MATHEMATICS

CLASS XII

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CHAPTER-I

RELATIONS AND FUNCTIONS

2 MARK QUESTIONS

1.If R = {(a, a^3): a is a prime number less than 5} be a relation. Find the range of R .

Answer:

Given, R = {{a, cd): a is a prime number less than 5}
We know that, 2 and 3 are the prime numbers less than 5.
So, a can take values 2 and 3.
Thus, R = {(2, 2^3), (3, 3^3)} = {(2, 8), (3, 27)}
Hence, the range of R is (8, 27}.

2.If f: {1,3, 4} → {1, 2, 5} and g: {1,2, 5} → {1, 3} given by f = {(1,2), (3, 5), (4,1)} and g = {(1,3), (2, 3), (5,1)}. Write down gof.

Answer:

Given, functions f:{1, 3, 4} → {1, 2, 5} and g : {1, 2, 5} → {1, 3} are defined as f = {(1, 2),(3, 5),(4, 1)} and g = {(1, 3),(2, 3),(5, 1)}
Therefore, f(1) = 2, f(3) = 5, f(4) = 1
and g(1) = 3, g (2) = 3, g(5) = 1
Now, gof: {1,3,4} → {1,3} and it is defined as
gof (1) = g[(f(1)] = g(2) = 3

$gof(3) = g[f(3)] = g(5) = 1$
$gof\ (4) = g[f(4)] = g(1) = 3$
$\therefore gof = \{(1, 3), (3, 1), (4, 3)\}$

3.Let R is the equivalence relation in the set A = {0,1, 2, 3, 4, 5} given by R = {(a, b) : 2 divides (a – b)}. Write the equivalence class [0].

Answer:

Given, $R = \{(a, b): 2 \text{ divides} (a - b)\}$
and $A = \{ 0,1, 2, 3, 4, 5\}$
Clearly, $[0] = \{b \in A : (0, b) \in R\}$
$= \{b \in A: 2 \text{ divides } (0 - b)\}$
$= \{b \in A : 2\text{divides } (-b)\} = \{0, 2, 4\}$
Hence, equivalence class of $[0] = \{0,2,4\}$.

4.If R = {(x, y): x + 2y = 8} is a relation on N, then write the range of R.

Answer:

Given, the relation R is defined on the set of natural numbers, i.e. N as
$R= \{(x, y) : x + 2y = 8\}$
To find the range of R, $x + 2y = 8$ can be rewritten as $y = 8-x2$
On putting $x = 2$, we get $y = 8-22 = 3$
On puttmg $x = 4$, we get $y = 8-42 = 2$
On putting $x = 6$, we get $y = 8-62 = 1$
As, $x, y \in N$, therefore $R = \{(2, 3), (4, 2), (6, 1)\}$. Hence, the range of relation R is $\{3,2,1\}$.
Note: For $x = 1, 3, 5, 7, 9, \ldots\ldots\ldots$ we do not get y as natural number.

5.If $A = \{1, 2, 3\}$, $S = \{4, 5,6, 7\}$ and $f = \{(1, 4), (2, 5), (3, 6)\}$ is a function from A to B. State whether f is one-one or not. (All India 2011)

Answer:

Given, $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$
and $f:A \rightarrow B$is defined as $f = \{(1, 4), (2, 5), (3, 6)\}$
i.e. $f(1) = 4$, $f(2) = 5$and $f(3) = 6$.
It can be seen that the images of distinct elements of A under f are distinct. So, f is one-one.

4 MARK QUESTIONS

1. Show that the Signum Function f: R → R, given by

f(x)=
{1for x>00
for x=0 is neither one–one nor onto–1for x<0

Solution:

Check for one to one function:

For example:

f(0) = 0

f(-1) = -1

f(1) = 1

f(2) = 1

f(3) = 1

Since, the different elements say f(1), f(2) and f(3), shows the same image, then the function is not one to one function.

Check for Onto Function:

For the function,f: R →R

f(x)={1for x>00for x=0−1for x<0
In this case, the value of f(x) is defined only if x is 1, 0, -1

For any other real numbers(for example $y = 2$, $y = 100$) there is no corresponding element x.

Thus, the function "f" is not onto function.

Hence, the given function "f" is neither one-one nor onto.

2: If f: R → R is defined by $f(x) = x^2 - 3x + 2$, find $f(f(x))$.

Solution:

Given function:

$f(x) = x^2 - 3x + 2.$

To find $f(f(x))$

$f(f(x)) = f(x)^2 - 3f(x) + 2.$

$= (x^2 - 3x + 2)^2 - 3(x^2 - 3x + 2) + 2$

By using the formula $(a-b+c)^2 = a^2 + b^2 + c^2 - 2ab + 2ac - 2ab$, we get

$= (x^2)^2 + (3x)^2 + 2^2 - 2x^2 (3x) + 2x^2(2) - 2x^2(3x) - 3(x^2 - 3x + 2) + 2$

Now, substitute the values

$= x^4 + 9x^2 + 4 - 6x^3 - 12x + 4x^2 - 3x^2 + 9x - 6 + 2$

$= x^4 - 6x^3 + 9x^2 + 4x^2 - 3x^2 - 12x + 9x - 6 + 2 + 4$

Simplify the expression, we get,

$f(f(x)) = x^4 - 6x^3 + 10x^2 - 3x$

7 MARK QUESTIONS

1: Let A = N × N and * be the binary operation on A defined by (a, b) * (c, d) = (a + c, b + d). Show that * is commutative and associative. Find the identity element for * on A, if any.

Solution:

Check the binary operation * is commutative :

We know that, * is commutative if (a, b) * (c, d) = (c, d) * (a, b) ∀ a, b, c, d ∈ R

L.H.S =(a, b) * (c, d)

=(a + c, b + d)

R. H. S = (c, d) * (a, b)

=(a + c, b + d)

Hence, L.H.S = R. H. S

Since (a, b) * (c, d) = (c, d) * (a, b) ∀ a, b, c, d ∈ R

* is commutative (a, b) * (c, d) = (a + c, b + d)

Check the binary operation * is associative :

We know that * is associative if (a, b) * ((c, d) * (x, y)) = ((a, b) * (c, d)) * (x, y) ∀ a, b, c, d, x, y ∈ R

L.H.S = (a, b) * ((c, d) * (x, y)) = (a+c+x, b+d+y)

R.H.S = ((a, b) * (c, d)) * (x, y) = (a+c+x, b+d+y)

Thus, L.H.S = R.H.S

Since $(a, b) * ((c, d) * (x, y)) = ((a, b) * (c, d)) * (x, y)\ \forall\ a, b, c, d, x, y \in R$

Thus, the binary operation * is associative

Checking for Identity Element:

e is identity of * if $(a, b) * e = e * (a, b) = (a, b)$

where $e = (x, y)$

Thus, $(a, b) * (x, y) = (x, y) * (a, b) = (a, b)\ (a + x, b + y)$

$= (x + a , b + y) = (a, b)$

Now, $(a + x, b + y) = (a, b)$

Now comparing these, we get:

$a+x = a$

$x = a - a = 0$

Next compare: $b + y = b$

$y = b-b = 0$

Since $A = N \times N$, where x and y are the natural numbers. But in this case, x and y is not natural number. Thus, the identity element does not exist.

Therefore, operation * does not have any identity element.

2: Let $f : N \to Y$ be a function defined as $f(x) = 4x + 3$, where, $Y = \{y \in N: y = 4x + 3$ for some $x \in N\}$. Show that f is invertible. Find the inverse.

Solution:

Checking for Inverse:

$f(x) = 4x + 3$

Let $f(x) = y$

$y = 4x + 3$

$y - 3 = 4x$

$4x = y - 3$

$x = (y - 3)/4$

Let $g(y) = (y - 3)/4$

where $g: Y \rightarrow N$

Now find gof:

$gof = g(f(x))$

$= g(4x + 3) = [(4x + 3) - 3]/4$

$= [4x + 3 - 3]/4$

$=4x/4$

$= x = I_N$

Now find fog:

$fog = f(g(y))$

$= f\ [(y - 3)/4]$

$=4[(y - 3)/4] +3$

$= y - 3 + 3$

$= y + 0$

$= y = I_y$

Thus, $gof = I_N$ and $fog = I_y$,

Hence, f is invertible

Also, the Inverse of $f = g(y) = [y - 3]/4$

3: Let A = R {3} and B = R – {1}. Consider the function f: A →B defined by f (x) = (x- 2)/(x -3). Is f one-one and onto? Justify your answer.

Solution:

Given function:

$f(x) = (x-2)/(x-3)$

Checking for one-one function:

$f(x_1) = (x_1 - 2)/(x_1 - 3)$

$f(x_2) = (x_2 - 2)/(x_2 - 3)$

Putting $f(x_1) = f(x_2)$

$(x_1-2)/(x_1-3) = (x_2-2)/(x_2-3)$

$(x_1-2)(x_2-3) = (x_1-3)(x_2-2)$

$x_1(x_2-3) - 2(x_2-3) = x_1(x_2-2) - 3(x_2-2)$

$x_1 x_2 - 3x_1 - 2x_2 + 6 = x_1 x_2 - 2x_1 - 3x_2 + 6$

$-3x_1 - 2x_2 = -2x_1 - 3x_2$

$3x_2 - 2x_2 = -2x_1 + 3x_1$

$x_1 = x_2$

Hence, if $f(x_1) = f(x_2)$, then $x_1 = x_2$

Thus, the function f is one-one function.

Checking for onto function:

$f(x) = (x-2)/(x-3)$

Let $f(x) = y$ such that y B i.e. $y \in R - \{1\}$

So, $y = (x-2)/(x-3)$
$y(x-3) = x-2$

$xy - 3y = x - 2$

$xy - x = 3y - 2$

$x(y-1) = 3y-2$

$x = (3y-2)/(y-1)$

For $y = 1$, x is not defined But it is given that. $y \in R - \{1\}$

Hence, $x = (3y-2)/(y-1) \in R - \{3\}$ Hence, f is onto.