3.1. Statistics in Python

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Requirements

- Standard scientific Python environment (numpy, scipy, matplotlib)
- Pandas
- Statsmodels
- Seaborn

To install Python and these dependencies, we recommend that you download Anaconda Python or Enthought Canopy, or preferably use the package manager if you are under Ubuntu or other linux.

See also: Bayesian statistics in Python
This chapter does not cover tools for Bayesian statistics. Of particular interest for Bayesian modelling is PyMC, which implements a probabilistic programming language in Python.

Why Python for statistics?
R is a language dedicated to statistics. Python is a general-purpose language with statistics modules. R has more statistical analysis features than Python, and specialized syntaxes. However, when it comes to building complex analysis pipelines that mix statistics with e.g. image analysis, text mining, or control of a physical experiment, the richness of Python is an invaluable asset.

Contents

In this document, the Python inputs are represented with the sign “>>>”.

Disclaimer: Gender questions

Some of the examples of this tutorial are chosen around gender questions. The reason is that on such questions controlling the truth of a claim actually matters to many people.

3.1.1. Data representation and interaction

3.1.1.1. Data as a table

The setting that we consider for statistical analysis is that of multiple *observations* or *samples* described by a set of different *attributes* or *features*. The data can than be seen as a 2D table, or matrix, with columns giving the different attributes of the data, and rows the observations. For instance, the data contained in **examples/brain_size.csv**:

```
"";"Gender";"FSIQ";"VIQ";"PIQ";"Weight";"Height";"MRI_Count
    "
"1";"Female";133;132;124;"118";"64.5";816932
"2";"Male";140;150;124;".";"72.5";1001121
"3";"Male";139;123;150;"143";"73.3";1038437
"4";"Male";133;129;128;"172";"68.8";965353
"5";"Female";137;132;134;"147";"65.0";951545
```

3.1.1.2. The pandas data-frame

We will store and manipulate this data in a **pandas.DataFrame**, from the pandas module. It is the Python equivalent of the spreadsheet table. It is different from a 2D numpy array as it has named columns, can contain a mixture of different data types by column, and has elaborate selection and pivotal mechanisms.

3.1.1.2.1. Creating dataframes: reading data files or converting arrays

Reading from a CSV file: Using the above CSV file that gives observations of brain size and weight and IQ (Willerman et al. 1991), the data are a mixture of numerical and categorical values:

Separator

It is a CSV file, but the separator is ";"

```
>>> import pandas
>>> data = pandas.read_csv('examples/brain_size.csv', sep='
    ;', na_values=".")
>>> data
    Unnamed: 0  Gender  FSIQ  VIQ  PIQ  Weight  Height  MRI
    _Count
0            1  Female   133  132  124     118    64.5
    816932
1            2    Male   140  150  124     NaN    72.5    1
    001121
```

```
2            3    Male    139   123   150      143     73.3     1
      038437
3            4    Male    133   129   128      172     68.8
      965353
4            5  Female    137   132   134      147     65.0
      951545
...
```

Missing values

The weight of the second individual is missing in the CSV file. If we don't specify the missing value (NA = not available) marker, we will not be able to do statistical analysis.

Creating from arrays: A `pandas.DataFrame` can also be seen as a dictionary of 1D 'series', eg arrays or lists. If we have 3 numpy arrays:

```
>>> import numpy as np
>>> t = np.linspace(-6, 6, 20)
>>> sin_t = np.sin(t)
>>> cos_t = np.cos(t)
```

We can expose them as a `pandas.DataFrame`:

```
>>> pandas.DataFrame({'t': t, 'sin': sin_t, 'cos': cos_t})
        cos       sin         t
0   0.960170  0.279415 -6.000000
1   0.609977  0.792419 -5.368421
2   0.024451  0.999701 -4.736842
3  -0.570509  0.821291 -4.105263
4  -0.945363  0.326021 -3.473684
5  -0.955488 -0.295030 -2.842105
6  -0.596979 -0.802257 -2.210526
7  -0.008151 -0.999967 -1.578947
8   0.583822 -0.811882 -0.947368
...
```

Other inputs: pandas can input data from SQL, excel files, or other formats. See the pandas documentation.

3.1.1.2.2. Manipulating data

data is a **pandas.DataFrame**, that resembles R's dataframe:

```
>>> data.shape    # 40 rows and 8 columns
(40, 8)

>>> data.columns  # It has columns
Index([u'Unnamed: 0', u'Gender', u'FSIQ', u'VIQ', u'PIQ', u
      'Weight', u'Height', u'MRI_Count'], dtype='object')

>>> print(data['Gender'])  # Columns can be addressed by na
      me
0     Female
1       Male
2       Male
3       Male
4     Female
...

>>> # Simpler selector
>>> data[data['Gender'] == 'Female']['VIQ'].mean()
109.45
```

Note: For a quick view on a large dataframe, use its *describe* method: **pandas.DataFrame.describe()**.

groupby: splitting a dataframe on values of categorical variables:

```
>>> groupby_gender = data.groupby('Gender')
>>> for gender, value in groupby_gender['VIQ']:
...     print((gender, value.mean()))
('Female', 109.45)
('Male', 115.25)
```

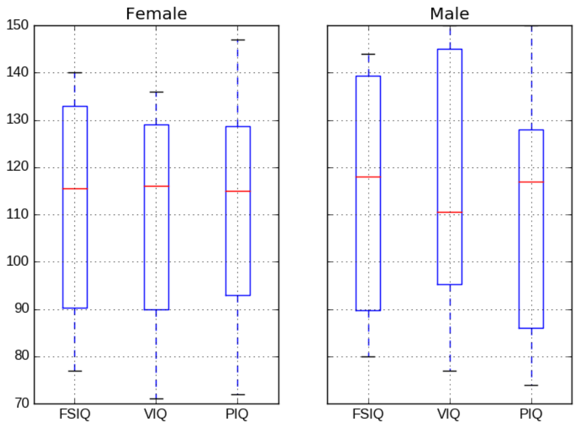
groupby_gender is a powerful object that exposes many operations on the resulting group of dataframes:

```
>>> groupby_gender.mean()
```

```
        Unnamed: 0   FSIQ     VIQ     PIQ      Weight     H
      eight  MRI_Count
Gender
Female        19.65  111.9  109.45  110.45  137.200000  65.7
      65000   862654.6
Male          21.35  115.0  115.25  111.60  166.444444  71.4
      31579   954855.4
```

Use tab-completion on *groupby_gender* to find more. Other common grouping functions are median, count (useful for checking to see the amount of missing values in different subsets) or sum. Groupby evaluation is lazy, no work is done until an aggregation function is applied.

Exercise



- What is the mean value for VIQ for the full population?
- How many males/females were included in this study?

  Hint use 'tab completion' to find out the methods that can be called, instead of 'mean' in the above example.
- What is the average value of MRI counts expressed in log units, for males and females?

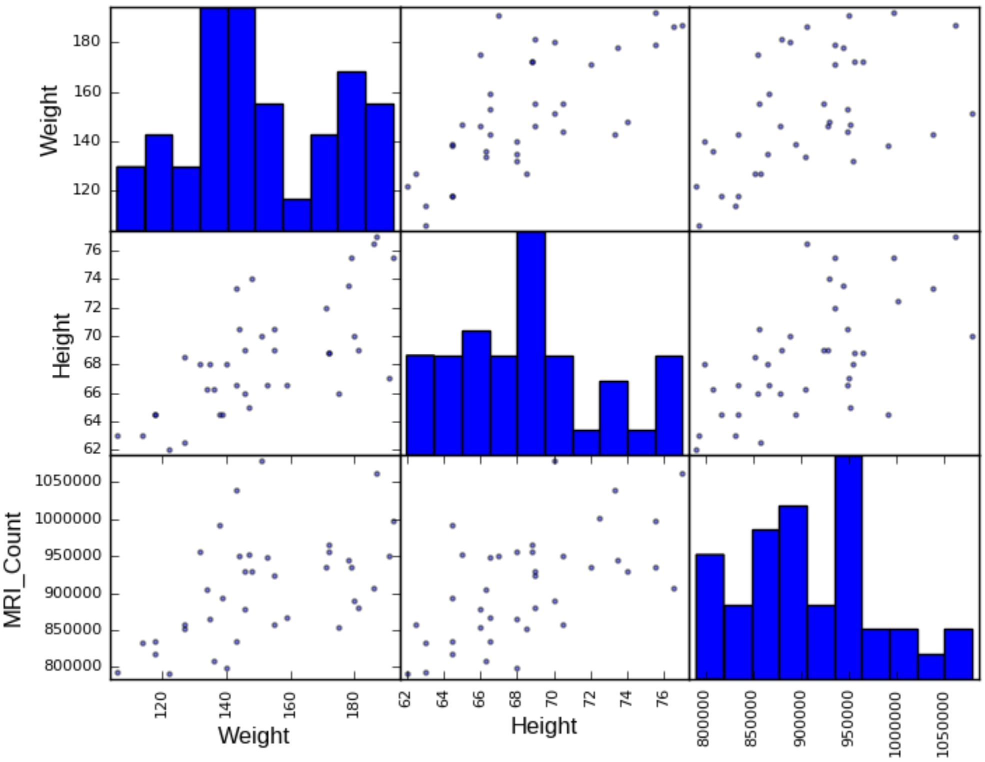
Note: *groupby_gender.boxplot* is used for the plots above (see this example).

3.1.1.2.3. Plotting data

Pandas comes with some plotting tools (**`pandas.tools.plotting`**, using matplotlib behind the scene) to display statistics of the data in dataframes:

Scatter matrices:

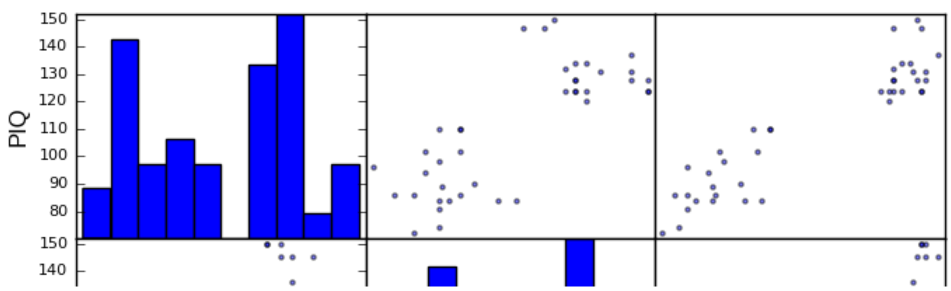
```
>>> from pandas.tools import plotting
>>> plotting.scatter_matrix(data[['Weight', 'Height', 'MRI_
    Count']])
```

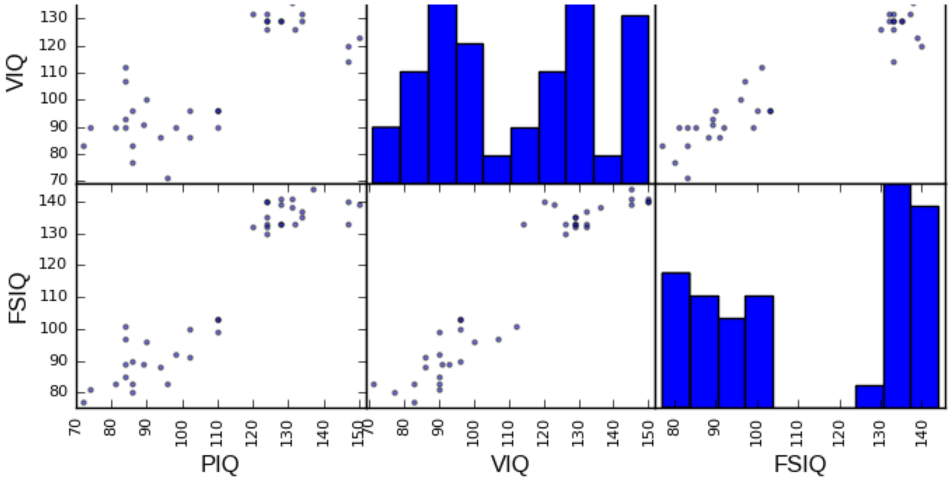



```
>>> plotting.scatter_matrix(data[['PIQ', 'VIQ', 'FSIQ']])
```

Two populations

The IQ metrics are bimodal, as if there are 2 sub-populations.





Exercise

Plot the scatter matrix for males only, and for females only. Do you think that the 2 sub-populations correspond to gender?

3.1.2. Hypothesis testing: comparing two groups

For simple statistical tests, we will use the **scipy.stats** sub-module of scipy:

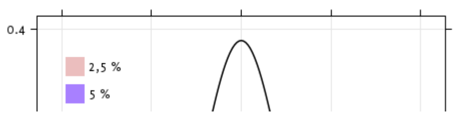
```
>>> from scipy import stats
```

See also: Scipy is a vast library. For a quick summary to the whole library, see the scipy chapter.

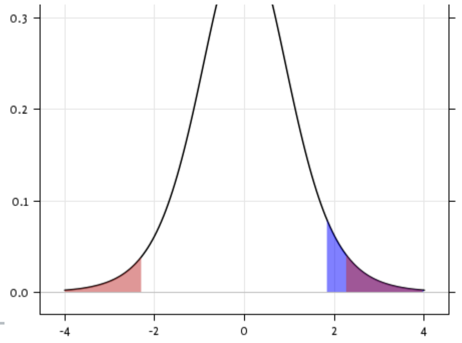
3.1.2.1. Student's t-test: the simplest statistical test

3.1.2.1.1. 1-sample t-test: testing the value of a population mean

scipy.stats.ttest_1samp() tests if the



population mean of data is likely to be equal to a given value (technically if observations are drawn from a Gaussian distributions of given population mean). It returns the T statistic, and the p-value (see the function's help):




```
>>> stats.ttest_1samp(data['VIQ'], 0)
(...30.088099970..., 1.32891964...e-28)
```

With a p-value of 10^-28 we can claim that the population mean for the IQ (VIQ measure) is not 0.

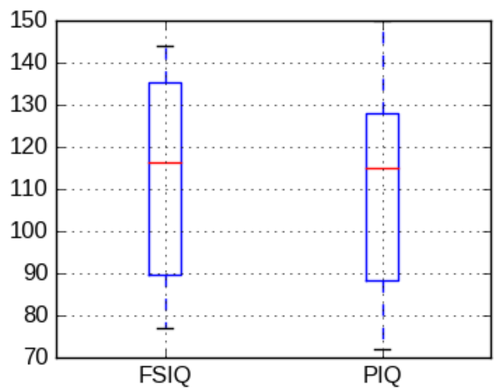
3.1.2.1.2. 2-sample t-test: testing for difference across populations

We have seen above that the mean VIQ in the male and female populations were different. To test if this is significant, we do a 2-sample t-test with **scipy.stats.ttest_ind()**:

```
>>> female_viq = data[data['Gender'] == 'Female']['VIQ']
>>> male_viq = data[data['Gender'] == 'Male']['VIQ']
>>> stats.ttest_ind(female_viq, male_viq)
(...-0.77261617232..., 0.4445287677858...)
```

3.1.2.2. Paired tests: repeated measurements on the same indivuals

PIQ, VIQ, and FSIQ give 3 measures of IQ. Let us test if FISQ and PIQ are significantly different. We can use a 2 sample test:



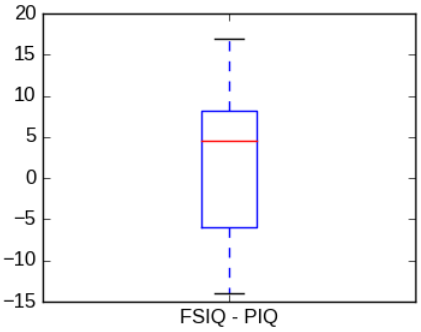

```
>>> stats.ttest_ind(data['FSIQ'], data['PIQ'])
(...0.46563759638..., 0.64277250...)
```

The problem with this approach is that it forgets that there are links between observations: FSIQ and PIQ are measured on the same individuals. Thus the variance due to inter-subject variability is confounding, and can be removed, using a "paired test", or "repeated measures test":

```
>>> stats.ttest_rel(data['FSIQ'], data['PIQ'])
(...1.784201940..., 0.082172638183...)
```

This is equivalent to a 1-sample test on the difference:

```
>>> stats.ttest_1samp(data['FSIQ']
      - data['PIQ'], 0)
(...1.784201940..., 0.082172638...)
```



T-tests assume Gaussian errors. We can use a Wilcoxon signed-rank test, that relaxes this assumption:

```
>>> stats.wilcoxon(data['FSIQ'], data['PIQ'])
(274.5, 0.106594927...)
```

Note: The corresponding test in the non paired case is the Mann–Whitney U test, **`scipy.stats.mannwhitneyu()`**.

Exercise

- Test the difference between weights in males and females.
- Use non parametric statistics to test the difference between VIQ in males and females.

Conclusion: we find that the data does not support the hypothesis that males and females have different VIQ.

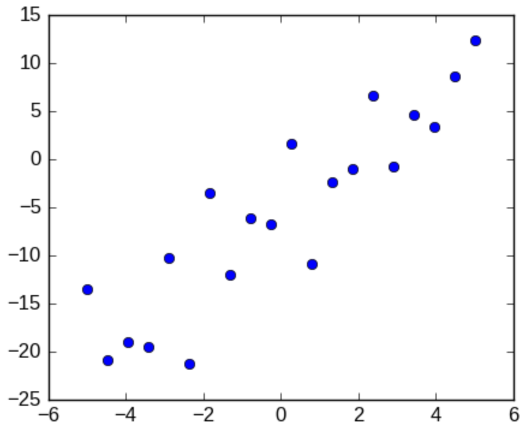
3.1.3. Linear models, multiple factors,

and analysis of variance

3.1.3.1. “formulas” to specify statistical models in Python

3.1.3.1.1. A simple linear regression

Given two set of observations, *x* and *y*, we want to test the hypothesis that *y* is a linear function of *x*. In other terms:



$$y = x * coef + intercept + e$$

where *e* is observation noise. We will use the statsmodels module to:

1. Fit a linear model. We will use the simplest strategy, ordinary least squares (OLS).
2. Test that *coef* is non zero.

First, we generate simulated data according to the model:

```
>>> import numpy as np
>>> x = np.linspace(-5, 5, 20)
>>> np.random.seed(1)
>>> # normal distributed noise
>>> y = -5 + 3*x + 4 * np.random.normal(size=x.shape)
>>> # Create a data frame containing all the relevant varia
    bles
>>> data = pandas.DataFrame({'x': x, 'y': y})
```

"formulas" for statistics in Python

See the statsmodels docu-

Then we specify an OLS model and fit it:

```
>>> from statsmodels.formula.api
      import ols
>>> model = ols("y ~ x", data).fit()
```

We can inspect the various statistics derived from the fit:

```
>>> print(model.summary())
                            OLS Regression Results
==========================...
Dep. Variable:                       y   R-squared:
      0.804
Model:                             OLS   Adj. R-squared:
      0.794
Method:                  Least Squares   F-statistic:
      74.03
Date:                 ...                Prob (F-statistic):
      8.56e-08
Time:                         ...        Log-Likelihood:
      -57.988
No. Observations:                   20   AIC:
      120.0
Df Residuals:                       18   BIC:
      122.0
Df Model:                            1
==========================...
                 coef    std err          t      P>|t|
      [95.0% Conf. Int.]
------------------------------------------...
Intercept     -5.5335      1.036     -5.342      0.000
      -7.710    -3.357
x              2.9369      0.341      8.604      0.000
      2.220     3.654
==========================...
Omnibus:                         0.100   Durbin-Watson:
      2.956
Prob(Omnibus):                   0.951   Jarque-Bera (JB):
      0.322
Skew:                           -0.058   Prob(JB):
      0.851
```

```
Kurtosis:                       2.390   Cond. No.
      3.03
==========================...
```

Terminology:

Statsmodels uses a statistical terminology: the *y* variable in statsmodels is called 'endogenous' while the *x* variable is called exogenous. This is discussed in more detail here.

To simplify, *y* (endogenous) is the value you are trying to predict, while *x* (exogenous) represents the features you are using to make the prediction.

Exercise

Retrieve the estimated parameters from the model above. **Hint**: use tab-completion to find the relevent attribute.

3.1.3.1.2. Categorical variables: comparing groups or multiple categories

Let us go back the data on brain size:

```
>>> data = pandas.read_csv('examples/brain_size.csv', sep=
      ;', na_values=".")
```

We can write a comparison between IQ of male and female using a linear model:

```
>>> model = ols("VIQ ~ Gender + 1", data).fit()
>>> print(model.summary())
                            OLS Regression Results
==========================...
Dep. Variable:                    VIQ   R-squared:
      0.015
Model:                            OLS   Adj. R-squared:
      -0.010
Method:                 Least Squares   F-statistic:
```

```
      0.5969
Date:                ...                  Prob (F-statistic):
      0.445
Time:                        ...          Log-Likelihood:
      -182.42
No. Observations:                  40    AIC:
      368.8
Df Residuals:                      38    BIC:
      372.2
Df Model:                           1
==========================...
                  coef    std err          t      P>|t|
      [95.0% Conf. Int.]
------------------------------------------------------------
      ------------...
Intercept       109.4500     5.308     20.619     0.000
      98.704   120.196
Gender[T.Male]    5.8000     7.507      0.773     0.445
      -9.397    20.997
==========================...
Omnibus:                       26.188   Durbin-Watson:
      1.709
Prob(Omnibus):                  0.000   Jarque-Bera (JB):
      3.703
Skew:                           0.010   Prob(JB):
      0.157
Kurtosis:                       1.510   Cond. No.
      2.62
==========================...
```

Tips on specifying model

Forcing categorical: the 'Gender' is automatically detected as a categorical variable, and thus each of its different values are treated as different entities.

An integer column can be forced to be treated as categorical using:

```
>>> model = ols('VIQ ~ C(Gender)', data).fit()
```

Intercept: We can remove the intercept using *- 1* in the formula, or force the use of an intercept using *+ 1*.

By default, statsmodels treats a categorical variable with K possible values as K-1 'dummy' boolean variables (the last level being absorbed into the intercept term). This is almost always a good default choice - however, it is possible to specify different encodings for categorical variables (http://statsmodels.sourceforge.net/devel/contrasts.html).

Link to t-tests between different FSIQ and PIQ

To compare different types of IQ, we need to create a "long-form" table, listing IQs, where the type of IQ is indicated by a categorical variable:

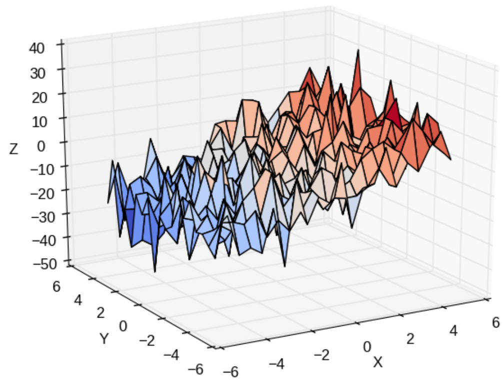
```
>>> data_fisq = pandas.DataFrame({'iq': data['FSIQ'], 'type': 'fsiq'})
>>> data_piq = pandas.DataFrame({'iq': data['PIQ'], 'type': 'piq'})
>>> data_long = pandas.concat((data_fisq, data_piq))
>>> print(data_long)
         iq  type
    0   133  fsiq
    1   140  fsiq
    2   139  fsiq
    ...
    31  137   piq
    32  110   piq
    33   86   piq
    ...

>>> model = ols("iq ~ type", data_long).fit()
>>> print(model.summary())
                            OLS Regression Results
...
==========================...
                 coef    std err          t      P>|t|
      [95.0% Conf. Int.]
---------------------------------------------...
Intercept     113.4500      3.683     30.807      0.000
      106.119   120.781
type[T.piq]    -2.4250      5.208     -0.466      0.643
      -12.793     7.943
...
```

We can see that we retrieve the same values for t-test and corresponding p-values for the effect of the type of iq than the previous t-test:

```
>>> stats.ttest_ind(data['FSIQ'], data['PIQ'])
(...0.46563759638..., 0.64277250...)
```

3.1.3.2. Multiple Regression: including multiple factors

Consider a linear model explaining a variable *z* (the dependent variable) with 2 variables *x* and *y*:



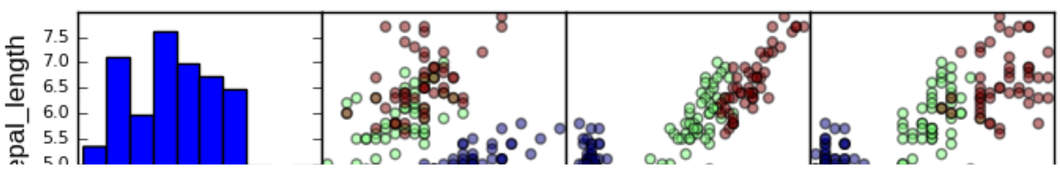
$$z = x\,c_1 + y\,c_2 + i + e$$

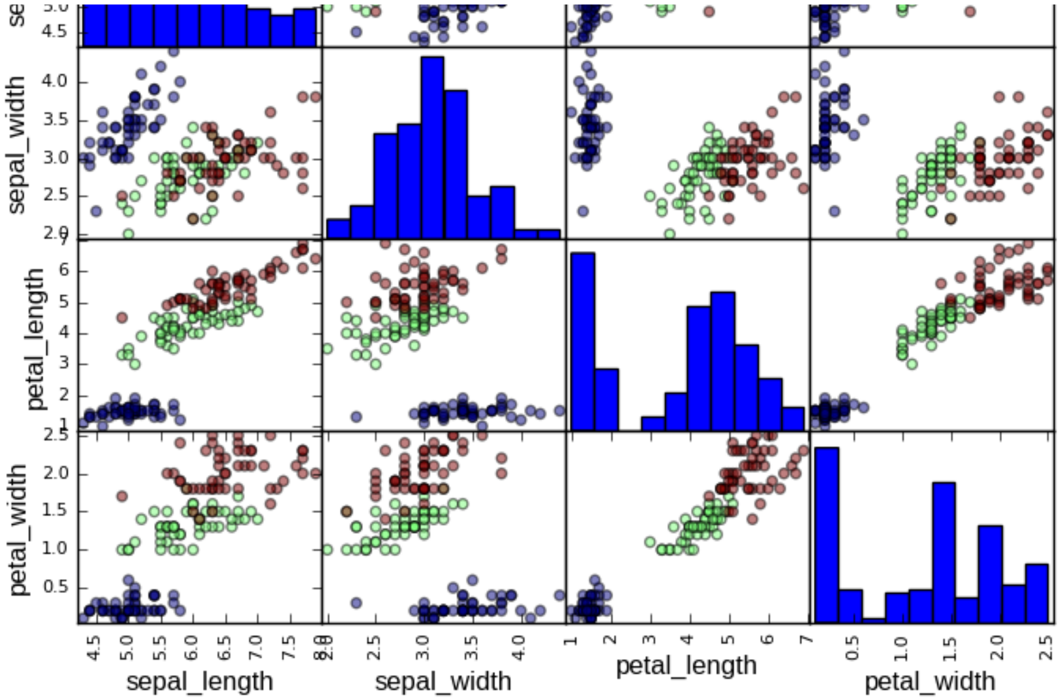
Such a model can be seen in 3D as fitting a plane to a cloud of (*x*, *y*, *z*) points.

Example: the iris data (`examples/iris.csv`)

Sepal and petal size tend to be related: bigger flowers are bigger! But is there in addition a systematic effect of species?

blue: setosa, green: versicolor, red: virginica






```
>>> data = pandas.read_csv('examples/iris.csv')
>>> model = ols('sepal_width ~ name + petal_length', data).
      fit()
>>> print(model.summary())
                            OLS Regression Results
==========================...
Dep. Variable:                sepal_width   R-squared:
      0.478
Model:                                OLS   Adj. R-squared:
      0.468
Method:                     Least Squares   F-statistic:
      44.63
Date:                     ...               Prob (F-statistic):
      1.58e-20
Time:                             ...       Log-Likelihood:
      -38.185
No. Observations:                     150   AIC:
      84.37
Df Residuals:                         146   BIC:
      96.41
Df Model:                               3
==========================...
                         coef    std err          t      P>|
      t|  [95.0% Conf. Int.]
------------------------------------------...
Intercept              2.9813      0.099     29.989      0.0
      00      2.785      3.178
```

```
name[T.versicolor]    -1.4821      0.181     -8.190     0.0
      00     -1.840    -1.124
name[T.virginica]     -1.6635      0.256     -6.502     0.0
      00     -2.169    -1.158
petal_length           0.2983      0.061      4.920     0.0
      00      0.178     0.418
==========================...
Omnibus:                        2.868   Durbin-Watson:
      1.753
Prob(Omnibus):                  0.238   Jarque-Bera (JB):
      2.885
Skew:                          -0.082   Prob(JB):
      0.236
Kurtosis:                       3.659   Cond. No.
      54.0
==========================...
```

3.1.3.3. Post-hoc hypothesis testing: analysis of variance (ANOVA)

In the above iris example, we wish to test if the petal length is different between versicolor and virginica, after removing the effect of sepal width. This can be formulated as testing the difference between the coefficient associated to versicolor and virginica in the linear model estimated above (it is an Analysis of Variance, ANOVA). For this, we write a **vector of 'contrast'** on the parameters estimated: we want to test `"name[T.versicolor] - name[T.virginica]"`, with an F-test:

```
>>> print(model.f_test([0, 1, -1, 0]))
<F test: F=array([[ 3.24533535]]), p=[[ 0.07369059]], df_d
      enom=146, df_num=1>
```

Is this difference significant?

Exercise

Going back to the brain size + IQ data, test if the VIQ of male and female are different after removing the effect of brain size, height and weight.

3.1.4. More visualization: seaborn for statistical exploration

Seaborn combines simple statistical fits with plotting on pandas dataframes.

Let us consider a data giving wages and many other personal information on 500 individuals (Berndt, ER. The Practice of Econometrics. 1991. NY: Addison-Wesley).

The full code loading and plotting of the wages data is found in corresponding example.

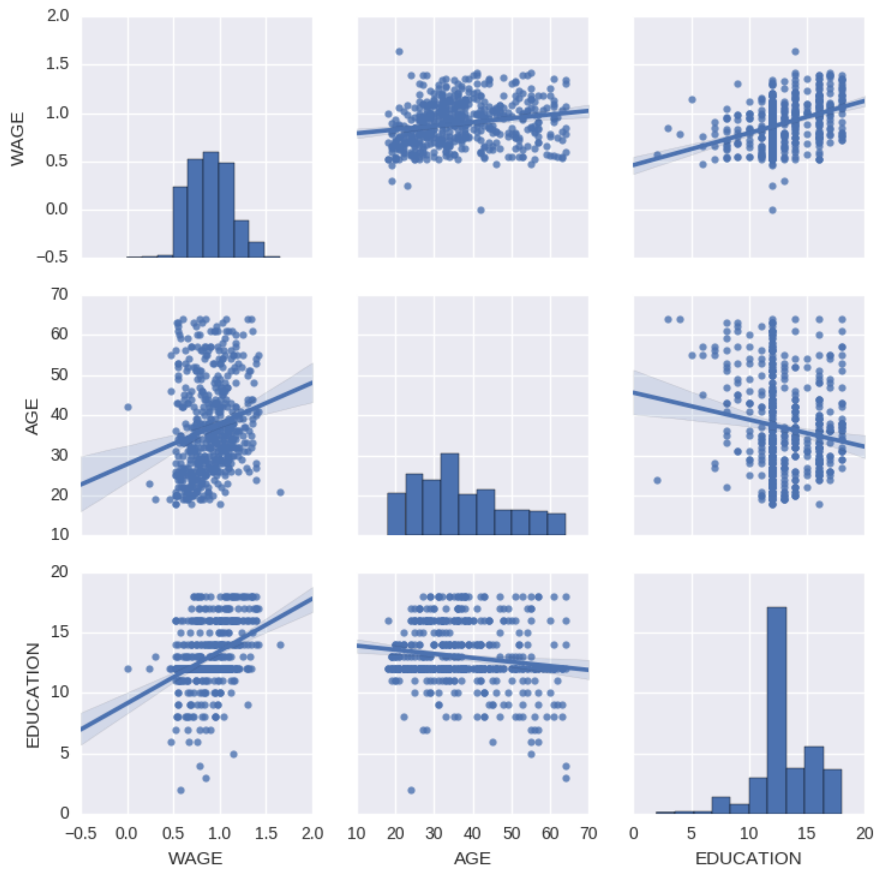
```
>>> print data
     EDUCATION  SOUTH  SEX  EXPERIENCE  UNION      WAGE  AG
      E  RACE  \
0            8      0    1          21      0  0.707570   3
      5     2
1            9      0    1          42      0  0.694605   5
      7     3
2           12      0    0           1      0  0.824126   1
      9     3
3           12      0    0           4      0  0.602060   2
      2     3
...
```

3.1.4.1. Pairplot: scatter matrices

We can easily have an intuition on the interactions between continuous variables using **seaborn.pairplot()** to display a scatter matrix:

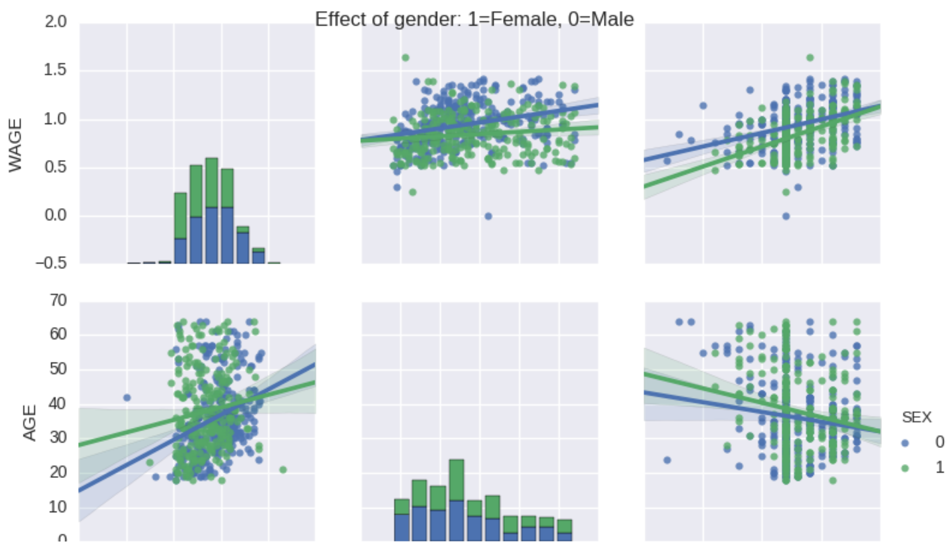
```
>>> import seaborn
```

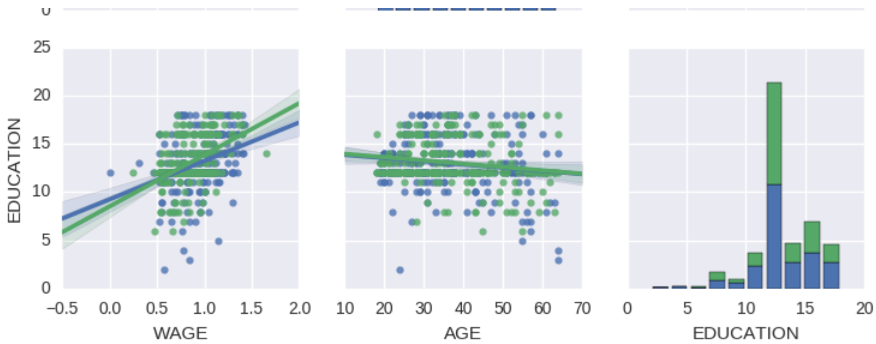
```
>>> seaborn.pairplot(data, vars=['WAGE', 'AGE', 'EDUCATION'],
...                  kind='reg')
```



Categorical variables can be plotted as the hue:

```
>>> seaborn.pairplot(data, vars=['WAGE', 'AGE', 'EDUCATION'],
...                  kind='reg', hue='SEX')
```





Look and feel and matplotlib settings

Seaborn changes the default of matplotlib figures to achieve a more "modern", "excel-like" look. It does that upon import. You can reset the default using:

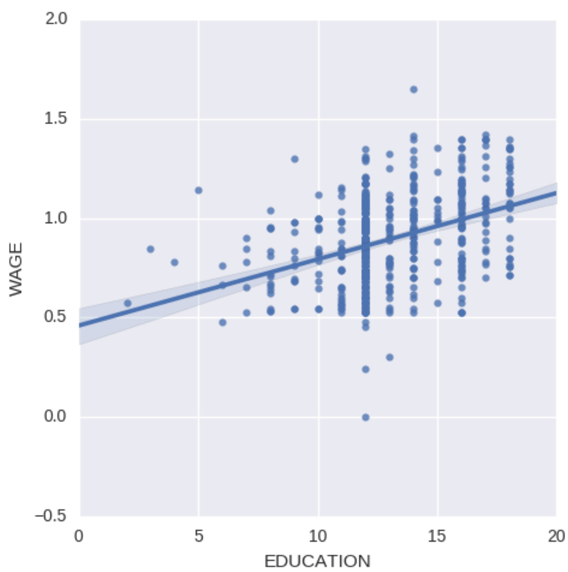
```
>>> from matplotlib import pyplot as plt
>>> plt.rcdefaults()
```

To switch back to seaborn settings, or understand better styling in seaborn, see the relevent section of the seaborn documentation.

3.1.4.2. lmplot: plotting a univariate regression

A regression capturing the relation between one variable and another, eg wage and eduction, can be plotted using `seaborn.lmplot()`:

```
>>> seaborn.lmplot(y='WAGE', x='EDUCATION', data=data)
```

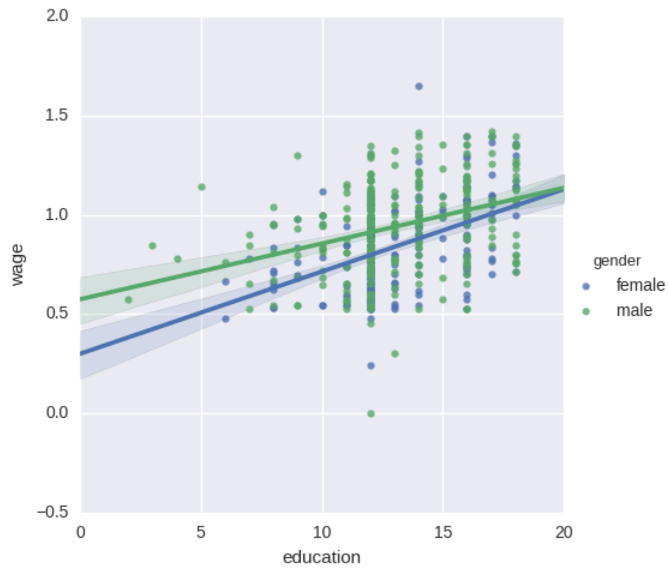


Robust regression

Given that, in the above plot, there seems to be a couple of data points that are outside of the main cloud to the right, they might be outliers, not representative of the population, but driving the regression.

To compute a regression that is less sentive to outliers, one must use a robust model. This is done in seaborn using `robust=True` in the plotting functions, or in statsmodels by replacing the use of the OLS by a "Robust Linear Model", **`statsmodels.formula.api.rlm()`**.

3.1.5. Testing for interactions



Do wages increase more with education for males than females?

The plot above is made of two different fits. We need to formulate a single model that tests for a variance of slope across the to population. This is done via an "interaction".

```
>>> result = sm.ols(formula='wage ~ education + gender + education * gender',
...                 data=data).fit()
>>> print(result.summary())
...
```

```
                            coef    std err      t      P>|t|
      [95.0% Conf. Int.]
------------------------------------------------------------
      -------------------
Intercept                   0.2998    0.072    4.173    0.000
      0.159   0.441
gender[T.male]              0.2750    0.093    2.972    0.003
      0.093   0.457
education                   0.0415    0.005    7.647    0.000
      0.031   0.052
education:gender[T.male]   -0.0134    0.007   -1.919    0.056
      -0.027   0.000
==========================...
...
```

Can we conclude that education benefits males more than females?

Take home messages

- Hypothesis testing and p-value give you the **significance** of an effect / difference
- **Formulas** (with categorical variables) enable you to express rich links in your data
- **Visualizing** your data and simple model fits matters!
- **Conditionning** (adding factors that can explain all or part of the variation) is important modeling aspect that changes the interpretation.

3.1.6. Full code examples

3.1.6.1. Code behind the figures

3.1.6.2. Solutions to this chapter’s exercises