Original Research

Minimizing TE in Moment-nulled or Flow-encoded Two- and Three-dimensional Gradient-Echo Imaging[1]

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A method for minimizing field-echo delay in moment-nulled gradient-echo imaging is presented. Even though ramps are accounted for, the analysis yields simple closed-form solutions. The method is then generalized to the section-select waveform for three-dimensional volume imaging and to flow encoding for phase-contrast imaging. Three strategies for first-moment selection in phase-contrast imaging are discussed, including a new strategy that always yields the minimum echo delay. Trapezoidal and triangular gradient lobe shapes are analyzed.

Index terms: Gradient waveforms • Model, mathematical • Phase imaging • Pulse sequences • Rapid imaging • Three-dimensional imaging • Vascular studies

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Abbreviations: RF = radio frequency, 3D = three-dimensional, 2D = two-dimensional, VENC = velocity encoding that produces a phase difference of π radian.

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ECHO-DELAY MINIMIZATION is valuable in gradient-echo imaging for the reduction of susceptibility effects and flow artifacts due to intravoxel dephasing (1–4). This report describes a flexible algorithm for calculating gradient waveforms with minimized field-echo delay.

• METHODS

The zeroth and first moments of the gradient waveform, M_0 and M_1, respectively (5 and references therein), are defined as

$$M_0 = \int_{-\infty}^{\infty} G(t')\,dt' \qquad (1)$$

and

$$M_1 = \int_{-\infty}^{\infty} G(t')t'\,dt', \qquad (2)$$

where $G(t')$ is an arbitrary gradient waveform. The zeroth moment M_0 is the gradient area.

To further simplify the analysis, the algorithm exploits the translational property of the gradient first moment. If a gradient lobe with first moment M_1 and area M_0 is time shifted by t_0, then, from Equation (2), the new first moment is

$$M_1' = M_1 + M_0 t_0. \qquad (3)$$

Note that $M_1' = M_1$ whenever $M_0 = 0$. Thus, for gradient waveforms with zero net area, we can select any temporal origin ($t = 0$) for calculating the first moment M_1 (5,6). Waveforms with nulled zero- and first-order moments remain moment nulled when shifted in time.

For a symmetric lobe (eg, a trapezoid with ramps of equal duration) centered at the origin ($t = 0$)—in other words, an even function—the first moment M_1 is zero. From Equation (3), the first moment for a symmetric lobe centered at t_0 is

$$M_1' = M_0 t_0 \qquad (4)$$

Thus (as far as the first moment is concerned), a symmetric lobe centered at $t = t_0$ acts as if all its area is at $t = t_0$, and Equation (2) need not be calculated explicitly. The first moment is calculated as the product of the lobe area and "center of mass" t_0. In the present formalism, gradient waveforms are composed of symmetric lobes (eg, trapezoids) whenever possible to exploit the simplicity of Equation (4).

Calculation of a Flow-compensated Section-Select Waveform

Figure 1a shows a first-moment-nulled section-select waveform composed of three lobes labeled "*S*," "*1*," and "*2*." No delay is inserted between the gradient lobes, so the duration of the total waveform is minimized. The flat part of the "*S*" lobe is applied concurrently with the selective excitation RF pulse, so its amplitude and duration are determined by the bandwidth and duration of the RF pulse and by the desired section thickness. Thus, the area A_s and first moment M_s of the "*S*" lobe are determined by the imaging requirements and are calculated explicitly with Equations (1) and (2). As is well known (7), the portion of the "*S*" lobe to the left of a point near the peak of the RF pulse is excluded in the calculation of M_0 and M_1 because the transverse magnetization behaves as if it were created instantaneously at that time point. The goal is to solve for the area A and width w (including ramps) for the two unknown trapezoids "*1*" and "*2*" so that M_0 and M_1 for the entire waveform are zero.

Let the "*1*" and "*2*" lobes have widths w_1 and w_2 and areas A_1 and A_2, respectively (with the signs of A_1 and A_2 chosen so that both lobes shown in Fig 1a are positive). Nulling M_0 imposes

$$A_1 = A_2 + A_s. \qquad (5)$$

Since $M_0 = 0$, we are free to choose the $t = 0$ point arbitrarily (see Eq [3]). For convenience, we choose $t = 0$ at the junction of the "*S*" and "*1*" lobes, as shown in Figure 1a. Since lobes "*1*" and "*2*" are centered at $t = 0.5\,w_1$ and $w_1 + 0.5\,w_2$, respectively, nulling the first moment leads to

$$-M_s - 0.5A_1w_1 + A_2(w_1 + 0.5w_2) = 0. \qquad (6)$$

To minimize the duration of the waveform, we desire that the lobes have minimum width. The performance limitations of the gradient subsystem (rise time and peak amplitude), however, must be obeyed. Assuming that the maximum gradient strength is h (gauss per centimeter) and the minimum rise time from zero to h is r (milliseconds), the minimum width w for a trapezoidal lobe of (positive) area A is

$$w = A/h + r. \qquad (7)$$

If $A > hr$, Equation (7) will lead to trapezoidal lobes with slew rate–limited ramps. If, however, $0 < A < hr$, the optimal lobe is triangular and w as expressed in Equation (7) will be slightly greater than optimal. Use of Equation (7) guarantees a physically realizable waveform, and the minimum TE typically will not suffer, since if $A < hr$, TE is likely to be limited by a different gradient axis. Further, the echo-delay penalty, if any, is small, at most r per trapezoid. If this penalty is considered excessive, the analysis can be extended to allow triangular gradient lobes, as described in Appendix A. For simplicity, trapezoidal lobes are used in this analysis. Combining Equations (5), (6), and (7) to eliminate A_1, w_1, and w_2 yields

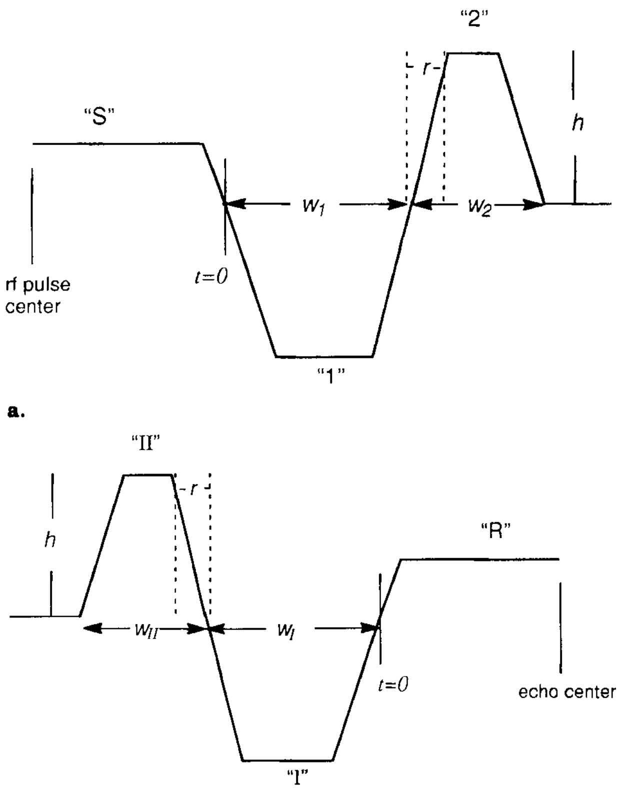


Figure 1. **(a)** Flow-compensated section-select gradient waveform with three lobes, "*S*," "*1*," and "*2*." For the purpose of the calculation, the "*S*" lobe starts near the center of the radio-frequency (RF) pulse (not shown). **(b)** Flow-compensated readout gradient waveform with three lobes, "*II*," "*I*," and "*R*." For the purpose of the calculation, the flat top of the "*R*" lobe extends from the beginning of data sampling until the echo peak (*echo center*). h = peak amplitude, r = duration of slew-rate–limited ramp from 0 to h, w = width of lobe.

$$A_2^2 + A_2hr - (hM_s + 0.5hrA_s + 0.5A_s^2) = 0. \qquad (8)$$

Equation (8) has only one physically significant root that is both real and positive:

$$A_2 = \left[-hr + \sqrt{(hr)^2 + 2(hrA_s + A_s^2 + 2hM_s)}\right]/2. \qquad (9)$$

Once A_2 is known, A_1 and the widths w_1 and w_2 can be solved with Equations (5) and (7).

These parameters provide the minimum-duration, flow-compensated section-select gradient waveform. The calculation for the readout waveform is analogous. As shown in Figure 1, the readout gradient ("*R*," "*I*," and "*II*" lobes) is a time-reversed version of the section-select gradient. Thus,

$$A_{II} = \left[-hr + \sqrt{(hr)^2 + 2(hrA_r + A_r^2 + 2hM_r)}\right]/2. \qquad (10)$$

Three-dimensional (3D) Volume Imaging

For 3D imaging, the previous readout waveform solution (Eq [10]) can be used directly. Henceforth, we emphasize the calculation of the section-select

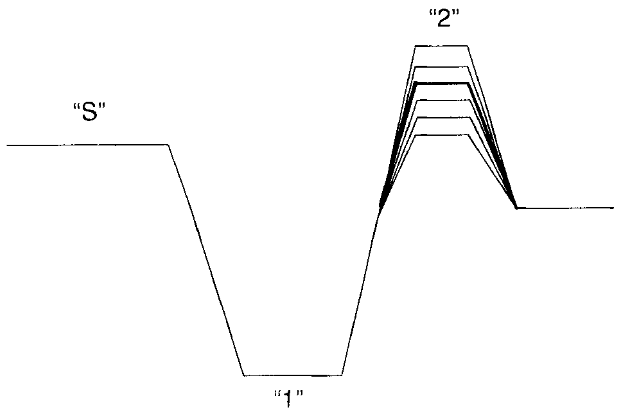
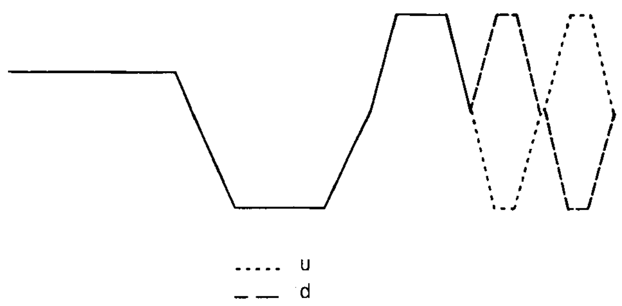
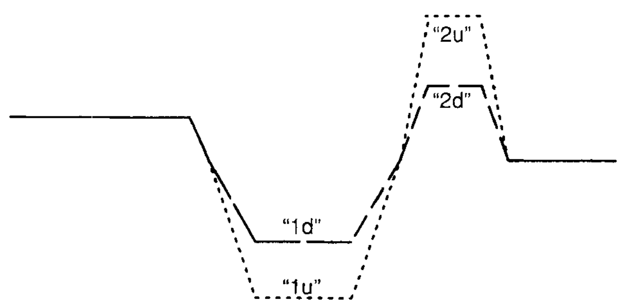


Figure 2. A flow-compensated section-select gradient waveform with lobes "S," "*1*," and "2," modified to allow section encoding for 3D volume imaging. The stepped section-encoding gradient area is combined with the "2" lobe. The waveform has zero area and a zero first moment only for the center section-encoding step (bold).



a.



b.

Figure 3. Two-sided flow encoding. **(a)** Bipolar flow-encoding lobes added to a flow-compensated section-select gradient waveform. The flow experiments *u* and *d* signify *up* and *down*, respectively. **(b)** The five lobes in **a** are combined into three lobes to minimize echo delay.

waveform, since only that waveform must be modified for the 3D case, specifically to include the phase-encoding function in the section-select direction (section encoding). To minimize waveform duration, the section-encoding function can be combined with the "*1*" or "2" gradient lobe, rather than performed by a dedicated lobe. It is shown in Appendix B that the minimum duration is obtained when the section-encoding area is combined with the "2" rather than the "*1*" lobe. This choice has the additional benefit of minimizing misregistration artifacts (8,9) when there is motion along an oblique section-readout direction.

Let $\pm A_e$ be the range of areas required for section encoding. For a section thickness Δz, $\Delta z = \pi/(\gamma A_e)$, where γ is the gyromagnetic ratio in units of radians per second per gauss. To ensure that the gradient amplitude limit h is not exceeded, amplitude "headroom" is incorporated for gradient lobe "2." Thus,

$$w_2 = (A_2 + A_e)/h + r. \tag{11}$$

The section-select waveform is designed to be area and first-moment nulled for the central section-encoding step (Fig 2). Equations (5), (6), and (11) are combined to yield

$$A_2 = \left[-(hr + A_e/2) + \sqrt{(hr + A_e/2)^2 + 2(hrA_s + A_s^2 + 2hM_s)}\right]/2. \tag{12}$$

Note that for $A_e \to 0$, Equation (12) reduces to the two-dimensional (2D) case (Eq [9]). The area A_1 and the widths w_1 and w_2 can then be solved for the 3D case with Equations (5), (7), and (11).

For section-encoding steps other than the central one, the amplitude of the "2" lobe is incremented so that the area A_2' satisfies $A_2 - A_e \le A_2' \le A_2 + A_e$. The widths w_1 and w_2 are held constant for all section-encoding steps. For section-encoding steps other than the central one, the area and first moment are both nonzero, just as in the phase-encoding direction.

Flow Encoding for Phase Contrast: "Two-sided" Case

It is of interest to find simple closed-form expressions for widths and amplitudes of gradient lobes that minimize TE for phase-contrast angiography (10,11). One possible way to implement flow encoding is to add bipolar gradient lobes to a flow-compensated waveform, as shown in Figure 3a. This is an example of what can be called "two-sided" flow encoding: gradient moments that are symmetric about $M_1 = 0$. Two-sided encoding can be alternatively obtained with shorter minimum echo delays by combining gradient lobes, as shown in Figure 3b. This combined-lobe strategy can be analyzed by extension of the techniques used in the previous two sections.

If the desired total change in the first moment is ΔM ($\Delta M = \pi/(\gamma$VENC, where VENC is the velocity encoding that produces a phase difference of π radian) in two-sided flow encoding, the waveforms have first moments of $\pm\Delta M/2$. For the worst of the two cases (in terms of minimum duration) Equation (6) becomes

$$-M_s - 0.5A_{1u}w_1 + A_{2u}(w_1 + 0.5w_2) = \Delta M/2. \tag{13}$$

Here the subscript u (signifying *up*) indicates one of the two flow experiments. Equation (13), combined with the zero area constraint and Equation (11), yields

$$A_{2u} = \left\{-(hr + A_e/2) + \sqrt{(hr + A_e/2)^2 + 2[hrA_s + A_s^2 + 2h(M_s + 0.5\Delta M)]}\right\}/2. \tag{14}$$

The 3D situation, which includes A_e, has been used, so that Equation (14) is valid for both 3D and 2D ($A_e = 0$) phase-contrast imaging. A_{1u} is calculated with an expression analogous to Equation (5). Once the widths w_1 and w_2 are solved with Equation (7), the lobe areas for the other flow experiment, A_{2d} (*d* signifying *down*), can be solved. For simplicity, and without TE penalty, the same widths are used in both flow experiments. From the first-moment constraint of the down flow experiment, it can be shown that

$$A_{2d} = \frac{(2M_s - \Delta M + A_s w_1)}{(w_1 + w_2)}, \tag{15}$$

and, as before, $A_{1d} = A_{2d} + A_s$.

As long as $\Delta M > 0$, it is certain that $|A_{1d}| > |A_{2d}|$. Therefore, the peak gradient amplitude h will never be exceeded if w_1 and w_2 are calculated to accommodate the up flow experiment.

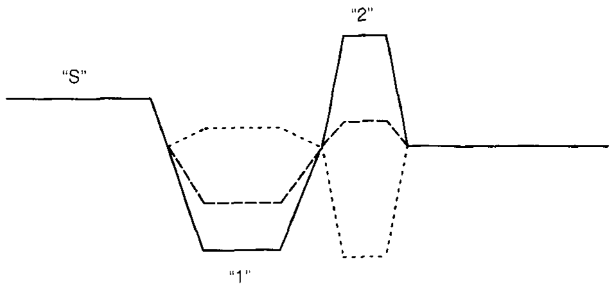


Figure 4. Section-select gradient waveform useful for analyzing the minimum-TE and one-sided flow-encoding strategies. The dotted line, used in the minimum-TE flow-encoding calculation, represents an inverted version of lobe "2," indicated by the solid line The dashed line is used in the one-sided flow-encoding calculation. (See text.)

Minimum-TE Flow-Encoding Strategies

The two-sided strategy achieves the desired moment change ΔM with the constraint that the first moments of the two flow experiments be symmetrically placed about $M_1 = 0$. If this constraint is relaxed, shorter echo delays can be obtained. The minimum-TE method (Fig 4) uses flow measurements without regard to their absolute first moments M_{1u} and M_{1d}; the only requirements are their separation ($\Delta M = M_{1u} - M_{1d}$) and a further constraint that the total waveform duration be minimized.

In the change from up to down, lobes "*1*" and "*2*" must change in area by the same amount. The larger the area change, the larger ΔM is. Since lobe "2" is narrower (with finite peak-gradient amplitudes), its width limits the strength of the flow encoding. This limit is reached when lobe "2" switches from full positive amplitude in the first experiment to full negative amplitude in the second. Conversely, given a desired ΔM, the minimum-duration gradient is the one for which the "2" lobe reverses in this manner (see solid and dotted lines in Fig 4). The first moments for the two flow experiments are

$$-M_s - 0.5A_{1u}w_1 + A_{2u}(w_1 + 0.5w_2) = M_{1u} \tag{16a}$$

and

$$-M_s - 0.5A_{1d}w_1 + A_{2d}(w_1 + 0.5w_2) = M_{1d}. \tag{16b}$$

Subtraction of Equation (16b) from Equation (16a), in conjunction with $\Delta M = M_{1u} - M_{1d}$, and Equations (5) and (11), produces a quadratic equation in A_{2u}, which yields

$$A_{2u} = \Big[-(A_s + A_e + 2hr) + \sqrt{(A_s + A_e + 2hr)^2 + 8h\Delta M}\Big]/4. \tag{17}$$

Note that Equation (17) is independent of M_s, the first moment of the section-select lobe, since the absolute first moment is not of interest. Again, this same minimum-TE formalism can be applied to the readout gradient.

"One-sided" Flow Encoding

The final flow-encoding strategy presented here obtains one flow experiment (eg, up) with $M_1 = 0$ (ie, moment nulled) (12). The first moment of the down flow experiment is then determined by the desired flow encoding.

This "one-sided" flow encoding can be implemented by first solving the flow-compensated problem, as already discussed (solid line, Fig 4). For the second experiment, a negative excursion from the moment-nulled case is selected (ie, $M_{1d} = -\Delta M$), since the positive excursion will always lengthen the gradient duration. To adjust M_{1d}, and therefore ΔM, lobe "*1*" is made less negative and lobe "2" less positive by the same incremental area (dashed line, Fig 4). As before, since the width of lobe "2" is less than that of lobe "*1*," its amplitude will change more and its width sets the limit on the magnitude of ΔM that can be achieved. Since the separation of the centers of these two lobes is $(w_1 + w_2)/2$, the largest achievable moment change ΔM_{max} occurs when the change in area of each lobe is $2A_{2u}$:

$$\Delta M_{max} = A_{2u}(w_1 + w_2), \tag{18}$$

which corresponds to

$$\mathrm{VENC}_{min} = \pi/[\gamma A_{2u}(w_1 + w_2)]. \tag{19}$$

For $\Delta M \leq \Delta M_{max}$ or $\mathrm{VENC} \geq \mathrm{VENC}_{min}$, the lobe widths optimized for flow compensation can be used. If stronger encoding is needed, the lobe widths must be increased. One successful approach in this case is to use the expression for the first-order moment-nulled case (Eq [9]), with M_s replaced by $M_s + \Delta M - \Delta M_{max}$. Intuitively, lobe widths are selected that are capable of generating a first moment equal to that of the section-select lobe (M_s), plus the amount by which the one-sided encoding (with moment-nulled widths) falls short ($\Delta M - \Delta M_{max}$). This increases the calculated widths and ensures that the amplitude of A_{2d} will not be less than $-h$.

• RESULTS

Tables 1 and 2 give the total lobe width $w_1 + w_2$ obtainable with the three methods, as a function of flow-encoding strength for section thicknesses of 3 and 20

Table 1
Duration (w_1 + w_2; See Fig 1a) of Gradient Lobes (in Milliseconds) for Various VENC Values and Four Flow-Encoding Strategies

VENC (cm/sec)	Flow-Encoding Method: Two-sided	Min-TE	One-sided	Nonoptimized (Fig 3a)
0.5	25.3	23.9	33.5	30.0
1	19.0	17.4	24.0	23.5
2	14.6	12.8	17.1	18.9
5	11.0	8.80	10.5	14.8
10	9.39	6.81	7.22	12.8
20	8.41	5.45	7.22	11.4
50	7.73	4.32	7.22	10.1
100	7.48	3.82	7.22	9.47
200	7.36	3.53	7.22	9.04
400	7.29	3.36	7.22	8.76
1,000	7.25	3.25	7.22	8.55
∞	7.22	. . .	7.22	. . .

Note.—The strength of the flow encoding is inversely proportional to the VENC value. The calculation was performed for a 2D section-select gradient waveform based on a 3-mm-thick section, 3.2-msec RF duration, and 1,250-Hz bandwidth RF pulse. The parameters h and r are 0.95 G/cm and 570 μsec, respectively. The last column compares the duration of the last four lobes in the nonoptimized two-sided method shown in Figure 3a. The last row represents the moment-nulled case for the one-sided and two-sided methods. Min-TE = minimum TE.

Table 2
Duration of Gradient Lobes As in Table 1, but for 20-mm Section Thickness

VENC (cm/sec)	Flow-Encoding Method: Two-sided	Min-TE	One-sided	Nonoptimized (Fig 3a)
0.5	23.1	23.0	32.3	25.0
1	16.6	16.4	23.0	18.5
2	12.3	11.8	16.5	13.9
5	7.99	7.77	10.7	9.78
10	5.98	5.73	7.76	7.73
20	4.59	4.29	5.66	6.29
50	3.42	3.04	3.74	5.02
100	2.89	2.46	2.70	4.40
200	2.56	2.02	2.16	3.98
400	2.37	1.76	2.16	3.70
1,000	2.25	1.56	2.16	3.48
∞	2.16	. . .	2.16	. . .

Note.—Since the section thickness is 20 mm instead of 3 mm, a larger proportion of the gradient area is used for flow encoding (rather than refocusing the section-select gradient) (see text).

mm, respectively. A calculation of the duration of the four rightmost lobes in the nonoptimized two-sided waveform (Fig 3a) is included for comparison.

As expected, the minimum-TE method always provides the most compact waveform (hence shortest echo delay), but typically by only 1–4 msec in the VENC range of 5–400 cm/sec for the chosen parameters. TE reduction is greatest when the "inherent" moment M_s is largest (eg, narrow section thickness) and when weak flow encoding (large VENC value) is used.

The one-sided and two-sided methods yield almost identical TEs at weak flow encoding (large VENC value), since in these cases both are nearly moment nulled. For $\Delta M < \Delta M_{max}$, the echo delay for the one-sided method is exactly that of the moment-nulled solution, while the two-sided method requires more time for the up flow experiment. Conversely, at strong encoding (small VENC value) and weak inherent moments, the one-sided method imposes a longer TE to achieve a doubled first moment in the down flow experiment and can be even less efficient than the nonoptimized two-sided calculation.

• DISCUSSION

The advantage of the derived formalism is the simple closed-form expressions that result. It is specifically designed to be a noniterative, noninteractive method. Given user inputs of section thickness, field of view, VENC, and so forth, one can directly generate the optimized lobes. An alternative spreadsheet matrix-inversion method (13) has been proposed. That method requires user interaction, although it does allow the user to account for additional concerns such as higher gradient moments. Also, the simple closed-form expressions occasionally lead to some broader insight. For example, the preference of lobe "2" for combined section encoding in 3D imaging (Appendix B) was not intuitive but is a general result. It should be noted that although simple closed-form expressions have been presented for some optimized waveform design problems (eg, reference 9), to our knowledge, the method presented here is the first to include ramps in the analysis.

Like the method in reference 13, the proposed method can be described as "lobe based"; in other words, lobes of specified shapes (eg, trapezoids or triangles) form the basis of the waveforms. A more general approach was taken by Simonetti et al (14,15), who used constrained optimization to calculate continuously varying waveforms. Although that method allows optimization of a wider set of parameters than described here, it is more computationally intensive. To simply minimize echo delay, given maximum slew rate, gradient amplitude, and desired first moment, the proposed method suffices.

Applications in which an echo delay that is longer than the minimum may be desired, for example, to place fat and water in (or out) of phase. If that is the case, delays can be inserted at the end of the section-select waveform and prior to the readout waveform.

In conclusion, algorithms for calculating minimum-duration gradient waveforms have been presented for flow compensation and flow encoding in 2D and 3D MR imaging. The section-select gradient waveform has been emphasized, but the results are immediately applicable to the readout direction. The phase-encoding gradient waveform presents a simpler problem and has been omitted, although the techniques developed here apply. These algorithms offer flexibility and relative simplicity, while ensuring minimized (or, in rare cases, nearly minimized) TE for any choice of section thickness, field of view, spatial resolution, flow-encoding strength, and so forth. In such wide-ranging applications, tabulated solutions are cumbersome.

• APPENDIX A

Triangular-shaped Gradient Lobes

If $0 < A < hr$, the minimum-duration lobe is triangular rather than trapezoidal and

$$w = 2\sqrt{\frac{Ar}{h}}. \qquad \text{(A1)}$$

For the moment-nulled case, Equation (A1) is substituted into Equation (6) as needed and a quartic expression results. If lobe "2" is a triangle and lobe "*1*" a trapezoid, the resulting equation has a single physically significant solution:

$$A_2 = \Big\{hr + 2\sqrt{A_s(A_s + hr) + 2M_sh} - hr\sqrt{1 + [4/(hr)]\sqrt{A_s(A_s + hr) + 2M_sh}}\Big\}/2. \qquad \text{(A2)}$$

When both lobes are slew-rate–limited triangles, there is still a closed-form solution to the quartic equation, but several cases must be considered. This complication, along with simplicity and negligible echo-delay penalty, makes the solution that assumes trapezoidal lobes more attractive.

• APPENDIX B

Lobe Choice for Combined Section Encoding

It is shown here that shorter TEs are obtained when section encoding is combined with lobe "2" rather than lobe "*1*." Since the "S" lobe is fixed, the echo delay depends on the sum $w_1 + w_2$.

For trapezoidal lobes, regardless of which lobe performs the section encoding, the total width is given by

$$\begin{aligned} w_1 + w_2 &= (A_1 + A_2 + A_e)/h + 2r \\ &= (2A_2 + A_s + A_e)/h + 2r. \qquad \text{(B1)} \end{aligned}$$

Thus, the method that gives the smaller A_2 leads to a more compact waveform.

Paralleling the derivation of Equation (12), but with section encoding on lobe "*1*,"

$$A_2 = \Big[-(hr + A_e/2) + \sqrt{(hr + A_e/2)^2 + 2(hrA_s + A_s^2 + 2hM_s + A_sA_e)}\Big]/2. \qquad \text{(B2)}$$

The positive term $(A_s A_e)$ is present in Equation (B2) but absent in Equation (12). Thus, incorporating section encoding into lobe "2" *always* leads to a more compact waveform. ●

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