

Lista 1

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1. Introdução

Nossa implementação dos algoritmos foi feita na linguagem C++. Além do arquivo `main.cpp`, organizamos o código em outros dois arquivos: `state.cpp`, que implementa uma classe que representa um estado do problema 8 puzzle ou 15 puzzle, e `search.cpp`, que implementa os algoritmos de busca. Baseamos os algoritmos em uma função de busca em grafos generalizada, onde executando um algoritmo mudamos apenas a estrutura de dados da fronteira de busca, e implementamos as filas ou listas de prioridades de cada algoritmo. Dessa forma pudemos facilitar a adição de novos algoritmos no programa, mantendo as mesmas entradas e saídas esperadas. A performance de execução das instâncias de 8 puzzle foi bastante satisfatória, porém para as instâncias de 15 puzzle o programa consumiu bastante tempo e espaço de memória. Acreditamos que, em troca da generalização de código que acelerou a implementação, perdemos em questão da otimização de espaço, e para instâncias do problema 15 puzzle o conjunto de estados expandidos cresceu além do que esperávamos.

2. Resultados

O código-fonte foi compilado usando o compilador de C++ GNU (g++ 11.3.0), em um subsistema Ubuntu 22.04 operando sob um sistema Windows 10. A máquina da execução possuía uma CPU AMD Ryzen 3 3200G de 3.6 GHz, e 16 GB de memória RAM. Para cada algoritmo todas as instâncias de 8-puzzle ou 15-puzzle foram executadas uma vez, e os resultados foram armazenados em um arquivo CSV. O comando utilizado para executar o programa em todas as instancias foi `xargs -a ./15puzzle_instances.txt -I{} -d'\n' ./main -astar {} > 15puzzle-astar.csv`. Na tabela a seguir apresentamos as médias das métricas calculadas para cada algoritmo. Em seguida apresentamos todos os resultados de cada instância para cada algoritmo.

Algoritmo	Nº Nodos	Tamanho Solução	Tempo (s)	Média h(s)	h(s) Inicial
BFS-Graph	96822,81	22,16	0,06405808563	0	13,88
Iterative Deepening	2578290,56	22,16	0,3341968899	0	13,88
A*	895	22,16	0,00046032079	10,0277522	13,88
IDA*	2373,03	22,16	0,00062721205	10,4150672	13,88
GBFS	392,42	140,52	0,00019755027	6,8930464	13,88
A* (15-puzzle)	9506440,74	52,70	18,81081575	25,81806701	36,85

2.1. BFS-Graph

Instância	Nº Nodos	Tamanho Solução	Tempo (s)	Média h(s)	h(s) Inicial
5 6 2 7 1 8 3 4 0	70436	22	0.0238807000	0	12
5 3 2 8 7 1 4 0 6	98412	23	0.0344589000	0	15
1 0 7 5 2 4 3 8 6	11718	17	0.0030292000	0	13
3 6 4 2 8 0 5 7 1	134664	25	0.0507454010	0	17
0 6 1 7 4 2 3 8 5	5209	16	0.0012406000	0	10
0 3 8 2 1 4 7 6 5	59830	22	0.0184665000	0	12
0 8 2 1 5 6 7 3 4	71614	22	0.0233370000	0	14
5 0 2 6 4 8 1 7 3	42704	21	0.0125782000	0	11
3 0 8 4 1 2 6 7 5	10486	17	0.0025411000	0	7
7 2 6 3 1 5 8 0 4	55230	21	0.0165776000	0	13
2 4 7 0 3 6 8 1 5	86288	23	0.0280493000	0	15
2 1 6 0 4 3 7 5 8	87902	23	0.0292331000	0	11
1 2 8 0 3 7 5 6 4	24271	19	0.0066745000	0	13
1 4 8 2 7 6 0 3 5	67445	22	0.0216364010	0	14
5 0 2 1 6 8 7 3 4	21294	19	0.0057169000	0	13
6 2 1 5 0 4 3 8 7	67976	22	0.0211834000	0	10
1 8 0 4 5 2 6 7 3	30932	20	0.0087703000	0	10
7 6 4 8 1 3 5 0 2	51926	21	0.0154127000	0	19
1 8 7 5 0 4 2 3 6	74010	22	0.0237303000	0	18
2 8 0 4 1 7 5 6 3	57200	22	0.0173578000	0	16
7 8 1 3 6 2 4 5 0	6175	16	0.0015035000	0	14
1 4 7 3 6 2 0 8 5	4890	16	0.0010873000	0	10
5 2 3 4 7 1 8 0 6	128087	25	0.0465070000	0	15
5 1 3 2 6 4 7 8 0	59140	22	0.0183232000	0	14
1 0 2 5 8 6 4 7 3	86044	23	0.0291641010	0	13
4 7 6 0 5 2 3 8 1	124428	25	0.0461572000	0	15
5 8 2 1 7 6 3 0 4	55828	21	0.0172760000	0	15
3 0 2 6 1 4 5 7 8	1904	13	0.0004236000	0	7
2 0 1 7 5 3 8 6 4	89286	23	0.0301545000	0	13
8 1 0 5 3 7 2 4 6	96167	24	0.0323363000	0	16
0 2 6 5 4 7 1 3 8	108997	24	0.0391493000	0	14
4 5 0 1 7 2 3 8 6	6800	16	0.0017495000	0	12
0 6 3 5 8 2 1 7 4	12237	18	0.0030908000	0	16
1 4 8 0 7 5 2 6 3	91221	23	0.0310486000	0	13
1 5 0 8 4 3 2 7 6	146642	26	0.0571480000	0	14
2 3 6 8 1 0 5 4 7	166169	27	0.0701069010	0	17
1 6 0 7 3 8 4 5 2	61999	22	0.0190899000	0	14
8 4 3 1 6 5 0 7 2	149038	26	0.0585498000	0	14
3 8 2 7 5 1 0 6 4	66394	22	0.0267202010	0	12
7 2 3 6 4 8 1 0 5	134964	25	0.0515792000	0	13
3 1 2 4 0 8 5 7 6	3528	14	0.0007811000	0	8

8 5 3 1 2 6 4 0 7	141058	25	0.0548922000	0	19
2 0 8 6 1 3 4 7 5	51334	21	0.0158261010	0	11
3 6 7 1 2 8 4 5 0	147235	26	0.0579715000	0	16
0 3 6 1 2 8 4 7 5	96629	24	0.0321118000	0	14
4 8 6 5 0 7 1 3 2	117820	24	0.0422223010	0	20
7 2 0 1 8 6 3 4 5	12721	18	0.0031686000	0	14
4 0 7 1 3 2 5 8 6	82402	23	0.0268742000	0	15
1 4 3 0 5 8 7 6 2	55950	21	0.0172838000	0	11
1 3 6 5 8 4 0 2 7	170358	28	0.0732455000	0	16
3 1 6 4 0 8 7 5 2	41346	20	0.0123691000	0	12
8 0 5 1 4 3 7 2 6	163708	27	0.0707879000	0	15
1 0 2 8 7 3 4 6 5	25948	19	0.0073001000	0	11
5 8 1 4 7 3 2 6 0	146431	26	0.0584572010	0	16
6 4 1 2 0 3 7 8 5	16644	18	0.0042852000	0	12
6 1 7 4 8 3 5 2 0	113398	24	0.0408610000	0	16
0 1 5 3 8 7 6 4 2	12763	18	0.0034726000	0	8
7 8 2 6 4 3 0 1 5	102477	24	0.0358052000	0	12
6 0 8 7 4 3 5 2 1	91593	23	0.0313287010	0	17
5 4 0 7 6 2 8 3 1	116022	24	0.0422914000	0	16
5 6 1 8 7 0 2 4 3	170975	27	0.0738326000	0	19
1 6 7 4 2 0 3 5 8	132375	25	0.0498580000	0	13
5 3 6 4 7 2 1 0 8	43631	21	0.0131976000	0	15
0 8 7 4 2 6 1 3 5	61343	22	0.0191112000	0	18
7 2 0 5 4 3 8 6 1	173107	28	0.0741226010	0	14
5 3 0 6 7 1 8 2 4	37323	20	0.0105703000	0	16
5 7 2 8 3 6 4 0 1	166627	27	0.0699893000	0	17
3 7 8 1 4 6 5 0 2	165544	27	0.0707585010	0	15
7 4 5 8 6 3 2 0 1	136365	25	0.0519480000	0	19
0 4 1 8 2 5 3 7 6	101428	24	0.0353975010	0	10
5 1 7 2 3 6 8 4 0	99337	24	0.0341860000	0	16
6 3 4 8 2 1 5 7 0	69657	22	0.0220192000	0	16
2 0 1 5 7 6 4 8 3	169153	27	0.0719568010	0	15
7 8 0 6 3 1 2 5 4	146614	26	0.0567947000	0	18
6 2 4 3 7 0 5 1 8	136769	25	0.0525046000	0	11
7 5 4 8 3 6 0 2 1	144168	26	0.0551530000	0	20
3 8 4 5 7 1 0 6 2	32178	20	0.0090125000	0	14
5 6 2 8 7 1 0 3 4	149505	26	0.0601459000	0	16
3 2 4 6 5 8 0 7 1	6810	16	0.0017625000	0	10
7 5 1 6 8 4 2 0 3	165286	27	0.0708087010	0	17
3 5 2 0 8 1 4 6 7	20088	19	0.0054998000	0	11
8 0 6 7 1 3 5 2 4	134133	25	0.0523063000	0	21
2 5 8 4 1 0 6 3 7	3471	15	0.0008005000	0	11
7 0 2 1 8 6 3 4 5	9452	17	0.0025548000	0	13
2 1 5 3 8 0 6 7 4	4001	15	0.0009166000	0	7
3 8 6 4 7 0 1 5 2	134421	25	0.0514021000	0	17
1 0 7 2 4 3 6 8 5	10951	17	0.0027626000	0	11
3 6 7 5 8 0 4 1 2	165164	27	0.0711064010	0	17

6 2 0 3 1 4 8 7 5	32194	20	0.0093864000	0	8
7 1 5 4 0 3 2 6 8	37083	20	0.0108822000	0	12
8 0 7 4 6 1 5 2 3	163614	27	0.0670292000	0	21
8 2 0 5 7 6 1 3 4	145000	26	0.0557741010	0	18
3 8 5 7 0 1 6 4 2	68791	22	0.0222633000	0	12
3 5 4 1 7 6 0 2 8	69185	22	0.0221562000	0	14
0 2 8 5 4 1 7 3 6	66051	22	0.0212806000	0	12
4 3 7 0 6 1 5 8 2	25603	19	0.0071873000	0	17
2 1 3 4 6 5 0 7 8	30499	20	0.0084542000	0	8
1 0 5 4 6 2 7 3 8	45472	21	0.0132723000	0	9
7 2 3 1 5 4 8 6 0	106698	24	0.0407068010	0	14
5 7 2 6 4 0 8 3 1	130566	25	0.0481584000	0	13

2.2. Iterative Deepening

Instância	Nº Nodos	Tamanho Solução	Tempo (s)	Média h(s)	h(s) Inicial
5 6 2 7 1 8 3 4 0	809224	22	0.1660324400	0	12
5 3 2 8 7 1 4 0 6	1659716	23	0.3361890670	0	15
1 0 7 5 2 4 3 8 6	56332	17	0.0108979920	0	13
3 6 4 2 8 0 5 7 1	3832745	25	0.8245119390	0	17
0 6 1 7 4 2 3 8 5	20305	16	0.0062137460	0	10
0 3 8 2 1 4 7 6 5	572929	22	0.1203127540	0	12
0 8 2 1 5 6 7 3 4	840836	22	0.1755698610	0	14
5 0 2 6 4 8 1 7 3	336804	21	0.0738202870	0	11
3 0 8 4 1 2 6 7 5	48030	17	0.0111341570	0	7
7 2 6 3 1 5 8 0 4	523656	21	0.1128828370	0	13
2 4 7 0 3 6 8 1 5	1183127	23	0.2412914090	0	15
2 1 6 0 4 3 7 5 8	1229083	23	0.2580326620	0	11
1 2 8 0 3 7 5 6 4	144153	19	0.0291273200	0	13
1 4 8 2 7 6 0 3 5	726908	22	0.1550685670	0	14
5 0 2 1 6 8 7 3 4	119854	19	0.0255590450	0	13
6 2 1 5 0 4 3 8 7	746437	22	0.1559863600	0	10
1 8 0 4 5 2 6 7 3	201422	20	0.0479386480	0	10
7 6 4 8 1 3 5 0 2	463439	21	0.0965939090	0	19
1 8 7 5 0 4 2 3 6	880730	22	0.2889956780	0	18
2 8 0 4 1 7 5 6 3	529433	22	0.1129323300	0	16
7 8 1 3 6 2 4 5 0	25389	16	0.0050089060	0	14
1 4 7 3 6 2 0 8 5	18803	16	0.0050339560	0	10
5 2 3 4 7 1 8 0 6	3233556	25	0.7456225220	0	15
5 1 3 2 6 4 7 8 0	559966	22	0.1266916660	0	14
1 0 2 5 8 6 4 7 3	1161412	23	0.2434183710	0	13
4 7 6 0 5 2 3 8 1	3012022	25	0.6540714070	0	15
5 8 2 1 7 6 3 0 4	533665	21	0.1192534080	0	15
3 0 2 6 1 4 5 7 8	6472	13	0.0012565000	0	7
2 0 1 7 5 3 8 6 4	1258485	23	0.2802458380	0	13

8 1 0 5 3 7 2 4 6	1506581	24	0.3152866860	0	16
0 2 6 5 4 7 1 3 8	2082556	24	0.4574162000	0	14
4 5 0 1 7 2 3 8 6	28816	16	0.0061635700	0	12
0 6 3 5 8 2 1 7 4	57963	18	0.0124422390	0	16
1 4 8 0 7 5 2 6 3	1339032	23	0.2885835590	0	13
1 5 0 8 4 3 2 7 6	5227219	26	1.1733282310	0	14
2 3 6 8 1 0 5 4 7	10582097	27	2.3182193130	0	17
1 6 0 7 3 8 4 5 2	618550	22	0.1391113570	0	14
8 4 3 1 6 5 0 7 2	5647709	26	1.2264032890	0	14
3 8 2 7 5 1 0 6 4	707091	22	0.1965260770	0	12
7 2 3 6 4 8 1 0 5	3906491	25	0.8946849770	0	13
3 1 2 4 0 8 5 7 6	13166	14	0.0039152020	0	8
8 5 3 1 2 6 4 0 7	4826279	25	1.0611260120	0	19
2 0 8 6 1 3 4 7 5	445186	21	0.0979491430	0	11
3 6 7 1 2 8 4 5 0	5301125	26	1.1607984680	0	16
0 3 6 1 2 8 4 7 5	1521264	24	0.3326156330	0	14
4 8 6 5 0 7 1 3 2	2550556	24	0.5403079660	0	20
7 2 0 1 8 6 3 4 5	60744	18	0.0135334580	0	14
4 0 7 1 3 2 5 8 6	1071876	23	0.2290364340	0	15
1 4 3 0 5 8 7 6 2	542261	21	0.1247298390	0	11
1 3 6 5 8 4 0 2 7	13426501	28	2.9414974440	0	16
3 1 6 4 0 8 7 5 2	323493	20	0.0720937810	0	12
8 0 5 1 4 3 7 2 6	9617303	27	2.1283892230	0	15
1 0 2 8 7 3 4 6 5	160297	19	0.0363235610	0	11
5 8 1 4 7 3 2 6 0	5150682	26	1.1021570540	0	16
6 4 1 2 0 3 7 8 5	86884	18	0.0201859560	0	12
6 1 7 4 8 3 5 2 0	2296791	24	0.5231932710	0	16
0 1 5 3 8 7 6 4 2	60654	18	0.0136185770	0	8
7 8 2 6 4 3 0 1 5	1715624	24	0.3880041000	0	12
6 0 8 7 4 3 5 2 1	1336827	23	0.2845038160	0	17
5 4 0 7 6 2 8 3 1	2536222	24	0.5650554900	0	16
5 6 1 8 7 0 2 4 3	15171649	27	3.4996155310	0	19
1 6 7 4 2 0 3 5 8	3587527	25	0.8222646220	0	13
5 3 6 4 7 2 1 0 8	344582	21	0.0729628050	0	15
0 8 7 4 2 6 1 3 5	599226	22	0.1294691480	0	18
7 2 0 5 4 3 8 6 1	15073854	28	4.0151483630	0	14
5 3 0 6 7 1 8 2 4	277227	20	0.0656799800	0	16
5 7 2 8 3 6 4 0 1	10843669	27	3.0805376040	0	17
3 7 8 1 4 6 5 0 2	10295974	27	2.4667308600	0	15
7 4 5 8 6 3 2 0 1	4095213	25	0.9353491320	0	19
0 4 1 8 2 5 3 7 6	1665813	24	0.3737612210	0	10
5 1 7 2 3 6 8 4 0	1583030	24	0.3288421260	0	16
6 3 4 8 2 1 5 7 0	792904	22	0.1732748660	0	16
2 0 1 5 7 6 4 8 3	12496979	27	3.1376473120	0	15
7 8 0 6 3 1 2 5 4	5225955	26	1.1993678230	0	18
6 2 4 3 7 0 5 1 8	4088312	25	0.9689706240	0	11
7 5 4 8 3 6 0 2 1	4873453	26	1.1188244310	0	20

3 8 4 5 7 1 0 6 2	214527	20	0.0539222350	0	14
5 6 2 8 7 1 0 3 4	5735592	26	1.2987518050	0	16
3 2 4 6 5 8 0 7 1	28881	16	0.0059396110	0	10
7 5 1 6 8 4 2 0 3	10128430	27	2.3696119060	0	17
3 5 2 0 8 1 4 6 7	110829	19	0.0237883220	0	11
8 0 6 7 1 3 5 2 4	3745531	25	0.8674164140	0	21
2 5 8 4 1 0 6 3 7	12608	15	0.0028431590	0	11
7 0 2 1 8 6 3 4 5	42323	17	0.0100484940	0	13
2 1 5 3 8 0 6 7 4	15054	15	0.0029220410	0	7
3 8 6 4 7 0 1 5 2	3814064	25	0.8471337960	0	17
1 0 7 2 4 3 6 8 5	51080	17	0.0116710350	0	11
3 6 7 5 8 0 4 1 2	10083429	27	2.2651046680	0	17
6 2 0 3 1 4 8 7 5	216000	20	0.0465596830	0	8
7 1 5 4 0 3 2 6 8	269408	20	0.0600377160	0	12
8 0 7 4 6 1 5 2 3	9608157	27	2.1372809250	0	21
8 2 0 5 7 6 1 3 4	4963354	26	1.0741641230	0	18
3 8 5 7 0 1 6 4 2	761900	22	0.1710867760	0	12
3 5 4 1 7 6 0 2 8	779904	22	0.1711376420	0	14
0 2 8 5 4 1 7 3 6	713444	22	0.1551618230	0	12
4 3 7 0 6 1 5 8 2	156601	19	0.0356671450	0	17
2 1 3 4 6 5 0 7 8	197513	20	0.0422433500	0	8
1 0 5 4 6 2 7 3 8	365116	21	0.0793671900	0	9
7 2 3 1 5 4 8 6 0	1905883	24	0.4082052940	0	14
5 7 2 6 4 0 8 3 1	3411288	25	0.7389257380	0	13

2.3. A*

Instância	Nº Nós	Tamanho Solução	Tempo (s)	Média h(s)	h(s) Inicial
5 6 2 7 1 8 3 4 0	835	22	0.0006071490	9.08244	12
5 3 2 8 7 1 4 0 6	1024	23	0.0008473970	10.0667	15
1 0 7 5 2 4 3 8 6	72	17	0.0000629320	9.15079	13
3 6 4 2 8 0 5 7 1	953	25	0.0007617740	11.6783	17
0 6 1 7 4 2 3 8 5	91	16	0.0001429450	6.72561	10
0 3 8 2 1 4 7 6 5	652	22	0.0004395930	9.37182	12
0 8 2 1 5 6 7 3 4	350	22	0.0002695180	10.4395	14
5 0 2 6 4 8 1 7 3	612	21	0.0004726340	9.81136	11
3 0 8 4 1 2 6 7 5	232	17	0.0001222440	7.8625	7
7 2 6 3 1 5 8 0 4	177	21	0.0000922030	10.8271	13
2 4 7 0 3 6 8 1 5	511	23	0.0002630690	9.97655	15
2 1 6 0 4 3 7 5 8	1037	23	0.0005653980	10.3864	11
1 2 8 0 3 7 5 6 4	197	19	0.0001085040	8.38938	13
1 4 8 2 7 6 0 3 5	697	22	0.0004731540	9.28096	14
5 0 2 1 6 8 7 3 4	239	19	0.0001302640	7.90571	13
6 2 1 5 0 4 3 8 7	1622	22	0.0013026310	9.72591	10
1 8 0 4 5 2 6 7 3	315	20	0.0001669750	8.87339	10

764813502	30	21	0.0000409310	10.3269	19
187504236	373	22	0.0001915460	10.5618	18
280417563	550	22	0.0002870090	10.553	16
781362450	34	16	0.0000224910	8.22951	14
147362085	118	16	0.0000659030	8.47549	10
523471806	1449	25	0.0007641240	10.8645	15
513264780	873	22	0.0004787550	10.0839	14
102586473	808	23	0.0005660970	9.78436	13
476052381	1167	25	0.0006225300	10.5056	15
582176304	308	21	0.0001633750	9.42476	15
302614578	41	13	0.0000296710	6.08	7
201753864	533	23	0.0003334410	11.0371	13
810537246	346	24	0.0002117670	11.5	16
026547138	929	24	0.0007849750	10.2885	14
450172386	117	16	0.0000738120	8.69951	12
063582174	26	18	0.0000194410	9.02083	16
148075263	666	23	0.0004094130	10.136	13
150843276	1734	26	0.0010330320	10.7851	14
236810547	1824	27	0.0011510160	12.1416	17
160738452	231	22	0.0001392750	11.4182	14
843165072	1807	26	0.0010877440	10.7077	14
382751064	655	22	0.0004028730	9.45545	12
723648105	2298	25	0.0014084450	10.2244	13
312408576	76	14	0.0000602820	7.45185	8
853126407	786	25	0.0005775480	11.2696	19
208613475	609	21	0.0003656520	9.22804	11
367128450	2231	26	0.0013193010	10.6504	16
036128475	1757	24	0.0010830030	9.85137	14
486507132	299	24	0.0003815430	11.503	20
720186345	41	18	0.0000295910	9.95946	14
407132586	543	23	0.0004579440	10.6895	15
143058762	438	21	0.0002584980	10.0621	11
136584027	2864	28	0.0017140230	12.2559	16
316408752	221	20	0.0001281940	9.78497	12
805143726	2849	27	0.0016502420	11.0714	15
102873465	179	19	0.0001040830	8.82792	11
581473260	1075	26	0.0007419130	11.5799	16
641203785	177	18	0.0001031130	8.58117	12
617483520	752	24	0.0004198030	10.0917	16
015387642	182	18	0.0001207240	8.1865	8
782643015	1355	24	0.0007872240	10.0853	12
608743521	205	23	0.0001168930	11.0603	17
540762831	712	24	0.0003843220	10.5858	16
561870243	1421	27	0.0007785240	12.0228	19
167420358	2186	25	0.0012701490	9.91612	13
536472108	168	21	0.0000955230	10.6818	15

0 8 7 4 2 6 1 3 5	39	22	0.0000258810	12.0147	18
7 2 0 5 4 3 8 6 1	4038	28	0.0023962550	11.1541	14
5 3 0 6 7 1 8 2 4	90	20	0.0000522210	9.61688	16
5 7 2 8 3 6 4 0 1	2905	27	0.0016044410	10.8626	17
3 7 8 1 4 6 5 0 2	2301	27	0.0012811100	11.8772	15
7 4 5 8 6 3 2 0 1	698	25	0.0004104220	10.8711	19
0 4 1 8 2 5 3 7 6	2190	24	0.0012311380	9.48527	10
5 1 7 2 3 6 8 4 0	936	24	0.0005242070	11.8384	16
6 3 4 8 2 1 5 7 0	238	22	0.0001281240	9.92629	16
2 0 1 5 7 6 4 8 3	3038	27	0.0017090340	10.957	15
7 8 0 6 3 1 2 5 4	1632	26	0.0008847780	10.8184	18
6 2 4 3 7 0 5 1 8	4306	25	0.0024495870	10.3003	11
7 5 4 8 3 6 0 2 1	739	26	0.0003986120	11.9036	20
3 8 4 5 7 1 0 6 2	382	20	0.0003964220	9.36435	14
5 6 2 8 7 1 0 3 4	2084	26	0.0016857820	10.7022	16
3 2 4 6 5 8 0 7 1	99	16	0.0000702520	7.52841	10
7 5 1 6 8 4 2 0 3	2764	27	0.0014964270	11.4317	17
3 5 2 0 8 1 4 6 7	428	19	0.0003099100	8.66071	11
8 0 6 7 1 3 5 2 4	715	25	0.0003695910	13.0852	21
2 5 8 4 1 0 6 3 7	68	15	0.0000398110	7.35246	11
7 0 2 1 8 6 3 4 5	34	17	0.0000212310	9.27419	13
2 1 5 3 8 0 6 7 4	121	15	0.0000659120	7.37799	7
3 8 6 4 7 0 1 5 2	993	25	0.0005213660	10.7449	17
1 0 7 2 4 3 6 8 5	163	17	0.0000896630	8.92553	11
3 6 7 5 8 0 4 1 2	2389	27	0.0012892100	11.712	17
6 2 0 3 1 4 8 7 5	369	20	0.0002406770	8.33812	8
7 1 5 4 0 3 2 6 8	356	20	0.0002334970	8.59302	12
8 0 7 4 6 1 5 2 3	1461	27	0.0007797550	11.1426	21
8 2 0 5 7 6 1 3 4	854	26	0.0004534140	11.4814	18
3 8 5 7 0 1 6 4 2	730	22	0.0005879790	9.60016	12
3 5 4 1 7 6 0 2 8	257	22	0.0002076070	11.3218	14
0 2 8 5 4 1 7 3 6	414	22	0.0003450500	9.85248	12
4 3 7 0 6 1 5 8 2	181	19	0.0000982730	9.89644	17
2 1 3 4 6 5 0 7 8	336	20	0.0004156330	9.109	8
1 0 5 4 6 2 7 3 8	707	21	0.0003801010	8.57759	9
7 2 3 1 5 4 8 6 0	1467	24	0.0007914550	11.0388	14
5 7 2 6 4 0 8 3 1	1319	25	0.0007101120	10.7828	13

2.4. IDA*

Instância	Nº Nodos	Tamanho Solução	Tempo (s)	Média h(s)	h(s) Inicial
5 6 2 7 1 8 3 4 0	2631	22	0.0011576860	9.77497	12
5 3 2 8 7 1 4 0 6	3276	23	0.0013062610	10.5943	15
1 0 7 5 2 4 3 8 6	145	17	0.0000545010	9.95062	13
3 6 4 2 8 0 5 7 1	2720	25	0.0012766090	11.7755	17

061742385	165	16	0.0000599020	7.09677	10
038214765	1734	22	0.0008271460	9.23979	12
082156734	1741	22	0.0005565180	10.4312	14
502648173	439	21	0.0001466650	10.7479	11
308412675	464	17	0.0001549850	7.5844	7
726315804	611	21	0.0002972390	11.494	13
247036815	724	23	0.0002786390	11.7103	15
216043758	1659	23	0.0005117760	10.9092	11
128037564	372	19	0.0001779260	9.04984	13
148276035	1420	22	0.0004700050	9.65021	14
502168734	465	19	0.0001594150	8.18813	13
621504387	2130	22	0.0006650710	9.6343	10
180452673	794	20	0.0002756380	9.22578	10
764813502	71	21	0.0000305210	12.8624	19
187504236	456	22	0.0001578250	11.5224	18
280417563	368	22	0.0001247440	10.8775	16
781362450	27	16	0.0000142110	9.42857	14
147362085	76	16	0.0000306810	8.51639	10
523471806	2805	25	0.0009811840	11.1237	15
513264780	805	22	0.0002615390	10.1979	14
102586473	1666	23	0.0005305170	10.2797	13
476052381	2470	25	0.0007964250	10.3386	15
582176304	1294	21	0.0004230630	9.67426	15
302614578	100	13	0.0000409310	6.92655	7
201753864	1609	23	0.0005445070	11.2581	13
810537246	225	24	0.0000820330	13.9746	16
026547138	1820	24	0.0007922750	11.518	14
450172386	129	16	0.0000581920	9.44651	12
063582174	37	18	0.0000171010	10.0508	16
148075263	1500	23	0.0004827550	10.925	13
150843276	4075	26	0.0012478190	11.3632	14
236810547	5873	27	0.0020591640	12.1105	17
160738452	487	22	0.0001604350	11.7009	14
843165072	5296	26	0.0022577300	11.0939	14
382751064	2349	22	0.0011337450	9.76751	12
723648105	5296	25	0.0017724360	10.856	13
312408576	125	14	0.0000474020	7.92019	8
853126407	3436	25	0.0010636540	11.4408	19
208613475	1193	21	0.0004020520	9.66733	11
367128450	9239	26	0.0043848060	10.2424	16
036128475	2256	24	0.0007045920	10.429	14
486507132	626	24	0.0002068970	13.1367	20
720186345	92	18	0.0000363110	10.0403	14
407132586	1145	23	0.0003877020	11.0324	15
143058762	1201	21	0.0004012120	10.7242	11
136584027	5557	28	0.0017215730	12.3987	16
316408752	730	20	0.0002404570	9.96364	12

805143726	9355	27	0.0035750800	10.6335	15
102873465	661	19	0.0002974100	8.77668	11
581473260	2195	26	0.0006683610	12.0853	16
641203785	257	18	0.0000982430	9.28103	12
617483520	2440	24	0.0007764940	10.6625	16
015387642	339	18	0.0001161530	8.48955	8
782643015	2316	24	0.0007355330	10.8741	12
608743521	272	23	0.0001153440	13.1748	17
540762831	3401	24	0.0010633630	10.8254	16
561870243	9544	27	0.0029377310	11.4995	19
167420358	8845	25	0.0028630890	9.66801	13
536472108	360	21	0.0001255040	10.3406	15
087426135	103	22	0.0000442410	13.3125	18
720543861	11359	28	0.0039619340	11.5501	14
530671824	406	20	0.0001752150	9.65875	16
572836401	8302	27	0.0025280790	11.2233	17
378146502	6762	27	0.0020877150	11.8568	15
745863201	2421	25	0.0008557770	11.3534	19
041825376	3944	24	0.0012410190	9.62323	10
517236840	899	24	0.0003179800	11.5017	16
634821570	868	22	0.0002852290	10.4893	16
201576483	14476	27	0.0044289670	10.9017	15
780631254	3141	26	0.0009639700	11.7179	18
624370518	8979	25	0.0027413360	9.91441	11
754836021	1631	26	0.0006544590	11.5896	20
384571062	413	20	0.0001985070	9.48012	14
562871034	8936	26	0.0027007940	10.4408	16
324658071	166	16	0.0000621020	8.79371	10
751684203	7986	27	0.0024376260	11.3515	17
352081467	490	19	0.0002294270	8.27494	11
806713524	324	25	0.0001063930	14.1297	21
258410637	56	15	0.0000235810	7.91011	11
702186345	83	17	0.0000327810	9.68148	13
215380674	186	15	0.0000673520	7.26198	7
386470152	2713	25	0.0008469970	11.4693	17
107243685	206	17	0.0000988130	9.24069	11
367580412	5691	27	0.0017861350	11.7926	17
620314875	1408	20	0.0004741640	8.11884	8
715403268	573	20	0.0001882960	9.41962	12
807461523	2764	27	0.0009141990	11.3216	21
820576134	2791	26	0.0008759870	10.9685	18
385701642	2006	22	0.0006359400	9.71186	12
354176028	1238	22	0.0006103890	10.9558	14
028541736	1016	22	0.0005146860	10.9727	12
437061582	244	19	0.0001313840	9.90074	17
213465078	722	20	0.0003383210	9.40443	8

1 0 5 4 6 2 7 3 8	2213	21	0.0008107950	8.08958	9
7 2 3 1 5 4 8 6 0	2265	24	0.0007418230	11.2732	14
5 7 2 6 4 0 8 3 1	3613	25	0.0011215250	10.6734	13

2.5. GBFS

Instância	Nº Nós	Tamanho Solução	Tempo (s)	Média h(s)	h(s) Inicial
5 6 2 7 1 8 3 4 0	579	156	0.0004513340	7.00877	12
5 3 2 8 7 1 4 0 6	437	187	0.0003637620	6.86951	15
1 0 7 5 2 4 3 8 6	430	189	0.0003586510	6.61075	13
3 6 4 2 8 0 5 7 1	466	207	0.0003129500	6.94545	17
0 6 1 7 4 2 3 8 5	16	16	0.0000128100	5.5	10
0 3 8 2 1 4 7 6 5	407	152	0.0002140370	6.59085	12
0 8 2 1 5 6 7 3 4	422	168	0.0003375700	6.6126	14
5 0 2 6 4 8 1 7 3	96	27	0.0000676730	8.05422	11
3 0 8 4 1 2 6 7 5	576	87	0.0004694850	6.73002	7
7 2 6 3 1 5 8 0 4	634	119	0.0003311410	7.05254	13
2 4 7 0 3 6 8 1 5	90	57	0.0000634620	6.80982	15
2 1 6 0 4 3 7 5 8	560	141	0.0002988100	6.93239	11
1 2 8 0 3 7 5 6 4	83	47	0.0000466510	5.89116	13
1 4 8 2 7 6 0 3 5	502	146	0.0002665280	6.7343	14
5 0 2 1 6 8 7 3 4	66	39	0.0000388010	6.13445	13
6 2 1 5 0 4 3 8 7	501	146	0.0002806990	6.69877	10
1 8 0 4 5 2 6 7 3	81	44	0.0000443120	6.51064	10
7 6 4 8 1 3 5 0 2	46	37	0.0000297610	7.86905	19
1 8 7 5 0 4 2 3 6	386	160	0.0002154760	6.8064	18
2 8 0 4 1 7 5 6 3	429	190	0.0002587180	6.8082	16
7 8 1 3 6 2 4 5 0	540	132	0.0002860290	6.77443	14
1 4 7 3 6 2 0 8 5	453	166	0.0002409380	6.64375	10
5 2 3 4 7 1 8 0 6	464	181	0.0002588380	6.87607	15
5 1 3 2 6 4 7 8 0	651	198	0.0003531710	7.22193	14
1 0 2 5 8 6 4 7 3	406	189	0.0002217070	6.62845	13
4 7 6 0 5 2 3 8 1	423	187	0.0002434780	6.83737	15
5 8 2 1 7 6 3 0 4	453	203	0.0002386770	6.81716	15
3 0 2 6 1 4 5 7 8	17	15	0.0000158900	5.38235	7
2 0 1 7 5 3 8 6 4	405	165	0.0002677990	6.69777	13
8 1 0 5 3 7 2 4 6	542	140	0.0003025700	6.95327	16
0 2 6 5 4 7 1 3 8	135	38	0.0000897130	8.6913	14
4 5 0 1 7 2 3 8 6	67	34	0.0000372410	7.91453	12
0 6 3 5 8 2 1 7 4	452	194	0.0002697680	6.75905	16
1 4 8 0 7 5 2 6 3	432	195	0.0002924790	6.80921	13
1 5 0 8 4 3 2 7 6	521	162	0.0003258800	6.9145	14
2 3 6 8 1 0 5 4 7	111	79	0.0000725230	7.82741	17
1 6 0 7 3 8 4 5 2	590	98	0.0003723810	6.83349	14
8 4 3 1 6 5 0 7 2	549	148	0.0003503510	6.89877	14

3 8 2 7 5 1 0 6 4	447	184	0.0003906820	6.72819	12
7 2 3 6 4 8 1 0 5	460	211	0.0003580410	6.95679	13
3 1 2 4 0 8 5 7 6	394	140	0.0002791390	6.4721	8
8 5 3 1 2 6 4 0 7	65	45	0.0000464510	7.79832	19
2 0 8 6 1 3 4 7 5	510	155	0.0003270610	6.72898	11
3 6 7 1 2 8 4 5 0	525	222	0.0003363200	6.77065	16
0 3 6 1 2 8 4 7 5	533	130	0.0003647810	6.88877	14
4 8 6 5 0 7 1 3 2	416	184	0.0002854890	6.96346	20
7 2 0 1 8 6 3 4 5	19	18	0.0000176310	7.47222	14
4 0 7 1 3 2 5 8 6	428	181	0.0003004890	6.7876	15
1 4 3 0 5 8 7 6 2	552	149	0.0003551610	6.85234	11
1 3 6 5 8 4 0 2 7	461	198	0.0004074630	7.30037	16
3 1 6 4 0 8 7 5 2	496	174	0.0003916220	7.07913	12
8 0 5 1 4 3 7 2 6	88	49	0.0001723060	7.53896	15
1 0 2 8 7 3 4 6 5	601	101	0.0004165930	6.78992	11
5 8 1 4 7 3 2 6 0	452	206	0.0003398210	7.27904	16
6 4 1 2 0 3 7 8 5	560	132	0.0004032630	6.86391	12
6 1 7 4 8 3 5 2 0	43	36	0.0000353720	7.33333	16
0 1 5 3 8 7 6 4 2	480	180	0.0005097860	6.94675	8
7 8 2 6 4 3 0 1 5	459	216	0.0003234700	7.02829	12
6 0 8 7 4 3 5 2 1	80	55	0.0000561610	7.75887	17
5 4 0 7 6 2 8 3 1	68	46	0.0000486510	6.6281	16
5 6 1 8 7 0 2 4 3	525	219	0.0003418400	6.79652	19
1 6 7 4 2 0 3 5 8	401	153	0.0002913400	6.59748	13
5 3 6 4 7 2 1 0 8	632	195	0.0004132130	7.22292	15
0 8 7 4 2 6 1 3 5	416	188	0.0002858390	6.95522	18
7 2 0 5 4 3 8 6 1	496	210	0.0003208200	7.47886	14
5 3 0 6 7 1 8 2 4	20	20	0.0000156700	7.91667	16
5 7 2 8 3 6 4 0 1	659	129	0.0004370230	7.1188	17
3 7 8 1 4 6 5 0 2	474	197	0.0003932830	6.95585	15
7 4 5 8 6 3 2 0 1	62	41	0.0000449710	7.63393	19
0 4 1 8 2 5 3 7 6	423	170	0.0002774690	6.56225	10
5 1 7 2 3 6 8 4 0	514	158	0.0004220630	6.83297	16
6 3 4 8 2 1 5 7 0	394	166	0.0003319000	6.625	16
2 0 1 5 7 6 4 8 3	420	181	0.0002798890	6.91263	15
7 8 0 6 3 1 2 5 4	410	178	0.0002683480	6.70621	18
6 2 4 3 7 0 5 1 8	386	175	0.0002796290	6.51458	11
7 5 4 8 3 6 0 2 1	448	176	0.0002947290	6.87753	20
3 8 4 5 7 1 0 6 2	389	164	0.0002624080	6.53546	14
5 6 2 8 7 1 0 3 4	466	194	0.0003223600	6.90413	16
3 2 4 6 5 8 0 7 1	453	194	0.0003026290	6.61615	10
7 5 1 6 8 4 2 0 3	442	193	0.0003072790	6.83655	17
3 5 2 0 8 1 4 6 7	590	129	0.0003891320	6.82882	11
8 0 6 7 1 3 5 2 4	454	197	0.0002967990	7.0087	21
2 5 8 4 1 0 6 3 7	57	39	0.0000419010	5.57547	11
7 0 2 1 8 6 3 4 5	18	17	0.0000159600	7.25714	13

2 1 5 3 8 0 6 7 4	387	149	0.0003540620	6.43586	7
3 8 6 4 7 0 1 5 2	572	175	0.0003754820	7.08036	17
1 0 7 2 4 3 6 8 5	417	179	0.0002789390	6.6449	11
3 6 7 5 8 0 4 1 2	596	175	0.0004237230	7.10293	17
6 2 0 3 1 4 8 7 5	382	168	0.0002600380	6.38881	8
7 1 5 4 0 3 2 6 8	522	150	0.0003654320	6.74273	12
8 0 7 4 6 1 5 2 3	580	151	0.0005097760	7.11359	21
8 2 0 5 7 6 1 3 4	458	208	0.0004023830	6.90999	18
3 8 5 7 0 1 6 4 2	485	176	0.0004209530	6.98945	12
3 5 4 1 7 6 0 2 8	580	152	0.0004288240	6.98928	14
0 2 8 5 4 1 7 3 6	409	152	0.0002702090	6.71724	12
4 3 7 0 6 1 5 8 2	403	161	0.0003886520	6.64895	17
2 1 3 4 6 5 0 7 8	636	186	0.0004205330	7.13375	8
1 0 5 4 6 2 7 3 8	48	29	0.0000362110	5.71591	9
7 2 3 1 5 4 8 6 0	613	104	0.0004018320	6.96051	14
5 7 2 6 4 0 8 3 1	450	203	0.0003553710	6.84375	13

2.6. A* no 15-puzzle

Instância	Nº Nós	Tamanho Solução	Tempo (s)	Média h(s)	h(s) Inicial
14 13 15 7 11 12 9 5 6 0 2 1 4 8 10 3	14028344	57	25.0309811190	27.1869	41
13 5 4 10 9 12 8 14 2 3 7 1 0 15 11 6	4415279	55	6.7608166580	23.3948	43
14 7 8 2 13 11 10 4 9 12 5 0 3 6 1 15	30003145	59	57.5357843690	27.7517	41
5 12 10 7 15 11 14 0 8 2 1 13 3 4 9 6	5253685	56	8.3249848650	26.6208	42
4 7 14 13 10 3 9 12 11 5 6 15 1 2 8 0	2021382	56	2.7241898110	28.1532	42
14 7 1 9 12 3 6 15 8 11 2 5 10 0 4 13	969356	52	1.2728123560	27.4165	36

2 11 15 5 13 4 6 7 12 8 10 1 9 3 14 0	7170013	52	12.4333146230	23.3837	30
12 11 15 3 8 0 4 2 6 13 9 5 14 1 10 7	2618503	50	3.9910203200	24.4245	32
3 14 9 11 5 4 8 2 13 12 6 7 10 1 15 0	313211	46	0.3389646130	21.5037	32
13 11 8 9 0 15 7 10 4 3 6 14 5 12 2 1	17805045	59	31.6301212110	27.735	43
5 9 13 14 6 3 7 12 10 8 4 0 15 2 11 1	3525006	57	5.1775211940	28.7505	43
14 1 9 6 4 8 12 5 7 2 3 0 10 11 13 15	32334	45	0.0230592430	26.2362	35
3 6 5 2 10 0 15 14 1 4 13 12 9 8 11 7	831333	46	1.0888948970	19.5075	36
7 6 8 1 11 5 14 10 3 4 9 13 15 2 0 12	26088053	59	50.4820622380	28.499	41
13 11 4 12 1 8 9 15 6 5 14 2 7 3 10 0	25792756	62	46.6908237300	31.4546	44
1 3 2 5 10 9 15 6 8 14 13 11 12 4 7 0	909219	42	1.2490148540	18.6516	24
15 14 0 4 11 1 6 13 7 5 8 9 3 2 10 12	89166546	66	206.8514982270	31.9188	46
6 0 14 12 1 15 9 10 11 4 7 2 8 3 5 13	1267588	55	1.5113347560	27.2594	43

7 11 8 3 14 0 6 15 1 4 13 9 5 12 2 10	154092	46	0.1603728330	22.9871	36
6 12 11 3 13 7 9 15 2 14 8 10 4 1 5 0	1995813	52	2.8471241020	24.573	36
12 8 14 6 11 4 7 0 5 1 10 15 3 13 9 2	5668025	54	9.9638031400	26.4034	34
14 3 9 1 15 8 4 5 11 7 10 13 0 2 12 6	13338796	59	25.7691961490	28.2474	41
10 9 3 11 0 13 2 14 5 6 4 7 8 15 1 12	1398857	49	1.9348267130	24.2771	33
7 3 14 13 4 1 10 8 5 12 9 11 2 15 6 0	4691710	54	7.4660487250	26.3122	34
11 4 2 7 1 0 10 15 6 9 14 8 3 13 5 12	6227205	52	10.3534276950	24.7003	32
5 7 3 12 15 13 14 8 0 10 9 6 1 4 2 11	20161235	58	35.9528475580	27.5853	40
14 1 8 15 2 6 0 3 9 12 10 13 4 7 5 11	11278428	53	20.9811281760	25.4592	33
13 14 6 12 4 5 1 0 9 3 10 2 15 11 8 7	909442	52	1.0957774000	27.8727	36
9 8 0 2 15 1 4 14 3 10 7 5 11 13 6 12	3917549	54	6.0651213900	26.1191	38
12 15 2 6 1 14 4 8 5 3 7 0 10 13 9 11	215990	47	0.2304172570	24.8402	35

12 8 15 13 1 0 5 4 6 3 2 11 9 7 14 10	192534	50	0.2001435350	28.1648	38
14 10 9 4 13 6 5 8 2 12 7 0 1 3 11 15	17025056	59	29.2800337060	29.2737	43
14 3 5 15 11 6 13 9 0 10 2 12 4 1 7 8	18535767	60	33.9675774570	30.1101	42
6 11 7 8 13 2 5 4 1 10 3 9 14 0 12 15	2063117	52	2.9057994380	25.9825	36
1 6 12 14 3 2 15 8 4 5 13 9 0 7 11 10	4626661	55	6.9522545230	26.9748	39
12 6 0 4 7 3 15 1 13 9 8 11 2 14 5 10	2309697	52	3.3277629730	25.7915	36
8 1 7 12 11 0 10 5 9 15 6 13 14 2 3 4	20889986	58	38.9133734280	27.636	40
7 15 8 2 13 6 3 12 11 0 4 10 9 5 1 14	827864	53	0.9697090120	28.7101	41
9 0 4 10 1 14 15 3 12 6 5 7 11 13 8 2	1118025	49	1.4554118590	24.2808	35
11 5 1 14 4 12 10 0 2 7 13 3 9 15 6 8	6540243	54	11.2274292360	26.3806	36
8 13 10 9 11 3 15 6 0 1 2 14 12 5 4 7	5727374	54	9.2029511570	26.1219	36
4 5 7 2 9 14 12 13 0 3 6 11 8 1 15 10	48447	42	0.0354164430	23.3	30

11 15 14 13 1 9 10 4 3 6 2 12 7 5 8 0	7332466	64	10.5813104220	33.0528	48
12 9 0 6 8 3 5 14 2 4 11 7 10 1 15 13	2731989	50	4.1055798610	24.6075	32
3 14 9 7 12 15 0 4 1 8 5 6 11 10 2 13	539196	51	0.6010463450	27.3161	39
8 4 6 1 14 12 2 15 13 10 9 5 3 7 0 11	1430455	49	1.8815712150	22.9179	35
6 10 1 14 15 8 3 5 13 0 2 7 4 9 11 12	179689	47	0.1752960310	25.4753	35
8 11 4 6 7 3 10 9 2 12 15 13 0 1 5 14	119535	49	0.0980311710	25.1685	39
10 0 2 4 5 1 6 12 11 13 9 7 15 3 14 8	73083875	59	171.3026519800	28.4108	33
12 5 13 11 2 10 0 9 7 8 4 3 14 6 15 1	4399444	53	7.2884505620	25.0994	39
10 2 8 4 15 0 1 14 11 13 3 6 9 7 5 12	3031922	56	4.5330243940	28.0748	44
10 8 0 12 3 7 6 2 1 14 4 11 15 13 9 5	11641228	56	21.6904654650	26.8034	38
14 9 12 13 15 4 8 10 0 2 1 7 3 11 5 6	38607523	64	72.4937056420	30.4284	50
12 11 0 8 10 2 13 15 5 4 7 3 6 9 14 1	7221615	56	12.2164889500	27.8207	40

13 8 14 3 9 1 0 7 15 5 4 10 12 2 6 11	152123	41	0.1570406000	20.6961	29
3 15 2 5 11 6 4 7 12 9 1 0 13 14 10 8	46798812	55	107.5964502580	25.3307	29
5 11 6 9 4 13 12 0 8 2 15 10 1 7 3 14	484048	50	0.5865891250	26.1174	36
5 0 15 8 4 6 1 14 10 11 3 9 7 12 2 13	506538	51	0.5982629670	25.5453	37
15 14 6 7 10 1 0 11 12 8 4 9 2 5 13 3	49797791	57	112.4592551920	26.7052	35
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15	-	-	-	-	-
6 13 3 2 11 9 5 10 1 7 12 14 8 4 0 15	508957	45	0.5975787000	21.7271	31
4 6 12 0 14 2 9 13 11 8 3 15 7 10 1 5	1828847	57	2.3551877090	29.0846	43
8 10 9 11 14 1 7 15 13 4 0 12 6 2 5 3	21862510	56	46.7959719600	25.5476	40
5 2 14 0 7 8 6 3 11 12 13 15 4 10 9 1	5622335	51	9.7959300020	23.6183	31
7 8 3 2 10 12 4 6 11 13 5 15 0 1 9 14	920623	47	1.1744067970	22.6228	31
11 6 14 12 3 5 1 15 8 0 10 13 9 7 4 2	71601915	61	158.3937765660	28.8012	41

7 1 2 4 8 3 6 11 10 15 0 5 14 12 13 9	8430894	50	16.1068753660	23.9214	28
7 3 1 13 12 10 5 2 8 0 6 11 14 15 4 9	5471060	51	9.7631668800	25.118	31
6 0 5 15 1 14 4 9 2 13 8 10 11 12 7 3	4579005	53	7.1183068970	25.3428	37
15 1 3 12 4 0 6 5 2 8 14 9 13 10 7 11	8179227	52	14.2984796110	24.6644	30
5 7 0 11 12 1 9 10 15 6 2 3 8 4 13 14	557862	44	0.6819754380	20.6301	30
12 15 11 10 4 5 14 0 13 7 1 2 9 8 3 6	34437004	56	71.8795757050	25.3943	38
6 14 10 5 15 8 7 1 3 4 2 0 12 9 11 13	188717	49	0.1794497590	23.8731	37
14 13 4 11 15 8 6 9 0 7 3 1 2 10 12 5	1373939	56	1.7439578040	28.7716	46
14 4 0 10 6 5 1 3 9 2 13 15 12 7 8 11	3440513	48	5.6294637680	22.0602	30
15 10 8 3 0 6 9 5 1 14 13 11 7 2 12 4	7622602	57	12.8764540300	27.7297	41
0 13 2 4 12 14 6 9 15 1 10 3 11 5 8 7	2184110	54	2.9549660730	27.598	34
3 14 13 6 4 15 8 9 5 12 10 0 2 7 1 11	619299	53	0.6655711050	27.3807	41

0 1 9 7 11 13 5 3 14 12 4 2 8 6 10 15	68633	42	0.0551356460	21.7811	28
11 0 15 8 13 12 3 5 10 1 4 6 14 9 7 2	3951356	57	5.7496758500	26.9136	43
13 0 9 12 11 6 3 5 15 8 1 10 4 14 2 7	744209	53	0.8728179670	28.0517	39
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15	-	-	-	-	-
12 3 9 1 4 5 10 2 6 11 15 0 14 7 13 8	1900358	49	2.7496286620	24.1278	31
15 8 10 7 0 12 14 1 5 9 6 3 13 11 4 2	10972804	55	18.8605678600	26.659	37
4 7 13 10 1 2 9 6 12 8 14 5 3 0 11 15	158355	44	0.1538214460	20.8886	32
6 0 5 10 11 12 9 2 1 7 4 3 14 8 13 15	195222	45	0.2174640590	22.2905	35
9 5 11 10 13 0 2 1 8 6 14 12 4 7 3 15	4681483	52	7.3446028480	25.962	34
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15	-	-	-	-	-
11 1 7 4 10 13 3 8 9 14 0 15 6 5 2 12	3559259	54	5.2968394320	26.4619	38
5 4 7 1 11 12 14 15 10 13 8 6 2 0 9 3	820923	50	0.9493268100	26.0792	36

9 7 5 2 14 15 12 10 11 3 6 1 8 13 0 4	18150875	57	32.4733972790	26.8734	41
3 2 7 9 0 15 12 4 6 11 5 14 8 13 10 1	45112197	57	97.7104764910	25.8101	37
13 9 14 6 12 8 1 2 3 4 0 7 5 10 11 15	438960	46	0.5366619570	20.9652	34
5 7 11 8 0 14 9 13 10 12 3 15 6 1 4 2	276325	53	0.2738494850	27.7666	45
4 3 6 13 7 15 9 0 10 5 8 11 2 12 1 14	751475	50	0.9179434670	25.0634	34
1 7 15 14 2 6 4 9 12 11 13 3 0 8 5 10	1263033	49	1.6673644650	22.9222	35
9 14 5 7 8 15 1 2 10 4 13 6 12 0 11 3	191601	44	0.1929004960	21.2678	32
0 11 3 12 5 2 1 9 8 10 14 15 7 4 13 6	9499694	54	17.9389973350	25.1273	34
7 15 4 0 10 9 2 5 12 11 13 6 1 3 14 8	5342754	57	8.0933803730	28.7625	39
11 4 0 8 6 10 5 13 12 7 14 3 1 2 9 15	4484857	54	6.8428081570	25.1702	38