CHAPTER-X

VECTORS

2 MARK QUESTIONS

1.Write the direction ratios of the vector $3\vec{a} + 2\vec{b}$, where $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$

Answer:

Clearly, $3\vec{a} + 2\vec{b} = 3(\hat{i} + \hat{j} - 2\hat{k}) + 2(2\hat{i} - 4\hat{j} + 5\hat{k})$
$= (3\hat{i} + 3\hat{j} - 6\hat{k}) + (4\hat{i} - 8\hat{j} + 10\hat{k})$
$= 7\hat{i} - 5\hat{j} + 4\hat{k}$
Hence, direction ratios of vectors $3\vec{a} + 2\vec{b}$ are 7, – 5 and 4.

2.Find the unit vector in the direction of the sum of the vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $4\hat{i} - 3\hat{j} + 2\hat{k}$.

Answer:

Let $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = 4\hat{i} - 3\hat{j} + 2\hat{k}$
Now, sum of two vectors,
$\vec{a} + \vec{b} = (2\hat{i} + 3\hat{j} - \hat{k}) + (4\hat{i} - 3\hat{j} + 2\hat{k}) = 6\hat{i} + \hat{k}$

$$\therefore \text{Required unit vector} = \frac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|}$$

$$= \frac{6\hat{i}+\hat{k}}{\sqrt{6^2+1^2}} = \frac{6\hat{i}+\hat{k}}{\sqrt{36+1}} = \frac{6\hat{i}+\hat{k}}{\sqrt{37}} = \frac{6}{\sqrt{37}}\hat{i} + \frac{\hat{k}}{\sqrt{37}}$$

3.Write a vector in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9 units.

Answer:

$3\hat{i} - 6\hat{j} + 6\hat{k}$

4.If $\vec{a} = x\hat{i} + 2\hat{j} - z\hat{k}$ and $\vec{b} = 3\hat{i} - y\hat{j} + \hat{k}$ are two equal vectors, then write the value of $x + y + z$. (Delhi 2013)

Answer:

Two vectors are equal, if coefficients of their components are equal.
Given, $\vec{a} = \vec{b} \Rightarrow x\hat{i} + 2\hat{j} - z\hat{k} = \hat{i} - y\hat{j} + \hat{k}$

On comparing the coefficient of components, we get
$x = 3, y = -2, z = -1$
Now, $x + y + z = 3 - 2 - 1 = 0$

5.P and Q are two points with position vectors $3\vec{a} - 2\vec{b}$ and $\vec{a} + \vec{b}$, respectively. Write the position vector of a point R which divides the line segment PQ in the ratio 2 : 1 externally.

Answer:

$-\vec{a} + 4\vec{b}$

6.L and M are two points with position vectors $2\vec{a} - \vec{b}$ and $\vec{a} + 2\vec{b}$, respectively. Write the position vector of a point N which divides the line segment LM in the ratio 2 : 1 externally.

Answer:

$5\vec{b}$

7.Find the sum of the following vectors. $\vec{a} = \hat{i} - 3\hat{k}$, $\vec{b} = 2\hat{j} - \hat{k}$, $\vec{c} = 2\hat{i} - 3\hat{j} + 2\hat{k}$.

Answer:

$3\hat{i} - \hat{j} - 2\hat{k}$

4 MARK QUESTIONS

1: Find the unit vector in the direction of the sum of the vectors

a→=2i^−j^+2k^

b→=−i^+j^+3k^

Solution:

Let *c*→ be the sum of *a*→ and *b*→.

$$\vec{c} = (2\hat{\imath} - \hat{\jmath} + 2\hat{k}) + (-\hat{\imath} + \hat{\jmath} + 3\hat{k}) = \hat{\imath} + 5\hat{k}$$

$$|\vec{c}| = \sqrt{1^2 + 5^2} = \sqrt{26}$$

The unit vector is:

$$\hat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{\hat{\imath} + 5\hat{k}}{\sqrt{26}} = \frac{1}{\sqrt{26}}\hat{\imath} + \frac{5}{\sqrt{26}}\hat{k}$$

2: Find the vector joining the points P(2, 3, 0) and Q(− 1, − 2, − 4) directed from P to Q.

Solution:

Since the vector is to be directed from P to Q, clearly P is the initial point and Q is the terminal point.

P(2, 3, 0) = (x_1, y_1, z_1)

Q(-1, -2, -4) = (x_2, y_2, z_2)

Vector joining the points P and Q is:

$\overrightarrow{PQ} = (-1-2)\hat{\imath} + (-2-3)\hat{\jmath} + (-4-0)\hat{k}$

$$\overrightarrow{PQ} = -3\hat{\imath} - 5\hat{\jmath} - 4\hat{k}$$

3. No. 6: Show that the points A, B and C with position vectors

a→=3i^−4j^−4k^,b→=2i^−j^+k^
c→=i^−3j^−5k^
form the vertices of a right-angled triangle.
Solution:

Solution:

Position vectors of points A, B and C are respectively given as below.

a→=3*i*^−4*j*^−4*k*^,
b→=2*i*^−*j*^+*k*^
c→=*i*^−3*j*^−5*k*^

$\overrightarrow{AB} = \vec{b} - \vec{a} = (2-3)\hat{\imath} + (-1+4)\hat{\jmath} + (1+4)\hat{k} = -\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$

$\overrightarrow{BC} = \vec{c} - \vec{b} = (1-2)\hat{\imath} + (-3+1)\hat{\jmath} + (-5-1)\hat{k} = -\hat{\imath} - 2\hat{\jmath} - 6\hat{k}$

$\overrightarrow{CA} = \vec{a} - \vec{c} = (3-1)\hat{\imath} + (-4+3)\hat{\jmath} + (-4+5)\hat{k} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

$\therefore \left|\overrightarrow{AB}\right|^2 = (-1)^2 + 3^2 + 5^2 = 1 + 9 + 25 = 35$

$\left|\overrightarrow{BC}\right|^2 = (-1)^2 + (-2)^2 + (-6)^2 = 1 + 4 + 36 = 41$

$\left|\overrightarrow{CA}\right|^2 = 2^2 + (-1)^2 + 1^2 = 4 + 1 + 1 = 6$

$\left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{CA}\right|^2 = 35 + 6 = 41 = \left|\overrightarrow{BC}\right|^2$

Therefore, ABC is a right-angled triangle.

4: Find the area of the triangle with vertices A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5).

Solution:

Vertices of a triangle ABC are A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5).

Let AB and BC be the adjacent sides of triangle ABC.

$$\overrightarrow{AB} = (2-1)\hat{\imath} + (3-1)\hat{\jmath} + (5-2)\hat{k} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$$

$$\overrightarrow{BC} = (1-2)\hat{\imath} + (5-3)\hat{\jmath} + (5-5)\hat{k} = -\hat{\imath} + 2\hat{\jmath}$$

$$ar(\Delta\, ABC) = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{BC}|$$

$$|\overrightarrow{AB} \times \overrightarrow{BC}| = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 3 \\ -1 & 2 & 0 \end{vmatrix} = \hat{\imath}(-6) - \hat{\jmath}(3) + \hat{k}(2+2) = -6\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$$

$$\overrightarrow{AB} \times \overrightarrow{BC} = \sqrt{(-6)^2 + (-3)^2 + 4^2} = \sqrt{36+9+16} = \sqrt{61}$$

Hence, the area of triangle ABC is √61/2 sq.units

5.Find the sum of the vectors $\vec{a} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$ $\vec{b} = -2\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$ and $\vec{c} = \hat{\imath} - 6\hat{\jmath} - 7\hat{k}$.

Answer:

Given vectors are $\vec{a} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$ $\vec{b} = -2\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$ and $\vec{c} = \hat{\imath} - 6\hat{\jmath} - 7\hat{k}$.
Sum of the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ is
$\vec{a} + \vec{b} + \vec{c} = (\hat{\imath} - 2\hat{\jmath} + \hat{k}) + (-2\hat{\imath} + 4\hat{\jmath} + 5\hat{k}) + (\hat{\imath} - 6\hat{\jmath} - 7\hat{k})$
$= -4\hat{\jmath} - \hat{k}$

6.Write the direction cosines of vector -2î + ĵ – 5k̂.

Answer:

Direction cosines of the vector aî + bĵ + ck̂ are

$$\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}.$$

Let $\quad \vec{a} = -2\hat{i} + \hat{j} - 5\hat{k}$

$\therefore$ Direction cosines of $\vec{a}$ are

$$\frac{-2}{\sqrt{(-2)^2+(1)^2+(-5)^2}}, \frac{1}{\sqrt{(-2)^2+(1)^2+(-5)^2}}$$

and $$\frac{-5}{\sqrt{(-2)^2+(1)^2+(-5)^2}}$$

i.e. $\frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-5}{\sqrt{30}}$

7 MARK QUESTIONS

1.Find the magnitude of each of the two vectors a⃗ and b⃗, having the same magnitude such that the angle between them is 60° and their scalar product is 92.

Answer:

Given, two vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a}|=|\vec{b}|$,

$\vec{a}\cdot\vec{b}=\frac{9}{2}$ and angle between them is 60°.

We know that

$$\vec{a}\cdot\vec{b}=|\vec{a}|\,|\vec{b}|\cos\theta,$$

where θ is angle between $\vec{a}$ and $\vec{b}$.

$\therefore \quad \frac{9}{2}=|\vec{a}|\cdot|\vec{a}|\cos 60^\circ$ **(1/2)**

$\Rightarrow \quad \frac{1}{2}\cdot|\vec{a}|^2=\frac{9}{2}$ $\left[\because \cos 60^\circ=\frac{1}{2}\right]$

$\Rightarrow \quad |\vec{a}|^2=9$

$\Rightarrow \quad |\vec{a}|=3$

[$\because$ magnitude cannot be negative]

Thus, $\quad |\vec{a}|=|\vec{b}|=3$ **(1/2)**

2.If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular vectors of the same magnitude, then prove that $\vec{a}+\vec{b}+\vec{c}$ is equally inclined with the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$.

Answer:

If three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular to each other, then $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$ and if all three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are equally inclined with the vector $(\vec{a}+\vec{b}+\vec{c})$ that means each vector $\vec{a}$, $\vec{b}$ and $\vec{c}$ makes equal angle with $(\vec{a}+\vec{b}+\vec{c})$ by using formula

$\cos\theta = a^{\to} \cdot b^{\to}\ |a^{\to}|\,|b^{\to}|.$

Given, $|\vec{a}| = |\vec{b}| = |\vec{c}| = \lambda$ (say) ...(i)

and $\vec{a}\cdot\vec{b} = 0, \vec{b}\cdot\vec{c} = 0$ and $\vec{c}\cdot\vec{a} = 0$ ...(ii)

Now, $|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2$

$$+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})$$

$$= \lambda^2+\lambda^2+\lambda^2+2(0+0+0) = 3\lambda^2$$

$\Rightarrow \quad |\vec{a}+\vec{b}+\vec{c}| = \sqrt{3}\lambda$

[length cannot be negative]

Suppose $(\vec{a}+\vec{b}+\vec{c})$ is inclined at angles θ_1, θ_2 and θ_3 respectively with vectors $\vec{a}, \vec{b}$ and $\vec{c}$, then

$$(\vec{a}+\vec{b}+\vec{c})\cdot\vec{a} = |\vec{a}+\vec{b}+\vec{c}|\ |\vec{a}|\cos\theta_1$$

$[\because \vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta]$

$\Rightarrow \quad |\vec{a}|^2 + \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c} = \sqrt{3}\lambda\times\lambda\cos\theta_1$

$\Rightarrow \quad \lambda^2+0+0 = \sqrt{3}\lambda^2\cos\theta_1$

[from Eqs. (i) and (ii)]

$\therefore \quad \cos\theta_1 = \dfrac{1}{\sqrt{3}}$

$$(\vec{a}+\vec{b}+\vec{c})\cdot\vec{b} = |\vec{a}+\vec{b}+\vec{c}||\vec{b}|\cos\theta_2$$

$\Rightarrow \quad \vec{a}\cdot\vec{b} + |\vec{b}|^2 + \vec{c}\cdot\vec{b} = \sqrt{3}\lambda\cdot\lambda\cos\theta_2$

$\Rightarrow \quad 0+\lambda^2+0 = \sqrt{3}\lambda^2\cos\theta_2$

[from Eqs. (i) and (ii)]

$\Rightarrow \quad \cos\theta_2 = \dfrac{1}{\sqrt{3}}$

Similarly, $(\vec{a}+\vec{b}+\vec{c})\cdot\vec{c} = |\vec{a}+\vec{b}+\vec{c}||\vec{c}|\cos\theta_3$

$\Rightarrow \quad \cos\theta_1 = \dfrac{1}{\sqrt{3}}$

Thus, $\cos\theta_1 = \cos\theta_2 = \cos\theta_3 = \dfrac{1}{\sqrt{3}}$

Hence, it is proved that $(\vec{a}+\vec{b}+\vec{c})$ is equally inclined with the vectors $\vec{a}, \vec{b}$ and $\vec{c}$.

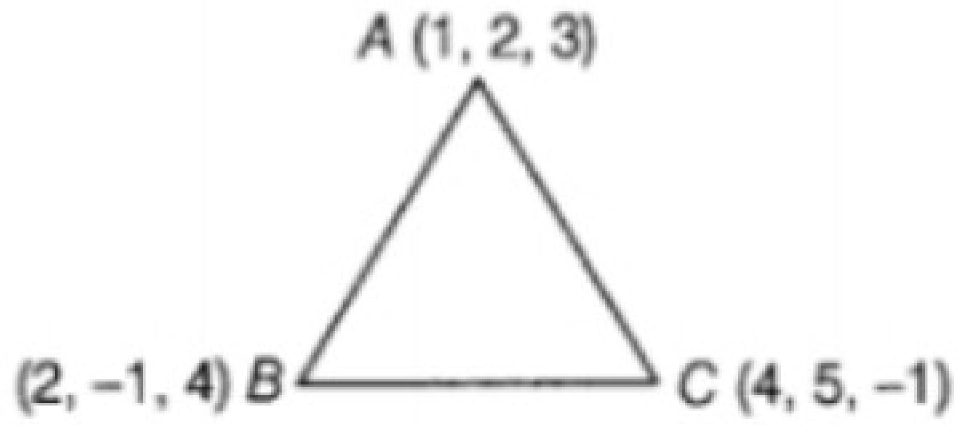
3.Using vectors, find the area of the ΔABC, whose vertices are A(1, 2, 5), 5(2, -1, 4) and C(4, 5, -1).

Answer:

Let the position vectors of the verices A, B and C of ΔABC be

$$\overrightarrow{OA} = \hat{i} + 2\hat{j} + 3\hat{k},\ \overrightarrow{OB} = 2\hat{i} - \hat{j} + 4\hat{k}$$

and $\overrightarrow{OC} = 4\hat{i} + 5\hat{j} - \hat{k}$, respectively.



Then, $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$$= (2\hat{i} - \hat{j} + 4\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= \hat{i} - 3\hat{j} + \hat{k}$$

and $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$

$$= (4\hat{i} + 5\hat{j} - \hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= (3\hat{i} + 3\hat{j} - 4\hat{k})$$

Now, $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 3 & 3 & -4 \end{vmatrix}$

$$= \hat{i}(12-3) - \hat{j}(-4-3) + \hat{k}(3+9)$$

$$= 9\hat{i} + 7\hat{j} + 12\hat{k}$$

and $|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(9)^2 + (7)^2 + (12)^2}$

$$= \sqrt{81 + 49 + 144} = \sqrt{274}$$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

$$= \frac{1}{2}\sqrt{274} \text{ sq units}$$

4.Show that the points A, B, C with position vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 5\hat{j} - 5\hat{k}$ and $5\hat{i} - 4\hat{j} - 4\hat{k}$ respectively, are the vertices of a right-angled triangle. Hence find the

We have,

$\overrightarrow{AB}$ = (Position vector of B) − (Position vector of A)

$$= (\hat{i} - 3\hat{j} - 5\hat{k}) - (2\hat{i} - \hat{j} + \hat{k})$$

$$= -\hat{i} - 2\hat{j} - 6\hat{k}$$

$$\overrightarrow{BC} = (3\hat{i} - 4\hat{j} - 4\hat{k}) - (\hat{i} - 3\hat{j} - 5\hat{k})$$

$$= 2\hat{i} - \hat{j} + \hat{k}$$

and $\overrightarrow{CA} = (2\hat{i} - \hat{j} + \hat{k}) - (3\hat{i} - 4\hat{j} - 4\hat{k})$

$$= -\hat{i} + 3\hat{j} + 5\hat{k}$$

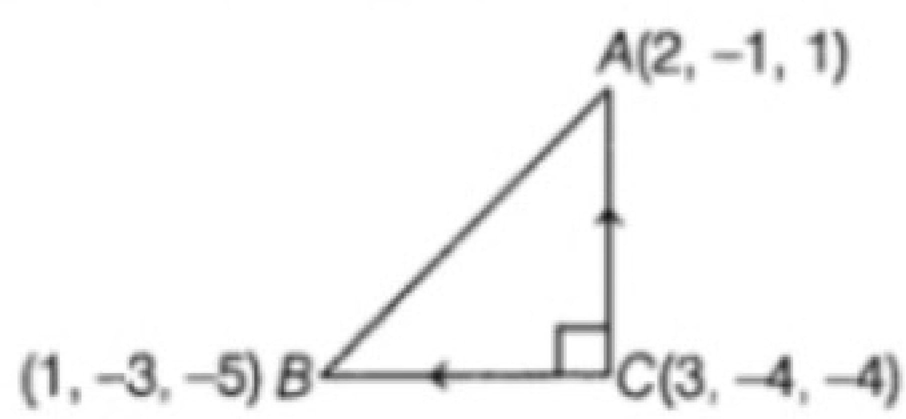
Here, $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0$

$\Rightarrow$ A, B and C are the vertices of a triangle.

Now, $\overrightarrow{BC} \cdot \overrightarrow{CA} = (2\hat{i} - \hat{j} + \hat{k}) \cdot (-\hat{i} + 3\hat{j} + 5\hat{k})$

$$= -2 - 3 + 5 = 0$$

$\Rightarrow \overrightarrow{BC} \perp \overrightarrow{CA} \Rightarrow \angle C = 90°$



Now, area of $\Delta ABC = \frac{1}{2}|\overrightarrow{CB} \times \overrightarrow{CA}|$

$$= \frac{1}{2}|(-8\hat{i} - 11\hat{j} + 5\hat{k})| = \frac{1}{2}\sqrt{210} \text{ sq units}$$