**Subject4: Implement and Analysis of Algorithms**

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**Project1: Implement and Analysis of Common Sorting Algorithms**

In this project, we tried 8 different algorithms, which are insertion-sort, bubble-sort, selection-sort, counting-sort, quick-sort, merge-sort, heap-sort and intro-sort. Here give a table to compare these good algorithms.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | Best Time Complexity | Worst Time Complexity | Average Time Complexity | Stability | Advantage | Disadvantage |
| Insertion sort | O(n) | O(n^2) | Θ(n^2) | T | Quick in small test cases | O(n^2) is not acceptable when n is big |
| Bubble sort | O(n^2) | O(n^2) | Θ(n^2) | T | Easy to write | O(n^2) is not acceptable when n is big |
| Selection sort | O(n^2) | O(n^2) | Θ(n^2) | F | Easy to understand | O(n^2) is not acceptable when n is big |
| Counting sort | O(n) | O(n) | O(n) | T | The fastest algorithm when sorting bounded integers | Needs too much space, is not suit for abstract elements |
| Quick sort | O(nlogn) | O(n^2) | O(nlogn) | F | Quick | Time complexity can worsen to O(n^2) when data is special |
| Merge sort | O(nlogn) | O(nlogn) | Θ(nlogn) | T | Stable and quick | Needs O(n) extra space |
| Heap sort | O(nlogn) | O(nlogn) | Θ(nlogn) | F | Time complexity is stable and quick | Coding complexity is higher, not quick as quick-sort |
| Intro sort | O(nlogn) | O(nlogn) | Θ(nlogn) | F | The complexity will not worsen as quick-sort and even faster in big test cases | Coding complexity is very high |
| Radix sort | O(nlogk) | O(nlogk) | Θ(nlogk) | T | When integers are small, it can sort quickly | Cost a large sum of memory |

After writing codes of these algorithms, we made a test for them.

Here goes the result (***seconds, running 100 times***):

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** |
| **Algorithm / testcases** | **[3,1,10]** | **[7,1,100]** | **[8,1,100]** | **[10,1,100]** | **[50,1,100]** | **[100,1,100]** | **[1e3,1,1e4]** | **[1e4,1,1e4]** | **[1e5,1,1e5]** | **[1e6,1,1e6]** | **[1e4,1,1]** | **[1e4,1,2]** | **[1e6,1,100]** |
| **Bubble sort** | 3.4e-05 | 3.8e-05 | 3.9e-05 | 6e-05 | 0.000152 | 0.000741 | 0.081903 | 12.9937 | N/A | N/A | 6.00655 | 12.2812 | N/A |
| **Counting sort** | 0.000346 | 0.000283 | 0.000358 | 0.000266 | 0.000297 | 0.00035 | 0.003551 | 0.009406 | 0.106647 | 1.71637 | 0.002507 | 0.002031 | 0.132891 |
| **Heap sort** | 5.2e-05 | 3.8e-05 | 4.4e-05 | 3.9e-05 | 9.1e-05 | 0.000136 | 0.005266 | 0.076338 | 1.07909 | 15.251 | 0.003955 | 0.031763 | 9.12897 |
| **Insertion sort** | 6.8e-05 | 4.9e-05 | 3.4e-05 | 3.5e-05 | 9.4e-05 | 0.00022 | 0.016319 | 1.7313 | N/A | N/A | 0.001183 | 0.85815 | N/A |
| **Merge sort** | 4.5e-05 | 9.2e-05 | 6.2e-05 | 9.8e-05 | 0.000155 | 0.000382 | 0.005251 | 0.087687 | 1.00966 | 11.6311 | 0.020728 | 0.029245 | 7.54492 |
| **Quick sort** | 9.2e-05 | 4.1e-05 | 3.7e-05 | 4.2e-05 | 7.3e-05 | 0.000185 | 0.003582 | 0.066211 | 0.827287 | 9.33422 | 3.41382 | 1.69959 | 345.842 |
| **Selection sort** | 4.7e-05 | 3.8e-05 | 3.9e-05 | 4e-05 | 0.000127 | 0.000881 | 0.038085 | 3.85988 | N/A | N/A | 3.42656 | 3.38535 | N/A |
| **Radix sort** | 0.000123 | 0.000133 | 0.000199 | 0.000172 | 0.00068 | 0.003483 | 0.01631 | 0.151731 | 1.32204 | 12.8391 | 0.127762 | 0.13206 | 14.3869 |
| **STL sort** | 3.4e-05 | 5.5e-05 | 3.6e-05 | 3.9e-05 | 6.1e-05 | 9.1e-05 | 0.002389 | 0.057353 | 0.643091 | 7.82551 | 0.007178 | 0.014152 | 3.8654 |
| **Intro sort** | 3.3e-05 | 3.7e-05 | 6e-05 | 4.2e-05 | 5.8e-05 | 0.000165 | 0.002325 | 0.06138 | 0.708481 | 8.77974 | 0.008467 | 0.014123 | 4.29275 |

Two specially organized test cases:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Test14: [100000,1000000,1100000]+[100000,900000,1000000]+[100000,800000,900000]+[100000,700000,800000]+[100000,600000,700000]+[100000,500000,600000]+[100000,400000,500000]+[100000,300000,400000]+[100000,200000,300000]+[100000,100000,200000] | | | | | | | | | |
| **Bubble sort** | **Counting sort** | **Heap sort** | **Insertion sort** | **Merge sort** | **Quick sort** | **Selection sort** | **Radix sort** | **STL sort** | **Intro sort** |
| N/A | Error | 14.7471 | N/A | 11.2109 | 9.46302 | N/A | 13.0001 | 8.04907 | 8.65674 |
| Test15: [100000,100000,200000]+[100000,200000,300000]+[100000,300000,400000]+[100000,400000,500000]+[100000,500000,600000]+[100000,600000,700000]+[100000,700000,800000]+[100000,800000,900000]+[100000,900000,1000000]+ [100000,1000000,1100000] | | | | | | | | | |
| **Bubble sort** | **Counting sort** | **Heap sort** | **Insertion sort** | **Merge sort** | **Quick sort** | **Selection sort** | **Radix sort** | **STL sort** | **Intro sort** |
| N/A | Error | 14.5477 | N/A | 10.9559 | 8.80519 | N/A | 12.9844 | 7.67523 | 8.45927 |

*Note: 1. [n,l,r] means that test case contains n random integers between l and r.*

*2. Optimize option -O2 was active.*

We can see that, in some special testcases such as testcase 11,12,13, the unoptimized quick-sort is as slow as the O(n^2) sorting algorithms.

After analyzing the data above, we found that heap-sort and merge-sort are usually 1.2-2 times slower than quick-sort but the time cost still grows by O(nlogn).

The *Intro-sort* is an algorithm which combines quick-sort, heap-sort and insertion-sort together. At the beginning, this algorithm uses quick-sort to sort the array, when the recursion depth grows too much, the algorithm detects it and change to use heap-sort to sort the rest elements. When the length of the range becomes short, it turns to use insertion-sort when O(n^2) is faster than O(nlogn). And the optimized quick-sort can run faster.

We have tried many ways to optimize the speed of the code. And spent a lot of time reading STL files. However, our code is still a little bit slower than std::sort function, but has made a great improvement compared with the other algorithms.

And we found an interesting skill from the STL: when we try to calculate middle value of two integers, we write as:

Mid=(l+r)>>1;

But this has a problem, that is, if l and r are both very big may be 2e9, then 2e9+2e9 will cause error of integer exceeding. Therefore, we should write this way:

Mid=l+((r-l)>>1);

This will make you avoid exceeding easyly.

**Project2: Friend Filter**

First, we read info into an array. And get tree copy of the info. Sort three arrays, one by hash value of name, one by height and the other one by weight. Find info by dichotomy.

Following operations are provided:

*f name : find a person by name*

*qh l r : query l-th to r-th tall person*

*qw l r : query l-th to r-th heavy person*

*gh x y : find all persons whose height is between x and y*

*gw x y : find all persons whose weight is between x and y*

*Note：For operation qh and qw, for example, in series [1,1,2,4,4,4,5,5,6], 1s are considered as the first, 4s are considered as the third etc..*

To deal with the rank, we prepared a suffix-array, to store the ranks of each elements.

Because the array was sorted, all query operations are O(logn). So, the time complexity of initializing is O(nlogn+len), where len is the total length of names(Hash algorithm is O(len)), and the time complexity of each query if O(logn).

However, though the query operation is quick enough, when n becomes large, printing the result can cost a large sum of time. Printing all of the result cost O(m\*n) time. Therefore, the total time complexity is O(mn+nlogn+n)=O(n^2).

We made a test and analyzed the time-cost with Matlab. As we can see, the time grows by O(n^2) with a small coefficient.

Linear model:

f(x) = a\*x\*x + b\*x\*log(x)/log(2) + c\*x

Coefficients (with 95% confidence bounds):

a = 5.274e-08 (5.031e-08, 5.516e-08)

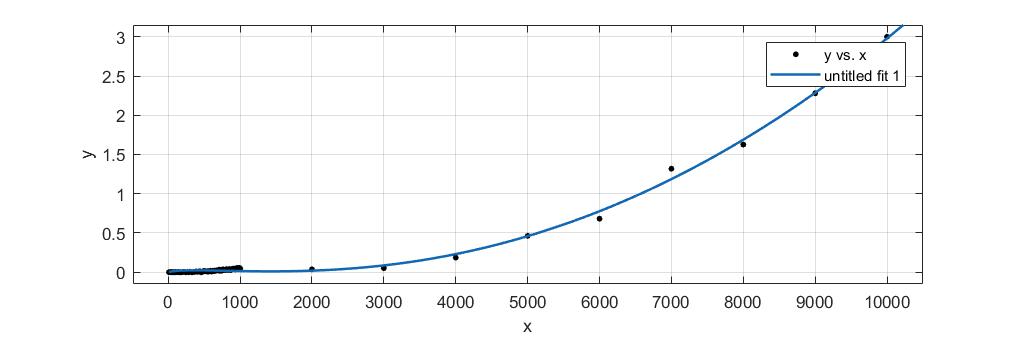
b = -5.766e-05(-6.406e-05, -5.126e-05)

c = 0.0005368 (0.0004723, 0.0006013)

Goodness of fit:

SSE: 0.06523 R-square: 0.9964

Adjusted R-square: 0.9963 RMSE: 0.02469



Fitting curve