

# Lie group notes

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# Conventions

- By Lie group, we mean either real or complex.
- A Lie subgroup of a Lie group is called a closed Lie subgroup (see Theorem 3).

# Chapter 1

## Lie groups: basic definitions

The following theorem allows us to reduce the study of Lie groups to the study of finite groups and connected Lie groups, since for a general Lie group  $G$  we have

$$G = G^0 \times G/G^0,$$

where  $G^0$  is the identity component of  $G$ .

**Theorem 1** (Theorem 2.6, [Kir08]). *Let  $G$  be a real or complex Lie group and  $G^0$  its identity component. Then  $G^0$  is a normal subgroup of  $G$  and  $G/G^0$  is a discrete group.*

In fact, we can reduce the case of connected Lie groups to simply connected Lie groups:

**Theorem 2** (Theorem 2.7, [Kir08]). *Let  $G$  be a connected Lie group. Then its universal cover  $\tilde{G}$  has a canonical structure of a Lie group such that the covering map  $p : \tilde{G} \rightarrow G$  is a homomorphism of Lie groups whose kernel is isomorphic to the fundamental group of  $G$ . Moreover, in this case,  $\ker p$  is a discrete central subgroup in  $\tilde{G}$ .*

We have the following connection between subgroups and Lie subgroups (i.e. subgroups that are also submanifolds):

**Theorem 3** (Theorem 2.8, [Kir08]).

- *Any Lie subgroup of a Lie group is closed in the topology of the ambient group.*

- *Any closed subgroup of a Lie group is a real Lie subgroup.*

# Bibliography

- [Kir08] Alexander A Kirillov. *An introduction to Lie groups and Lie algebras*. Vol. 113. Cambridge University Press, 2008 (cit. on p. 3).