CHAPTER-VII

INTEGRALS

2 MARK QUESTIONS

1:Write the anti-derivative of the following function: $3x^2+4x^3$

Solution:

Given: $3x^2+4x^3$

The antiderivative of the given function is written as:

$\int 3x^2+4x^3\,dx = 3(x^3/3) + 4(x^4/4)$

$= x^3 + x^4$

Thus, the antiderivative of $3x^2+4x^3 = x^3 + x^4$

2.Write the value of ∫dxx2+16

Answer:

$$\text{Let } I = \int \frac{dx}{x^2+16} = \int \frac{dx}{x^2+(4)^2}$$

$$= \frac{1}{4}\tan^{-1}\frac{x}{4} + C$$

$$\left[\because \int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C\right]$$

4 MARK QUESTIONS

1:Evaluate: $\int 3ax/(b^2+c^2x^2)\ dx$

Solution:

To evaluate the integral, $I = \int 3ax/(b^2+c^2x^2)\ dx$

Let us take $v = b^2+c^2x^2$, then

$dv = 2c^2x\ dx$

Thus, $\int 3ax/(b^2+c^2x^2)\ dx$

$= (3ax/2c^2x)\int dv/v$

Now, cancel x on both numerator and denominator, we get

$= (3a/2c^2)\int dv/v$

$= (3a/2c^2)\log|b^2+c^2x^2| + C$

Where C is an arbitrary constant

2:Determine $\int \tan^8x \sec^4 x\ dx$

Solution:

Given: $\int \tan^8x \sec^4 x\ dx$

Let $I = \int \tan^8x \sec^4 x\ dx$ — (1)

Now, split $\sec^4x = (\sec^2x)(\sec^2x)$

Now, substitute in (1)

$I = \int \tan^8x (\sec^2x)(\sec^2x)\ dx$

$= \int \tan^8x (\tan^2 x+1)(\sec^2x)\ dx$

It can be written as:

$= \int \tan^{10}x \sec^2 x \, dx + \int \tan^8 x \sec^2 x \, dx$

Now, integrate the terms with respect to x, we get:

$I = (\tan^{11} x / 11) + (\tan^9 x / 9) + C$

Hence, $\int \tan^8 x \sec^4 x \, dx = (\tan^{11} x / 11) + (\tan^9 x / 9) + C$

3.Write the value of $\int 2-3\sin x \cos 2x \, dx$.

Answer:

Let $I = \int 2-3\sin x \cos 2x \, dx$
$= \int (2\cos 2x - 3\sin x \cos 2x) \, dx$
$= \int (2 \sec^2 x - 3 \sec x \tan x) \, dx$
$= 2\int \sec^2 x \, dx - 3\int \sec x \tan x \, dx$
$= 2 \tan x - 3 \sec x + C$

7 MARK QUESTIONS

1.Determine the antiderivative F of "f" , which is defined by f (x) = $4x^3 - 6$, where F (0) = 3.

Solution:

Given function: f (x) = $4x^3 - 6$

Now, integrate the function:

$\int 4x^3 - 6\, dx = 4(x^4/4) - 6x + C$

$\int 4x^3 - 6\, dx = x^4 - 6x + C$

Thus, the antiderivative of the function, F is $x^4 - 6x + C$, where C is a constant

Also, given that, F(0) = 3,

Now, substitute x = 0 in the obtained antiderivative function, we get:

$(0)^4 - 6(0) + C = 3$

Therefore, C = 3.

Now, substitute C = 3 in antiderivative function

Hence, the required antiderivative function is $x^4 - 6x + 3$.

2.Integrate the given function using integration by substitution: $2x \sin(x^2+ 1)$ with respect to x:

Solution:

Given function: $2x \sin(x^2+ 1)$

We know that, the derivative of $x^2 + 1$ is $2x$.

Now, use the substitution method, we get

$x^2 + 1 = t$, so that $2x\, dx = dt$.

Hence, we get $\int 2x \sin(x^2+1)\, dx = \int \sin t\, dt$

$= -\cos t + C$

$= -\cos(x^2 + 1) + C$

Where C is an arbitrary constant

Therefore, the antiderivative of $2x \sin(x^2+ 1)$ using integration by substitution method is $= -\cos(x^2 + 1) + C$

3.Integrate: $\int \sin^3 x \cos^2 x\, dx$

Solution:

Given that, $\int \sin^3 x \cos^2 x\, dx$

This can be written as:

$\int \sin^3 x \cos^2 x\, dx = \int \sin^2 x \cos^2 x (\sin x)\, dx$

$=\int (1 - \cos^2 x) \cos^2 x (\sin x)\, dx$ —(1)

Now, substitute $t = \cos x$,

Then $dt = -\sin x\, dx$

Now, equation can be written as:

Thus, $\int \sin^3 x \cos^2 x\, dx = -\int (1-t^2)t^2\, dt$

Now, multiply t^2 inside the bracket, we get

$= -\int (t^2-t^4)\, dt$

Now, integrate the above function:

$= -[(t^3/3) - (t^5/5)] + C$ —(2)

Where C is a constant

Now, substitute $t = \cos x$ in (2)

$= -(⅓)\cos^3 x + (1/5)\cos^5 x + C$

Hence, $\int \sin^3 x \cos^2 x\, dx = -(⅓)\cos^3 x + (1/5)\cos^5 x + C$

4.Find ∫sin2x−cos2xsinxcosx dx.

Answer:

$$\text{Let } I = \int \frac{\sin^2 x - \cos^2 x}{\sin x \cdot \cos x} dx$$
$$= \int \left[\frac{\sin^2 x}{\sin x \cdot \cos x} - \frac{\cos^2 x}{\sin x \cdot \cos x} \right] dx$$
$$= \int \left[\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x} \right] dx$$

$= \int(\tan x - \cot x)\, dx$
$= \int \tan x\, dx - \int \cot x\, dx$
$= \log|\sec x| - [-\log|\operatorname{cosec} x|] + C$
$= \log|\sec x| + \log|\operatorname{cosec} x| + C$
$= \log|\sec x \cdot \operatorname{cosec} x| + C$

5.Given, $\int e^x (\tan x + 1) \sec x \, dx = e^x f(x) + C$.
Write f(x) satisfying above. (All India 2012;

Answer:

Use the relation $\int e^x [f(x) + f'(x)dx = e^x f(x) + C$ and simplify it.

Given that $\int e^x (\tan x + 1) \sec x \, dx = e^x \cdot f(x) + C$
$\Rightarrow \int e^x (\sec x + \sec x \tan x)dx = e^x f(x) + C$
$\Rightarrow e^x . \sec x + C = e^x f(x) + C$
$[\because e^x \{f(x) + f'(x)\}dx = e^x f(x) + C$ and here $\frac{d}{dx} (\sec x) = \sec x \tan x]$
On comparing both sides, we get
$f(x) = \sec x$