CHAPTER-XI

THREE DIMENSIONAL GEOMETRY

2 MARK QUESTIONS

1: If a line makes angles 90°, 135°, 45° with the x, y and z-axes respectively, find its direction cosines.

Solution:

Let the direction cosines of the line be **l**, m, and n.

$l = \cos 90° = 0$

$m = \cos 135° = -1/\sqrt{2}$

$n = \cos 45° = 1/\sqrt{2}$

Hence, the direction cosines of the line are 0, $-1/\sqrt{2}$, and $1/\sqrt{2}$.

2.Write the vector equation of the line given by x−53=y+47=z−62 (Delhi 2011)

Answer:

$\vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$

3.Find the equation of the plane passing through the line of intersection of the planes $\vec{r} \cdot (\hat{i} + 3\hat{j}) - 6 = 0$ and $\vec{r} \cdot (3\hat{i} - \hat{j} - 4\hat{k}) = 0$, whose perpendicular distance from origin is unity. (All India 2013C)

Answer:

$\vec{r} \cdot (2\hat{i} + \hat{j} - 2\hat{k}) - 3 = 0$

and $\vec{r} \cdot (-\hat{i} + 2\hat{j} + 2\hat{k}) - 3 = 0$

4 .Find the coordinates of the point, where the line through (3, – 4, – 5) and (2, – 3, 1) crosses the plane, passing through the points (2, 2, 1), (3, 0, 1) and (4, – 1, 0).

Answer:

(1, – 2, 7)

4 MARK QUESTIONS

1: Show that the points A (2, 3, – 4), B (1, – 2, 3) and C (3, 8, – 11) are collinear.

Solution:

We know that the direction ratios of the line passing through two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are given by:

$x_2 - x_1, y_2 - y_1, z_2 - z_1$ or $x_1 - x_2, y_1 - y_2, z_1 - z_2$

Given points are A (2, 3, – 4), B (1, – 2, 3) and C (3, 8, – 11).

Direction ratios of the line joining A and B are:

$1 - 2, -2 - 3, 3 + 4$

i.e. $-1, -5, 7$.

The direction ratios of the line joining B and C are:

$3 - 1, 8 + 2, -11 - 3$

i.e., $2, 10, -14$.

From the above, it is clear that direction ratios of AB and BC are proportional.

That means AB is parallel to BC. But point B is common to both AB and BC.

Hence, A, B, C are collinear points.

2: Find the angle between the pair of lines given by

$$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$$

$$\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$$

Solution:

From the given,

$$\overrightarrow{b_1} = (\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$$

$$\overrightarrow{b_2} = (3\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$$

Let θ be the angle between the given pair of lines.

$$\cos\theta = \left|\frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right| = \left|\frac{(\hat{i}+2\hat{j}+2\hat{k})\cdot(3\hat{i}+2\hat{j}+6\hat{k})}{\sqrt{1+4+4}\,\sqrt{9+4+36}}\right|$$

$$= \left|\frac{3+4+12}{3\times 7}\right| = \frac{19}{21}$$

$$\theta = \cos^{-1}\left(\frac{19}{21}\right)$$

3: Show that the lines (x – 5)/7 = (y + 2)/-5 = z/1 and x/1 = y/2 = z/3 are perpendicular to each other.

Solution:

Given lines are:

(x – 5)/7 = (y + 2)/-5 = z/1 and x/1 = y/2 = z/3

The direction ratios of the given lines are 7, -5, 1 and 1, 2, 3, respectively.

We know that,

Two lines with direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 are perpendicular to each other if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Therefore, 7(1) + (-5) (2) + 1 (3)

= 7 – 10 + 3

=0

Hence, the given lines are perpendicular to each other.

4: Find the intercepts cut off by the plane 2x + y – z = 5.

Solution:

Given plane is $2x + y - z = 5$ ……(i)

Dividing both sides of the equation (i) by 5,

$(2/5)x + (y/5) - (z/5) = 1$

$$\frac{x}{\frac{5}{2}} + \frac{y}{5} + \frac{z}{-5} = 1 \;\ldots\ldots\; (ii)$$

We know that,

The equation of a plane in intercept form is $(x/a) + (y/b) + (z/c) = 1$, where a, b, c are intercepts cut off by the plane at x, y, z-axes respectively.

For the given equation,

$a = 5/2$, $b = 5$, $c = -5$

Hence, the intercepts cut off by the plane are 5/2, 5 and -5.

5.If a line makes angles 90°, 60° and θ with X, Y and Z-axis respectively, where θ is acute angle, then find θ. (Delhi 2015)

Answer:

Let l, m and n be the direction cosines of the given line. Then, we have
$l = \cos 90° = 0$,
$m = \cos 60° = 12$
and $n = \cos \theta$
$\because l^2 + m^2 + n^2 = 1$

$$\therefore \quad 0 + \left(\frac{1}{2}\right)^2 + \cos^2\theta = 1$$

$$\Rightarrow \qquad \cos^2\theta = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\Rightarrow \qquad \cos\theta = \pm\frac{\sqrt{3}}{2}$$

[$\because \cos\theta$ cannot be negative as θ is an acute angle]
$\Rightarrow \cos\theta = \cos 30°$
$\therefore \theta = 30°$

7 MARK QUESTIONS

1: Find the equations of the planes that passes through three points (1, 1, 0), (1, 2, 1), and (– 2, 2, – 1).

Solution:

Given points are (1, 1, 0), (1, 2, 1), and (– 2, 2, – 1).

$$\begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ -2 & 2 & -1 \end{vmatrix} = 1(-2-2) - 1(-1+2) + 0 = -5 \neq 0$$

Therefore, the plane will pass through the given three points.

We know that,

The equation of the plane through the points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y-1 & z \\ 0 & 1 & 1 \\ -3 & 1 & -1 \end{vmatrix} = 0$$

$(x - 1)(-2) - (y - 1)(3) + z(3) = 0$

$-2x + 2 - 3y + 3 + 3z = 0$

$-2x - 3y + 3z + 5 = 0$

$-2x - 3y + 3z = -5$

Therefore, $2x + 3y - 3z = 5$ is the required Cartesian equation of the plane.

2.The equations of a line is 5x – 3 = 15y + 7 = 3 – 10z. Write the direction cosines of the line.

Answer:

Given equation of a line is
5x – 3 = 15y + 17 = 3 – 10z ……. (i)
Lei us first convert the equation in standard form
x−x1a= y−y1b= z−z1c ……. (iii)
Let us divide Eq. (i) by LCM (coefficients of x, y and z), i.e. LCM (5, 15, 10) = 30
Now, the Eq. (i) becomes

$$\frac{5x-3}{30}=\frac{15y+7}{30}=\frac{3-10z}{30}$$

$$\Rightarrow \frac{5\left(x-\frac{3}{5}\right)}{30}=\frac{15\left(y+\frac{7}{15}\right)}{30}=\frac{-10\left(z-\frac{3}{10}\right)}{30}$$

$$\Rightarrow \frac{x-\frac{3}{5}}{6}=\frac{y+\frac{7}{15}}{2}=\frac{z-\frac{3}{10}}{-3}$$

On comparing the above equation with Eq.(ii), we get 6, 2, – 3 are the direction ratios of the given line.
Now, the direction cosines of given line are

$$\frac{6}{\sqrt{6^2+2^2+(-3)^2}}, \frac{2}{\sqrt{6^2+2^2+(-3)^2}} \text{ and } \frac{-3}{\sqrt{6^2+2^2+(-3)^2}}$$

i.e. $\frac{6}{7}, \frac{2}{7}, \frac{-3}{7}$.

3.If a line makes angles a,p,y with the position direction of coordinate axes, then write the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$.

Answer:

Given, if a line makes angles a,y with the coordinate axes.
Then, direction cosine of a line are
$\cos\alpha, \cos\beta, \cos\gamma$
$\therefore \sin^2\alpha + \sin^2\beta + \sin^2\gamma$
$= 1 - \cos^2\alpha + 1 - \cos^2\beta + 1 - \cos^2\gamma$
$= 3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$
$= 3 - 1 = 2$ [$\because \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$]

4.Write the equation of the straight line through the point (α β γ) and parallel to Z-axis.

Answer:

The vector equation of a line parallel to Z-axis is $\vec{m} = 0\hat{i} + 0\hat{j} + 0\hat{k}$ Then, the required line passes through the point A(α β γ) whose position vector is $\vec{r}1 = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ and is parallel to the vector $\vec{m} = (0\hat{i} + 0\hat{j} + \hat{k})$.
$\therefore$ The equation is $\vec{r} = \vec{r1} + \lambda\vec{m}$
$= (\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}) + \lambda(0\hat{i} + 0\hat{j} + 0\hat{k})$
$= (\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}) + \lambda(\hat{k})$