

# MAT332 - Fall 2016

## Midterm Topics

Here are the important theorems and algorithms we have learned in class so far:

### **Important theorems:**

- A graph has an even number of odd-degree vertices
- If a simple graph  $G$  has at least 3 vertices, and every vertex has degree  $\geq |V(G)|/2$ , then  $G$  has a Hamiltonian cycle
- A connected graph has an Eulerian circuit iff every vertex has even degree
- Every tournament has a Hamiltonian path
- The following are equivalent:  $G$  is connected and has no cycles;  $G$  is connected and has  $n - 1$  edges;  $G$  has  $n - 1$  edges and no cycles; between any two vertices of  $G$  exists exactly one path.
- König's theorem
- Hall's Theorem
- Every regular bipartite graph has a perfect matching
- Every latin rectangle is extendable to a latin square
- Tutte's Theorem (the proof of this is too long for me to expect you to reproduce it on a test, but you should certainly know the statement of the theorem)

### **Important algorithms:**

- How to construct a tree using a Prüfer code, how to decode a tree.
- Kruskal's greedy algorithm for finding a minimum-weight spanning tree.
- Dijkstra's algorithm for finding shortest distance between two vertices.
- Recursive algorithm for counting spanning trees using edge contraction/deletion.
- Gale Shapley algorithm for preference matching

On the midterm, you will be asked to prove one (or more) of these theorems or apply one (or more) of these algorithms to a certain graph. In fact, 18 out of the possible 56 points on the midterm will be questions of this type. That means that if you know the above theorems and algorithms, you are guaranteed to get at least 32% on your exam.

The rest of the exam will consist of questions like the ones you see on your homework: short answer questions and proofs. I realize that some homework questions probably take you an hour (or more) of quiet reflection before you have the right idea for the answer. I tried to make sure that the exam questions are not this difficult.