Homework 4, Medical Physics/BME 574 Released: March 13, 2019

Due: April 1, 2019

Mathematical Mappings:

1. Let a, b, d, and d be constants such that ad-bc $\neq$ 0, and consider a mapping, F(u, v) = (f(u,v), g(u,v)) = (au+bv, cu+dv).

    a. Setup this problem in matrix form in terms of coordinates (u, v) mapped to coordinates (x, y). What is the nature of the mapping in terms of possible affine degrees of freedom: translation, scale, rotation, or shear.

    b. Solve the system of linear equations to express the mappings in terms of u(x,y) and v(x,y).

    c. Show in general for this mapping that a line L(u,v) = Au+Bv+C = 0 in the uv-plane is a line in the xy-plane, and that images of parallel lines must be parallel.

2. The magnitude of the cross product $\vec{A} \times \vec{B}$ for two vectors $\vec{A}$ and $\vec{B}$ is the area of the parallelogram spanned by $\vec{A}$ and $\vec{B}$.

    a. Use this fact and the definition of the cross-product to show that the area of the parallelogram produced by a linear mapping with scalar constants such that lines in uv are mapped to $\vec{A} = a_1\vec{\imath} + a_2\vec{\jmath}$ and $\vec{B} = b_1\vec{\imath} + b\vec{\jmath}$ is expressed as the determinant:

    $$\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = |a_1 b_2 - a_2 b_1|.$$

    b. Let F(u,v) = (2u+3v, u+2v). Let R be the rectangle $u_o \leq u_o + \Delta u$, $v_o \leq v_o + \Delta v$. Find the magnification of the mapping. Is the magnification dependent on position in the uv-plane?

    c. Take the general case in Problem 1 above, show the image of a rectangle is a parallelogram and that the image of the rectangle is $|ad - bc|$ times as large.

    d. Comment on your results in part c above given the definition of the Jacobian discussed in class.

3. Let F(u,v) = (u cos(v), u sin(v)) for u > 0 and 0 $\leq$v < 2π. Find the magnification of F at (u, v) using the Jacobian. Is the magnification dependent on position in the uv-plane?

Matrix operations:

4. We want to rotate an object by some angle θ about some fixed point in the object. In this instance this point is not at the origin of the coordinate system, but is instead at the point (x_o, y_o), so a rotation of the object about the origin will not do what we want. However, if we first translate the point (x_o, y_o) to the origin, perform the rotation about the origin, and then translate the origin back to the point (x_o, y_o), we get the desired result.

    a. Write down the three-matrix product that will accomplish this. Be sure to remember the relationship between the order of the three matrices and the order of the three operations.

    b. Now perform the matrix multiplication to give the single 3x3 matrix that will accomplish this rotation in a single step.

    c. Now use this operator on the point:

$$p = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

to get

$$p' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

Write down the expressions for the new coordinates as functions of x, y, x_o, y_o, and θ.

    d. Paying careful attention to the order of operations…Load "HW4-Image-Fixed" and apply the transform in Matlab for specific values of x_o = xim/2, y_o = ydim/2, and $\theta = 15^o$. That is you wish to rotate about the center of the image.
        i. First use the script titled, "Affine_transform.m," and
        ii. Second use the built-in Matlab function "tform()." Describe and explain any differences observed.

    e. Save and write out the result from the tform() function as "HW4-Image-Moving." You will be registering images in Problem 6.

Registration Problems (These problems require the Computer Vision System Toolbox in Matlab):

5. Use the test images (HW4-Image-Fixed and HW4-Image-Moving) provided on the website and from Problem 4.

a. Perform a control point registration (See “Registration_script.m”) and describe any artifacts and their source(s) that appear in the registered images.

b. Repeat using feature-based registration (use built-in Matlab functions) using the commands as outlined in the Lecture 18 slides (also see Matlab documentation for “help matchFeatures”). The L1 and L2-norms are used (L2 is default) as similarity metric (“cost function”); report this metric for the registration result using both. Are there appreciable differences in the registration result? Describe any artifacts and their source that appear in the registered images.

c. Consider a reflection of the right-side shaded checkerboard image about the y-axis. Repeat Feature based registration on a rotated version of the reflected checkerboard image. Speculate on whether this symmetry transform would be “easy” or “hard” for an automated registration algorithm to resolve in general.