CHAPTER-IV

DETERMINANTS

2 MARK QUESTIONS

1.Find |AB|, if A = [00–12] and B = [3050]. (All India 2019)

Answer:

Given, $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$

Now, $AB = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\therefore \quad |AB| = 0$

2.In the interval it π/2 < x < π, find the value of x for which the matrix [2sinx132sinx] is singular. (All India 2015C)

Answer:

Let $A = \begin{bmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{bmatrix}$

$\because$ A is a singular matrix.

$\therefore \quad |A| = 0 \Rightarrow \begin{vmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{vmatrix} = 0$

$\Rightarrow \quad 4\sin^2 x - 3 = 0 \Rightarrow \sin^2 x = \frac{3}{4}$

$\Rightarrow \quad \sin x = \frac{\sqrt{3}}{2} \qquad \begin{bmatrix} \text{taking positive square} \\ \text{root because } \frac{\pi}{2} < x < \pi \end{bmatrix}$

$\therefore \quad x = \frac{2\pi}{3}$

3.If A is a square matrix satisfying A'A = I, write the value of |A|. (All India 2019)

Answer:

We have, A'A = I

$\Rightarrow |A'A| = |I| \Rightarrow |A'||A| = 1$ [$\because$ $|AB| = |A|\ |B|$]

$\Rightarrow |A|^2 = 1$ [$\because$ $|A'| = |A|$]

$\Rightarrow |A| = \pm 1$

4.If A and B are square matrices of the same order 3, such that $|A| = 2$ and AB = 27. Write the value of $|B|$. (Delhi 2019)

Answer:

We know that,
(i) $|kA| = k^n|A|$, if A is square matrix of nth order
(ii) $|AB| = |A| \times |B|$
Here, we have AB = 21 and n – 3
$\therefore |AB| = |2I| = 2^3|I| = 8.1 = 8$ [$\because |I| = 1$]
$\Rightarrow |4||B| = 8$
$\Rightarrow 2.|B| = 8 \Rightarrow |B| = 4$

4 MARK QUESTIONS

1.Write the value of Δ = ||||x+yz−3y+zx−3z+xy−3||||. (All India 2015)

Answer:

Given, $\Delta = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$

On applying $R_1 \rightarrow R_1 + R_2$, we get

$$\Delta = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$$

On taking (x + y + z) common from R_1 and -3

Δ = (x + y + z)(-3)||||1z11x11y1||||

= (x + y + z)(-3) × 0 [∵ R_1 and R_3 are identical]

= 0

2.Let A be a square matrix of order 3 × 3. Write the value of |2A|, where |A| = 4.

Answer:

We know that, for a square matrix A of order n,

$|kA| = k^n - |A|$

Here, $|2A| = 2^3.|A|$ [∵ order of A is 3 × 3]

$= 2^3 \times 4 = 8 \times 4 = 32$ [put |A| = 4]

3.If the determinant of matrix A of order 3×3 is of value 4, then write the value of $|3A|$.

Answer:

We know that, for a square matrix A of order n,
$|kA| = k^n - |A|$
Here, $|3A| = 3^3.|A|$ [$\because$ order of A is 3×3]
$= 108$

4.If A is a square matrix of order 3 such that $|adjA| = 64$,then find $|A|$.

Answer:

We know that, for a square matrix of order n,
$|adj\ (A)| = |A|^{n-1}$
Here, the order of A is 3×3 therefore n- 3
Now, $|adj\ (A)| = |A|^{3-1} = |A|^2$
Given, $|adj\ (A)| = 64 \Rightarrow 64 = |A|^2$
$\Rightarrow (8)^2 = |A|^2$
$\Rightarrow |A| = \pm 8$ [taking square root]

7MARK QUESTIONS

1.|A|=2≠0|A|=2≠0
Therefore A^{-1} exists

AB = I
A^{-1} AB = A^{-1}I
B = A^{-1}
adjA=[2143]$adjA$=[2413]
A−1=1|A|(adjA)A−1=1|A|($adjA$)
=12[2143]=12[2413]
=[112232]=[121232]
Hence B=[112232]

2.Consider the following system of linear equations; x + y + z = 6, x − y + z = 2, 2x + y + z = 1
(i) Express this system of equations in the Standard form AXB
(ii) Prove that A is non-singular.
(iii) Find the value of x, y and z satisfying the above equation.

Answer:

(i) Let AX = B,

Where $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}$

(ii) $|A| = 1(-1-1) - 1(1-2) + 1(1+2) = 2$

(iii) $C_{11} = -2, C_{12} = 1, C_{13} = 3, C_{21} = 0, C_{22} = -1$

$C_{23} = 1, C_{31} = 2, C_{32} = 0, C_{33} = -2$

$$adj(A)=\begin{bmatrix}-2 & 1 & 3\\ 0 & -1 & 1\\ 2 & 0 & -2\end{bmatrix}^T=\begin{bmatrix}-2 & 0 & 2\\ 1 & -1 & 0\\ 3 & 1 & -2\end{bmatrix}$$

$$A^{-1}=\frac{adj(A)}{|A|}=\frac{1}{2}\begin{bmatrix}-2 & 0 & 2\\ 1 & -1 & 0\\ 3 & 1 & -2\end{bmatrix}$$

$$X=A^{-1}B=\frac{1}{2}\begin{bmatrix}-2 & 0 & 2\\ 1 & -1 & 0\\ 3 & 1 & -2\end{bmatrix}\times\begin{bmatrix}6\\ 2\\ 1\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}-10\\ 4\\ 18\end{bmatrix}=\begin{bmatrix}-5\\ 2\\ 9\end{bmatrix}$$

$$\Rightarrow x=-5, y=2, z=9$$

3.(i) If |||x532|||=5, then x = ………..
(ii) Prove that
||||y+kyyyy+kyyyy+k||||=k2(3y+k)
(iii) Solve the following system of linear Equations, using matrix method; 5x + 2y = 3, 3x + 2y = 5 (March – 2012)

Answer:

(i) $\begin{vmatrix} x & 3 \\ 5 & 2 \end{vmatrix} = 5 \Rightarrow 2x - 15 = 5 \Rightarrow x = 10$

(ii) $$\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} \overset{C_1 \to C_1 + C_2 + C_3}{=} \begin{vmatrix} 3y+k & y & y \\ 3y+k & y+k & y \\ 3y+k & y & y+k \end{vmatrix}$$

$$= (3y+k)\begin{vmatrix} 1 & y & y \\ 1 & y+k & y \\ 1 & y & y+k \end{vmatrix}$$

$$\overset{R_2 \to R_2 - R_1, R_3 \to R_3 - R_1}{=} (3y+k) \begin{vmatrix} 1 & y & y \\ 0 & k & 0 \\ 0 & 0 & k \end{vmatrix} = (3y+k)k^2$$

(iii) Let AX = B,

Where $A=\begin{bmatrix}5 & 2\\ 3 & 2\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}3\\ 5\end{bmatrix}$

$|A|=10-6=4$

$C_{11}=2, C_{12}=-3, C_{21}=-2, C_{22}=5$

$adj(A)=\begin{bmatrix}2 & -3\\ -2 & 5\end{bmatrix}^T=\begin{bmatrix}2 & -2\\ -3 & 5\end{bmatrix}$

$A^{-1}=\frac{adj(A)}{|A|}=\frac{1}{4}\begin{bmatrix}2 & -2\\ -3 & 5\end{bmatrix}$

$X=A^{-1}B=\frac{1}{4}\begin{bmatrix}2 & -2\\ -3 & 5\end{bmatrix}\times\begin{bmatrix}3\\ 5\end{bmatrix}$

$=\frac{1}{4}\begin{bmatrix}-4\\ 16\end{bmatrix}=\begin{bmatrix}-1\\ 4\end{bmatrix}\Rightarrow x=-1, y=4$

4.Consider the matrices A=[2435]
(i) Find $A^2-7A-21=0$
(ii) Hence find A-1
(iii) Solve the following system of equations using matrix method 2x + 3y = 4; 4x + 5y = 6

Answer:

(i) $A^2 = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 16 & 21 \\ 28 & 37 \end{bmatrix}$

$$A^2 - 7A - 2I = \begin{bmatrix} 16 & 21 \\ 28 & 37 \end{bmatrix} - 7\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 21 \\ 28 & 37 \end{bmatrix} - \begin{bmatrix} 14 & 21 \\ 28 & 37 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(ii) We have; $A^2 - 7A = 2I$

$$\Rightarrow (A^2 - 7A)A^{-1} = 2I \times A^{-1}$$

$$\Rightarrow (A - 7I) = 2A^{-1} \Rightarrow A^{-1} = \frac{1}{2}(A - 7I)$$

$$\Rightarrow A^{-1} = \frac{1}{2}\left(\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} -5 & 3 \\ 4 & -2 \end{bmatrix}$$

(iii) The given system of equations can be converted into matrix form AX = B

$$X = A^{-1}B = \frac{1}{2}\begin{bmatrix} -5 & 3 \\ 4 & -2 \end{bmatrix} \times \begin{bmatrix} 4 \\ 6 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$x = -1, y = 2$

5.(i) Let A be a square matrix of order 2 x 2 then |KA| is equal to

(a) K|A|

(b) K^2|A|

(c) K^3|A|

(d) 2K|A|

(ii) Prove that

||||a−b−c2 b2c2a b−c−a2c2a2 bc−a−b||||=(a+b+c)3

(iii) Examine the consistency of the system of Equations. 5x + 3y = 5; 2x + 6y = 8

Answer:

(i) K^2 |A|

(ii) $$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} \quad R_1 \to R_1 + R_2 + R_3$$

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -b-c-a & 0 \\ 2c & 0 & -c-a-b \end{vmatrix} \quad \begin{matrix} C_2 \to C_2 - C_1 \\ C_3 \to C_3 - C_1 \end{matrix}$$

$$= (a+b+c)(a+b+c)^2 - 0 = (a+b+c)^3$$

(iii) The given system of equation can be written in matrix form as

$$A = \begin{bmatrix} 5 & 3 \\ 2 & 6 \end{bmatrix}; X = \begin{bmatrix} x \\ y \end{bmatrix}; B = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 5 & 3 \\ 2 & 6 \end{vmatrix} = 30 - 6 = 24 \neq 0$$

solution exist and hence it is consistent.

6.(a) Choose the correct statement related to the matnces A=[1001],B=[0110]

(i) A3=A,B3≠B (ii) A3≠A,B3=B (iii) A3=A,B3=B (iv) A3≠A,B3≠B

(b) If M=[7253] then verity the equation $M^2 - 10M + 11\, I_2 = O$

(c) Inverse of the matrix ⎡⎣⎢001110212⎤⎦⎥

Answer:

(a) (iii) $A^3 = A, B^3 = B$

(b) $M^2 = \begin{bmatrix} 7 & 5 \\ 2 & 3 \end{bmatrix} \times \begin{bmatrix} 7 & 5 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 59 & 50 \\ 20 & 19 \end{bmatrix}$

$$10M = \begin{bmatrix} 70 & 50 \\ 20 & 30 \end{bmatrix}; 11I = \begin{bmatrix} 11 & 0 \\ 0 & 11 \end{bmatrix}$$

$$M^2 - 10M + 11I_2$$

$$= \begin{bmatrix} 59 & 50 \\ 20 & 19 \end{bmatrix} - \begin{bmatrix} 70 & 50 \\ 20 & 30 \end{bmatrix} + \begin{bmatrix} 11 & 0 \\ 0 & 11 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(c) Cofactor matrix = $\begin{bmatrix} 2 & 1 & -1 \\ -2 & -2 & 1 \\ -1 & 0 & 0 \end{bmatrix}$

$$AdjA = \begin{bmatrix} 2 & -2 & -1 \\ 1 & -2 & 0 \\ -1 & 1 & 0 \end{bmatrix}; |A| = \begin{vmatrix} 0 & 1 & 2 \\ 0 & 1 & 1 \\ 1 & 0 & 2 \end{vmatrix} = -1$$

$$A^{-1} = \frac{AdjA}{|A|} = \frac{1}{-1}\begin{bmatrix} 2 & -2 & -1 \\ 1 & -2 & 0 \\ -1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 2 & 1 \\ -1 & 2 & 0 \\ 1 & -1 & 0 \end{bmatrix}$$