MP 710
Image Reconstruction & Partial Fourier Imaging

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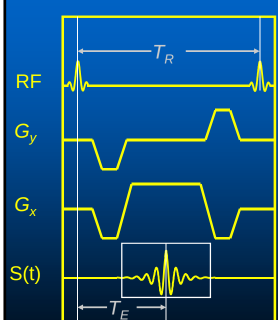
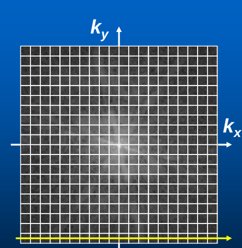
Overview

Spinwarp Imaging with less Data
- Sampling Basics
- Phase Resolution
- Phase FOV
- Partial Fourier

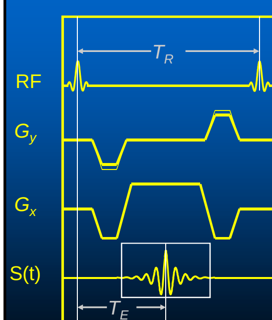
Data Acquisition – k-space

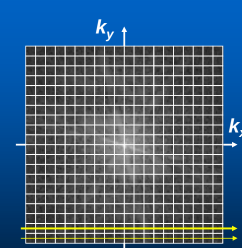




2D Gradient Echo

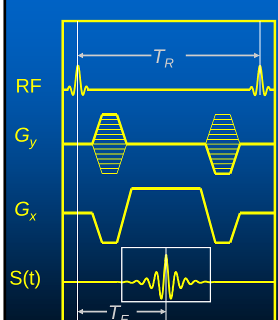
Data Acquisition – k-space

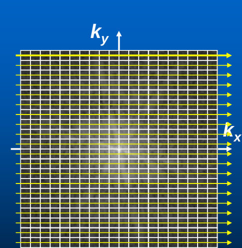




2D Gradient Echo

Data Acquisition – k-space



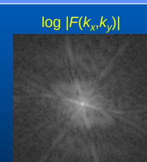


2D Gradient Echo

2D Fourier Transform

$$F(k_x,k_y) = \frac{1}{M \cdot N} \sum_{0}^{M-1} \sum_{0}^{N-1} f(x,y) e^{-j(2\pi/M)k_x x} e^{-j(2\pi/N)k_y y}$$

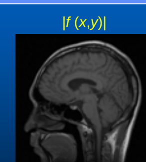
log $|F(k_x,k_y)|$



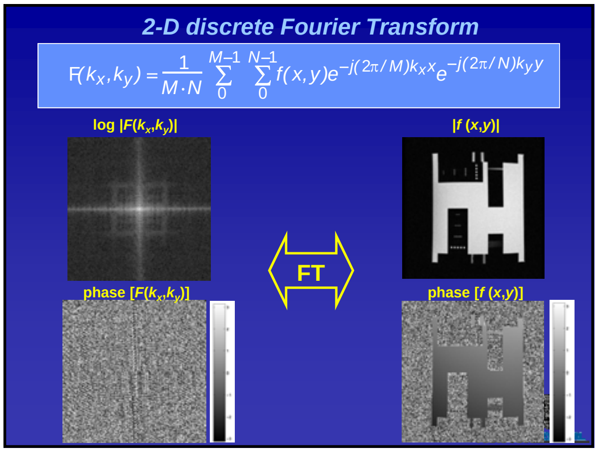
M x N data
M: # of readout points
N: # of phase encodes
Scan time: N x TR

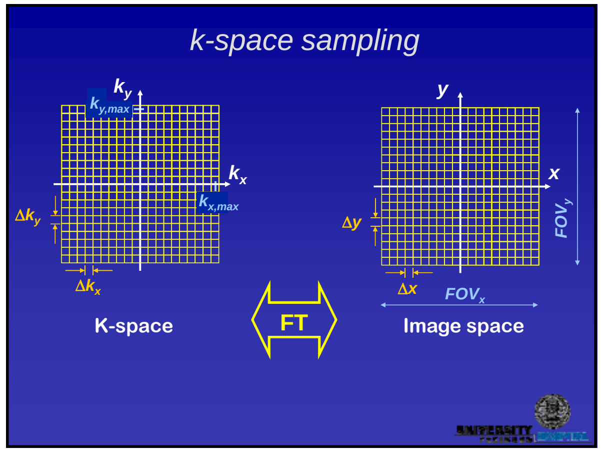
2D FT

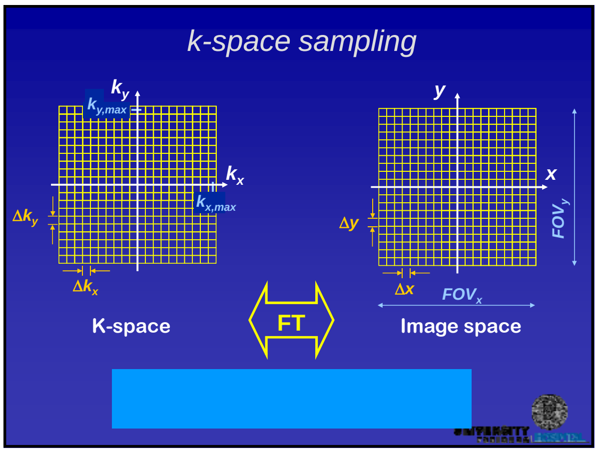
$|f\,(x,y)|$



M x N data


2-D discrete Fourier Transform
$F(k_x,k_y)=\frac{1}{M\cdot N}\sum_{0}^{M-1}\sum_{0}^{N-1} f(x,y)e^{-j(2\pi/M)k_x x}e^{-j(2\pi/N)k_y y}$
log $|F(k_x,k_y)|$
$|f(x,y)|$
FT
phase $[F(k_x,k_y)]$
phase $[f(x,y)]$


k-space sampling
k_y
$k_{y,max}$
k_x
$k_{x,max}$
Δk_y
Δk_x
y
x
Δy
Δx
FOV_y
FOV_x
K-space
FT
Image space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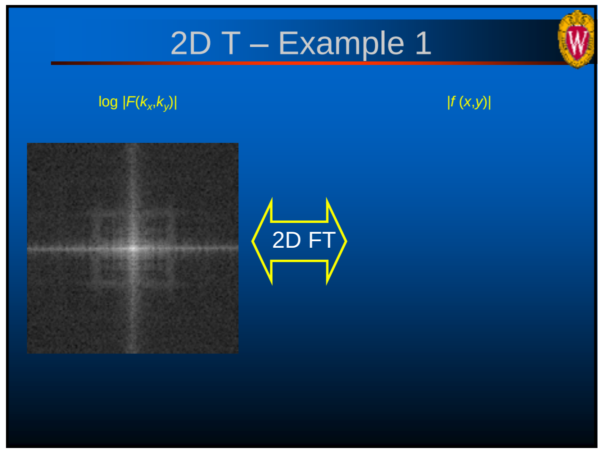
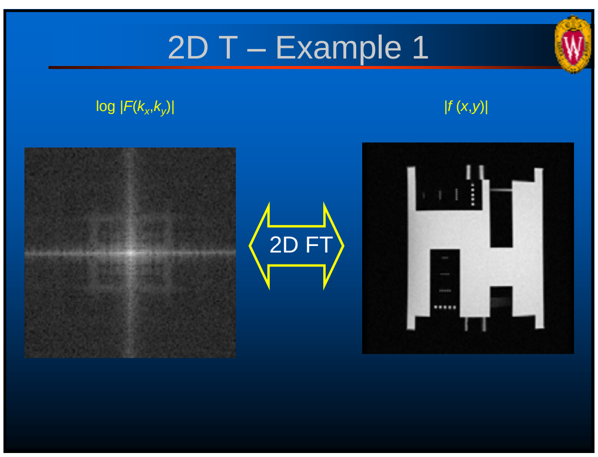

2D T – Example 1
log $|F(k_x,k_y)|$
$|f(x,y)|$
2D FT


2D T – Example 1
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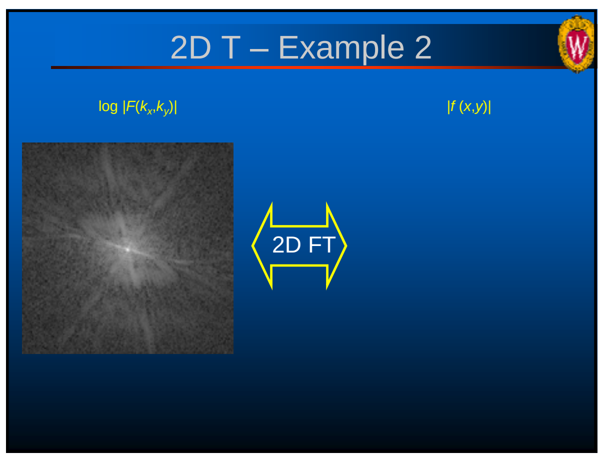
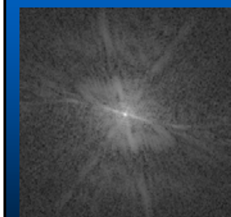

2D T – Example 2
log $|F(k_x,k_y)|$
$|f(x,y)|$
2D FT

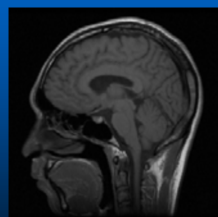
2D T – Example 2

log $|F(k_x,k_y)|$



2D FT

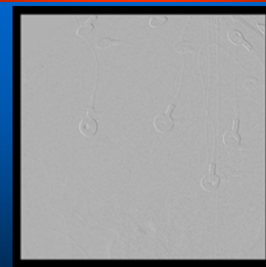
$|f(x,y)|$



Motivation

X-ray DSA

Image resolution: 768x768
Frame rate: 30 fps
High SNR



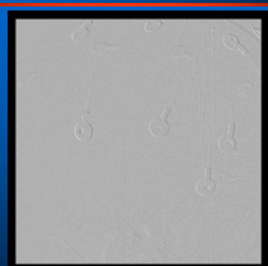
2D MRI on the rocks
Spoiled GRE, single echo TR = 4 ms

Image resolution: 256x256 9

Motivation

X-ray DSA

Image resolution: 768x768
Frame rate: 30 fps
High SNR



2D MRI on the rocks
Spoiled GRE, single echo TR = 4 ms

Image resolution: 256x256 9
Frame rate: 1 fps 30

Fully Sampled MRI - Tradeoffs

Spatial resolution ↑
Imaging Time ↑
SNR ↓

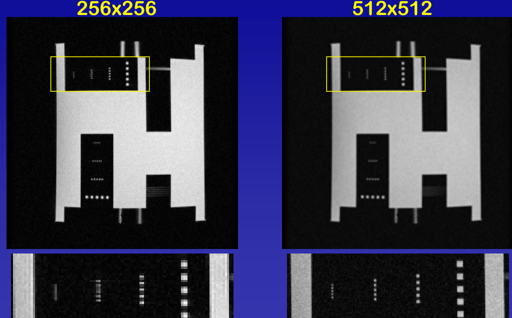
Imaging Time ↓
Spatial Resolution ↓
SNR ↓

SNR ↑
Spatial Resolution ↓
Imaging Time ↓

$$SNR \sim \underbrace{(\Delta x)(\Delta y)(\Delta z)}_{\text{Voxel size}} \underbrace{\sqrt{N_y \times N_z \times T_{measurement}}}_{\text{Total readout interval}} \underbrace{f(\rho, T_1, T_2)}_{\text{Tissue parameter}}$$

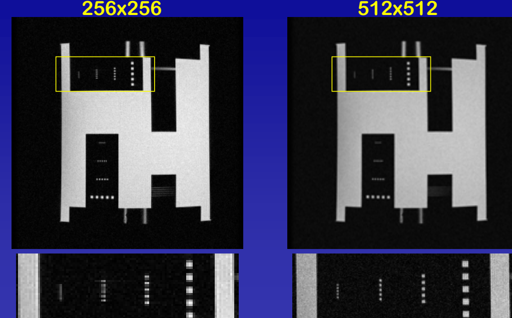
Increased spatial resolution

256x256 512x512

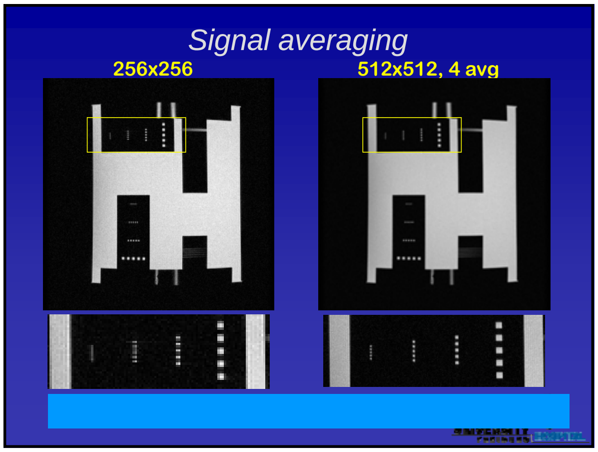


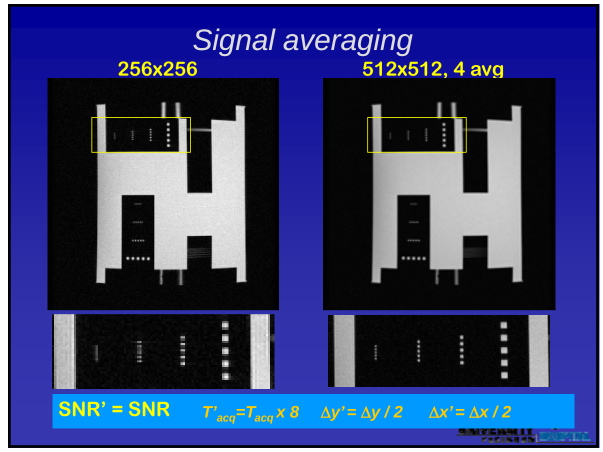
Increased spatial resolution

256x256 512x512



$N'_y = N_y \times 2$ $T'_{acq} = T_{acq} \times 2$ $\Delta y' = \Delta y / 2$ $\Delta x' = \Delta x / 2$ **SNR' = SNR /2**





MR Toolboxes for Rapid Imaging

Hardware
- fast gradients,
- multi receiver coils, ...

Rapid Sequences
- EPI, GRE, FSE, ...

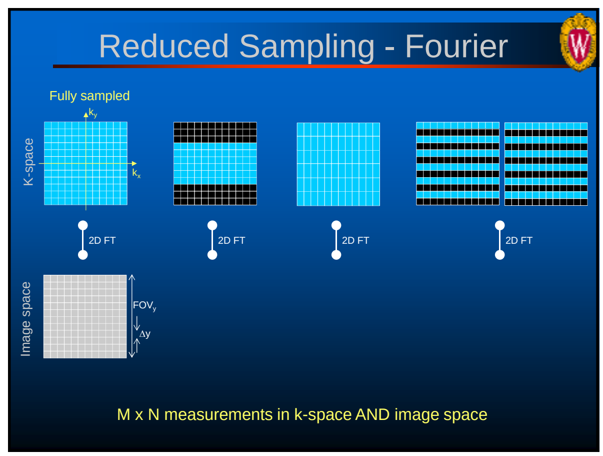
Parallel MRI
- Sense, Smash,
- Grappa, PILS, ...

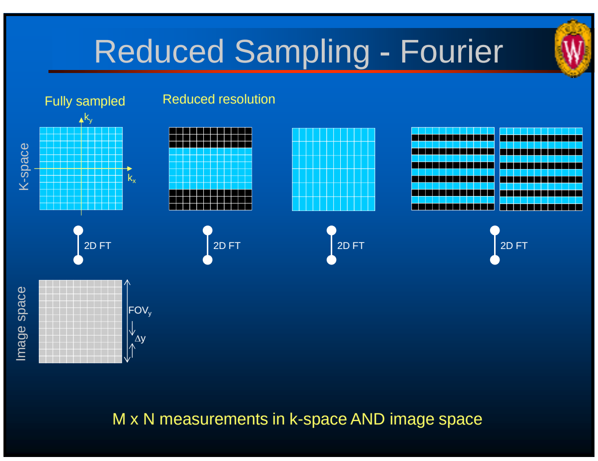
Trajectory Design
- spiral imaging
- radial sampling
- wavelets
- rosettes, ...

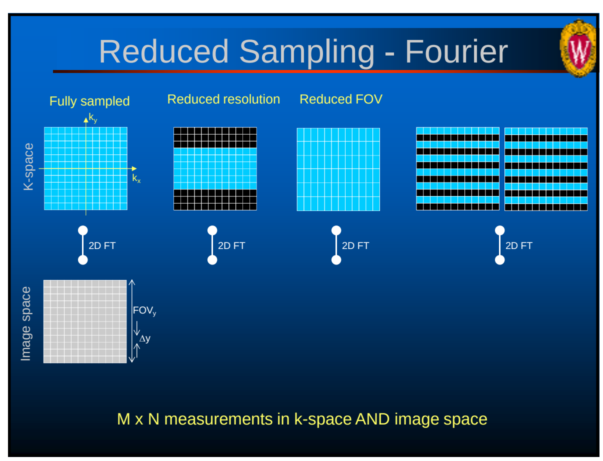
Partial Fourier MRI
- Zero-Filling, Homodyne, POCS

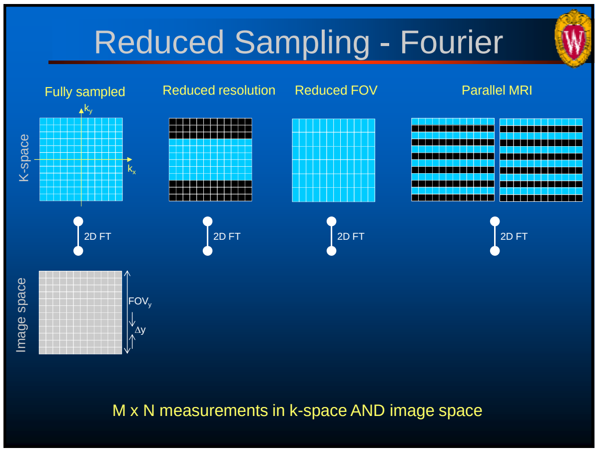
K-space Undersampling
- single image
  - partial Fourier imaging
  - radial undersampling
  - compressed sensing
- image series (e.g. dynamic series)
  - assumptions about object
    - DIME, UNFOLD, TSENSE,
    - k-t BLAST, k-t SENSE,
    - k-t GRAPPA, ...
  - heuristic k-t sampling pattern
    - *sliding window, view sharing,*
    - *keyhole, RIGR, BRISK,*
    - *TRICKS, HYPR, ...*

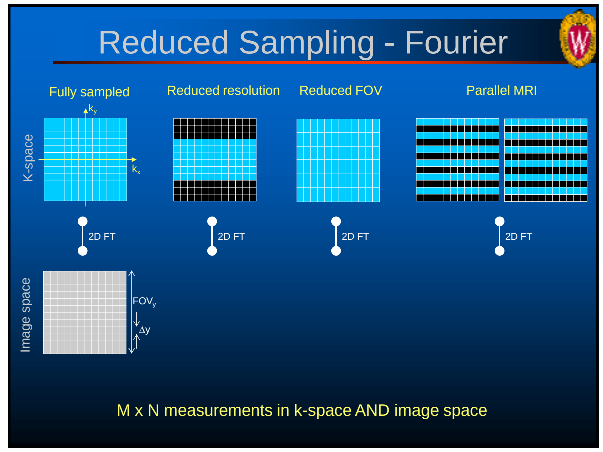
Constrained reconstruction








Reduced Sampling - Fourier
Fully sampled
Reduced resolution
Reduced FOV
Parallel MRI
K-space
2D FT
Image space
M x N measurements in k-space AND image space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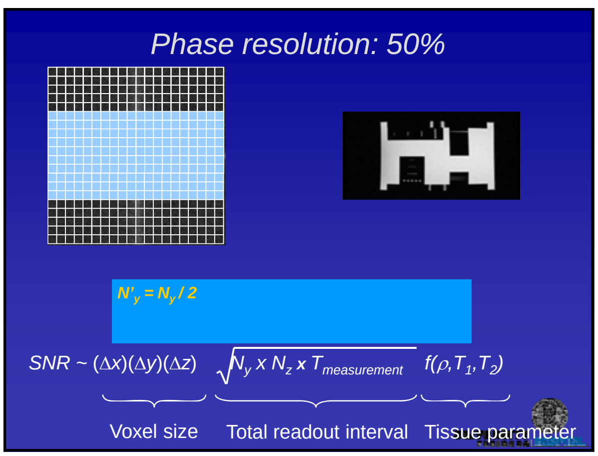
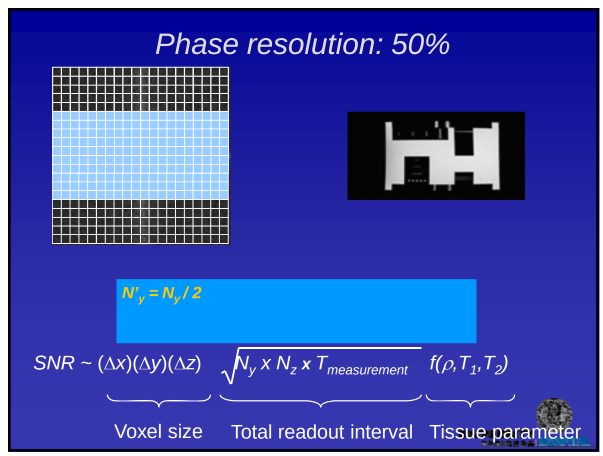

Phase resolution: 50%
N'y = Ny / 2
SNR ~ (Δx)(Δy)(Δz) √(Ny x Nz x Tmeasurement) f(ρ,T1,T2)
Voxel size
Total readout interval
Tissue parameter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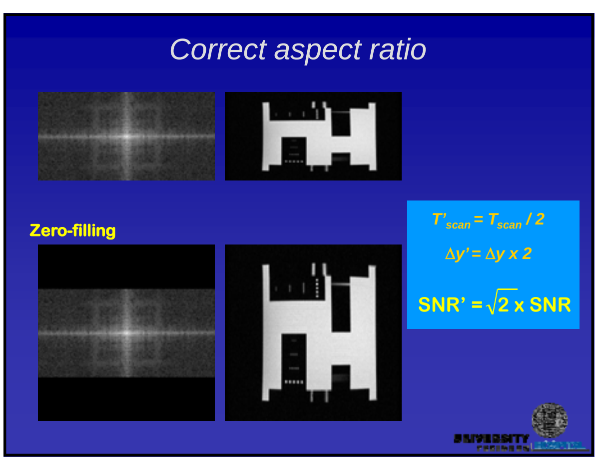
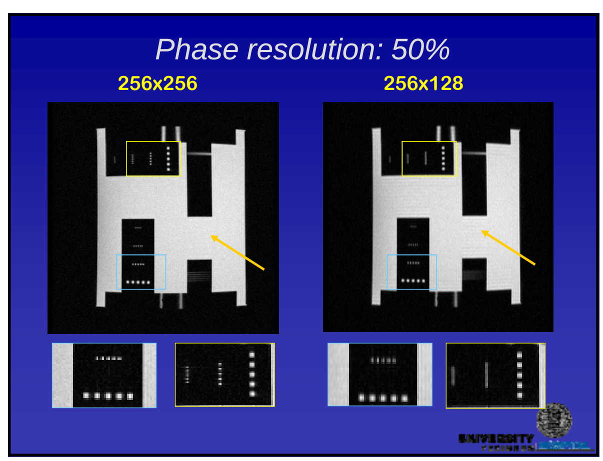
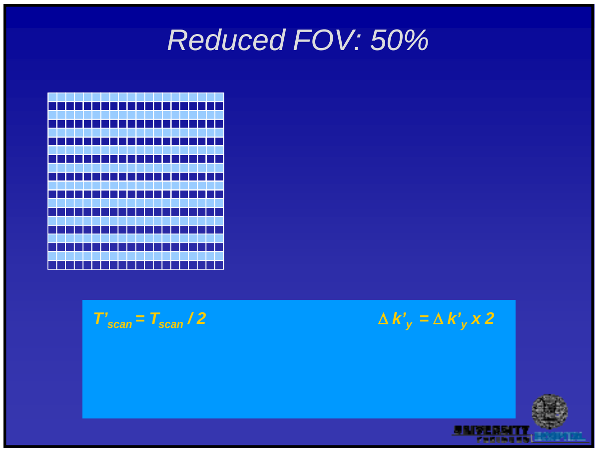
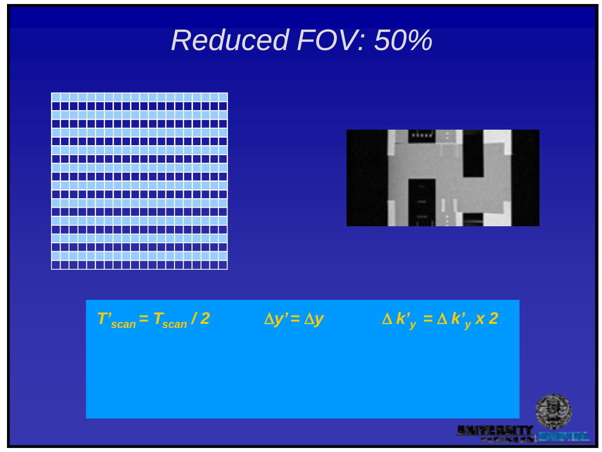

Correct aspect ratio
Zero-filling
T'scan = Tscan / 2
Δy' = Δy x 2
SNR' = √2 x SNR


Phase resolution: 50%
256x256
256x128


Reduced FOV: 50%
T'scan = Tscan / 2
Δ k'y = Δ k'y x 2


Reduced FOV: 50%
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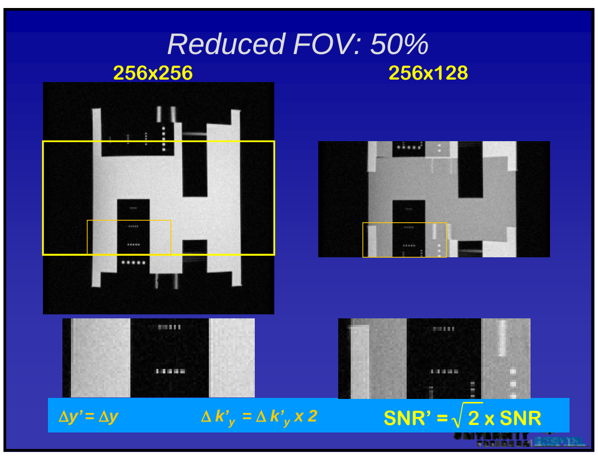
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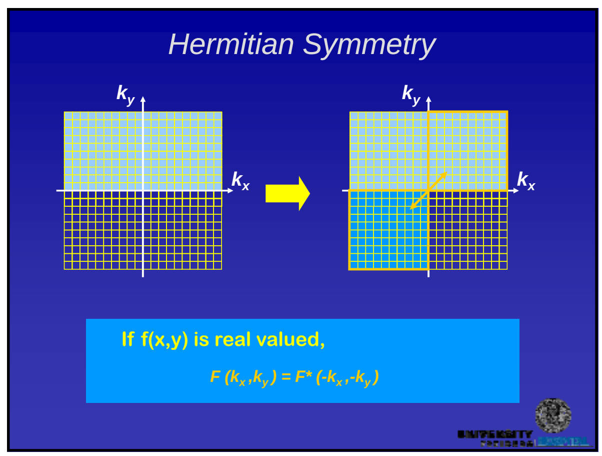
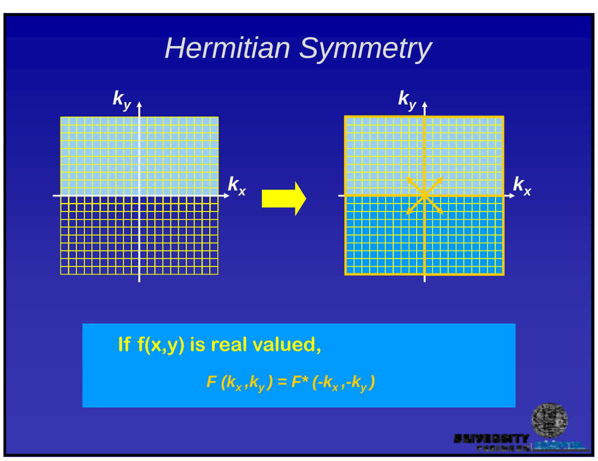
256x256
256x128
Δy' = Δy
Δ k'y = Δ k'y x 2
SNR' = √2 x SNR


Hermitian Symmetry
ky
kx
ky
kx
If f(x,y) is real valued,
F (kx,ky) = F* (-kx,-ky)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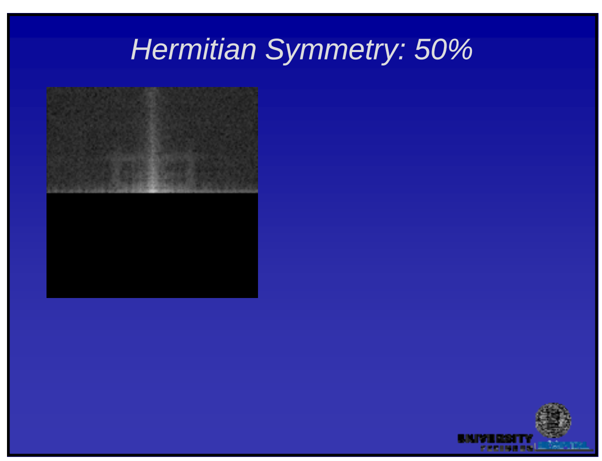
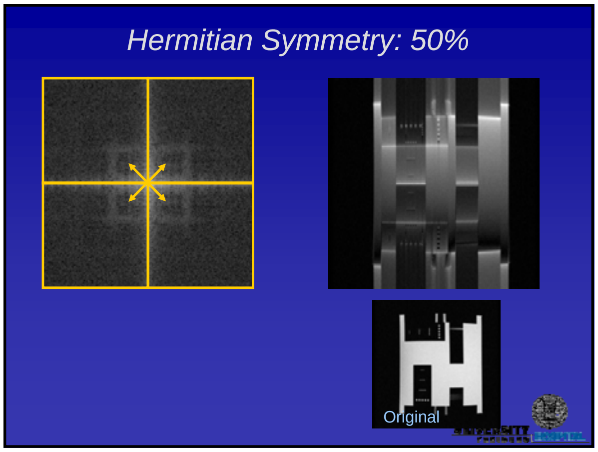
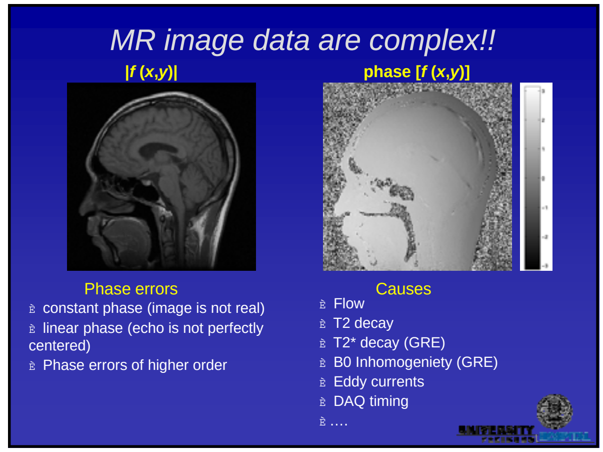
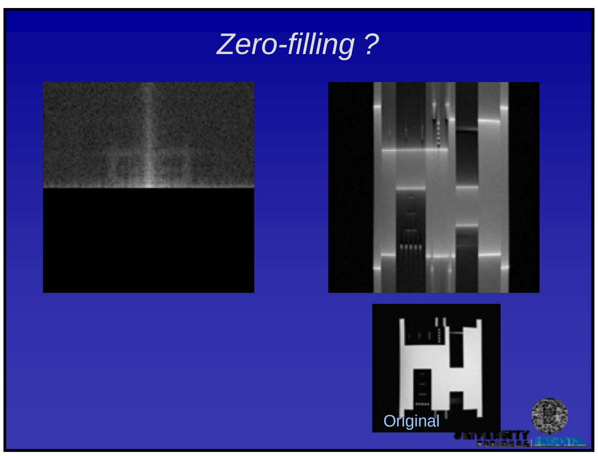
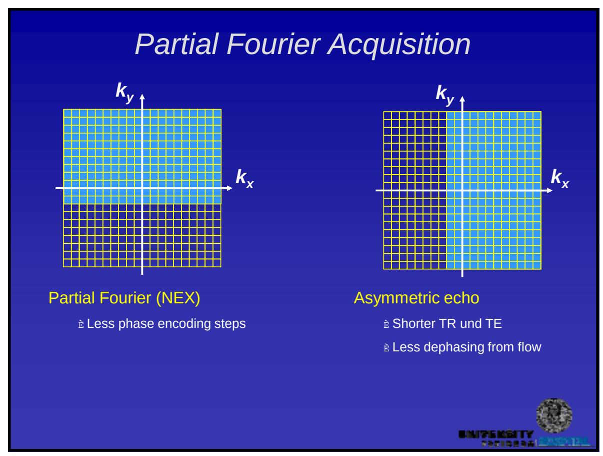
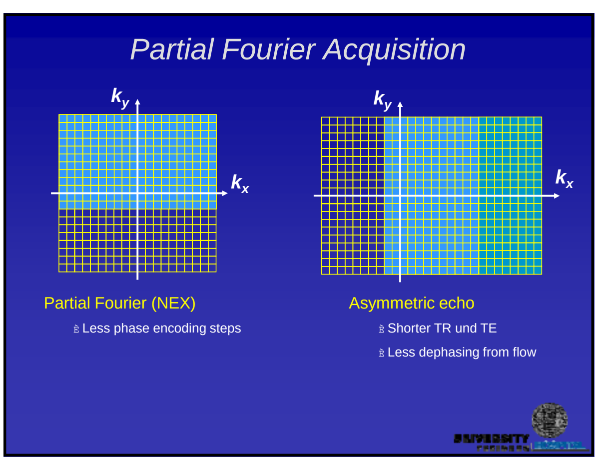

Hermitian Symmetry: 50%


Hermitian Symmetry: 50%
Original


MR image data are complex!!
|f (x,y)|
phase [f (x,y)]
Phase errors
constant phase (image is not real)
linear phase (echo is not perfectly centered)
Phase errors of higher order
Causes
Flow
T2 decay
T2* decay (GRE)
B0 Inhomogeniety (GRE)
Eddy currents
DAQ timing
....


Zero-filling ?
Original


Partial Fourier Acquisition
ky
kx
ky
kx
Partial Fourier (NEX)
Less phase encoding steps
Asymmetric echo
Shorter TR und TE
Less dephasing from flow


Partial Fourier Acquisition
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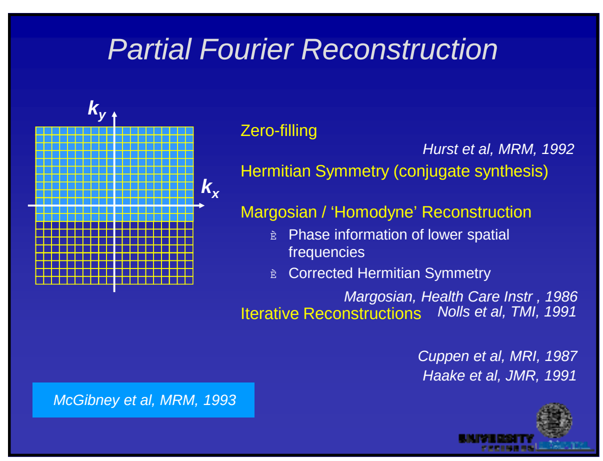
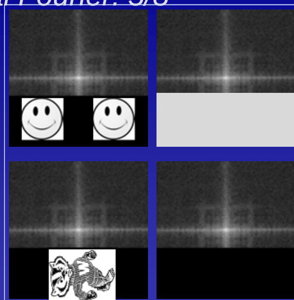
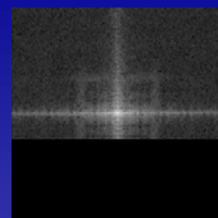

Partial Fourier Reconstruction
ky
kx
Zero-filling
Hurst et al, MRM, 1992
Hermitian Symmetry (conjugate synthesis)
Margosian / 'Homodyne' Reconstruction
Phase information of lower spatial frequencies
Corrected Hermitian Symmetry
Margosian, Health Care Instr , 1986
Iterative Reconstructions
Nolls et al, TMI, 1991
Cuppen et al, MRI, 1987
Haake et al, JMR, 1991
McGibney et al, MRM, 1993

Partial Fourier: 5/8

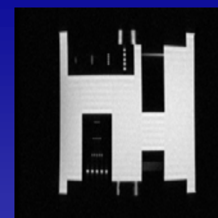


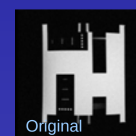
Possible solutions

Partial Fourier: 5/8

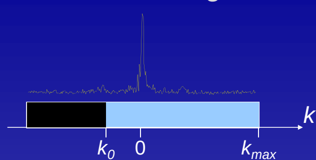


Zero-filling





Zero-filling



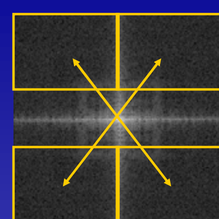
The lower k_0
- The higher the SNR
- The smaller the spatial resolution

Keep distance k_0 to k_{max} constant
- SNR stays unchanged
- Increases / decreases spatial resolution

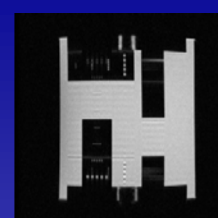
k_0 / k_{max} > ½ (e.g.. partial Fourier = 6/8)
- Small loss in spatial resolution

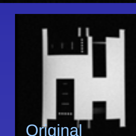
k_0 / k_{max} < 1/5 (e.g. partial Fourier = 5/8)
- Increased Gibb's ringing

Partial Fourier: 5/8



Hermitian Symmetry





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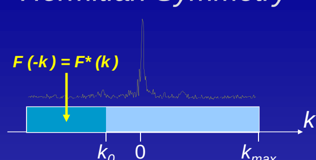
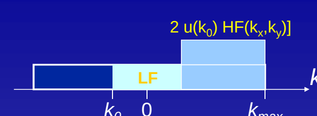


Poor image quality because of complex data
- Image has phase contributions

SNR loss: up to $\sqrt{2}$
- Shortened acquisition time

Margosian / Homodyne



Divide k-space into low (LF) and high (HF) spatial frequencies
- Low spatial frequencies are fully sampled
- Estimate phase in image from low spatial frequencies only: $\phi'(x)$

Double the magnitude of HF:
- Ensure constant k-space density

$$I_c(x,y) = \Im\, [LF(k_x,k_y) + 2\, u(k_0)\, HF(k_x,k_y)]$$
$$I(x,y) = \mathrm{Re}\, [I_c(x,y)]\, \exp(-i\, \phi'(x))$$

Margosian / Homodyne Rekonstruktion

1. Estimate phase from low spatial frequencies

$$\phi'(x,y) = \arg\left[\Im\,[LF(k_x,k_y)]\right]$$

2. Weight k-space data

$$M'(k_x,k_y) = W(k_x) \times [LF(k_x,k_y) + 2\,u(k_0)\,HF(k_x,k_y)]$$

3. Calculate image via Fourier transform

$$I_c(x,y) = \Im\,[M'(k_x,k_y)]$$

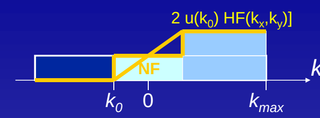
4. Calculate phase modulation from phase estimate

$$I'_c(x,y) = I_c(x,y)\,\exp(-i\,\phi'(x,y))$$

5. Use real component

$$I_c(x,y) = \mathrm{Re}\left[I'_c(x,y)\right]$$

Margosian / Homodyne



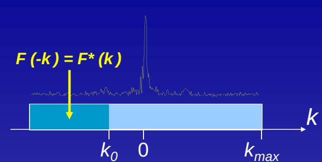
Weighting Function

- Ramp filter to reduce Gibb's ringing
- Reduce SNR

Problems

- Assumption: Phase varies slowly
- Doubling of HF increases the noise

Iterative Reconstruction



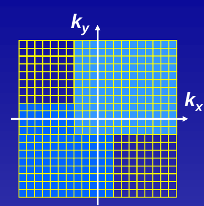
Allows for better phase estimation

converges after <5 iterations

Best image quality

Not implemented as product

2D Partial Fourier



Assymetric echo in k_x

partial Fourier in k_y

Not all data can be generated

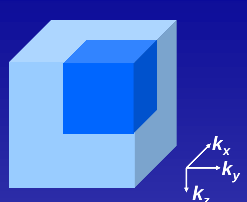
- Some higher spatial frequencies can be filled with zeros only
- Potential loss of diagnostic information

2D Iterative Reconstruction (POCS) *Haake et al, JMRI, 2001*

Iterative homodyne Reconstruction for 2D partial Fourier

Good results for $T_{scan} = 0.5\ T_{scan}$

3D Partial Fourier



Assymetric Echo in k_x

Partial Fourier in k_y

Partial Fourier in k_z

Few data and lots of zeros!!!

- Many higher spatial frequencies can be filled with zeros only
- (Potential) loss of diagnostic information
- 'Imaging at 0 Tesla'

Please remember

Acquisitions matrix is the critical parameter !!!!!

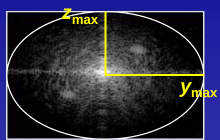
- Nread = read (Nx)
- Ny = Phase encodings in y
- Nz = Phase encodings in z
- Homodyne / zero-filling makes a difference

Reconstruction matrix is less important !!!!!

- One can always add many zeros

K-space corners

- Goal: reduce scan time



$A_{rectangle} = 2y_{max} \times 2z_{max} = 4y_{max}\, z_{max}$

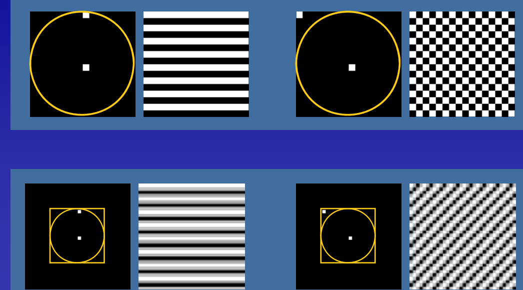
$A_{ellipse} = \pi\, y_{max}\, z_{max}$

reduction: $\pi/4$ = 78.5 %

- Sampling of corners
  - Higher spatial frequencies in the diagonales
  - Zero-filling becomes beneficial
  - Watch for Fermi filter (switched off)

M. Bernstein, MRM (2001)

K-spce corners



JOURNAL OF MAGNETIC RESONANCE IMAGING 14:270–280 (2001)

Original Research

Effect of Windowing and Zero-Filled Reconstruction of MRI Data on Spatial Resolution and Acquisition Strategy

Matt A. Bernstein, PhD,* Sean B. Fain, PhD, and Stephen J. Riederer, PhD

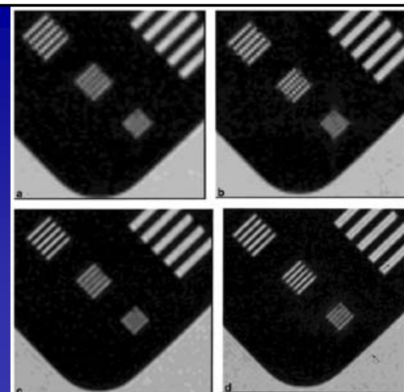


Figure 6. Magnified portions of θ = 45° phantom images. **a:** Radially windowed, and without zero-filling. **b:** No window, again without zero-filling. Note the banding artifact on both images, and the improved SNR on a. **c:** Radially windowed with zero filling. **d:** No window, again with zero-filling. Note the smallest group of five bars is resolved only in d. Also note the increase conspicuity of the truncation artifact (arrows) and noise on d.