Introduction to Dependent Types

Eagan Technology Unconference

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Section Outline

Quick Question

How many are familiar with this topic?

A Joke

This is not a `m-` tutorial.

A Joke

This is not a `m-` tutorial. Nor is it a `lens` tutorial

A Joke

This is not a `m-` tutorial. Nor is it a `lens` tutorial (aka the new new `m-` tutorial. . .

A Joke

This is not a `m-` tutorial. Nor is it a `lens` tutorial (aka the new new `m-` tutorial......because `arrows` *were* the new `m-` tutorials).

About This Talk

`Agda`, `Idris`, `Coq` and co* have full support for dependent types.

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Because of that, it's harder to see the build up, so we won't be directly using them in this talk.

Honestly though, it's because they're way over my head :(

(*) There was another mini joke here...

About This Talk

But we will be using `Haskell` though :)

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It's not truely dependent, but we can do more and more with each `language extension` that comes along.

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But we will be using `Haskell` though :)

It's not truely dependent, but we can do more and more with each `language extension` that comes along.

For the examples, there will also be *very* loose translation to imperative/OOP. Though please keep in mind that these are merely syntax translations, the actual concepts can differ vastly.

Section Outline

2 Review of Basics
- Values and Types
- Defining Data Types
- Functions

Values and Types

Values has Types, or Values are classified by Types.

```
..., -1, 0, 1, 2, 3, ... :: Int
```

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True, False :: Bool
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'a', 'b', 'c' :: Char
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Values and Types

Values has Types, or Values are classified by Types.

```
..., -1, 0, 1, 2, 3, ... :: Int

True, False :: Bool

'a', 'b', 'c' :: Char

"abc" :: String ~ [Char]
```

Values are also called Terms

About Types

How are data types defined?

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  - We can define equivalent non-sugar version ourselves

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How are data types defined?

- Some are built in magic: Int, Char, functions
- Some are built in sugar: list, tuples
    - We can define equivalent non-sugar version ourselves
- Rest can be user defined: Bool, String, Maybe

About Types

What are the data types like?

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- Multiple Value constructors

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- Paremetrize over another type

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About Types

What are the data types like?

- Multiple Value constructors
- Paremetrize over another type
- Recursive definition
- Synonyms of other types
- A combination of the above

Defining Data Types

Define new data type with data.

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- Left hand side (LHS) - Type constructor
- Right hand side (RHS) - Value constructor

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Type and Value constructors are capticalized.

Our First Example!

Define a person:

```
-- | params for firstname , lastname , age respectively
data Person = Person String String Int
```

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A loose translation:

```
enum Person {
  Person(String firstname, String lastname, Int age)
}
```

Our First Example!

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A loose translation:

```
enum Person {
  Person(String firstname, String lastname, Int age)
}
```

In this example, the Type and Value constructor have the same name. The Type of the Person constructor:

```
Person :: String -> String -> Int -> Person

bobby :: Person
bobby = Person "Bobby" "Smith" 23
```

Our First Example!

Define a person:

```
-- | params for firstname, lastname, age respectively
data Person = Person String String Int
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A loose translation:

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enum Person {
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In this example, the Type and Value constructor have the same name. The Type of the Person constructor:

```
Person :: String -> String -> Int -> Person

bobby :: Person
bobby = Person "Bobby" "Smith" 23

-- a loose translation:
Person bobby = new Person("Bobby", "Smith", 23)
```

Multiple Value Constructors

Data can have multiple Value constructors:

```
data Bool = False | True

data Weekdays = Sunday | Monday | Tuesday | Wednesday
              | Thursday | Friday | Saturday
```

Does this remind you of anything?

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data Bool = False | True

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Does this remind you of anything?

A loose translation:

```
enum Bool { False, True }

enum Weekdays {
  Sunday, Monday, Tuesday, Wednesday, Thursday, Friday,
      Saturday
}
```

Multiple Value Constructor

You can do type aliasing with type:

```
type Side = Double
type Radius = Double
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For example:

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data Shape = Triangle Side Side Side
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           | Circle Radius
```

Multiple Value Constructor

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type Side = Double
type Radius = Double
```

For example:

```
data Shape = Triangle Side Side Side
           | Rectangle Side Side
           | Circle Radius
```

A loose translation:

```
enum Shape {
  Triangle(Double side1, Double side2, Double side3),
  Rectangle(Double length, Double width),
  Circle(Double radius)
}
```

Multiple Value Constructor

Recall Side $\sim$ Radius $\sim$ Double:

```
data Shape = Triangle Side Side Side
           | Rectangle Side Side
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Multiple Value Constructor

Recall Side ∼ Radius ∼ Double:

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data Shape = Triangle Side Side Side
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```

Types of the Value constructors:

```
Triangle  :: Side -> Side -> Side -> Shape
Rectangle :: Side -> Side -> Shape
Circle    :: Radius -> Shape
```

Multiple Value Constructor

Recall Side ∼ Radius ∼ Double:

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data Shape = Triangle Side Side Side
           | Rectangle Side Side
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```

Types of the Value constructors:

```
Triangle  :: Side -> Side -> Side -> Shape
Rectangle :: Side -> Side -> Shape
Circle    :: Radius -> Shape
```

Example Shapes:

```
myTri, myRect, myCir :: Shape
myTri  = Triangle 2.1 3.2 5
myRect = Rectangle 4 4
myCir  = Circle 7.2
```

Parametrization

Types can parametrize over another type:

```
data Identity a = Identity a
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A loose translation:

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enum Identity<A> {
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data Identity a = Identity a
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The Type of the Identity constructor:

```
Identity :: a -> Identity a

intIdwrtSum :: Indentity Int
intIdwrtSum = Identity 0
```

Tuple

Parametrize over 2 types - 2-tuple!

```
data Tuple a b = Tuple a b
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```

A loose translation:

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enum Tuple<A, B> {
  Tuple(A a, B b)
}
```

Tuple

Parametrize over 2 types - 2-tuple!

```
data Tuple a b = Tuple a b
```

A loose translation:

```
enum Tuple<A, B> {
  Tuple(A a, B b)
}
```

With:

```
Tuple :: a -> b -> Tuple a b
```

Tuple

Actual built-in sugar:

```
   data Tuple a b = Tuple a b
=> data (,) a b = (,) a b
=> data (a, b) = (a, b)
```

Tuple

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   data Tuple a b = Tuple a b
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```

An example:

```
type Employed = Bool

barbara, chet, luffy :: (Person, Employed)
barbara = (Person "Barbara" "Sakura" 30, True)
chet    = (Person "Chet" "Awesome-Laser" 2, False)
luffy   = (Person "Luff D." "Monkey" 19, False)
```

Maybe

Like Bool, but parametrize an *a* over the True part:

```
data Maybe a = Nothing | Just a
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```
data Maybe a = Nothing | Just a
```

A loose translation:

```
enum Maybe<A> {
  Nothing,
  Just(A a)
}
```

The Types of the two Value constructors:

```
Nothing :: Maybe a
Just    :: a -> Maybe a
```

Maybe

From previous slide:

```
data Maybe a = Nothing | Just a
```

Maybe

From previous slide:

```
data Maybe a = Nothing | Just a
```

Say more with the occupation:

```
type Occupation = Maybe String

barbara, chet, luffy :: (Person, Occupation)
barbara = (Person "Barbara" "Sakura" 30, Just "dancer")
chet    = (Person "Chet" "Awesome-Laser" 2, Nothing)
luffy   = (Person "Luff D." "Monkey" 19, Just "pirate")
```

Either

Like Bool, but parametrize over both True and False:

```
data Either a b = Left a | Right b
```

Either

Like Bool, but parametrize over both True and False:

```
data Either a b = Left a | Right b
```

A loose translation:

```
enum Either<A, B> {
  Left(A a),
  Right(B b)
}
```

Either

Like Bool, but parametrize over both True and False:

```
data Either a b = Left a | Right b
```

A loose translation:

```
enum Either<A, B> {
  Left(A a),
  Right(B b)
}
```

The two Value constructors have Types:

```
Left  :: a -> Either a b
Right :: b -> Either a b
```

Either

From previous slide:

```
data Either a b = Left a | Right b
```

Either

From previous slide:

```
data Either a b = Left a | Right b
```

Refine with more details:

```
type Earning = Either String Int

barbara, chet, luffy :: (Person, Earning)
barbara = (Person "Barbara" "Sakura" 30,
           Right 100000)
chet    = (Person "Chet" "Awesome-Laser" 2,
           Left "Is a baby")
luffy   = (Person "Luff D." "Monkey" 19,
           Right 2000000)
```

Types with Recursion

Natural number:

```
data Nat = Z | S Nat

Z :: Nat
S :: Nat -> Nat
```

Types with Recursion

Natural number:

```
data Nat = Z | S Nat

Z :: Nat
S :: Nat -> Nat
```

A loose translation:

```
enum Nat {
  Z,
  S(Nat n)
}
```

Types with Recursion

Natural number:

```
data Nat = Z | S Nat

Z :: Nat
S :: Nat -> Nat

0 ~ Z
1 ~ S Z
2 ~ S (S Z)
3 ~ S (S (S Z))
```

Types with Recursion

List - recursive type while parametrize over another type:

```
data List a = Nil | Cons a (List a)

Nil :: List a
Cons :: a -> List a -> List a
```

Types with Recursion

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```
data List a = Nil | Cons a (List a)

Nil :: List a
Cons :: a -> List a -> List a
```

A loose translation:

```
enum List<A> {
  Nil,
  Cons(A a, List<A> as)
}
```

Types with Recursion

Actual built-in sugar is something like:

```
   data List a = Nil | Cons a (List a)
=> data [] a = [] | (:) a ([] a)
=> data [a] = [] | (:) a [a]
```

Types with Recursion

Actual built-in sugar is something like:

```
    data List a = Nil | Cons a (List a)
=> data [] a = [] | (:) a ([] a)
=> data [a] = [] | (:) a [a]
```

De-sugar that list:

```
ints :: List Int
ints = Cons 1 (Cons 2 (Cons 3 (Cons 4 Nil)))

-- built-in sugar
ints :: [] Int
ints = 1 : 2 : 3 : 4 : []

-- 2x the sugar!
ints :: [Int]
ints = [1, 2, 3, 4]
```

Functions

Maps Values of a Type to another Type:

```
even :: Int -> Bool
even 0 = True
even n = if rem n 2 == 0
         then True
         else False
```

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even :: Int -> Bool
even 0 = True
even n = if rem n 2 == 0
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```

Not as loose translation:

```
Bool even (Int n) {
  switch n:
    case n == 0:
      return True;
    default:
      if rem(n, 2) == 0
        return True;
      else
        return False;
}
```

Functions with Recursion

Use recursion for recursive types:

```
toInt :: Nat -> Int
toInt Z     = 0
toInt (S n) = 1 + toInt n
```

Functions with Recursion

Use recursion for recursive types:

```
toInt :: Nat -> Int
toInt Z     = 0
toInt (S n) = 1 + toInt n
```

Not as loose translation:

```
Int toInt (Nat n) {
  switch n:
    case Z:
      return 0;
    case (S m):   -- n ~ (S m)
      return 1 + toInt(m);
}
```

Type of Functions

Q: Actual type of functions?

Type of Functions

Q: Actual type of functions?
A: Built-in magic, it's called the *function arrow*, something like:

```
data (->) a b = implementation

even  :: Int -> Bool
toInt :: Nat -> Int
```

Functions with Parametric Polymorphism

Functions can be parametric:

```
id :: a -> a
id a = a
```

Functions with Parametric Polymorphism

Functions can be parametric:

```
id :: a -> a
id a = a
```

Not as loose translation:

```
A id<A>(A a) {
  return a;
}
```

Functions with Parametric Polymorphism

Functions can be parametric:

```
append :: [a] -> [a] -> [a]
append []     ys = ys
append (x:xs) ys = x : append xs ys
```

Functions with Parametric Polymorphism

Functions can be parametric:

```
append :: [a] -> [a] -> [a]
append []     ys = ys
append (x:xs) ys = x : append xs ys
```

A translation:

```
List<A> append(List<A> l1, List<A> l2) {
  switch l1:
    case Nil:
      return l2;
    case Cons(x, xs):
      List<A> rest = append(xs, l2);
      return Cons(x, rest);
}
```

Higher-order Functions

Functions that take functions as params:

```
-- actual name is ($)
apply :: (a -> b) -> a -> b
apply f x = f x

-- acutal name is (.)
compose :: (b -> c) -> (a -> b) -> (a -> c)
compose f g = \x -> f (g x)
```

Higher-order Functions

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apply :: (a -> b) -> a -> b
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compose :: (b -> c) -> (a -> b) -> (a -> c)
compose f g = \x -> f (g x)
```

Yay translations:

```
B apply(Func<A,B> f, A a) {
  return f(a);
}

Func<A,C> compose(Func<B,C> f, Func<A,B> g) {
  return x => f(g(x));
}
```

More Functions Examples

map:

```
map :: (a -> b) -> [a] -> [b]
map f []       = []
map f (x:xs) = f x : map f xs
```

More Functions Examples

map:

```
map :: (a -> b) -> [a] -> [b]
map f []     = []
map f (x:xs) = f x : map f xs
```

A translation:

```
List<B> map(Func<A,B> f, List<A> la) {
  switch la:
    case Nil:
      return Nil;
    case Cons(a, as):
      B b = f(a)
      List<B> rest = map(f, as);
      return Cons(b, rest);
}
```

More Functions Examples

zip:

```
zip :: [a] -> [b] -> [(a,b)]
zip []     ys     = []
zip xs     []     = []
zip (x:xs) (y:ys) = (x,y) : zip xs ys
```

More Functions Examples

zip:

```
zip :: [a] -> [b] -> [(a,b)]
zip []     ys     = []
zip xs     []     = []
zip (x:xs) (y:ys) = (x,y) : zip xs ys
```

A translation:

```
List<Tuple<A,B>> zip(List<A> l1, List<A> l2) {
  switch l1:
    case Nil:
      return Nil;
    case Cons(a, as):
      switch l2:
        case Nil:
          return Nil;
        case Cons(b, bs):
          Tuple<A,B> front = Tuple(a, b);
          List<Tuple<A,B>> rest = zip(as, bs);
          return Cons(front, rest);
}
```

Section Outline

3 What is Dependent Type
- λ-Calculus
- Extensions on λ-calculus

λ-Calculus

So far, we have seen:

λ-Calculus

So far, we have seen:

- function application
- function abstraction (aka higher-order functions)

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- substitution

λ-Calculus

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- function application
- function abstraction (aka higher-order functions)
- variable binding
- substitution

=> basis for simply typed λ-calculus.

λ-Calculus

Q: Sure, but can we have more?

λ-Calculus

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A: Yes, extend λ-calculus so we can have more forms of abstractions.

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Q: But how?
A: What if I told you. . .

λ-Calculus

Q: Sure, but can we have more?
A: Yes, extend λ-calculus so we can have more forms of abstractions.

Q: But how?
A: What if I told you...
...you already know at least 2 axes of extension :)

Subtype Polymorphism

Given data types T and P, if there is a relation between T and P by some notion of substitutability with T in place of P, then we say T is a *subtype* of the *supertype* P, denoted by $T <: P$.

Subtype Polymorphism

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The name for this extension of λ-calculus is called *subtype polymorphism* and is denoted by $\lambda_{<:}$.

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=> Object Oriented Programming.

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The name for this extension of λ-calculus is called *subtype polymorphism* and is denoted by $\lambda_{<:}$.
=> Object Oriented Programming.

Though this is not an axis that we are interested in.

Parametric Polymorphism

Introduce a mechanism of universal quantification over Types: Types can abstract over Types, allows for *generic data types* and *generic functions*.

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Parametric Polymorphism

Introduce a mechanism of universal quantification over Types: Types can abstract over Types, allows for *generic data types* and *generic functions*.
=> Generic Programming.

Recall:

```
data [a] = [] | (:) a [a]

id :: a -> a
map :: (a -> b) -> [a] -> [b]
```

Parametric Polymorphism

Introduce a mechanism of universal quantification over Types: Types can abstract over Types, allows for *generic data types* and *generic functions*.
=> Generic Programming.

Recall:

```
data [a] = [] | (:) a [a]

id :: a -> a
map :: (a -> b) -> [a] -> [b]
```

The name for this extension is formally *second order λ-calculus*, aka *System F*, denoted by $\lambda 2$,

Value and Type Interdependency

Re-thinking functions:

```
even :: Int -> Bool
even 0 = True
even n = if rem n 2 == 0
         then True
         else False
```

f maps numbers to True and False.

Value and Type Interdependency

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=> Values on RHS depends on the Values on LHS

Value and Type Interdependency

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=> Values on RHS depends on the Values on LHS

=> Values depending on Values

Value and Type Interdependency

Re-thinking parametrized data types:

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data Maybe a = Nothing | Just a

data List a = Nil | Cons a (List a)
```

Value and Type Interdependency

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List takes a Type and return Value constructors
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=> Values depending on Types
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Are we seeing a pattern yet?

Value and Type Interdependency

Then what about the other cases of dependencies?

Value and Type Interdependency

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- Values depending on Values: λ-calculus

Value and Type Interdependency

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Value and Type Interdependency

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- Types depending on Types:

Value and Type Interdependency

Then what about the other cases of dependencies?

- Values depending on Values: λ-calculus
- Values depending on Types: $\lambda 2$, *System F*
- Types depending on Types: $\lambda \underline{\omega}$
  => Type-level programming via type operators

Value and Type Interdependency

Then what about the other cases of dependencies?

- Values depending on Values: λ-calculus
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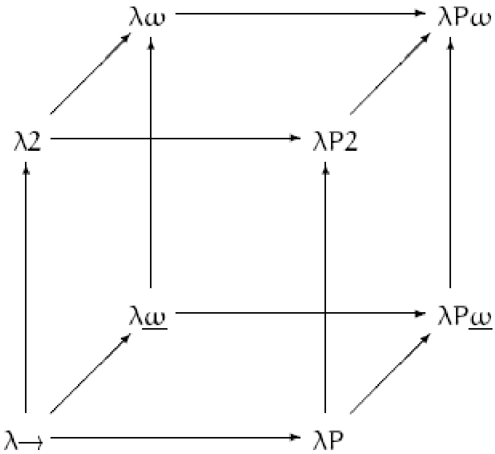
- Types depending on Types: $\lambda \underline{\omega}$
  => Type-level programming via type operators
- Types depending on Values:

Value and Type Interdependency

Then what about the other cases of dependencies?

- Values depending on Values: λ-calculus
- Values depending on Types: $\lambda 2$, *System F*
- Types depending on Types: $\lambda\underline{\omega}$
  => Type-level programming via type operators
- Types depending on Values: $\lambda\Pi$
  => Dependent types

Lambda Cube



Pi and Sigma Types

Π-type - quantification of values on type level

Pi and Sigma Types

Π-type - quantification of values on type level

Σ-type - dependent pair

System F_c

Currently, Haskell as of GHC 7.10.2 doesnot have true type operators. Achieves type-level programming through *type families* and equalities on Types. This axis of extension on $\lambda 2$ is termed System F_c.

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This, plus a Kind system, and a handful of *language extensions*, we are ready fake dependent types in Haskell.

Teaser

Example please:

```
data Vec (n :: Nat) a where
  VNil :: Vec 0 a
  (:>) :: a -> Vec n a -> Vec (n + 1) a

vs :: Vec 6 Int
vs = 4 :> 8 :> 15 :> 16 :> 23 :> 42 :> VNil
```

Teaser

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vs :: Vec 6 Int
vs = 4 :> 8 :> 15 :> 16 :> 23 :> 42 :> VNil
```

Translation* please:

```
enum Vec<Nat n, A> {
  Vec<0, n> VNil,
  Vec<n + 1, n> VCons(A a, Vec<n, A> va)
}

Vec<6, Int> vs = VCons(4, VCons(8, VCons(15, VCons(16,
    VCons(23, VCons(42, VNil))))));
```

(*) supreme looseness and totally made-up syntax

Section Outline

Kinds

Q: Types classify Values, but what classifies Types?
A: Kinds

Introducing ⋆

```
-- built-in magic: infinitely many value constructors
data Int = ... | -1 | 0 | 1 | 2 | ...
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```
Int :: *
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Int :: *
Bool ::
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Int :: *
Bool :: *
[Int] :: *
[] :: * -> *
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```
Int :: *
Bool :: *
[Int] :: *
[] :: * -> *
Maybe Person ::
```

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Introducing Constraint

Haskell has *typeclasses* that very loosely resemble interfaces in OOP. A basic Typeclass consists of a collection of function signatures for a Type to implement. Afterward, this typeclass instance can be used to provide context for functions.

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```
Show  -- types that can be serialized to String
Eq    -- types that can be compared for equality
Ord   -- types that can be ordered
Num   -- types that are like numbers: +, -, *, ...
      -- and many others
```

Introducing Constraint

An example:

```
data Ordering = LT | EQ | GT

show      :: Show a => a -> String  -- toString()
(==)      :: Eq a => a -> a -> Bool
compare   :: Ord a => a -> a -> Ordering
(+)       :: Num a => a -> a -> a
sequenceA :: (Applicative f, Traversable t) => t (f a)
    -> f (t a)
```

Introducing Constraint

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```

A loose translation with : for implements:

```
enum Ordering { LT, EQ, GT }

String show<A>(A a) where A : Show
Bool equals<A>(A a, A a) where A : Eq
Ordering compare<A>(A a, A a) where A : Ordering
A plus<A>(A a, A a) where A : Num
F<T<_>> sequenceA<F,T>(T<F<_>> tfa) where F :
    Applicative, T : Traversable
```

Introducing Constraint

These Typeclass contexts have Kind Constraint.

```
{-# LANGUAGE ConstraintKinds #-}

type ShowCxt a b = (Show a, Show b)

sameSerialization :: ShowContxt a b => a -> b -> Bool
sameSerialization a b = show a == show b

ShowCxt :: * -> * -> Constraint
```

Other Kinds

There are other Kinds aside from * and Constraint

```
import GHC.Prim

(*)           -- kind of fully realized type
(#)           -- kind of unboxed stuff used internally
Constraint    -- kind of constraints and type equality
OpenKind      -- superkind of (*) and (#)
AnyK          -- polymorphic kind for flexible arity
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AnyK          -- polymorphic kind for flexible arity

-- the only sort, sorts classify kinds
(*), (#), Constraint, OpenKind, AnyK :: BOX
BOX :: BOX
```

All these Kinds are built-in and inferred as of GHC 7.10.2.

Language Extensions

Compiler extensions that enable a variety of new functionalities:

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- Syntax extension
- Type-level programming
- Generic programming
- FFI
- Type disambiguation
- Typeclass extension

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Each extension has a name, and is enabled with the LANGUAGE pragma.

GADTs

KindSignatures

ConstraintKinds

Type Operators

DataKinds

Type Families

Section Outline

Questions?