

$$a^{100} = \underbrace{(a * a * a * \cdots * a)}_{100 \text{ times multiplication}}$$

$$(100)_{10} = (1100100)_2$$

$$100 = 2^6 + 2^5 + 2^2 = 64 + 32 + 4$$

$$a^{100} = a^{(2^6+2^5+2^2)} = a^{(64+32+4)} = a^{2^6} * a^{2^5} * a^{2^2}$$

$$\underbrace{a^{(2^1)} \xrightarrow[\text{multiply once}]{\text{square}} a^{(2^2)} \xrightarrow[\text{multiply once}]{\text{square}} a^{(2^3)} \xrightarrow[\text{multiply once}]{\text{square}} a^{(2^4)} \cdots}_{\log_2(n) \text{ times multiplication}}$$

$$O(n + \sum_{k=0}^{\log_2(n)-1} 2^k) = O(n + n) = O(n)$$

$$\frac{O(n)}{n} = O(1)$$

$$\begin{array}{l} \boxed{2} \longrightarrow \boxed{3x^2} \longrightarrow \boxed{4x^5} + \boxed{4} \longrightarrow \boxed{3x} \longrightarrow \boxed{-2x^2} \longrightarrow \boxed{5x^6} \\ \xrightarrow{\text{link}} \boxed{2} \longrightarrow \boxed{3x^2} \longrightarrow \boxed{4x^5} \longrightarrow \boxed{4} \longrightarrow \boxed{3x} \longrightarrow \boxed{-2x^2} \longrightarrow \boxed{5x^6} \\ \xrightarrow{\text{sort}} \boxed{2} \longrightarrow \boxed{4} \longrightarrow \boxed{3x} \longrightarrow \boxed{3x^2} \longrightarrow \boxed{-2x^2} \longrightarrow \boxed{4x^5} \longrightarrow \boxed{5x^6} \\ \xrightarrow{\text{merge}} \boxed{6} \longrightarrow \boxed{3x} \longrightarrow \boxed{x^2} \longrightarrow \boxed{4x^5} \longrightarrow \boxed{5x^6} \end{array}$$