Electromagnetism

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seriesColumb's law. Let q_1, q_2 be two point charges at positions $\boldsymbol{r}_1, \boldsymbol{r}_2$, respectively. Then the force $\boldsymbol{F}_2$ experienced by q_2 in the vicinity of q_1 is given by

$$\boldsymbol{F}_{12} = kq_1q_2\frac{\boldsymbol{r}_1 - \boldsymbol{r}_2}{\|\boldsymbol{r}_1 - \boldsymbol{r}_2\|^3},$$

where $k = \frac{1}{4\pi\varepsilon_0}$ and $\varepsilon_0 = 8,854\ F/m$ is the vacuum permittivity.

seriesElectric field. We define the *electric field* $\boldsymbol{E}$ as the force per unit of charge. For a point charge, we have that the electric field created by q_1 i the position of r_2 is

$$\boldsymbol{F}_2 = q_2\boldsymbol{E}_1(\boldsymbol{r}_2), \quad \boldsymbol{E}_1(\boldsymbol{r}_2) = kq_1\frac{\boldsymbol{r}_1 - \boldsymbol{r}_2}{\|\boldsymbol{r}_1 - \boldsymbol{r}_2\|^3}.$$

seriesSuperposition principle. ARREGLAR Let $\rho(\boldsymbol{r}) = \frac{dq}{d\mathcal{V}}$ be the volume charge density of an object. Then we have that

$$\boldsymbol{F} = \int_{\mathcal{V}} \rho(\boldsymbol{r})\boldsymbol{E}(\boldsymbol{r})d\mathcal{V}, \quad \boldsymbol{E}(\boldsymbol{r}) = k\int_{\mathcal{V}} \rho(\boldsymbol{r}')\frac{\boldsymbol{r} - \boldsymbol{r}'}{\|\boldsymbol{r} - \boldsymbol{r}'\|^3}d\mathcal{V}.$$ [1]

seriesElectric field superposition principle. Let $\rho(\boldsymbol{r}) = dq/d\mathcal{V}$, $\sigma(\boldsymbol{r}) = dq/d\mathcal{A}$ and $\lambda(\boldsymbol{r}) = dq/d\ell$ be the volume, surface and linear charge densities of an object, respectively. Then the resulting electric field at a point $\boldsymbol{r}$ is

$$\boldsymbol{E}(\boldsymbol{r}) = k\int_{\mathcal{V}} \rho(\boldsymbol{r}')\frac{\boldsymbol{r} - \boldsymbol{r}'}{\|\boldsymbol{r} - \boldsymbol{r}'\|^3}d\mathcal{V}'.$$
$$\boldsymbol{E}(\boldsymbol{r}) = k\int_{\mathcal{A}} \sigma(\boldsymbol{r}')\frac{\boldsymbol{r} - \boldsymbol{r}'}{\|\boldsymbol{r} - \boldsymbol{r}'\|^3}d\mathcal{A}'.$$
$$\boldsymbol{E}(\boldsymbol{r}) = k\int_{L} \lambda(\boldsymbol{r}')\frac{\boldsymbol{r} - \boldsymbol{r}'}{\|\boldsymbol{r} - \boldsymbol{r}'\|^3}d\ell.$$

seriesGauß's theorem.

$$\nabla\cdot\boldsymbol{E} = \frac{\rho}{\varepsilon_0} \iff \oint_{\mathcal{S}} \boldsymbol{E}\cdot\boldsymbol{n}d\mathcal{S} = \frac{1}{\varepsilon_0}\int_{\mathcal{V}} \rho(r)d\mathcal{V} = \frac{Q_T}{\varepsilon_0}, \quad (1)$$

where Q_T is the total charge enclosed within $\mathcal{V}$.

seriesWork. The work required to move a point charge q from a to b is

$$W_{b\to a} = -q\int_a^b \boldsymbol{E}\cdot d\ell,$$

where the negative sign indicates that the work is done against the field.

seriesElectric potential. The electric potential ϕ in a point $\boldsymbol{r}$ is defined as:

$$\boldsymbol{E} = -\nabla\phi, \quad \phi(\boldsymbol{r}) = k\int \frac{\rho(\boldsymbol{r}')}{\|r - r'\|}d\mathcal{V}. \quad (2)$$

And then,

$$\phi_a - \phi_b = -\int_b^a \boldsymbol{E}\cdot d\ell = \int_a^b \boldsymbol{E}\cdot d\ell.$$

Alternatively we can define the potential from the electric energy. And also if we consider the point P as a reference point we can define the *electric energy* U_a as follows

$$U_a - U_b = -q\int_P^a \boldsymbol{E}\cdot d\ell + q\int_P^b \boldsymbol{E}\cdot d\ell = -q\int_b^a \boldsymbol{E}\cdot d\ell = W_{b\to a}.$$

And finally we get

$$\Delta U(\boldsymbol{r}) = q\Delta\phi(\boldsymbol{r}).$$

seriesPoisson and Laplace equations. Taking into account formulas 1, 2, we get Poisson's equation

$$\nabla^2\phi = -\frac{\rho}{\varepsilon_0}.$$

If $\rho = 0$, we obtain Laplace's equation:

$$\nabla^2\phi = 0.$$

Definition 1.1. A conductor is a material in which charges can move freely.

seriesFaraday's cage. Inside a cavity with no charge of a conductor we have $\boldsymbol{E} = 0$ no matter how many charges and the potential are outside. This cavity is known as *Faraday's cage.*

System of N conductors

FALTA COSA.

Definition 1.2 (Capacitor). A capacitor is formed by two conductors of charges $\pm q$ and a potential difference $\Delta\phi$ not depending on the charge of other conductors. As a result, we have the following equality:

$$\Delta\phi = \frac{q}{C},$$

where C is the *capacitance* of the capacitor an it unit is the *farad* ($[C] = F$).

seriesCapacitors in series and parallel. The total capacitance of n capacitors in series is

$$\frac{1}{C_{\text{total}}} = \sum_{j=1}^{n}\frac{1}{C_i}.$$

The total capacitance of n capacitors in parallel is

$$C_{\text{total}} = \sum_{j=1}^{n} C_i.$$

[1] Analogously we can define $\sigma(\boldsymbol{r}) = \frac{dq}{dS}$ to be the surface charge density and $\lambda(\boldsymbol{r}) = \frac{dq}{d\ell}$ to be the linear charge density, and the integrals become as expected.

Potential energy of a charge distribution

seriesDiscrete charge distribution. Consider a distribution of n charges q_i. If ϕ_{ij} is the potential caused by the charge j on the point where the charge i is located, we have that the energy of the distribution W is

$$W = \sum_{i>j} q_i \phi_{ij} = \frac{1}{2} \sum_{i=1}^{n} q_i \phi_i,$$

where $\phi_i = \sum_{i \neq j} \phi_{ij}$.

seriesContinuous charge distribution. Consider a continuous charge distribution of density ρ. Then

$$W = \frac{1}{2} \int_{\mathcal{V}} \rho \phi d\mathcal{V} = \frac{\varepsilon_0}{2} \int_{\mathbb{R}^3} E^2 d\mathcal{V}.$$

Definition 1.3. The radius in which the electrostatic energy equals the rest energy of an electron is called *classical electron radius* and it's equal to:

$$r_0 = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{m_e c^2}.$$