- matrix(c(SStrt1, SStrt_x, SStrt_x, SStrt2), nrow = 2, ncol = 2)
B
```

```
##      [,1] [,2]
## [1,]   36   48
## [2,]   48   84
```

```r
dfB <- nrow(mat1)-1
dfB
```

```
## [1] 2
```

```r
cat("Residuals \n")
```

```
## Residuals
```

```r
W <- matrix(c(SSres1, SSres_x, SSres_x, SSres2), nrow = 2, ncol = 2)
W
```

```
##      [,1] [,2]
## [1,]   18  -13
## [2,]  -13   18
```

```r
dfW <- 12 - 3
dfW
```

```
## [1] 9
```

```r
cat("Total \n")
```

```
## Total
```

```r
B + W
```

```
##      [,1] [,2]
## [1,]   54   35
## [2,]   35  102
```

```r
#df
12 - 1
```

```
## [1] 11
```

c.)

```r
WLambda <- det(W) / det(W + B)
WLambda
```

```
## [1] 0.03618959
```

```r
#Test stat
((12 - nrow(mat1) - 1) / (nrow(mat1) - 1)) * ((1 - sqrt(WLambda)) / sqrt(WLambda))
```

```
## [1] 17.02656
```

```r
#Critical value
qf(0.01, df1 = 2 * nrow(tr1), df2 = 2 *(12 - nrow(tr1) - 2), lower.tail = FALSE)
```

```
## [1] 4.772578
```

Since our critical value is less than our test statistic, we reject H_0 when $\alpha = 0.01$. We have statistical evidence to conclude at least one τ_i is not 0.

Barlett's correction

```r
#Test statistic
- (12 - 1 - ((nrow(tr1) + nrow(mat1))/2)) * log(WLambda)
```

```
## [1] 28.21136
```

```r
#Critical Value
qchisq(0.01, df = nrow(tr1) * (nrow(mat1)-1), lower.tail = FALSE)
```

```
## [1] 13.2767
```

Since our critical value is once again less than our test statistic, we reject H_0. This conclusion lines up with the conclusion we made using Wilk's Lambda. While both are adequite in the case of small sample sizes, Bartlett's correction improves accuracy for large sample sizes.

P2 6.13.) a.) and b.)

```r
# tr1 <- matrix(c(6, 4, 8, 2, 3, -3, 4, -4, -3, -4, 3, -4), nrow = 3, ncol = 4, byrow = TRUE)
# tr1
#
# tr2 <- matrix(c(8,6,12,6,8,2,3,3,2,-5,-3,-6), nrow = 3, ncol = 4, byrow = TRUE)
# tr2
#
# xbar1 <- mean(tr1)
# xbar1
#
# xbar2 <- mean(tr2)
# xbar2
#
# xbar <- matrix(xbar1, nrow = 3, ncol = 4)
# xbar
#
# factef1 <- rowMeans(tr1) - matrix(xbar1, nrow = 3, ncol = 4)
# factef1
#
# factef2 <- matrix(c(rep(colMeans(tr1) - xbar1, 3)), nrow = 3, ncol = 4, byrow = TRUE)
# factef2
#
# resid1 <- tr1 - factef1 - factef2 - xbar
# resid1
```

```r
tr1 <- matrix(c(6, 4, 8, 2, 3, -3, 4, -4, -3, -4, 3, -4), nrow = 3, ncol = 4, byrow = TRUE)
tr1
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    6    4    8    2
## [2,]    3   -3    4   -4
## [3,]   -3   -4    3   -4
```

```r
tr2 <- matrix(c(8,6,12,6,8,2,3,3,2,-5,-3,-6), nrow = 3, ncol = 4, byrow = TRUE)
tr2
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    8    6   12    6
## [2,]    8    2    3    3
## [3,]    2   -5   -3   -6
```

```r
xbar1 <- mean(tr1)
xbar1
```

```
## [1] 1
```

```r
xbar2 <- mean(tr2)
xbar2
```

```
## [1] 3
```

```r
xbar <- matrix(xbar1, nrow = 3, ncol = 4)
xbar
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    1    1    1
## [2,]    1    1    1    1
## [3,]    1    1    1    1
```

```r
factef1_1 <- rowMeans(tr1) - matrix(xbar1, nrow = 3, ncol = 4)
factef1_1
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    4    4    4    4
## [2,]   -1   -1   -1   -1
## [3,]   -3   -3   -3   -3
```

```r
factef2_1 <- matrix(c(rep(colMeans(tr1) - xbar1, 3)), nrow = 3, ncol = 4, byrow = TRUE)
factef2_1
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1   -2    4   -3
## [2,]    1   -2    4   -3
## [3,]    1   -2    4   -3
```

```r
resid1 <- tr1 - factef1_1 - factef2_1 - xbar
resid1
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0    1   -1    0
## [2,]    2   -1    0   -1
## [3,]   -2    0    1    1
```

```r
SSmean1 <- sum(xbar^2)
SSmean1
```

```
## [1] 12
```

```r
SSfac1_1 <- sum(factef1_1^2)
SSfac1_1
```

```
## [1] 104
```

```r
SSfac2_1 <- sum(factef2_1^2)
SSfac2_1
```

```
## [1] 90
```

```r
SSres1 <- sum(resid1 ^2)
SSres1
```

```
## [1] 14
```

```r
SStol <- SSmean1 + SSfac1_1 + SSfac2_1 + SSres1
SStol
```

```
## [1] 220
```

```r
xbar <- matrix(xbar2, nrow = 3, ncol = 4)
xbar
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    3    3    3    3
## [2,]    3    3    3    3
## [3,]    3    3    3    3
```

```r
factef1 <- rowMeans(tr2) - matrix(xbar2, nrow = 3, ncol = 4)
factef1
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    5    5    5    5
## [2,]    1    1    1    1
## [3,]   -6   -6   -6   -6
```

```
factef2 <- matrix(c(rep(colMeans(tr2) - xbar2, 3)), nrow = 3, ncol = 4, byrow = TRUE)
factef2
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    3   -2    1   -2
## [2,]    3   -2    1   -2
## [3,]    3   -2    1   -2
```

```
resid2 <- tr2 - factef1 - factef2 - xbar
resid2
```

```
##      [,1] [,2] [,3] [,4]
## [1,]   -3    0    3    0
## [2,]    1    0   -2    1
## [3,]    2    0   -1   -1
```

```
SSmean2 <- sum(xbar^2)
SSmean2
```

```
## [1] 108
```

```
SSfac1_2 <- sum(factef1^2)
SSfac1_2
```

```
## [1] 248
```

```
SSfac2_2 <- sum(factef2^2)
SSfac2_2
```

```
## [1] 54
```

```
SSres2 <- sum(resid2 ^2)
SSres2
```

```
## [1] 30
```

```
SStol <- SSmean2 + SSfac1_2 + SSfac2_2 + SSres2
SStol
```

```
## [1] 440
```

```
SCPmean <-
  sum(matrix(xbar2, nrow = 3, ncol = 4) * matrix(xbar1, nrow = 3, ncol = 4))
SCPmean
```

```
## [1] 36
```

```r
SCPfac1 <-
  sum((rowMeans(tr2) - matrix(xbar2, nrow = 3, ncol = 4)) * (rowMeans(tr1) - matrix(xbar1, nrow = 3, nc
SCPfac1
```

```
## [1] 148
```

```r
SCPfac2 <-
  sum((matrix(c(rep(colMeans(tr1) - xbar1, 3)), nrow = 3, ncol = 4, byrow = TRUE) * (matrix(c(rep(colMea
SCPfac2
```

```
## [1] 51
```

```r
SCPres <- sum(resid1 * resid2)
SCPres
```

```
## [1] -8
```

```r
SCPtot <- SCPmean + SCPfac1 + SCPfac2 + SCPres
SCPtot
```

```
## [1] 227
```

c.)

```r
# Manova table
cat("Factor 1 \n")
```

```
## Factor 1
```

```r
fac1 <- matrix(c(SSfac1_1, SCPfac1, SCPfac1, SSfac1_2), nrow = 2, ncol = 2)
fac1
```

```
##      [,1] [,2]
## [1,]  104  148
## [2,]  148  248
```

```r
dfFac1 <- nrow(tr1)-1
dfFac1
```

```
## [1] 2
```

```r
cat("Factor 2 \n")
```

```
## Factor 2
```

```r
fac2 <- matrix(c(SSfac2_1, SCPfac2, SCPfac2, SSfac2_2), nrow = 2, ncol = 2)
fac2
```

```
##      [,1] [,2]
## [1,]   90   51
## [2,]   51   54
```

```r
dfFac2 <- ncol(tr1)-1
dfFac2
```

```
## [1] 3
```

```r
cat("Residuals \n")
```

```
## Residuals
```

```r
resid <- matrix(c(SSres1, SCPres, SCPres, SSres2), nrow = 2, ncol = 2)
resid
```

```
##      [,1] [,2]
## [1,]   14   -8
## [2,]   -8   30
```

```r
dfresid <- (nrow(tr1)-1) * (ncol(tr1)-1)
dfresid
```

```
## [1] 6
```

```r
cat("Total \n")
```

```
## Total
```

```r
fac1 + fac2 + resid
```

```
##      [,1] [,2]
## [1,]  208  191
## [2,]  191  332
```

```r
#df
nrow(tr1) * ncol(tr1) - 1
```

```
## [1] 11
```

d.)

```r
-1 * ((nrow(tr1) - 1)*(ncol(tr1) - 1) - (((2 + 1) - (nrow(tr1) - 1))/2) ) * log(det(resid)/ det(fac1 + r
```

```
## [1] 19.87339
```

```r
#Critical Value
qchisq(.05, 4, lower.tail = FALSE)
```

```
## [1] 9.487729
```

Since our test statistic is greater than our critical value, we reject H_0, supporting the claim that there are factor 1 effects.

```
-1 * ((nrow(tr2) - 1)*(ncol(tr2) - 1) - (((2 + 1) - (ncol(tr2) - 1))/2) ) * log(det(resid)/ det(fac2 + r
```

```
## [1] 17.77476
```

```
#Critical Value
qchisq(.05, (nrow(tr2) - 1)*(ncol(tr2) - 1), lower.tail = FALSE)
```

```
## [1] 12.59159
```

We once again reject the null hypothesis and conclude there are factor 2 effects as well.

6.14.)

```
tr1 <- matrix(c(14, 6, 8, 16, 1, 5, 0, 2, 3, -2, -11, -6), nrow = 3, ncol = 4, byrow = TRUE)
tr1
```

```
##      [,1] [,2] [,3] [,4]
## [1,]   14    6    8   16
## [2,]    1    5    0    2
## [3,]    3   -2  -11   -6
```

```
tr2 <- matrix(c(8, 2, 2, -4, 6, 12, 15, 7 , -2, 7, 1, 6), nrow = 3, ncol = 4, byrow = TRUE)
tr2
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    8    2    2   -4
## [2,]    6   12   15    7
## [3,]   -2    7    1    6
```

```
xbar1 <- mean(tr1)
xbar1
```

```
## [1] 3
```

```
xbar2 <- mean(tr2)
xbar2
```

```
## [1] 5
```

```
xbar <- matrix(xbar1, nrow = 3, ncol = 4)
xbar
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    3    3    3    3
## [2,]    3    3    3    3
## [3,]    3    3    3    3
```

```r
factef1_1 <- rowMeans(tr1) - matrix(xbar1, nrow = 3, ncol = 4)
factef1_1
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    8    8    8    8
## [2,]   -1   -1   -1   -1
## [3,]   -7   -7   -7   -7
```

```r
factef2_1 <- matrix(c(rep(colMeans(tr1) - xbar1, 3)), nrow = 3, ncol = 4, byrow = TRUE)
factef2_1
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    3    0   -4    1
## [2,]    3    0   -4    1
## [3,]    3    0   -4    1
```

```r
resid1 <- tr1 - factef1_1 - factef2_1 - xbar
resid1
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0   -5    1    4
## [2,]   -4    3    2   -1
## [3,]    4    2   -3   -3
```

```r
SSmean1 <- sum(xbar^2)
SSmean1
```

```
## [1] 108
```

```r
SSfac1_1 <- sum(factef1_1^2)
SSfac1_1
```

```
## [1] 456
```

```r
SSfac2_1 <- sum(factef2_1^2)
SSfac2_1
```

```
## [1] 78
```

```r
SSres1 <- sum(resid1 ^2)
SSres1
```

```
## [1] 110
```

```r
SStol <- SSmean1 + SSfac1_1 + SSfac2_1 + SSres1
SStol
```

```
## [1] 752
```

```r
SCPmean <-
  sum(matrix(xbar2, nrow = 3, ncol = 4) * matrix(xbar1, nrow = 3, ncol = 4))
SCPmean
```

```
## [1] 180
```

```r
SCPfac1 <-
  sum((rowMeans(tr2) - matrix(xbar2, nrow = 3, ncol = 4)) * (rowMeans(tr1) - matrix(xbar1, nrow = 3, nc
SCPfac1
```

```
## [1] -60
```

```r
SCPfac2 <-
  sum((matrix(c(rep(colMeans(tr1) - xbar1, 3)), nrow = 3, ncol = 4, byrow = TRUE) * (matrix(c(rep(colMea
SCPfac2
```

```
## [1] -27
```

```r
SCPres <- sum(resid1 * resid2)
SCPres
```

```
## [1] 8
```

```r
SCPtot <- SCPmean + SCPfac1 + SCPfac2 + SCPres
SCPtot
```

```
## [1] 101
```

b.)

```r
# tr1_13 <- matrix(c(6, 4, 8, 2, 3, -3, 4, -4, -3, -4, 3, -4), nrow = 3, ncol = 4, byrow = TRUE)
# tr1_13
#
# tr2_13 <- matrix(c(8,6,12,6,8,2,3,3,2,-5,-3,-6), nrow = 3, ncol = 4, byrow = TRUE)
# tr2_13
#
# tr1_14 <- matrix(c(14, 6, 8, 16, 1, 5, 0, 2, 3, -2, -11, -6), nrow = 3, ncol = 4, byrow = TRUE)
# tr1_14
#
# tr2_14 <- matrix(c(8, 2, 2, -4, 6, 12, 15, 7 , -2, 7, 1, 6), nrow = 3, ncol = 4, byrow = TRUE)
# tr2_14
#
#
# tr1 <- (tr1_13 + tr1_14) #/ 2
# tr1
#
# tr2 <- (tr2_13 + tr2_14) #/ 2
# tr2
#
#
```

```r
#
# tr1 <- matrix(c(14, 6, 8, 16, 1, 5, 0, 2, 3, -2, -11, -6), nrow = 3, ncol = 4, byrow = TRUE)
# tr1
# tr2 <- matrix(c(8, 2, 2, -4, 6, 12, 15, 7 , -2, 7, 1, 6), nrow = 3, ncol = 4, byrow = TRUE)
# tr2
```

```r
xbar1 <- mean(tr1)
xbar1
```

```
## [1] 3
```

```r
xbar2 <- mean(tr2)
xbar2
```

```
## [1] 5
```

```r
xbar <- matrix(xbar1, nrow = 3, ncol = 4)
xbar
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    3    3    3    3
## [2,]    3    3    3    3
## [3,]    3    3    3    3
```

```r
factef1_1 <- rowMeans(tr1) - matrix(xbar1, nrow = 3, ncol = 4)
factef1_1
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    8    8    8    8
## [2,]   -1   -1   -1   -1
## [3,]   -7   -7   -7   -7
```

```r
factef2_1 <- matrix(c(rep(colMeans(tr1) - xbar1, 3)), nrow = 3, ncol = 4, byrow = TRUE)
factef2_1
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    3    0   -4    1
## [2,]    3    0   -4    1
## [3,]    3    0   -4    1
```

```r
resid1 <- tr1 - factef1_1 - factef2_1 - xbar
resid1
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0   -5    1    4
## [2,]   -4    3    2   -1
## [3,]    4    2   -3   -3
```

```r
SSmean1 <- sum(xbar^2)
SSmean1
```

```
## [1] 108
```

```r
SSfac1_1 <- sum(factef1_1^2)
SSfac1_1
```

```
## [1] 456
```

```r
SSfac2_1 <- sum(factef2_1^2)
SSfac2_1
```

```
## [1] 78
```

```r
SSres1 <- sum(resid1 ^2)
SSres1
```

```
## [1] 110
```

```r
SStol <- SSmean1 + SSfac1_1 + SSfac2_1 + SSres1
SStol
```

```
## [1] 752
```

```r
xbar <- matrix(xbar2, nrow = 3, ncol = 4)
xbar
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    5    5    5    5
## [2,]    5    5    5    5
## [3,]    5    5    5    5
```

```r
factef1 <- rowMeans(tr2) - matrix(xbar2, nrow = 3, ncol = 4)
factef1
```

```
##      [,1] [,2] [,3] [,4]
## [1,]   -3   -3   -3   -3
## [2,]    5    5    5    5
## [3,]   -2   -2   -2   -2
```

```r
factef2 <- matrix(c(rep(colMeans(tr2) - xbar2, 3)), nrow = 3, ncol = 4, byrow = TRUE)
factef2
```

```
##      [,1] [,2] [,3] [,4]
## [1,]   -1    2    1   -2
## [2,]   -1    2    1   -2
## [3,]   -1    2    1   -2
```

```r
resid2 <- tr2 - factef1 - factef2 - xbar
resid2
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    7   -2   -1   -4
## [2,]   -3    0    4   -1
## [3,]   -4    2   -3    5
```

```r
SSmean2 <- sum(xbar^2)
SSmean2
```

```
## [1] 300
```

```r
SSfac1_2 <- sum(factef1^2)
SSfac1_2
```

```
## [1] 152
```

```r
SSfac2_2 <- sum(factef2^2)
SSfac2_2
```

```
## [1] 30
```

```r
SSres2 <- sum(resid2 ^2)
SSres2
```

```
## [1] 150
```

```r
SStol <- SSmean2 + SSfac1_2 + SSfac2_2 + SSres2
SStol
```

```
## [1] 632
```

```r
xbar <- matrix(xbar2, nrow = 3, ncol = 4)
xbar
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    5    5    5    5
## [2,]    5    5    5    5
## [3,]    5    5    5    5
```

```r
factef1 <- rowMeans(tr2) - matrix(xbar2, nrow = 3, ncol = 4)
factef1
```

```
##      [,1] [,2] [,3] [,4]
## [1,]   -3   -3   -3   -3
## [2,]    5    5    5    5
## [3,]   -2   -2   -2   -2
```

```
factef2 <- matrix(c(rep(colMeans(tr2) - xbar2, 3)), nrow = 3, ncol = 4, byrow = TRUE)
factef2
```

```
##      [,1] [,2] [,3] [,4]
## [1,]   -1    2    1   -2
## [2,]   -1    2    1   -2
## [3,]   -1    2    1   -2
```

```
resid2 <- tr2 - factef1 - factef2 - xbar
resid2
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    7   -2   -1   -4
## [2,]   -3    0    4   -1
## [3,]   -4    2   -3    5
```

```
SSmean2 <- sum(xbar^2)
SSmean2
```

```
## [1] 300
```

```
SSfac1_2 <- sum(factef1^2)
SSfac1_2
```

```
## [1] 152
```

```
SSfac2_2 <- sum(factef2^2)
SSfac2_2
```

```
## [1] 30
```

```
SSres2 <- sum(resid2 ^2)
SSres2
```

```
## [1] 150
```

```
SStol <- SSmean2 + SSfac1_2 + SSfac2_2 + SSres2
SStol
```

```
## [1] 632
```

c.)

```
-1 * ((nrow(tr1) - 1)*(ncol(tr1) - 1) - (((2 + 1) - (nrow(tr1) - 1))/2) ) * log(det(resid)/ det(fac1 + r
```

```
## [1] 19.87339
```

```r
#Critical Value
qchisq(.05, 4, lower.tail = FALSE)
```

```
## [1] 9.487729
```

```r
-1 * ((nrow(tr2) - 1)*(ncol(tr2) - 1) - (((2 + 1) - (ncol(tr2) - 1))/2) ) * log(det(resid)/ det(fac2 + r
```

```
## [1] 17.77476
```

```r
#Critical Value
qchisq(.05, (nrow(tr2) - 1)*(ncol(tr2) - 1), lower.tail = FALSE)
```

```
## [1] 12.59159
```

We reject the null hypothesis for factor 1 however we fail to reject factor 2. We can conclude that the factor 1 effect is non zero but we cannot conclude that factor 2 effects are non zero.

```r
cat("Factor 1 \n")
```

```
## Factor 1
```

```r
fac1 <- matrix(c(SSfac1_1, SCPfac1, SCPfac1, SSfac1_2), nrow = 2, ncol = 2)
fac1
```

```
##      [,1] [,2]
## [1,]  456  -60
## [2,]  -60  152
```

```r
dfFac1 <- nrow(tr1)-1
dfFac1
```

```
## [1] 2
```

```r
cat("Factor 2 \n")
```

```
## Factor 2
```

```r
fac2 <- matrix(c(SSfac2_1, SCPfac2, SCPfac2, SSfac2_2), nrow = 2, ncol = 2)
fac2
```

```
##      [,1] [,2]
## [1,]   78  -27
## [2,]  -27   30
```

```r
dfFac2 <- ncol(tr1)-1
dfFac2
```

```
## [1] 3
```

```r
cat("Residuals \n")
```

```
## Residuals
```

```r
resid <- matrix(c(SSres1, SCPres, SCPres, SSres2), nrow = 2, ncol = 2)
resid
```

```
##      [,1] [,2]
## [1,]  110    8
## [2,]    8  150
```

```r
dfresid <- (nrow(tr1)-1) * (ncol(tr1)-1)
dfresid
```

```
## [1] 6
```

```r
cat("Total \n")
```

```
## Total
```

```r
fac1 + fac2 + resid
```

```
##      [,1] [,2]
## [1,]  644  -79
## [2,]  -79  332
```

```r
#df
nrow(tr1) * ncol(tr1) - 1
```

```
## [1] 11
```

6.19.)

a.)

```r
X <- read.csv("T6-10.DAT", header = FALSE, sep = "")
names(X) <- c("fuel", "repair", "capital", "truck")

X1 <- X[X$truck == "gasoline", 1:3]
X2 <- X[X$truck == "diesel", 1:3]

n1 <- nrow(X1)
n2 <- nrow(X2)
p <- ncol(X1)

null <- rep(0, p)
xbar1 <- colMeans(X1)
xbar2 <- colMeans(X2)
```

```r
S1 <- cov(X1)
S2 <- cov(X2)
Spool <- ((n1-1)*S1+(n2-1)*S2)/(n1+n2-2)

T2 <- (n1*n2/(n1+n2))*t(xbar1-xbar2-null)%*%solve(Spool)%*%(xbar1-xbar2-null)
critval <- (n1 + n2 - 2)*p/(n1 + n2 - p - 1)*qf(0.01, p, n1 + n2 - p - 1, lower.tail = FALSE)

cat("Hotellings T2 is:", T2)
```

```
## Hotellings T2 is: 50.91279
```

```r
cat("\nCritical value is:", critval)
```

```
##
## Critical value is: 12.93096
```

Since our critical value is less than our T^2 value, we reject H_0, concluding there is statistical evidence that the cost vectors between groups are not zero vectors.

b.)

```r
solve(Spool)%*%(xbar1 - xbar2)
```

```
##               [,1]
## fuel     0.2547452
## repair  -0.1339036
## capital -0.3188296
```

We can see from our solution vector that the variable with the highest absolute influence would be capital, followed by fuel, and finally repair.

c.)

```r
lowerCI <- t(diag(p))%*%(xbar1 - xbar2) - sqrt(critval) * sqrt(diag((t(diag(p))%*%Spool%*%diag(p))*(1/n
upperCI <- t(diag(p))%*%(xbar1 - xbar2) + sqrt(critval) * sqrt(diag((t(diag(p))%*%Spool%*%diag(p))*(1/n

results <- as.data.frame(cbind(lowerCI, upperCI),
                     row.names = c("Fuel", "Repair", "Capital"))
results
```

```
##                 V1        V2
## Fuel     -1.704346  5.930264
## Repair   -7.022268  1.722920
## Capital -13.526479 -3.628618
```

d.)

```r
Xred <- X[-c(9, 21),]

X1red <- Xred[Xred$truck=="gasoline",1:3]
X2red <- Xred[Xred$truck=="diesel",1:3]
```

```r
n1 <- nrow(X1red)
n2 <- nrow(X2red)

xbar1 <- colMeans(X1red)
xbar2 <- colMeans(X2red)

S1 <- cov(X1red)
S2 <- cov(X2red)
Spool <- ((n1-1)*S1+(n2-1)*S2)/(n1+n2-2)

T2 <- (n1*n2/(n1+n2))*t(xbar1-xbar2-null)%*%solve(Spool)%*%(xbar1-xbar2-null)
critval <- (n1 + n2 - 2)*p/(n1 + n2 - p - 1)*qf(0.01, p, n1 + n2 - p - 1, lower.tail = FALSE)

cat("Hotellings T2 is:", T2)
```

```
## Hotellings T2 is: 52.3836
```

```r
cat("\nCritical value is:", critval)
```

```
##
## Critical value is: 12.99521
```

We can see here that our critical value is less than our T^2, we we still reject the null hypothesis, that is that the mean difference in cost vectors is not the zero vector.