Lecture 26
Segmentation: Active Contours

MP574: Applications

Sean B. Fain, PhD (sfain@wisc.edu)
Diego Hernando, PhD (dhernando@wisc.edu)
ITK/VTK Applications: Andrew Hahn, PhD (adhahn@wisc.edu)

Learning Objectives

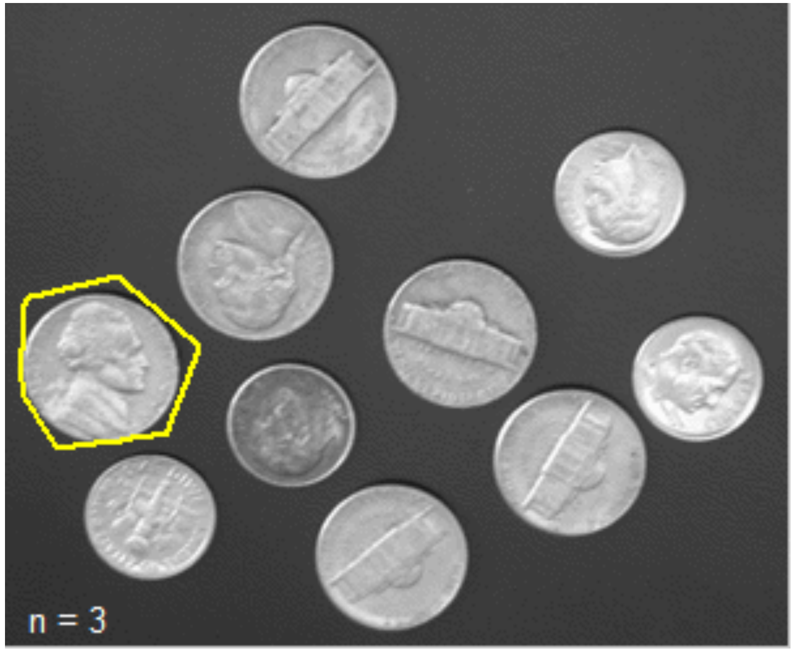
- Introduce active contour-based segmentation
  - Cost/energy function
  - Canonical iterative optimization algorithms
- Combination with levels sets
  - Review applications
- Strengths and Limitations
  - Can deal with complex boundaries
  - Labor-saving but typically supervised
  - Cannot “split” to find new object boundaries (unless used in combination with level-set or graph-cut techniques)

Active Contour Models (“Snakes”)

- First introduced in 1987 by Kass et al, and gained popularity since then.
- Represents an object boundary or some other salient image feature as a parametric curve.
- Minimizes energy associated with the curve to find object boundary.

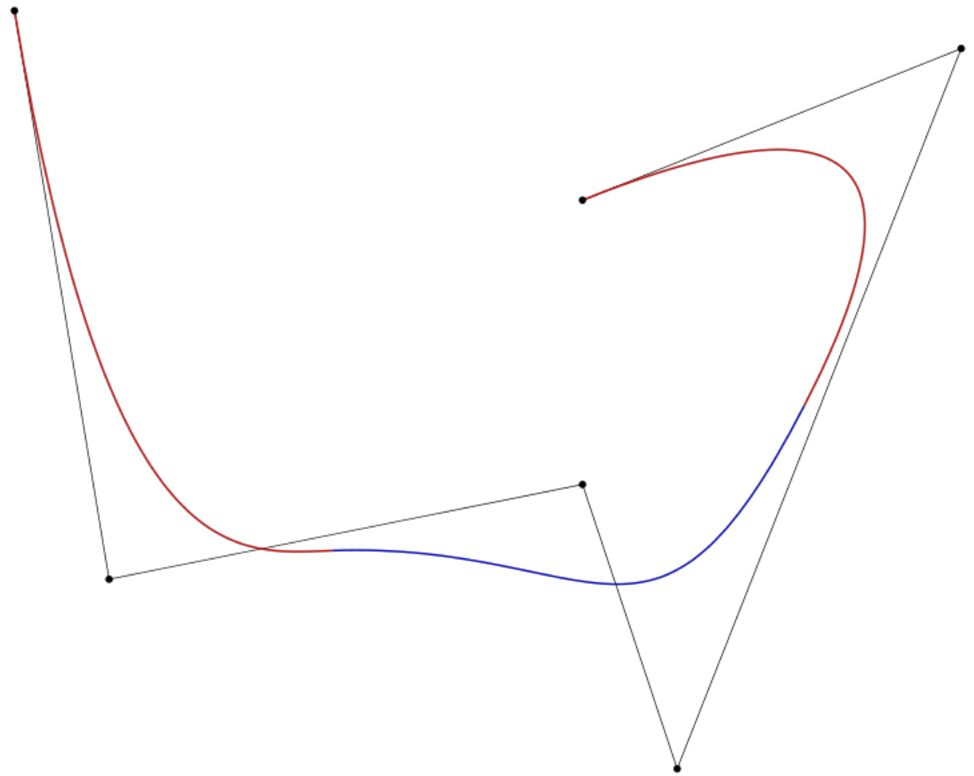
Active Contour Models

- Basic algorithm:
  - A higher level process or a user initializes any curve close to the object boundary.
  - The snake then deforms towards the object while iteratively minimizing the energy term.
  - In the end it completely shrink-wraps around the object.



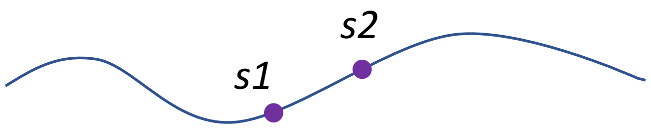
ACM: Snake-Model (Kaas et al.)

- Contour described by spline curve
  - Set of control points
  - Piecewise cubic polynomials
  - C2 continuous at breakpoints
    (2^{nd} derivatives are continuous)



Wojciech mula / Wikimedia Commons / Public Domain

Geometry of Curves



A curve can be represented by

1. y = f(x)

2. (x(s), y(s))

Length of the curve is given by

$$l = \int_{s2}^{s1} \sqrt{dx^2 + dy^2} = \int_{s2}^{s1} \sqrt{\left(\frac{dx}{ds}\right)^2 + \left(\frac{dy}{ds}\right)^2}\, ds$$

Snake Energy

A contour evolves to minimize energy

$$E_{snake} = \underbrace{E_{elas} + E_{bend}}_{\text{Internal}} + \underbrace{E_{img} + E_{const}}_{\text{External}}$$

E_{elas}: elastic energy of contour
E_{bend}: bending energy of contour
E_{img}: energy driven from image
E_{const}: energy constrained by user

- Internal energy enforces continuity and smoothness of contour
- External / image energy contains
  - Line energy: pulls contour towards bright or dark lines
  - Edge energy: pulls contour towards strong gradients
  - Termination energy: pulls contour towards corners
- Constraint energy: used to include interactive user input

Internal Energy

$$v: [a, b], \qquad v(a) = v(b)$$

$$E(v(s)) = \boxed{\int_a^b \alpha(s)|v'(s)|^2 ds + \int_a^b \beta(s)|v''(s)|^2 ds} + \int_a^b \lambda(s) g^2(|\nabla I(v(s))|) ds$$

$$v(s) = (x(s), y(s))$$

$$v'(s) = \left(\frac{dx}{ds}, \frac{dy}{ds}\right)$$

$$|v'(s)| = \sqrt{\left(\frac{dx}{ds}\right)^2 + \left(\frac{dy}{ds}\right)^2}$$

External Energy

$$E(v(s)) = \int_a^b \alpha(s)|v'(s)|^2 ds + \int_a^b \beta(s)|v''(s)|^2 ds + \boxed{\int_a^b \lambda(s) g^2(|\nabla I(v(s))|) ds}$$

$$g(\nabla I) = \frac{1}{1 + |\nabla I|^2}$$

Discretization

$$E(v(s)) = \int_a^b \alpha(s)|v'(s)|^2 ds + \int_a^b \beta(s)|v''(s)|^2 ds + \int_a^b \lambda(s) g^2(|\nabla I(v(s))|) ds$$

$$E_{snake} = \sum_{i=1}^{N} (\alpha_i {E^i}_{elas} + \beta_i {E^i}_{bend} + \lambda_i {E^i}_{img})$$

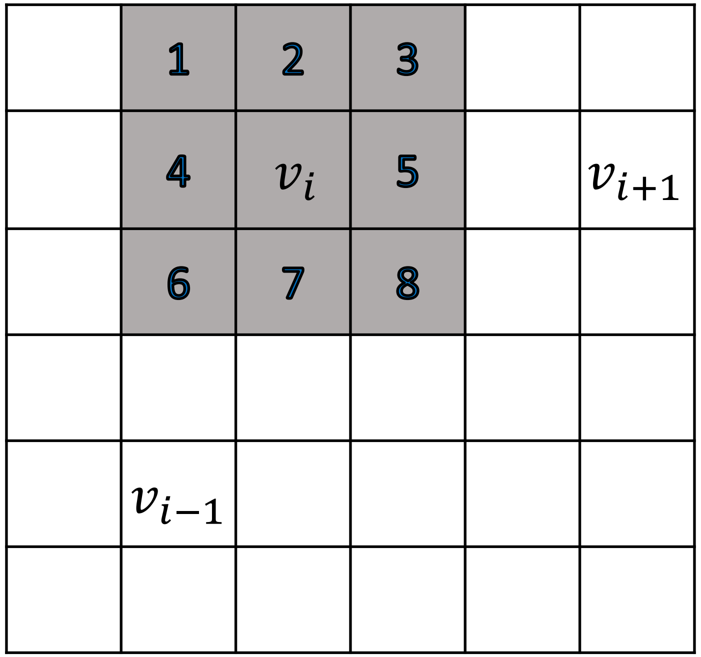
The internal energy terms are converted to discrete form with the derivatives substituted by finite differences

$${E^i}_{elas} = \|v_i - v_{i+1}\|^2$$

$${E^i}_{bend} = \|v_{i-1} - 2v_i + v_{i+1}\|^2$$

Deforming Contour (Internal Energy Term)

- Discrete approximation of curve geometry is calculated at control points, v_i, v_{i+1}
  - Finite differences are with respect to the coordinate Euclidian distances
  - <u>Not</u> the signal intensities



$$E^{i}{}_{elas} = \{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2\}$$

$$E^{i}{}_{bend} = \{x_{i-1} - 2x_i + x_{i+1})^2 + (y_{i-1} - 2y_i + y_{i+1})^2\}$$

Corners

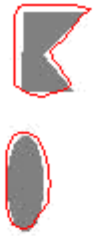
1. Move each control point iteratively.

2. Search for corners as curvature maxima along the contour.

3. Set $\beta_i = 0$ for points with maximal curvature.

4. Neglecting contribution from these points keep the contour piecewise smooth.

Impact of Parameter Regularization

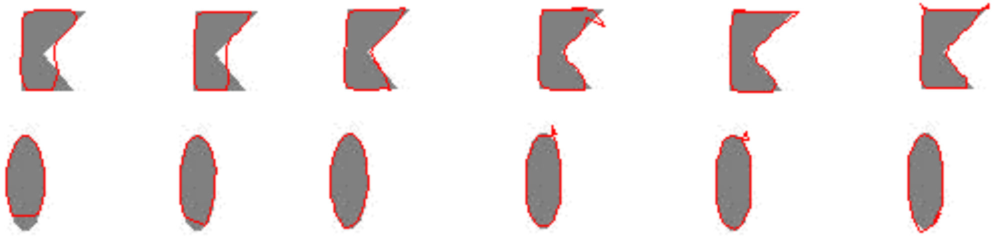


Initial Contour

$$E_{snake} = \sum_{i=1}^{N} (\alpha_i {E^i}_{elas} + \beta_i {E^i}_{bend} + \lambda_i {E^i}_{img})$$

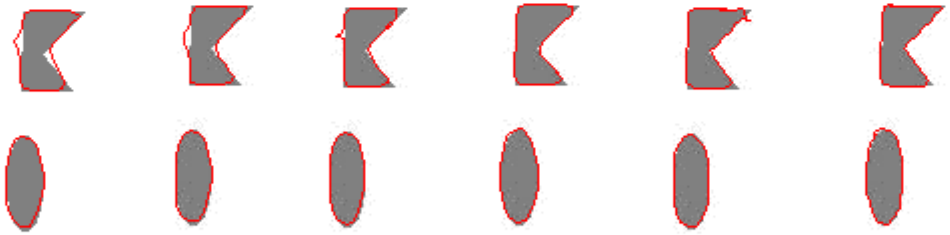
$\boxed{\alpha = 1 - \beta}$

$\lambda = 0.5$



$[\alpha, \beta]$: [0.0 1.0] [0.2 0.8] [0.4 0.6] [0.6 0.4] [0.8 0.2] [1.0 0.0]

$\lambda = 1.0$



$[\alpha, \beta]$: [0.0 1.0] [0.2 0.8] [0.4 0.6] [0.6 0.4] [0.8 0.2] [1.0 0.0]

By Liyan Zhang

Level Sets: In some ways a 3D Implementation of the ACM model

Implementation as a finite difference equation:

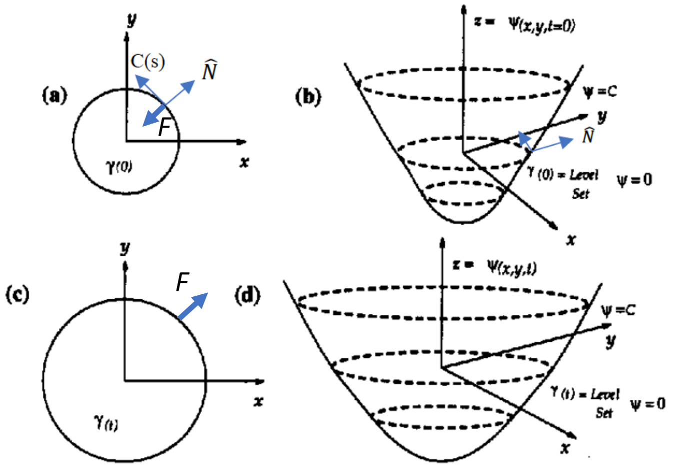
$$\frac{\varphi(i,j,t+\Delta t)-\varphi(i,j,t)}{\Delta t}+\max[F,0]\,\nabla^{+C}(i,j)+\min[F,0]\,\nabla^{-C}(i,j)=0$$

Updating the surface:

$$\varphi(i,j,t+\Delta t)=\varphi(i,j,t)-\Delta t[\max[F,0]\,\nabla^{+C}(i,j)+\min[F,0]\,\nabla^{-C}(i,j)]$$

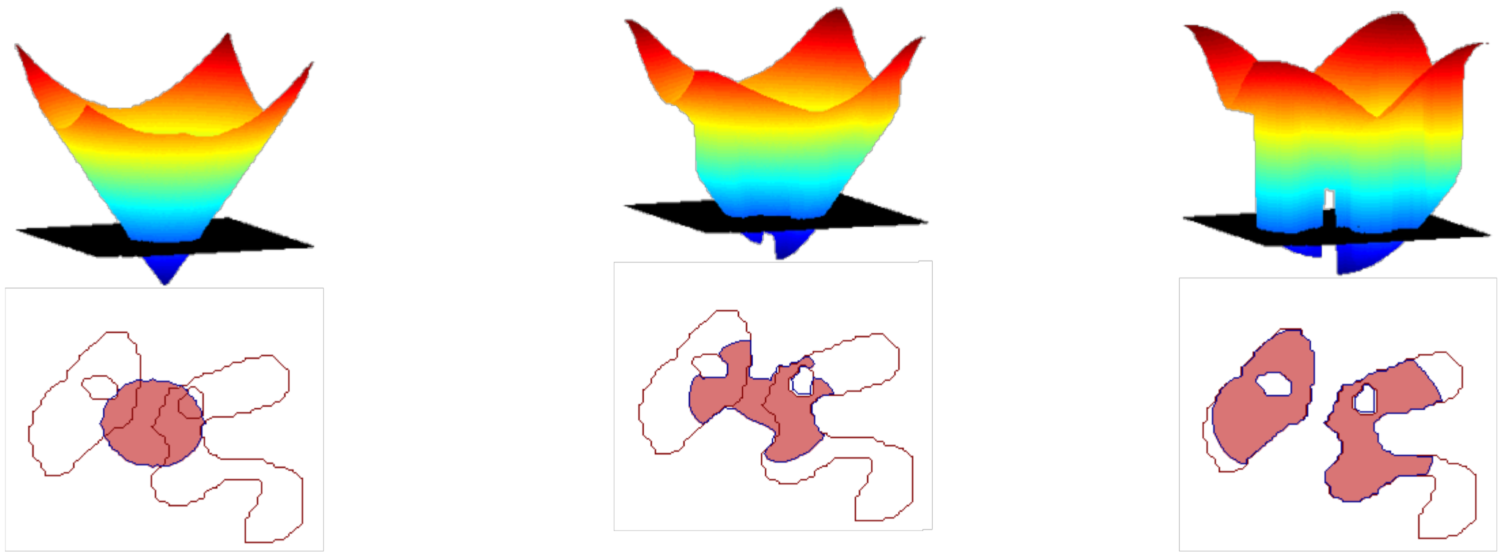
Updating with curvature term included:

$$\varphi(i,j,t+\Delta t)=\varphi(i,j,t)-\Delta t[\max[F,0]\,\nabla^{+C}(i,j)+\min[F,0]\,\nabla^{-C}(i,j)]+\boxed{\Delta t[\kappa(i,j)]}$$



$\varphi(C(x,y;t),t)=0$ at the level set.

Level Sets: Adaptive splitting of contours

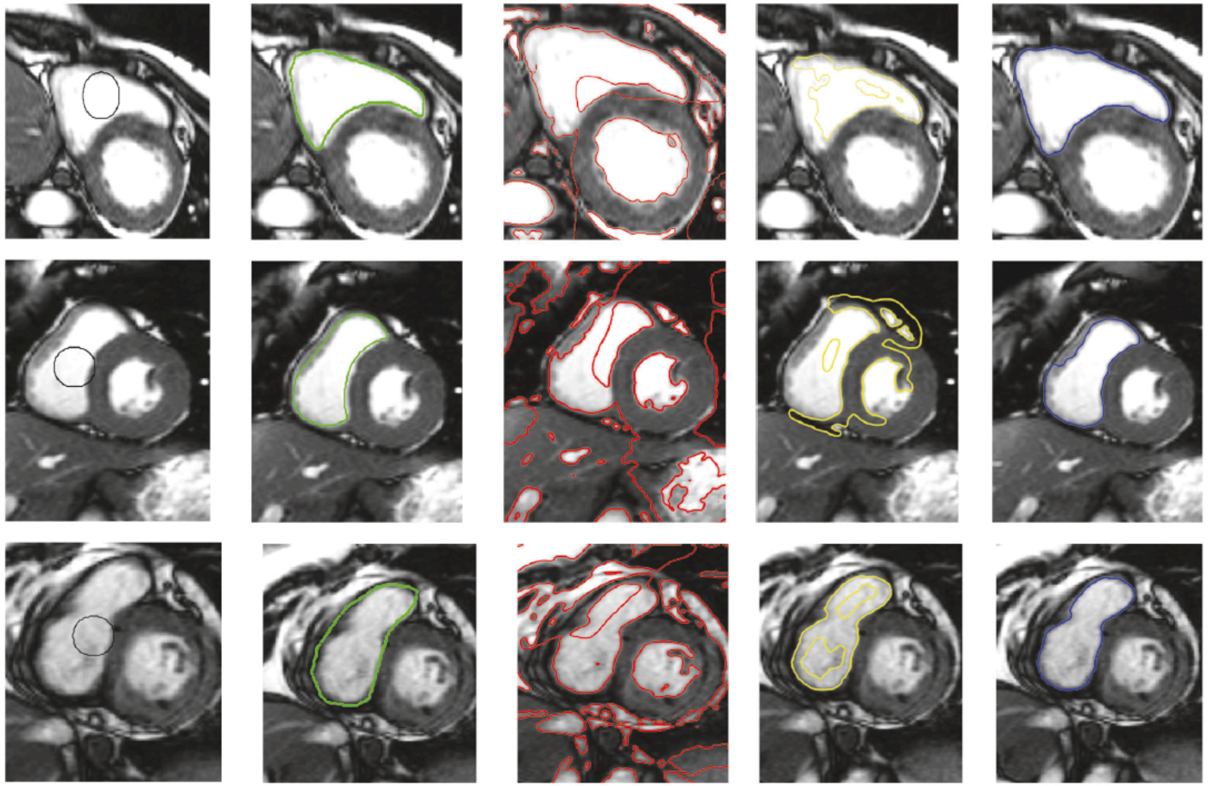


(a) initial contour (b) contour split (c) two new contours

Figure: An initial circle expands inside the form where F>0 and collapses outside the form (F<0), the corresponding surface is also shown with a plane intersecting at z=0

ACM in Cardiac Segmentation Applications

- (a) Original images with the initial contour
- (b) Manual segmentation
- (c) Edge detector
- (d) Incorporation of level set with ACM
- (e) Signed pressure force (SPF) cost function added.



(a) (b) (c) (d) (e)

Research Article

Segmentation of Left and Right Ventricles in Cardiac MRI Using Active Contours

Shafiullah Soomro,[1] Farhan Akram,[2] Asad Munir,[1] Chang Ha Lee,[1] and Kwang Nam Choi[1]

[1]*Department of Computer Science and Engineering, Chung-Ang University, Seoul 156-756, Republic of Korea*
[2]*Department of Computer Engineering and Mathematics, Rovira i Virgili University, 43007 Tarragona, Spain*

Hindawi
Computational and Mathematical Methods in Medicine
Volume 2017, Article ID 8350680, 16 pages
https://doi.org/10.1155/2017/8350680

ACM+ Levels Sets in Cardiac Segmentation Applications

- (a and d) Original images with the initial contour,
- (b and c) intermediate results using ACM + level sets
- (c and f) final segmentation results.

$$\frac{\partial \phi}{\partial t} = \text{spf}\,(I\,(x)) \cdot \alpha\,|\nabla \phi|$$

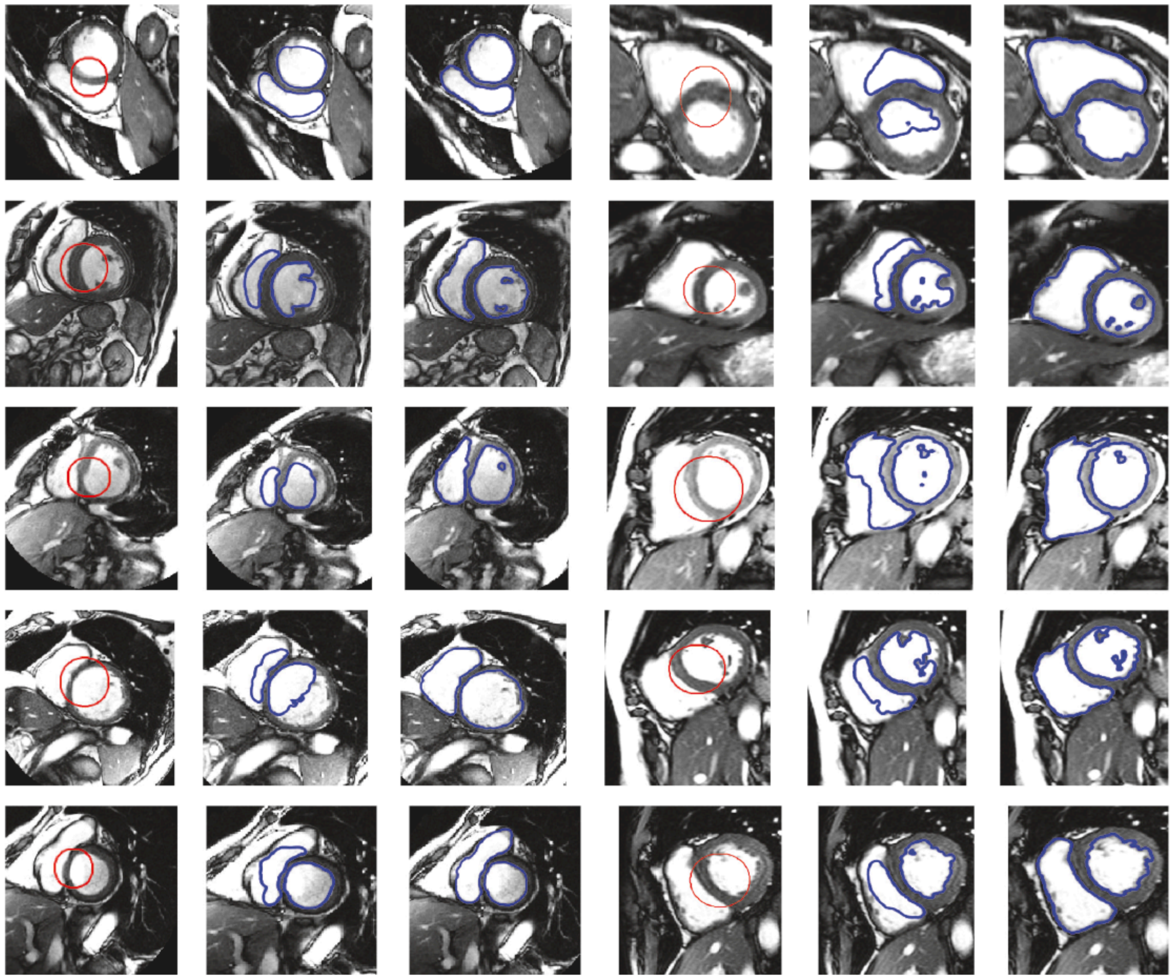
$$spf(I) = \frac{I(x) - \frac{c_1 + c_2}{2}}{\max(\left|I(x) - \frac{c_1 + c_2}{2}\right|)}$$

Research Article

Segmentation of Left and Right Ventricles in Cardiac MRI Using Active Contours

Shafiullah Soomro,[1] Farhan Akram,[2] Asad Munir,[1] Chang Ha Lee,[1] and Kwang Nam Choi[1]

[1]*Department of Computer Science and Engineering, Chung-Ang University, Seoul 156-756, Republic of Korea*
[2]*Department of Computer Engineering and Mathematics, Rovira i Virgili University, 43007 Tarragona, Spain*



(a) (b) (c) (d) (e) (f)

Hindawi
Computational and Mathematical Methods in Medicine
Volume 2017, Article ID 8350680, 16 pages
https://doi.org/10.1155/2017/8350680

Level Set Means

$$\frac{\partial \phi}{\partial t} = \mathrm{spf}\left(I\left(x\right)\right) \cdot \alpha \left|\nabla \phi\right|$$

$$c_1 = \frac{\int_\Omega I\left(x\right) H_\varepsilon\left(\phi\left(x\right)\right) dx}{\int_\Omega H_\varepsilon\left(\phi\left(x\right)\right) dx},$$

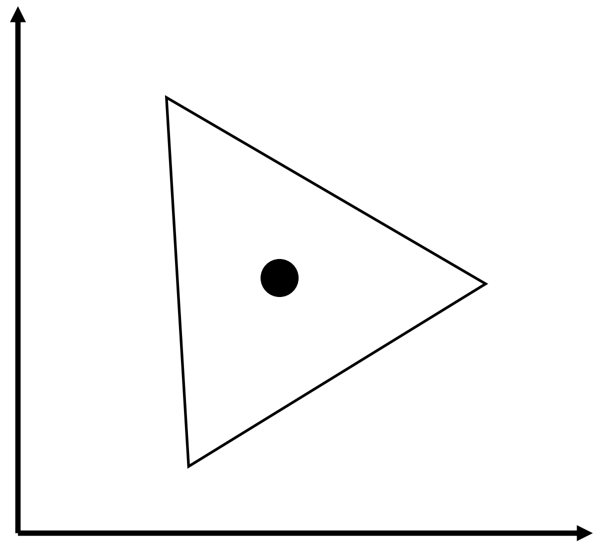
$$spf(I) = \frac{I(x) - \frac{c_1 + c_2}{2}}{\max\left(\left|I(x) - \frac{c_1 + c_2}{2}\right|\right)}$$

$$c_2 = \frac{\int_\Omega I\left(x\right)\left(1 - H_\varepsilon\left(\phi\left(x\right)\right)\right) dx}{\int_\Omega \left(1 - H_\varepsilon\left(\phi\left(x\right)\right)\right) dx},$$

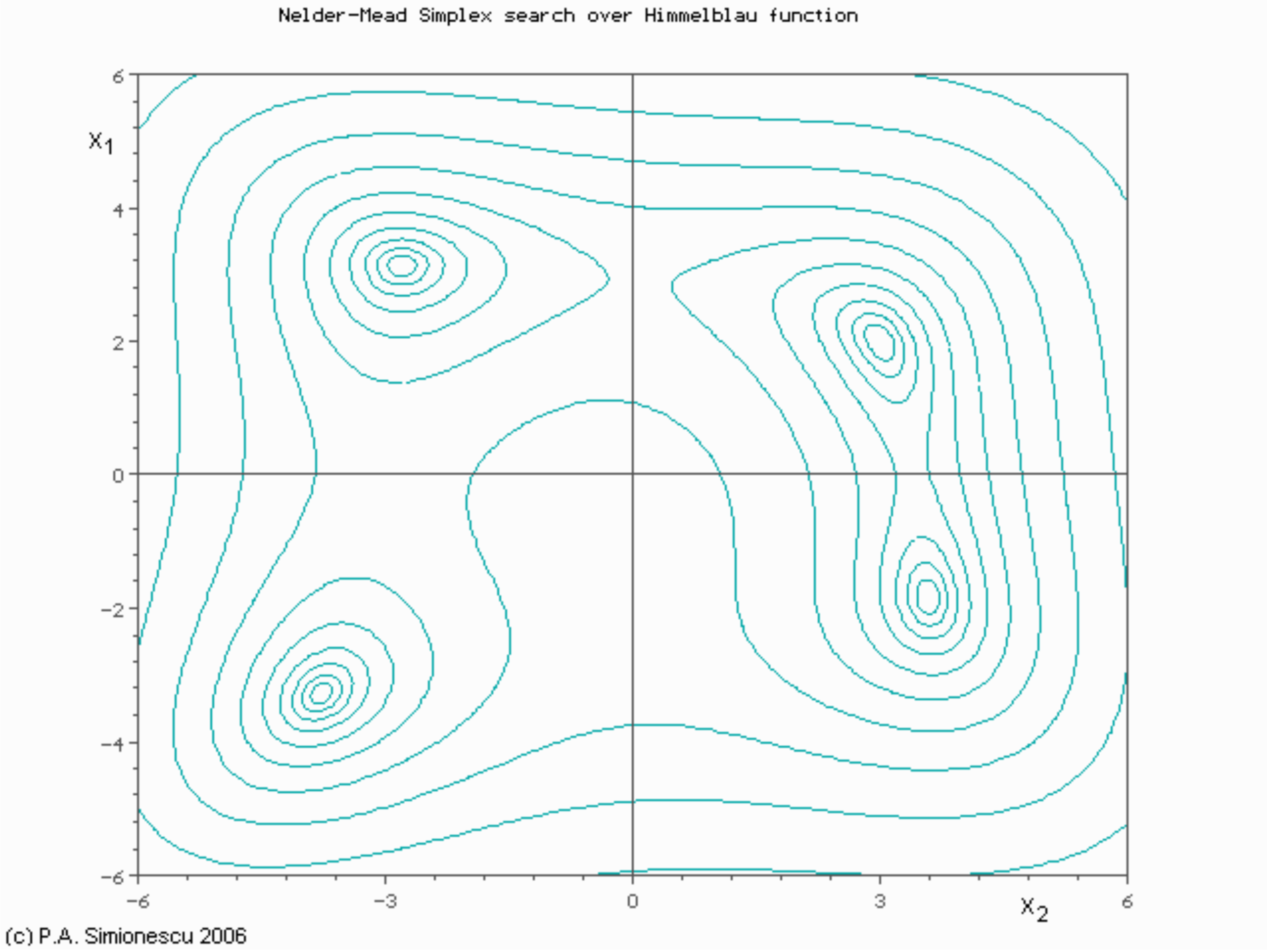
spf ∈ $[-1, 1]$; it shrinks the contour when it is defined outside and expands when defined inside of the object. c_1 and c_2 are the image intensity means.

Optimization Techniques (Examples)

- Gradient descent
  - Calculate partial derivatives for cost function
  - Move step into direction of negative gradient
  - Repeat until stopping criteria satisfied
- Nelder-Mead simplex
  - Initialize simplex around initial guess
  - Pick worst point of simplex
    - Perform reflection, expansion, shrinking or contraction
  - Repeat until stopping criteria satisfied



Nelder-Mead Simplex Search



Animated gif generated by P.A. Simionescu

$$b = a + \gamma \nabla F(a)$$

$$\nabla F(a) = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial y} + \ldots \bigg|_{x=a}$$

$$c = b + \gamma \nabla F(b)$$

$$F(a) > F(b) > F(c) \ldots$$

Summary

- Active Contours
- Strengths
  - Computationally straight-forward, canonical method
  - Well-established and widely available
  - Adapted to integrate with level-sets and graph-cut methods
  - Particularly well adapted to lesion or ventricle (heart) structure segmentation in dynamic settings – labor saving since user must place the control points only once.
- Limitations
  - As originally implemented constrained to segmentation of relatively simple single structures
  - Typically require user supervision (control point placement) in most real-world settings