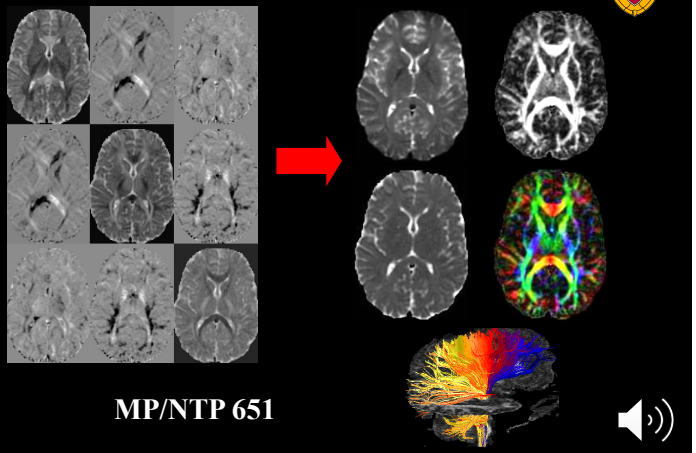
Quantitative DTI Maps



MP/NTP 651

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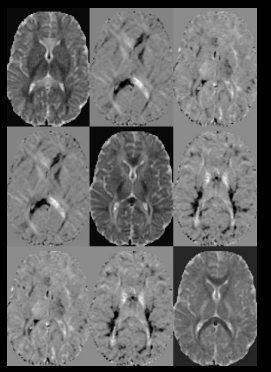
The Diffusion Tensor

$$\mathbf{D} = \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{yy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{pmatrix} =$$



Not Rotationally Invariant!

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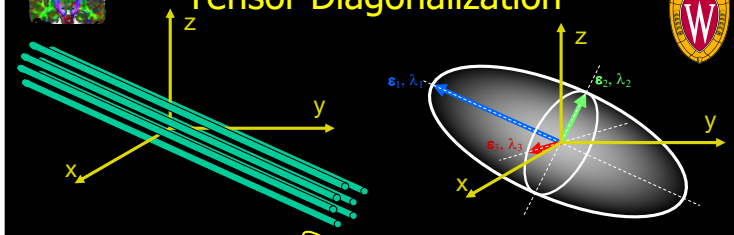
Tensor Diagonalization



$$\mathbf{D} = \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{yy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{pmatrix} = \mathbf{E}^{T} \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} \mathbf{E}$$

Diffusion Tensor

Eigenvalues

Matrix of 3 eigenvectors

$D_{ii}>0$

$D_{ij}=D_{ji}$

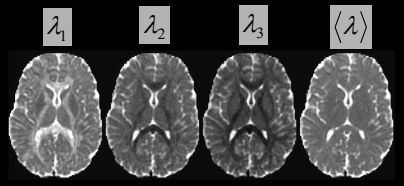
3

Tensor Diagonalization

$$\mathbf{DE} = \mathbf{E\Lambda}$$

$$\Lambda = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

$$\mathbf{E} = \begin{pmatrix} \hat{e}_{1x} & \hat{e}_{2x} & \hat{e}_{3x} \\ \hat{e}_{1y} & \hat{e}_{2y} & \hat{e}_{3y} \\ \hat{e}_{1z} & \hat{e}_{2z} & \hat{e}_{3z} \end{pmatrix}$$



•Rotationally invariant

$$trace(\mathbf{D}) = trace(\Lambda) = \lambda_1 + \lambda_2 + \lambda_3 = D_{xx} + D_{yy} + D_{zz} = 3\langle\lambda\rangle$$

Mean Diffusivity: MD = trace(D)/3

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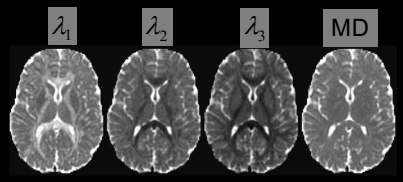
Diffusivities

- Three Eigenvalues of Diffusion Tensor
  - Amount of water diffusion in principal directions
  - Major, Medium, Minor – λ1, λ2, λ3
  - Da = Axial Diffusivity = λ1 ~ axonal degeneration
  - Dr = Radial Diffusivity = (λ2 + λ3)/2 ~ myelination
- Mean Diffusivity (MD) – average of eigenvalues
  - **Overall magnitude of Diffusion**
  - **Measure of tissue density**

mm²/s



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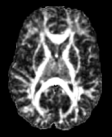
Fractional Anisotropy

$$FA = \sqrt{\frac{3((\lambda_1 - \langle\lambda\rangle)^2 + (\lambda_2 - \langle\lambda\rangle)^2 + (\lambda_3 - \langle\lambda\rangle)^2)}{2(\lambda_1^2 + \lambda_2^2 + \lambda_3^2)}}$$



- **Variance of diffusivity with direction**
- **Normalized between 0 and 1**
- **Ubiquitous measure of microstructure**
- **Extremely sensitive**
- **Low specificity**

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What Do We Measure?

Average: Radial Diffusivity - "Myelin"

Axial Diffusivity – "Axons"

Average: Mean Diffusivity - "Density"

Normalized Variance: Anisotropy - "White Matter Microstructure"

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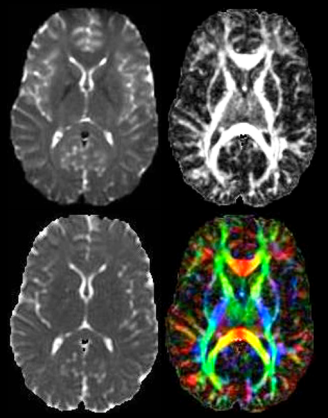
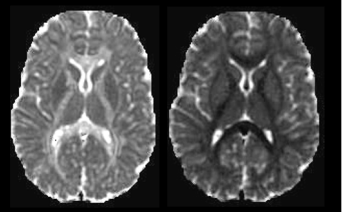
Diffusion Tensor Imaging





Parallel Diffusivity ('Axons')

Perpendicular Diffusivity ('Myelin')

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