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NMR Spin-Echo Flow Measurements

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Patterns of fluid flow are investigated by a technique utilizing NMR spin echoes. Spin-echo amplitudes are sensitive to the phase distribution of the resonating nuclei in the fluid, and the phase distribution is dependent upon the flow. A mathematical transformation of the spin-echo amplitudes observed provides a velocity distribution function which indicates the relative number of molecules flowing in any given velocity range when plotted. Experimental data are obtained from ordinary laminar flow, and blood flow in human fingers.

I. INTRODUCTION

Nuclear magnetic resonance techniques have heretofore been applied to measuring average and instantaneous flow velocities.[1–4] In the present work, we utilize NMR spin-echo techniques,[5] which have been utilized for measurement of diffusion in fluids,[6,7] and are here applied to the study of velocity distribution patterns.

NMR spin-echo amplitudes are sensitive to the phase distribution of the rotating nuclei. These phases, in turn, depend on the flow. The echo amplitude as a function of time to echo has sufficient information to determine the flow velocity distribution under the conditions used in the experiment. The measurement takes less than 0.1 sec and can be made responsive to very slow flows (about 10^{-2} cm/sec).

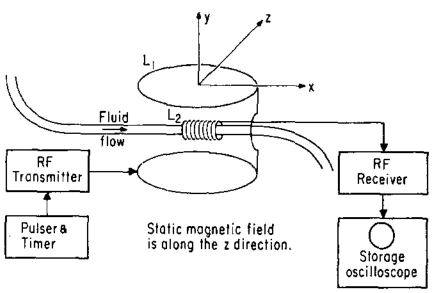

FIG. 1. Schematic of the NMR spin-echo flow measurement.

A schematic diagram of the apparatus is shown in Fig. 1. The transmitter driving coil L_1 produces two short pulses of rf field of proper length to induce spin echoes. The first pulse is a 90° pulse, and the second pulse is an 180° pulse. The resulting echoes are sensed by L_2, amplified and displayed. The coil L_2 encircles the flow to be measured. The coil L_1 is much larger than L_2, external to the flow, approximately Helmholtz and provides a nearly uniform rf field within L_2.

The pulser, with its timing circuitry, produces pulse pairs at 2- or 3-sec intervals (for fluids with relaxation times greater than 0.1 sec such as water, blood, etc.), with spacing t_1, between pulses. Typically, t_1 is between 0.5 and 100 msec. The echo amplitudes, as a function of t_1, are displayed and recorded. A relatively large number of echoes (20–100) are needed to define A, the echo amplitude as a function of t_1. The echo amplitude $A(2t_1)$ is measured as a function of $2t_1$.

We define a flow velocity distribution function $F(v)$ which is the density of nuclei within the receiver coil per velocity range. Another way of describing $F(v)$ is to define $F(v)dv$ as the number of nuclei with velocity v in the range v–$v+dv$. The even part of the $F(v)$, which is $f(v)$ can be found as the Fourier cosine transform of $A(2t_1)$ following a change of variable. Once $f(v)$ is obtained, it is plotted versus v to show the relative number of nuclei flowing in a given velocity range.

II. THEORY

Theoretically, the amplitude of an echo (due to one pulse pair) is proportional to

$$\int_{-\infty}^{\infty} \cos(\phi) g(\phi) d\phi. \tag{1}$$

The function $g(\phi)$ represents the distribution in a phase ϕ of the various "isochromatic" groups of nuclear moments. Each group's contribution is proportional to the cosine of its phase relative to the phase of the net dipole moment of the groups. This formula is used as the starting point in deriving a relation between a flow velocity distribution and $A(2t_1)$. We make the following assumptions and simplifications (refer to the schematic of Fig. 1).

(1) The transmitter coil provides a uniform rf field over a volume of the flow larger than the receiver coil.

(2) The dc magnetic field can be characterized by a linear gradient G in a direction parallel to the flow, at least in the region of the flow.

(3) The flow velocities of interest are much larger than the self-diffusion of the liquid so that the position of an isochromatic group of molecules is well approximated by

$$\mathbf{x}(2t_1) = \mathbf{x}(0) + 2vt_1\hat{x}, \tag{2}$$

where v is the group's velocity and $\hat{x}$ is a unit vector parallel to the flow.

(4) The liquid has been in the dc field for a time much longer than T_1.

(5) $2vt_1 \ll$ dimensions of receiver coil. Under these assumptions, the phase of a molecular group, $2t_1$ sec after the 1st pulse is given by

$$\phi = \gamma G v t_1^2, \tag{3}$$

where γ is the magnetogyric ratio.

If we interpret $g(\phi)$ as a density function then we can rewrite (1) as

$$A(2t_1)=K\int_{-\infty}^{\infty}\cos(\phi)g(\phi)d\phi \tag{4}$$

$$=2K\int_0^{\infty}\cos(\gamma Gvt_1^2)f(v)dv, \tag{5}$$

where K is a proportionality constant depending on geometry, receiver gain, etc. The function $f(v)$ is a density for the number of groups at a particular velocity. Note that the echo amplitude depends only on the even part of $F(v)$.

Equation (5) has the form of a Fourier cosine transform; we find $f(v)$ as the transform inverse.

$$u=\gamma Gt_1^2; \qquad 2t_1=2[u/(\gamma G)]^{1/2} \tag{6}$$

$$K^{-1}A(2t_1)=2\int_0^{\infty}\cos(uv)f(v)dv \tag{7}$$

$$f(v)=\pi^{-1}\int_0^{\infty}\cos(uv)K^{-1}A\left[2\left(\frac{u}{(\gamma G)}\right)^{1/2}\right]du \tag{8}$$

$$f(v)=\pi^{-1}\int_0^{\infty}\cos(uv)a(u)du. \tag{9}$$

In practice, it may be necessary to normalize $f(v)$ since some of the constants may not be known.

As an example, consider the case of laminar flow through a tube of circular cross section with average velocity $\bar{v}$.

$$F(v)=1/2\bar{v}, \qquad 0<v<2\bar{v}$$

$$=0, \qquad \text{otherwise} \tag{10}$$

$$A(2t_1)=\frac{K}{\bar{v}}\int_0^{\bar{v}}\cos(\gamma Gvt_1^2)dv \tag{11}$$

$$=K\sin(\gamma G\bar{v}t_1^2)/\gamma G\bar{v}t_1^2. \tag{12}$$

Note that the origin in (11) has been shifted from $v=0$ to $v=\bar{v}$ to make $f(v)$ an even function.

III. EXPERIMENTAL RESULTS

Experimental spin-echo observations of water flow in tubes providing Reynold's numbers between 10 and 100 show that we obtain good agreement with Eq. (12). Note that for laminar flow, $f(v)$ plotted versus v is a horizontal straight line both experimentally and theoretically.[8] Even though the stream lines of laminar flow show a paraboloid front, the number of molecules of fluid in any velocity range is a constant.

The blood flow in human fingers has been examined by this method. The apparatus consisted of a 2-cm-i.d. polystyrene cylinder supporting a 5-mm-long receiver coil and 10-cm-i.d. transmitter coils in a Helmholtz configuration. The proton resonance was detected at 10.5 MHz. No attempt was made to trigger the echoes relative to the heartbeat, so that a flow average is obtained over the time of observation. The field gradient

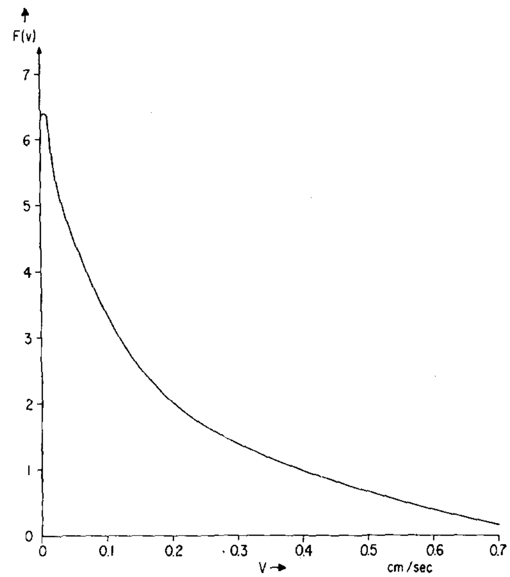

FIG. 2. Plot of $f(v)$ versus velocity for a human finger, $\gamma G=2\times10^4$, $T=1.9\times10^{-2}$.

was found by diffusion and laminar flow measurements of $A(2t_1)$.

The experimental data were well approximated by an exponential $A(2t_1)$, in all of the subjects. This form is convenient because it has a known cosine transform. If $A(2t_1)=\exp(t_1/T)$ is the form of the measurements, then

$$f(v)=[v^{-3/2}/(2\pi\gamma G)^{1/2}T]\{[\tfrac{1}{2}-C(x)]\cos(\pi x^2/2)$$
$$+[\tfrac{1}{2}-S(x)]\sin(\pi x^2/2)\},$$

where $C(x)$ and $S(x)$ are Fresnel integrals and $x=1/T(2\pi\gamma Gv)^{1/2}$. This complicated-looking function is partially tabulated

$$f(v)=(x/v)G(x),$$

where $G(x)$ is a tabulated function.[9] $f(v)$ has convenient asymptotic expansions and normalization

$$\int_0^{\infty}f(v)dv=\frac{(xv^{1/2})^{1/2}}{2}.$$

The numerical evaluation of $f(v)$ for a human finger is given in Fig. 2; experimental accuracy is about 10%.

It is important to point out that the method of obtaining velocity distributions as described above, gives the magnitude of the component of the velocity parallel to the field gradient. Thus for the case of laminar flow, this point is unimportant, but for the case of flow distributions in fingers and other such complex flow patterns, nuclei moving perpendicular to the field gradient contribute to $f(v)$ near $v=0$, independent of their speed. Therefore, in obtaining the flow distribution in fingers, the distribution plot is somewhat weighed near $v=0$ by the capillary flows perpendicular to the long dimension of the finger.

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Contactless Induction Method for Electric Resistivity Measurement

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A new absolute method is developed for measuring electric resistivity in the range of 10^{-8} to 10^{+8} Ω cm of a sample of almost arbitrary shape. This method takes into account the effect of self-induced conduction and polarization current. Measurements are made with the specimen placed in a constant rotating magnetic field in such an orientation that it experiences maximum torque and has minimum moment of inertia. This can be shown to be the same orientation for which, if the specimen were free to rotate, it would reach the angular velocity of the rotating field in minimum time. This method is applicable to homogeneous and isotropic conducting materials in both solid and liquid states, including ferromagnetic substances. The mathematical formula for the measured torque is developed and the technique for measuring electric resistivity is explained.

INTRODUCTION

The known methods for the measurement of electrical resistivity are not practical in application under all conditions[1,2] because of, for example, corrosive property of materials, enhanced electric conductivity, the injection of high-energy electrons of the probe contacts, or poor contacts due to the oxide layers in ceramic semiconductors.

This paper presents a novel solution to these problems. The new method is not dependent on electric contact since it utilizes the interaction of magnetic fields. The only quantity measured is a mechanical torque which depends upon the electric resistivity as a quantity to be evaluated together with other known parameters, such as strength of the magnetic induction field, the frequency of rotation of the field, the geometry, and apparent density of the sample.

In the first part of this paper, a theoretical discussion of the general principle is given and the general equation for the torque is developed. Next the method of measurement and the technique for eliminating the elastic constant of the system and shape of the sample are explained. Finally, there is a general presentation of supporting results and concluding remarks.

THEORETICAL

The method utilizes the interaction between a rotating magnetic field and electric current induced in a sample.[3,4] The resultant effect is a torque acting on a sample which can be measured and related to the electric resistivity of the sample. Therefore, one has to develop theoretically an expression for the torque acting on the sample which will contain parameters related to the sample and the rotating magnetic field.

In general, in order to compute the torque acting on the sample, one uses the fact that the power delivered by the rotating magnetic field is equal to the mechanical torque multiplied by the angular velocity. Therefore, it is essential to compute the power dissipated in the sample. This makes it necessary to obtain the mathematical expression for electric current density vector **j** in a sample produced by either the rotating constant magnetic induction **B**, or by rotating the sample in a stationary field **B**. Power dissipated is equal to[5]

$$P=\left(\frac{1}{2}\right)\int_{\tau}\frac{\mathbf{j}^2}{\sigma}\,d\tau, \tag{1}$$

where $\frac{1}{2}$ accounts for linear relationship between vector potential and current density. The evaluation of the electric current density is a very difficult task,[6] since it involves the solution of Maxwell equations which are applied to the sample, subject to boundary conditions which are different for each sample geometry. Such solutions are almost always in a form of an infinite sequence and therefore not the best for practical applications.

This new method, however, neglects boundary conditions and the experimental results show that boundary conditions are of no consequence in practical applications. Furthermore, any shape of the sample will give a correct result for the value of electric resistivity under the following conditions: The sample must be suspended in the constant rotating magnetic induction field **B** so as to render maximum torque and minimum