Statistical Methods for Decision Making

How to Assess Uncertainty Using Probability?

Introduction

- Managers will have to cope with uncertainty in many decision situations. Concepts of probability will help you measure uncertainty and perform associated analysis that are essential in making effective business decisions.

Probability – Meaning and Concepts

- Probability refers to chance or likelihood of a particular event – taking place.
- An event is an outcome of an experiment.
- An experiment is a process that is performed to understand and observe possible outcomes.
- Set of all outcomes of an experiment is called the sample space.

Example

- In a manufacturing unit three parts from the assembly are selected. You are observing whether they are defective or non-defective.

Determine:

a) The sample space.

b) the event of getting at least two defective parts.

Solution

a) Let S = Sample space. It is pictured as under

GGG	GGD	GDG	DGG
GDD	DGD	DDG	DDD

D – Defective

G – Non-defective

b) Let E denote the event of getting at least two defective parts. This implies that E will contain two defective, and three defectives. Looking at the sample diagram above, E = { GDD, DGD, DDG, DDD}. It is easy to see that E is a part of S and commonly called as a subset of S. Hence an event is always a subset of the sample space

Definition of probability

- Probability of an event A is defined as the ratio of two numbers m and n. In symbols

$$P(A) = \frac{m}{n}$$

Where m = number of ways that are favourable to the occurrence of A and n = the total number of outcomes of the experiment

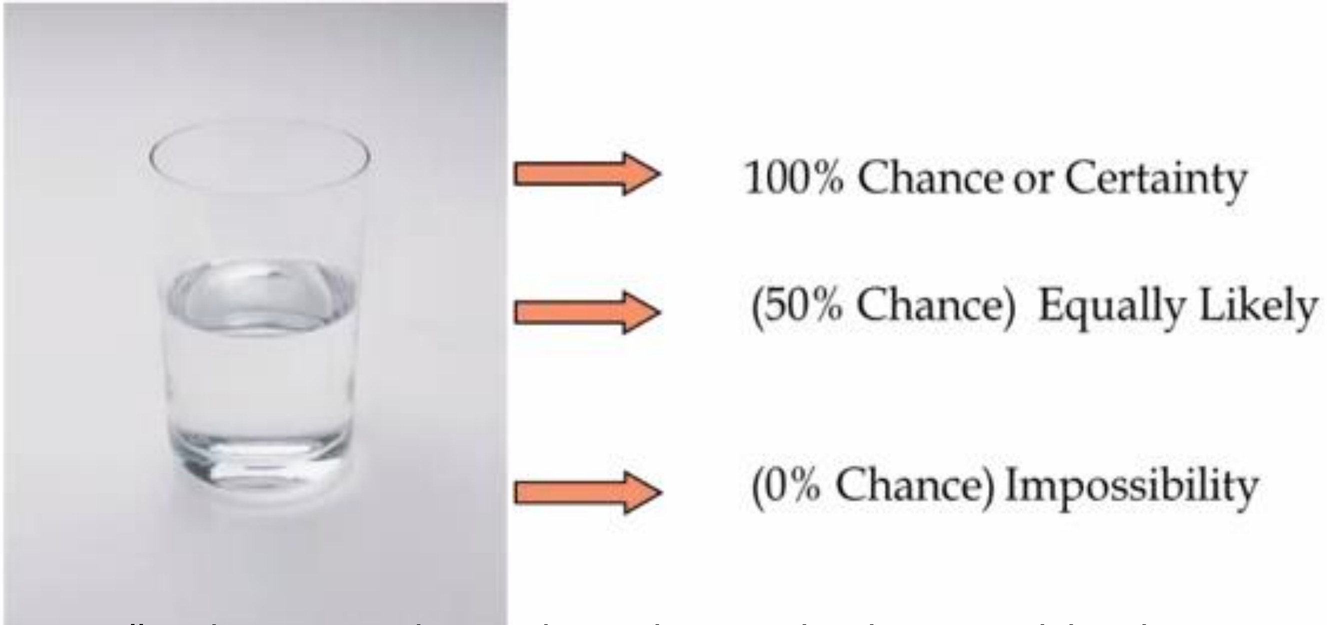
(All possible outcomes)

Please note that P(A) is always >= 0 and always <=1.

P(A) is a pure number.

Diagram explaining three extreme values of probability

- The range with in which probability of an event lies can be best understood by the following diagram. The glass shows three stages – Empty, half-full, and full to explain the properties of probability.

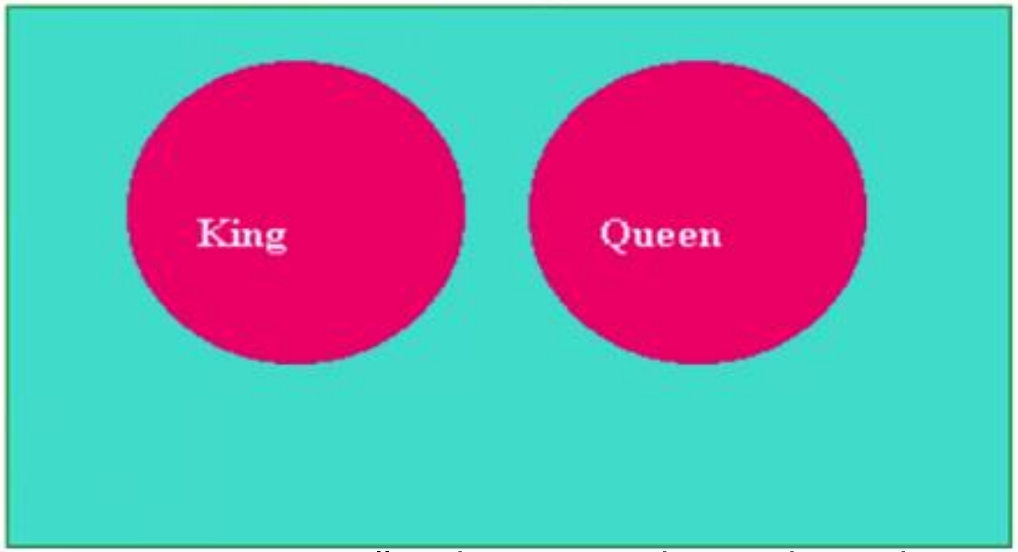


Types of Probability

- A Priori classical probability
- Empirical probability
- Subjective probability

Mutually exclusive events

Two events A and B are said to be mutually exclusive if the occurrence of A precludes the occurrence of B. For example, from a well shuffled pack of cards, if you pick up one card at random and would like to know whether it is a King or a Queen. The selected card will be either a King or a Queen. It cannot be both a King and a Queen. If King occurs, Queen will not occur and Queen occurs, King will not occur.



Independents events

Two events A and B are said to be independent if the occurrence of A is in no way influenced by the occurrence of B. Likewise occurrence of B is in no way influenced by the occurrence of A.

Rules for computing probability

- **1) Addition Rule – Mutually exclusive events**

$$P(A \cup B) = P(A) + P(B)$$

This rule says that the probability of the union of A and B is determined by adding the probability of the events A and B.

Here the symbol A U B is called A union B meaning A occurs, or B occurs or both A and B simultaneously occur.

When A and B are mutually exclusive, A and B cannot simultaneously occur.

Rules for computing probability

- **1) Addition Rule – Events are not Mutually exclusive events**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

This rule says that the probability of the union of A and B is determined by adding the probability of the event A and B and then subtracting the probability of the intersection of the events A and B.

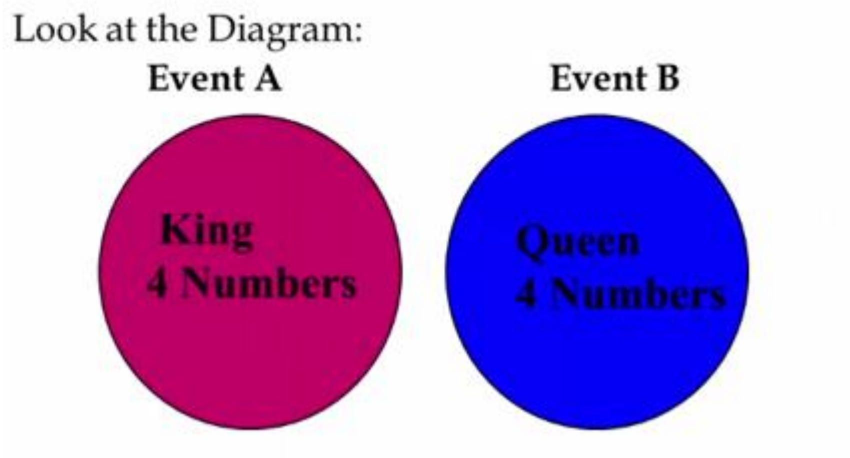
The symbol $A \cap B$ is called A intersection B meaning both A and B simultaneously occur.

Example of Addition Rules:

From a pack of well-shuffled cards, a card is picked up at random.

1) What is the probability of selected card is a King or a Queen?
2) What is the probability that the selected card is a King or a Diamond

Solution to part 1)



- Let A = getting a King
- Let B = getting a Queen

There are 4 kings and there are 4 Queens. The events are clearly mutually exclusive. Apply the formula.

$P(A \cup B) = P(A) + P(B) = 4/52 + 4/52 = 8/52 = 2/13$

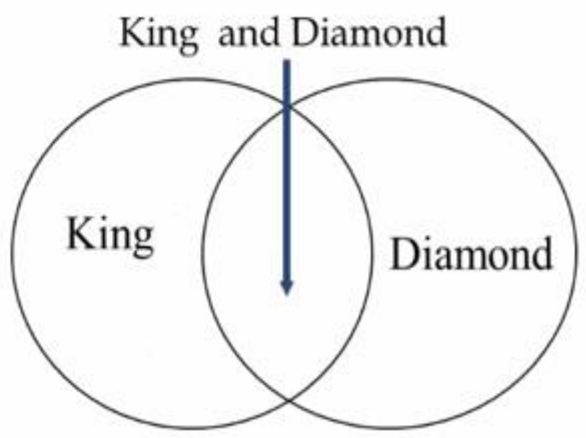
Solution to part 2)

- Look at the diagram:

There are totally 52 cards in the pack out of which 4 are kings and 13 are diamonds. Let the event A be getting a King and the event B be getting a Diamond. The two events here are not mutually exclusive because you can have a card, which is both king and a Diamond called King Diamond.

$$P(K \cup D) = P(K) + P(D) - P(K \cap D)$$

$$= 4/52 + 13/52 - 1/52 = 16/52 = 4/13$$



Multiplication rule

- Independent events

$P(A \cap B) = P(A).P(B)$

This rule says when the two events A and B are independent the probability of the simultaneous occurrence of A and B (also known as probability of intersection A and B) equals the product of the probability of A and the probability of B.

Of course this rule can be extended to more than two events.

Multiplication rule

Independent Events-Example

- Example:

The probability that you will get an A grade in Quantitative Methods is 0.7. The probability that you will get an A grade in Marketing is 0.5. Assuming these two courses are independent, compute the probability that you will get an A grade in both these subjects.

- Solution:

Let A = getting A grade in Quantitative Methods

Let B =getting A grade in Marketing

It is given that A and B are independent.

$P(A \cap B) = P(A).P(B) = 0.7.0.5 = 0.35$

Multiplication Rule

- **Events are not independent (conditional probability)**

$$P(A \cap B) = P(A).P(B/A)$$

- This rule says that the probability of the intersection of the events A and B equals the product of the probability of A and the probability of B given that A has happened or known to you. This is symbolized in the second term of the above expression as P (B/A). P(B/A) is called the conditional probability of B given the fact that A has happened.

- We can also write $P(A \cap B) = P(B).P(A/B)$ if B has already happened.

Multiplication rule

Events are not independent-Example

From a pack of cards, 2 cards are drawn in succession one after the other. After every draw, the selected card is not replaced. What is the probability that in both the draws you will get Spades?

Solution:

Let A = getting spade in the first draw

Let B = getting spade in the second draw.

The cards are not replaced.

This situation requires the use of conditional probability.

- P(A) = 13/52 (There are 13 Spades and 52 cards in a pack)
- P(B/A) = 12/51(There are 12 Spades and 51 cards because the first card selected
- $P(A \cap B) = P(B).P(A/B) = (13/52).(12/51)=156/2652= 1/17$.

Marginal Probability

- Contingency table consists of rows and columns of two attributes at different levels with frequencies or numbers in each of the cells. It is a matrix of frequencies assigned to rows and columns.

- The term marginal is used to indicate that the probabilities are calculated using contingency table (also called joint probability table).

Marginal Probability-Example

- A survey involving 200 families was conducted. Information regarding family income per year and whether the family buys a car are given in the following table.

Family	Income below Rs 10 Lakhs	Income of Rs. > =10 lakhs	Total
Buyer of Car	38	42	80
Non-Buyer	82	38	120
Total	120	80	200

- a) What is the probability that a randomly selected family is a buyer of the car?
- b) What is the probability that a randomly selected family is both a buyer of car and belonging to income of Rs.10 lakhs and above?
- c) A family selected at random is found to be belonging to income of Rs 10 lakhs and above. What is the probability that this family is buyer of car?

Solution

- a) What is the probability that a randomly selected family is a buyer of the Car?

  80/200=0.40.

- b) What is the probability that a randomly selected family is both a buyer of car and belonging to income of Rs.10 lakhs and above?

  42/200 =0.21

- c) A family selected at random is found to be belonging to income of Rs 10 lakhs and above. What is the probability that this family is buyer of car?

  42/80 =0.525. Note this is a case of conditional probability of buyer given income is Rs.10 lakhs and above.

Bayes' Theorem

- Bayes' Theorem is used to revise previously calculated probabilities based on new information.

- Developed by Thomas Bayes in the 18th Century.

- It is an extension of conditional probability.

Bayes’ Theorem

Given a hypothesis H and evidence E, Bayes' theorem states that the relationship between the probability of the hypothesis $P(H)$ before getting the evidence and the probability $P(H \mid E)$ of the hypothesis after getting the evidence is

$$P(H \mid E) = \frac{P(E \mid H)}{P(E)} P(H).$$

For simplicity, let us assume that the Hypothesis and the Evidence are two events.

Many modern machine learning techniques rely on Bayes' theorem. For instance, spam filters use Bayesian updating to determine whether an email is real or spam, given the words in the email. Additionally, many specific techniques in statistics, such as calculating p-values or interpreting medical results, are best described in terms of how they contribute to updating hypotheses using Bayes' theorem.

Bayes' Theorem

$$P(B_i \mid A) = \frac{P(A \mid B_i)P(B_i)}{P(A \mid B_1)P(B_1) + P(A \mid B_2)P(B_2) + \cdots + P(A \mid B_k)P(B_k)}$$

- Where:
  - Bi = i^{th} event of K mutually exclusive and collectively exhaustive events.
  - A = new event that might impact P(Bi)

Bayes Theorem Discussion Problem

- A drilling company has estimated a 40% chance of striking oil for their new well.

- A detailed test has been scheduled for more information. Historically, 60% of successful wells have had detailed tests, and 20% of unsuccessful wells have had detailed tests.

- Given that this well has been scheduled for a detailed test, what is the probability that the well will be successful?

Some interesting problems for discussion

- Of the cars on a used car lot, 70% have air conditioning (AC) and 40% have a CD player (CD). 20% of the cars have both.

- What is the probability that a car has a CD player, given that it has AC?
  - –i.e., we want to find P(CD | AC)

Association of Attributes

- Of 37 men and 33 women, 36 are teetotallers (completely abstain from alcoholic beverages). Nine of the women are non-smokers and 18 of the men smoke but do not drink. 13 of the men and seven of the women drink but do not smoke.

- How many, both drink and smoke? What is the associated probability?