# A Modified Hausdorff Distance for Object Matching *

Marie-Pierre Dubuisson and Anil K. Jain
Department of Computer Science
Michigan State University
East Lansing, MI 48824
{dubuisso,jain}@cps.msu.edu

## Abstract

*The purpose of object matching is to decide the similarity between two objects. This paper introduces 24 possible distance measures based on the Hausdorff distance between two point sets. These measures can be used to match two sets of edge points extracted from any two objects. Based on our experiments on synthetic images containing various levels of noise, we determined that one of these distance measures, called the modified Hausdorff distance (MHD) has the best performance for object matching. The advantages of MHD over other distances are also demonstrated on several edge maps of objects extracted from real images.*

## 1 Introduction

In the context of this paper, the operation of object matching consists of computing a measure of similarity between two objects based on their shape attributes. The shape matching method proposed here is easy to compute and does not require the solution of the correspondence problem. The proposed distance measure is adapted from the Hausdorff distance between two sets of edge maps (points) associated with the objects of interest. We assume that a segmentation algorithm has already been applied to the entire image to extract the edge points belonging to the object of interest. We investigated 24 different distance measures and, based on their behavior in the presence of noise, have determined the best distance measure, called the modified Hausdorff distance (MHD) for object matching.

## 2 Distance Between Point Sets

In the following discussion, we assume that the distance between two points $a$ and $b$ is defined as the Euclidean distance $d(a,b) = \|a-b\|$. The distance between a point $a$ and a set of points $\mathcal{B} = \{b_1, \ldots, b_{N_b}\}$ is commonly defined as $d(a,\mathcal{B}) = \min_{b\in\mathcal{B}} \|a-b\|$. There are many different ways to define the directed distance between two point sets $\mathcal{A} = \{a_1, \ldots, a_{N_a}\}$ and point set $\mathcal{B} = \{b_1, \ldots, b_{N_b}\}$. We consider the following 6 directed distance measures ($d_2$, $d_3$, and $d_4$ are the generalized Hausdorff distance proposed by Huttenlocher *et al.* [1]).

$$d_1(\mathcal{A},\mathcal{B}) = \min_{a\in\mathcal{A}} d(a,\mathcal{B}) \tag{1}$$

$$d_2(\mathcal{A},\mathcal{B}) = {}^{50}K^{th}_{a\in\mathcal{A}} d(a,\mathcal{B}) \tag{2}$$

$$d_3(\mathcal{A},\mathcal{B}) = {}^{75}K^{th}_{a\in\mathcal{A}} d(a,\mathcal{B}) \tag{3}$$

$$d_4(\mathcal{A},\mathcal{B}) = {}^{90}K^{th}_{a\in\mathcal{A}} d(a,\mathcal{B}) \tag{4}$$

$$d_5(\mathcal{A},\mathcal{B}) = \max_{a\in\mathcal{A}} d(a,\mathcal{B}) \tag{5}$$

$$d_6(\mathcal{A},\mathcal{B}) = \frac{1}{N_a}\sum_{a\in\mathcal{A}} d(a,\mathcal{B}) \tag{6}$$

where ${}^{x}K^{th}_{a\in\mathcal{A}}$ represents the $K^{th}$ ranked distance such that $K/N_a = x\%$. For example, ${}^{50}K^{th}_{a\in\mathcal{A}}$ corresponds to the median of the distances $d(a,\mathcal{B}), \forall a \in \mathcal{A}$.

The directed distances $d(\mathcal{A},\mathcal{B})$ and $d(\mathcal{B},\mathcal{A})$ between two point sets $\mathcal{A}$ and $\mathcal{B}$ can be combined in the following four ways to define an undirected distance measure.

$$f_1(d(\mathcal{A},\mathcal{B}), d(\mathcal{B},\mathcal{A})) = \min(d(\mathcal{A},\mathcal{B}), d(\mathcal{B},\mathcal{A})) \tag{7}$$

$$f_2(d(\mathcal{A},\mathcal{B}), d(\mathcal{B},\mathcal{A})) = \max(d(\mathcal{A},\mathcal{B}), d(\mathcal{B},\mathcal{A})) \tag{8}$$

$$f_3(d(\mathcal{A},\mathcal{B}), d(\mathcal{B},\mathcal{A})) = \frac{d(\mathcal{A},\mathcal{B}) + d(\mathcal{B},\mathcal{A})}{2} \tag{9}$$

$$f_4(d(\mathcal{A},\mathcal{B}), d(\mathcal{B},\mathcal{A})) = \frac{N_a d(\mathcal{A},\mathcal{B}) + N_b d(\mathcal{B},\mathcal{A})}{N_a + N_b} \tag{10}$$

These four ways (Eqs (7)-(10)) of combining six directed distance measures (Eqs (1)-(6)) result in 24 possible undirected distance measures between two point sets as summarized in Table 1. Note that $D_{18}$ corresponds to the well-known Hausdorff distance.

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| directed distance | function | | | |
|---|---|---|---|---|
| | $f_1$ | $f_2$ | $f_3$ | $f_4$ |
| $d_1$ | $D_1$ | $D_2$ | $D_3$ | $D_4$ |
| $d_2$ | $D_5$ | $D_6$ | $D_7$ | $D_8$ |
| $d_3$ | $D_9$ | $D_{10}$ | $D_{11}$ | $D_{12}$ |
| $d_4$ | $D_{13}$ | $D_{14}$ | $D_{15}$ | $D_{16}$ |
| $d_5$ | $D_{17}$ | $D_{18}$ | $D_{19}$ | $D_{20}$ |
| $d_6$ | $D_{21}$ | $D_{22}$ | $D_{23}$ | $D_{24}$ |

Table 1: 24 distance measures between two point sets.

## 3 Matching using Distance Measures

A distance measure for the purpose of object matching should have the following properties: (1) it should have a large discriminatory power, (2) its value should increase with the amount of difference between the two objects.

We first analyzed the metric properties of the 24 distance measures. It can be shown that the Hausdorff distance ($D_{18}$) is the only distance measure that is a metric. $D_1, \ldots, D_{16}$ are not metrics because $D(\mathcal{A}, \mathcal{B}) = 0$ does not imply that $\mathcal{A} = \mathcal{B}$. This could be a severe problem for object matching since two different objects might produce a matching distance of 0. All the other distance measures ($D_{17}$, $D_{19}, \ldots, D_{24}$) are not metrics because they violate the triangle inequality.

Several experiments were performed on synthetic images to determine the properties of these 24 distance measures. Binary images (256×256) were obtained by generating a random number of black points placed randomly (from a uniform distribution) on the grid. Based on matching pairs of these images, we determined that $D_1, \ldots, D_9, D_{13}, D_{17}$, and $D_{21}$ were equal to 0 most of the time, indicating that these distance measures did not have sufficient discriminatory power. There does not appear to be too much difference among the operators $f_2$, $f_3$, and $f_4$, although $f_2$ produces a larger spread, resulting in a more discriminatory power. These preliminary observations enabled us to eliminate most of the distance measures from further evaluation. We restrict our attention to only the following four distance measures: $D_{10}$, $D_{14}$, $D_{18}$, and $D_{22}$.

We now study the behavior of the 4 distance measures ($D_{10}, D_{14}, D_{18}, D_{22}$) in the presence of noise. For this experiment, random line patterns were generated in 20 256×256 images by randomly generating the locations of the end points of a random number of lines. A set of 50 noisy images was then generated from each of these line patterns using the following four different noise models: (i) randomly perturb the end points of each of the lines ($K_n$); (ii) randomly add line features ($K_a$); (iii) randomly delete line features ($K_d$); and (iv) randomly flip pixels ($K_u$). The 4 distance measures were then computed between noise-free images and noisy images for various noise levels.

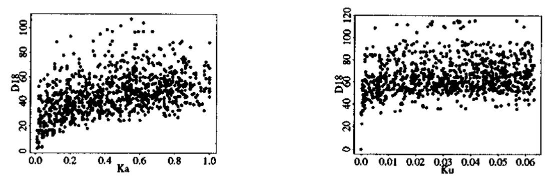


Figure 1: Behavior of the Hausdorff distance ($D_{18}$) as a function of the noise levels $K_a$ and $K_u$.

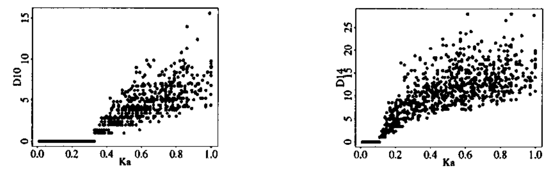


Figure 2: Behavior of the ranked distance measures ($D_{10}$ and $D_{14}$) as a function of the noise level $K_a$.

Figure 1 shows the behavior of the Hausdorff distance ($D_{18}$) as a function of $K_a$ and $K_u$. It can be seen that the distance values are large even in the presence of small amounts of noise and the values remain almost constant as the amount of noise varies. This is due to the fact that the Hausdorff distance value is set by the maximum distance among the two point sets. This maximum distance can be set by the slightest amount of noise and then the distance remains constant as the noise level is increased. In the context of object matching, this can be a real problem since a few outliers will perturb the distance measure greatly, even though the two objects might be very similar.

Huttenlocher *et al.* [1] proposed the generalized (ranked) Hausdorff distances ($D_{10}$ and $D_{14}$) to handle outliers and occlusions. Even these distance measures present some problems for object matching. These problems are most obvious when we plot the values of $D_{10}$ and $D_{14}$ as a function of $K_a$ (see Figure 2). The distances between the noise-free image and the noisy image stay equal to 0 even when a significant amount of noise has been added to the image. This is because parts of the images are allowed to be different to handle occlusions. The percentage of the image pixels (not necessarily contiguous) that should match depends on

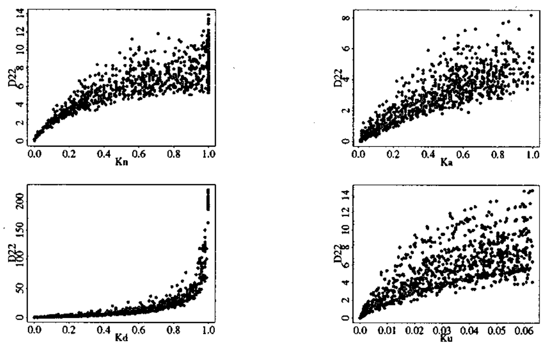
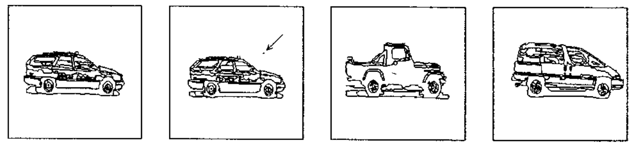


Figure 3: Behavior of the modified Hausdorff distance ($D_{22}$) as a function of the noise level for 4 types of noise models.

the choice of the rank $K$.

The distance measure $D_{22}$ is the only distance that has the desirable behavior for object matching purposes. It can be seen in Figure 3 that $D_{22}$ takes small values in the presence of low levels of noise and takes large values as the noise level increases. This distance, which we call the modified Hausdorff distance (MHD), is thus more appropriate than the other distance measures for object matching purposes.

The modified Hausdorff distance was applied to real edge images of several objects. Figure 4 shows the edge points for four of the objects in our test set. These edge images were obtained by applying a segmentation algorithm based on motion, color, and edge information to color images of moving vehicles [2] to retain only the edges belonging to the object of interest. Note that the two objects $\mathcal{A}$ and $\mathcal{B}$ are very similar in shape. For the purpose of this experiment, an outlier edge point representing a possible segmentation error was added to the image of $\mathcal{B}$ (see the arrow). The four distance measures, $D_{10}$, $D_{14}$, $D_{18}$, and $D_{22}$ were computed for pairs of objects and their values are reported in Table 2. The distance between the first two objects ($\mathcal{A}$ and $\mathcal{B}$) should be small since it is computed between objects known to be identical. It can be seen that the Hausdorff distance does not behave properly in the presence of an outlier since $D_{18}(\mathcal{A},\mathcal{B}) > D_{18}(\mathcal{A},\mathcal{C})$. The modified Hausdorff distance, however, correctly suggests that $D_{22}(\mathcal{A},\mathcal{B})$ is minimum. Let us now evaluate the ratio of the distance value for the true match and the distance value for the next best match. This ratio is 1 for $D_{10}$, 0.5 for $D_{14}$, and 0.25 for $D_{22}$. This shows that the modified Hausdorff distance ($D_{22}$) has more discriminatory power than the generalized Hausdorff distances.



$\mathcal{A}$ $\mathcal{B}$ $\mathcal{C}$ $\mathcal{D}$

Figure 4: Four real object edge images.

| $D_{10}$ | | | | |
|---|---|---|---|---|
| | $\mathcal{A}$ | $\mathcal{B}$ | $\mathcal{C}$ | $\mathcal{D}$ |
| $\mathcal{A}$ | 0 | **2** | 5 | 3 |
| $\mathcal{B}$ | **2** | 0 | 7 | 2 |
| $\mathcal{C}$ | 5 | 7 | 0 | 6 |
| $\mathcal{D}$ | 3 | 2 | 6 | 0 |

| $D_{14}$ | | | | |
|---|---|---|---|---|
| | $\mathcal{A}$ | $\mathcal{B}$ | $\mathcal{C}$ | $\mathcal{D}$ |
| $\mathcal{A}$ | 0 | **3** | 10 | 6 |
| $\mathcal{B}$ | **3** | 0 | 10 | 7 |
| $\mathcal{C}$ | 10 | 10 | 0 | 12 |
| $\mathcal{D}$ | 6 | 7 | 12 | 0 |

| $D_{18}$ | | | | |
|---|---|---|---|---|
| | $\mathcal{A}$ | $\mathcal{B}$ | $\mathcal{C}$ | $\mathcal{D}$ |
| $\mathcal{A}$ | 0 | **32** | 22 | 32 |
| $\mathcal{B}$ | **32** | 0 | 107 | 25 |
| $\mathcal{C}$ | 22 | 107 | 0 | 36 |
| $\mathcal{D}$ | 32 | 25 | 36 | 0 |

| $D_{22}$ (MHD) | | | | |
|---|---|---|---|---|
| | $\mathcal{A}$ | $\mathcal{B}$ | $\mathcal{C}$ | $\mathcal{D}$ |
| $\mathcal{A}$ | 0 | **1** | 6 | 4 |
| $\mathcal{B}$ | **1** | 0 | 6 | 4 |
| $\mathcal{C}$ | 6 | 6 | 0 | 6 |
| $\mathcal{D}$ | 4 | 4 | 6 | 0 |

Table 2: $D_{10}$, $D_{14}$, $D_{18}$, and $D_{22}$ for the four objects shown in Figure 4. Note that MHD has the best discriminatory power for object matching.

# 4 Concluding Remarks

We have determined that, among the class of distance measures based on the Hausdorff distance, the modified Hausdorff distance (MHD) is best for matching two objects based on their edge points. We have shown that MHD has the following desirable properties: (i) its value increases monotonically as the amount of difference between the two sets of edge points increases, and (ii) it is robust to outlier points that might result from segmentation errors. The MHD was applied to edge images of several real objects that were segmented from outdoor scenes and we observed that MHD is better than the other distance measures for the purpose of object matching.

## References


[1] D. P. Huttenlocher, G. A. Klanderman, and W. J. Rucklidge, "Comparing images using the Hausdorff distance", *IEEE Trans. PAMI*, vol. 15, pp. 850–863, 1993.

[2] M.-P. Dubuisson and A. K. Jain, "2D matching of 3D moving objects in color outdoor scenes", In. *Proc. IEEE Conf. CVPR*, Seattle, WA, pp. 887–891, 1994.