CHAPTER-II

INVERSE TRIGONOMETRIC FUNCTIONS

2 MARK QUESTIONS

1.Write the value of
$\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$. (All India 2019,13)
Answer:

We have, $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$
$= \tan^{-1}(\sqrt{3}) - \{\pi - \cot^{-1}(\sqrt{3})\}$ $[\because \cot^{-1}(-x) = \pi - \cot^{-1}x; x \in R]$
$= \tan^{-1}\sqrt{3} - \pi + \cot^{-1}\sqrt{3}$
$= (\tan^{-1}\sqrt{3} + \cot^{-1}\sqrt{3}) - \pi$
$= \pi 2 - \pi = -\pi 2$ $[\because \tan^{-1}x + \cot^{-1}x = \pi 2; x \in R]$
Which is the required principal value.

2.Find the principal value of
$\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$. (CBSE 2018 C; All India 2012)
Answer:
We have, $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$

$$= \tan^{-1}\left(\tan\frac{\pi}{3}\right) - \sec^{-1}\left(\sec\frac{2\pi}{3}\right)$$

$$\left[\because \tan\frac{\pi}{3} = \sqrt{3} \text{ and } \sec\frac{2\pi}{3} = -2\right]$$

$$= \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$$

$$\left[\begin{array}{l} \because \tan^{-1}(\tan\theta) = \theta; \forall\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \text{and} \\ \sec^{-1}(\sec\theta) = \theta; \forall\theta \in [0, \pi] - \left\{\frac{\pi}{2}\right\} \end{array}\right]$$

Which is the required principal value.

3: Determine the principal value of $\cos^{-1}(-1/2)$.

Solution:

Let us assume that, $y = \cos^{-1}(-1/2)$

We can write this as:

$\cos y = -1/2$

$\cos y = \cos(2\pi/3)$.

Thus, the Range of the principal value of $\cos^{-1}$ is $[0, \pi]$.

Therefore, the principal value of $\cos^{-1}(-1/2)$ is $2\pi/3$.

4: Find the value of $\cot(\tan^{-1}\alpha + \cot^{-1}\alpha)$.

Solution:

Given that: $\cot(\tan^{-1}\alpha + \cot^{-1}\alpha)$

$= \cot(\pi/2)$ (since, $\tan^{-1}x + \cot^{-1}x = \pi/2$)

$= \cot(180°/2)$ (we know that $\cot 90° = 0$)

$= \cot(90°)$

$= 0$

Therefore, the value of $\cot(\tan^{-1}\alpha + \cot^{-1}\alpha)$ is 0.

4 MARK QUESTIONS

1.If sin ($\sin^{-1}15 + \cos^{-1}x$) = 1, then find the value of x. (Delhi 2014)

Answer:

Given, $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$

$\Rightarrow \quad \sin^{-1}\frac{1}{5} + \cos^{-1}x = \sin^{-1}(1)$

$[\because \sin\theta = x \Rightarrow \theta = \sin^{-1}x]$

$\Rightarrow \quad \sin^{-1}\frac{1}{5} + \cos^{-1}x = \sin^{-1}\left(\sin\frac{\pi}{2}\right) \quad \left[\because \sin\frac{\pi}{2} = 1\right]$

$\Rightarrow \quad \sin^{-1}\frac{1}{5} + \cos^{-1}x = \frac{\pi}{2}$

$\Rightarrow \quad \sin^{-1}\frac{1}{5} = \frac{\pi}{2} - \cos^{-1}x$

$\Rightarrow \quad \sin^{-1}\frac{1}{5} = \sin^{-1}x$

$\left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}; x \in [-1, 1]\right]$

$\therefore \quad x = \frac{1}{5}$

2.Write the value of
$\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$. (All India 2019,13)

Answer:

We have, $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$
$= \tan^{-1}(\sqrt{3}) - \{\pi - \cot^{-1}(\sqrt{3})\}$ $[\because \cot^{-1}(-x) = \pi - \cot^{-1}x; x \in R]$
$= \tan^{-1}\sqrt{3} - \pi + \cot^{-1}\sqrt{3}$
$= (\tan^{-1}\sqrt{3} + \cot^{-1}\sqrt{3}) - \pi$
$= \pi 2 - \pi = -\pi 2$ $[\because \tan^{-1}x + \cot^{-1}x = \pi 2; x \in R]$
Which is the required principal value.

3.Find the principal value of
$\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$. (CBSE 2018 C; All India 2012)

Answer:

We have, $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$

$$= \tan^{-1}\left(\tan\frac{\pi}{3}\right) - \sec^{-1}\left(\sec\frac{2\pi}{3}\right)$$

$$\left[\because \tan\frac{\pi}{3} = \sqrt{3} \text{ and } \sec\frac{2\pi}{3} = -2\right]$$

$$= \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$$

$$\left[\begin{array}{l} \because \tan^{-1}(\tan\theta) = \theta; \forall\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \text{and} \\ \sec^{-1}(\sec\theta) = \theta; \forall\theta \in [0, \pi] - \left\{\frac{\pi}{2}\right\} \end{array}\right]$$

Which is the required principal value.

Question 4.
If $\sin(\sin^{-1}15 + \cos^{-1}x) = 1$, then find the value of x. (Delhi 2014)
Answer:

Given, $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$

$$\Rightarrow \quad \sin^{-1}\frac{1}{5} + \cos^{-1}x = \sin^{-1}(1)$$

$$[\because \sin\theta = x \Rightarrow \theta = \sin^{-1}x]$$

$$\Rightarrow \quad \sin^{-1}\frac{1}{5} + \cos^{-1}x = \sin^{-1}\left(\sin\frac{\pi}{2}\right) \quad \left[\because \sin\frac{\pi}{2} = 1\right]$$

$$\Rightarrow \quad \sin^{-1}\frac{1}{5} + \cos^{-1}x = \frac{\pi}{2}$$

$$\Rightarrow \quad \sin^{-1}\frac{1}{5} = \frac{\pi}{2} - \cos^{-1}x$$

$$\Rightarrow \quad \sin^{-1}\frac{1}{5} = \sin^{-1}x$$

$$\left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}; x \in [-1, 1]\right]$$

$$\therefore \quad x = \frac{1}{5}$$

5.If tan⁻¹x + tan⁻¹ y = π4; xy < 1, then write the value of x + y + xy. (All India 2014)
Answer:

Given, $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}, xy < 1$

We know that,

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right), xy < 1$$

$$\therefore \quad \tan^{-1}\left(\frac{x+y}{1-xy}\right) = \frac{\pi}{4} \Rightarrow \frac{x+y}{1-xy} = \tan\frac{\pi}{4}$$

$$\Rightarrow \quad \frac{x+y}{1-xy} = 1 \qquad \left[\because \tan\frac{\pi}{4} = 1\right]$$

$$\Rightarrow \quad x + y = 1 - xy$$

$$\therefore \quad x + y + xy = 1$$

6.Write the principal value of the following.
[cos−13√2+cos−1(−12)] (Delhi 2013C)
Answer:

We have, $\cos^{-1}\frac{\sqrt{3}}{2} + \cos^{-1}\left(-\frac{1}{2}\right)$

$$= \cos^{-1}\frac{\sqrt{3}}{2} + \left[\pi - \cos^{-1}\left(\frac{1}{2}\right)\right]$$

$$[\because \cos^{-1}(-x) = \pi - \cos^{-1} x, \forall x \in [-1, 1]]$$

$$= \cos^{-1}\left(\cos\frac{\pi}{6}\right) + \left[\pi - \cos^{-1}\left(\cos\frac{\pi}{3}\right)\right]$$

$$= \frac{\pi}{6} + \pi - \frac{\pi}{3} = \frac{\pi + 6\pi - 2\pi}{6} = \frac{5\pi}{6}$$

$$[\because \cos^{-1}(\cos\theta) = \theta; \forall \theta \in [0, \pi]]$$

which is the required principal value.

7.Write the value of tan (2 tan^{-1}15). (Delhi 2013)

Answer:

We have,

$$\tan\left(2\tan^{-1}\frac{1}{5}\right)=\tan\left[\tan^{-1}\left(\frac{2\times\frac{1}{5}}{1-\left(\frac{1}{5}\right)^2}\right)\right]$$

$$\left[\because 2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right);-1<x<1\right]$$

$$=\tan\left[\tan^{-1}\left(\frac{2\times5}{24}\right)\right]=\tan\left[\tan^{-1}\left(\frac{5}{12}\right)\right]=\frac{5}{12}$$

$$[\because \tan(\tan^{-1}x)=x;\ \forall x\in R]$$

7 MARK QUESTIONS

1: Prove that $\sin^{-1}(3/5) - \sin^{-1}(8/17) = \cos^{-1}(84/85)$.

Solution:

Let $\sin^{-1}(3/5) = a$ and $\sin^{-1}(8/17) = b$

Thus, we can write $\sin a = 3/5$ and $\sin b = 8/17$

Now, find the value of cos a and cos b

To find cos a:

$\text{Cos } a = \sqrt{[1 - \sin^2 a]}$

$= \sqrt{[1 - (3/5)^2]}$

$= \sqrt{[1 - (9/25)]}$

$= \sqrt{[(25-9)/25]}$

$= 4/5$

Thus, the value of $\cos a = 4/5$

To find cos b:

$\text{Cos } b = \sqrt{[1 - \sin^2 b]}$

$= \sqrt{[1 - (8/17)^2]}$

$= \sqrt{[1 - (64/289)]}$

$= \sqrt{[(289-64)/289]}$

$= 15/17$

Thus, the value of $\cos b = 15/17$

We know that cos (a- b) = cos a cos b + sin a sin b

Now, substitute the values for cos a, cos b, sin a and sin b in the formula, we get:

cos (a – b) = (4/5)x (15/17) + (3/5)x(8/17)

cos (a – b) = (60 + 24)/(17x 5)

cos (a – b) = 84/85

(a – b) = $\cos^{-1}$ (84/85)

Substituting the values of a and b $\sin^{-1}$ (3/5)- $\sin^{-1}$ (8/7) = $\cos^{-1}$ (84/85)

Hence proved.

2: Find the value of $\cos^{-1}$ (1/2) + 2 $\sin^{-1}$ (1/2).

Solution:

First, solve for $\cos^{-1}$ (1/2):

Let us take, y = $\cos^{-1}$ (1/2)

This can be written as:

cos y = (1/2)

cos y = cos (π /3).

Thus, the range of principal value of $\cos^{-1}$ is [0, π]

Therefore, the principal value of $\cos^{-1}$ (1/2) is π/3.

Now, solve for $\sin^{-1}$ (1/2):

Let y = $\sin^{-1}$ (1/2)

sin y = 1/2

sin y = sin (π/6)

Thus, the range of principal value of $\sin^{-1}$ is $[(-\pi)/2, \pi/2]$

Hence, the principal value of $\sin^{-1}(1/2)$ is $\pi/6$.

Now we have $\cos^{-1}(1/2) = \pi/3$ & $\sin^{-1}(1/2) = \pi/6$

Now, substitute the obtained values in the given formula, we get:

$= \cos^{-1}(1/2) + 2\sin^{-1}(1/2)$

$= \pi/3 + 2(\pi/6)$

$= \pi/3 + \pi/3$

$= (\pi+\pi)/3$

$= 2\pi/3$

Thus, the value of $\cos^{-1}(1/2) + 2\sin^{-1}(1/2)$ is $2\pi/3$.