CFD Course Formula Student Bizkaia 2017

Turbulence modelling

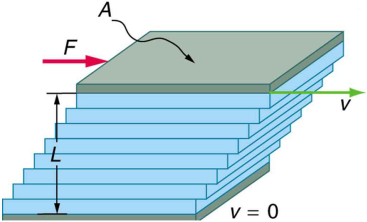
BCAM - Basque Center for Applied Mathematics
UPV/EHU

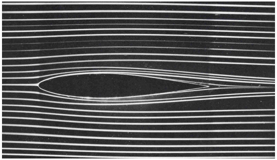
Fall 2017

1. What is “turbulence”?

Fluid flow: Laminar and Turbulent regimes

Laminar flow (or streamline flow): regime of fluid motion in **parallel layers**; There is no lateral mixing, no cross currents perpendicular to the direction of flow, nor eddies (or swirls) of fluids.



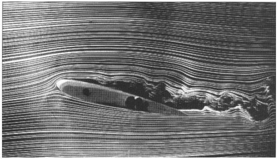
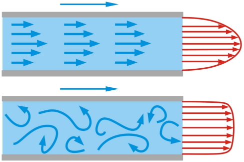
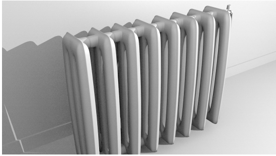
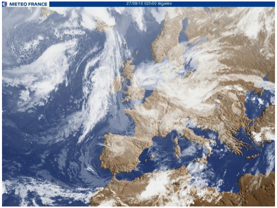


(Un)fortunately, almost all "interesting“ flows around us are **turbulent** . . .

Source: (left-up) http://philschatz.com/ (center-up) http://www.telegraph.co.uk/ (right-up) https://www.quora.com (bottom-left) Department of Engineering, University of Cambridge, multimedia video from Physics Education, 2003, by Holger Babinsky (bottom-right) https://novakdjokovicfoundation.org

Fluid flow: Laminar and Turbulent regimes

Turbulent flow: regime of fluid motion with **"random" and chaotic three-dimensional vorticity**;
We observe increased energy dissipation, mixing, heat transfer, and drag.


Source: (top-left) https://www.quora.com (top-center) http://happevanrijn.com (top-right) http://www.dustymars.net (bottom-left) https://dbaron.org (bottom-center) http://www.explainthatstuff.com/aerodynamics.html (bottom-right) Café Atico Bilbao

Why is it important to study turbulence?

At least, for **two types** of reasons:

1. **Engineering** applications:
   - airplanes must fly
   - weather must be forecast
   - sewage and water management systems must be built
   - society needs ever more energy-efficient hardware and gadgets
2. **Science**
   - understand the physics of the flow
   - complementary (or not) to engineering

Generation: some examples I

It is believed that the **inherent instability** of the physics of the flow plays the determinant role.

Examples:

1. Osborne Reynolds’ (1842-1912, Anglo-Irish) pipe experiment

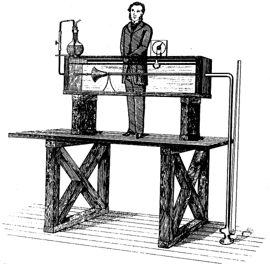
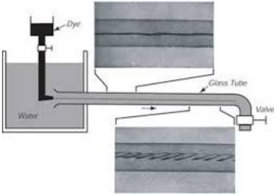
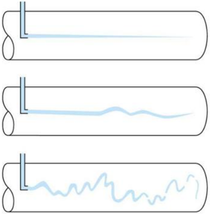


Fig. 9.1. Sketch of Reynolds’s dye experiment, taken from his 1883 paper



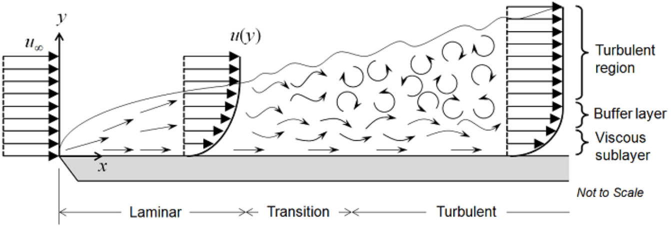


Source: (left) http://misclab.umeoce.maine.edu (right) https://analysisofflowinpipes.jimdo.com

$$\boxed{Re = \frac{\rho \boldsymbol{v} L}{\mu} \equiv \frac{\text{Intertial forces}}{\text{Viscous forces}}}$$

Generation: some examples II

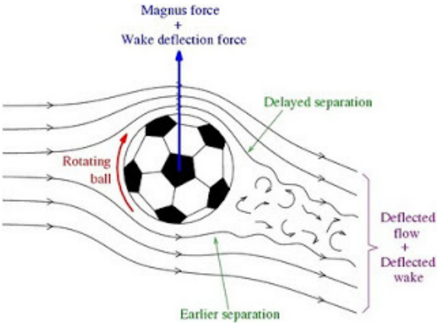
2. Instability in boundary layers (Ludwig Prandtl, 1875-1953, German)
   No-slip condition and theory on boundary layers (1904)



Source: https://www.comsol.com

Generation: some examples III

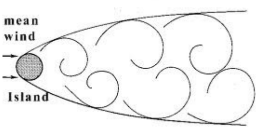
More on the boundary layers: the **Magnus effect** explains the deviation in the trajectory of a ball (i.e. soccer, tennis . . .)



Source: http://www.glogster.com

Generation: some examples IV

3. Theodore von Kármán (1881-1963, Hungarian-American) vortex street
(i.e. unsteady separation of flow around blunt bodies, frequency $\sim$ Reynolds number)



Source: (up-left) http://oiswww.eumetsat.org/ (up-center) https://nylander.wordpress.com/2005/01/11/von-karman-vortex-street/ (up-right) http://alg.umbc.edu/usaq/images/USA5.2006257.aqua.500m.crop.jpg (bottom-left) http://fox41blogs.typepad.com/ (bottom-right) https://en.wikipedia.org/

Complexity of turbulence

- **Werner Karl Heisenberg** (1901-1976, German, Nobel Prize in Physics in 1932):
  "When I meet God, I am going to ask him two questions: Why relativity? And why turbulence? I really believe he will have an answer for the first."
- Sir **Horace Lamb** (1849-1934, British): speech to the British Association for the Advancement of Science
  "I am an old man now, and when I die and go to heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of fluids. And about the former I am rather optimistic."

Characteristics of turbulence

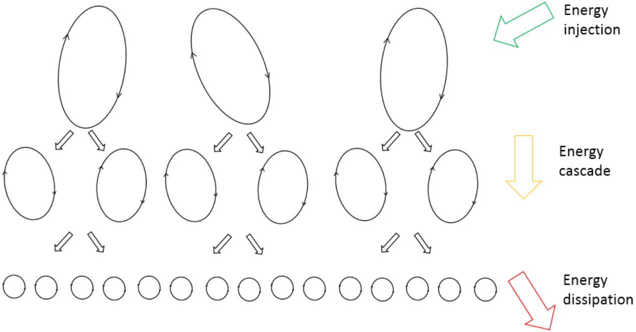
Characteristics of turbulence
(from *“Tennekes and Lumley: First Course in Turbulence”*):

1. “Random”: disorder and no-repeatability
2. Vortical: high concentration and intensity of vorticity
3. Non-linear and three-dimensional
4. Continuity of eddy structure (vortices are **strong structures**), reflected in a continuous spectrum of fluctuations over a range of frequencies (**wavenumber**)
5. Energy cascade, irreversibility and dissipativeness
6. **Intermittency**: turbulence can occupy only parts of the flow domain $\rightarrow$ typically we make use of **statistical** approach
7. **High diffusivity** of momentum, energy, species ...

Three hypothesis on turbulence

Hypotheses about turbulence: energy cascade I

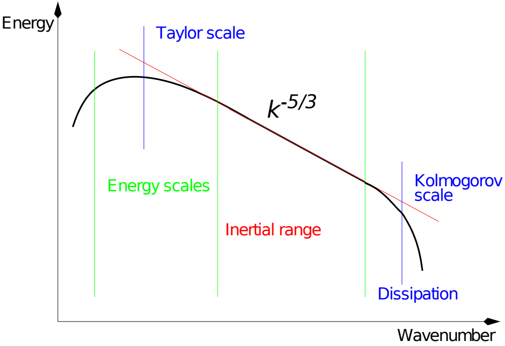
1. **Lewis Fry Richardson** (1881-1953, English) proposed the **turbulent cascade** for energy in 1920.
   Energy is transferred from the large scales of motion to the small scales. The dynamics of such system is **non-linear**.



Source: https://www.simscale.com

Hypotheses about turbulence: energy cascade II

Example. Think of a flow passing through a grate of size *L*. This will be the energy injection length scale (large scales), and from it the energy will be transported through an inertial scale towards the **smallest scales**, where it will be **dissipated**



“wavenumber” $k \equiv 2\pi/L$, where L the length scale of the vortex structure

Hypotheses about turbulence: frozen turbulence

2. Sir **Geoffrey Ingram Taylor** (1886-1975, British) hypothesis on “frozen turbulence”: as the mean flow $\boldsymbol{v}$ advects the eddies, the fundamental properties of the eddies **remain unchanged**, or “frozen”, since the turbulent fluctuations $\boldsymbol{v}'$ are much smaller $\boldsymbol{v}' \ll \boldsymbol{v}$

The **Taylor microscale** corresponds approximately to the **inertial subrange** of the energy spectrum. In such range, the energy is transferred down from the largest scales to the smallest ones **without dissipation**.

Hypotheses about turbulence: Kolmogorov scale

3. **Andrey Nikolaevich Kolmogorov** (1903-1987, Russian) **similarity hypothesis**: in every turbulent flow at sufficiently high Reynolds number, the statistics of the small scale motions have a **universal form** that is uniquely determined.
   - **Large scale** L vortices are **anisotropic**
   - **Small scale** η vortices loose all structure information about how they were created, they become **homogeneous** and **isotropic**, that is, “similar”
   - At the small length scale, the energy input and dissipation are in exact **balance**
   - NOTE: all this assuming **large Reynolds** number

What do we know for sure?

- We rely on **assumptions** formulated in the ’30-’40
- These hypotheses are made on the basis of logic and dimensional analysis of the flow, yet **we look for rigorous (mathematical) proof.**
- Of course, in many cases, there is extensive experimental evidence that these hypotheses are indeed **correct**.

Rigor

Source: "Full metal jacket"

2. Turbulence modelling

Why we need to model turbulence

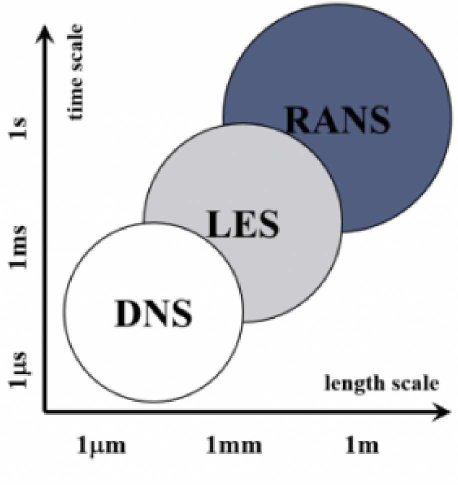
We can numerically simulate **inviscid** (i.e. Euler) or **laminar** flows using directly the governing equations that we derived.
But, despite the performance of supercomputers, turbulent flows represent a **significant problem**.

- The **direct** simulation of the governing NS equations (called "DNS") is possible only for **"simple"** cases and **low** *Re* number
- It can be shown that the number of mesh points scales as $Re^{9/4}$, while realistic applications have a $Re \sim 10^6 - 10^9$
- DNS is still **very useful** as research tool
- But in the majority of the cases we need to apply some **model for turbulence**

Three main approaches to turbulence

Three main approaches to turbulence

The basic **difference** among the 3 approaches is the **time and space resolution**.



Source: http://csyoo.unist.ac.kr

1) Direct Numerical Simulation (DNS)

1. **Direct Numerical Simulation (DNS)**
   - **No turbulence model at all**: it is regarded as the *analitic* solution to the N-S equations
   - Solves **all** scales of motion
   - **Extremely costly**, only used for scientific purpose (e.g. validate turbulence models, fundamental understanding of turbulence)
   - Limited to **simple** cases
   - It is “possible” only since the ’80 due to computers’ advancement (months of runs, hundreds of gigas of data to post-process)

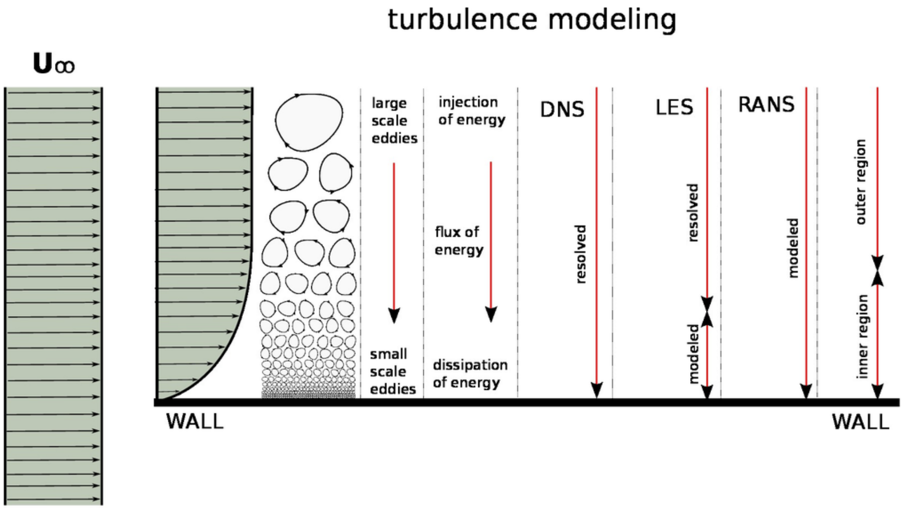
2) Large Eddy Simulation (LES)

2. **Large Eddy Simulation (LES)**
    - Solves **directly** the **filtered (in space)** NS equations (i.e. solves the large scale vortex structures while filters the smaller sub-grid resolution scales)
    - It uses a model for turbulence to dissipate the energy (usually) at the **sub-grid scales**
    - It is a clear trend in **engineering** since the last decades
    - Allows for more **complex** cases
    - Intermediate cost (weeks of runs)

3) Reynolds Averaged Navier-Stokes (RANS)

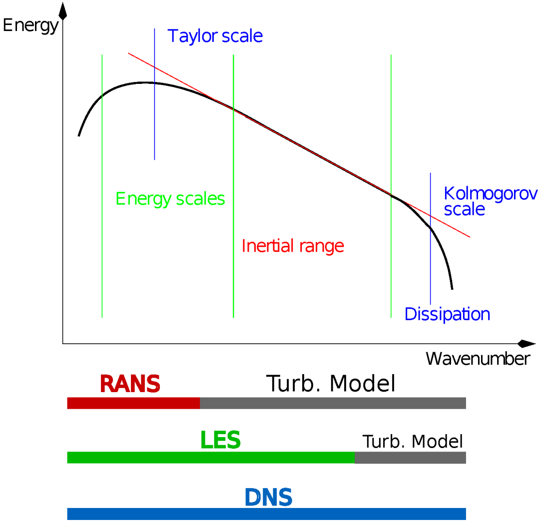
3. **Reynolds Averaged Navier-Stokes (RANS)**
   - Presented by Reynolds in 1885
   - Based on the **decomposition** of the variables into **mean** and **fluctuating** part $\boldsymbol{v} = \overline{\boldsymbol{v}} + \boldsymbol{v}'$
   - Then, a **time averaging** is operated and we have two **additional** terms in the NS equations
   - The dissipation due to turbulence is entirely modeled
   - Affordable computational cost: considerably coarser grids used compared to LES
   - Very popular: used in most of commercial/open software packages
   - Many models available (S-A, κ-ω, κ-ϵ, SST . . .)

Three approaches for turbulence



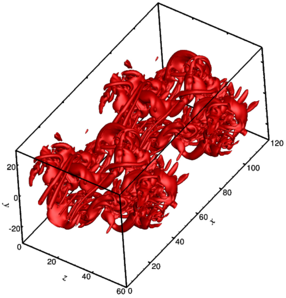
Source: https://www.cfdsupport.com

Three approaches for turbulence

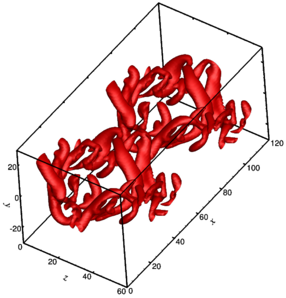


DNS vs LES

The LES capture the large scales, but misses some details and also some dynamics that the DNS is able to predict.
Turbulence structures in 3D:



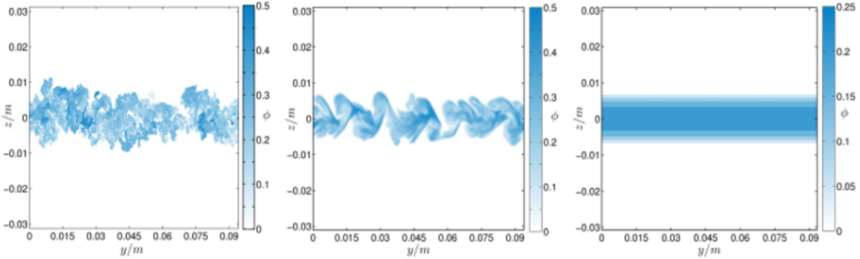
DNS



LES

DNS vs LES vs RANS

RANS simulations are way more affordable, but need to pay attention to their limitations: sometimes can give laminar flow, where the flow is actually turbulent!



DNS:
All turbulence scales are resolved

LES:
Only larger scales resolved

RANS:
we just see a laminar flow

Source: https://tu-freiberg.de

Turbulence modelling: main framework

1) We **cannot solve all turbulence scales** for practical applications

$\Downarrow$

2) We need to **model**, that is, **approximate the effects** of turbulence on the flow

$\Downarrow$

3) Among the observed physical characteristics of turbulence (enhanced vorticity, viscosity, convection, randomness . . .), it has been chosen the **enhanced viscosity** μ_t, called “eddy”, or “turbulent” viscosity, as **representative** property of turbulent regions in the domain, trying to **mimic the turbulent energy dissipation**

$\Downarrow$

4) The problem is now to **predict where and when** will turbulence occur in the domain, and “simply” **add** the eddy viscosity μ_t in those cells

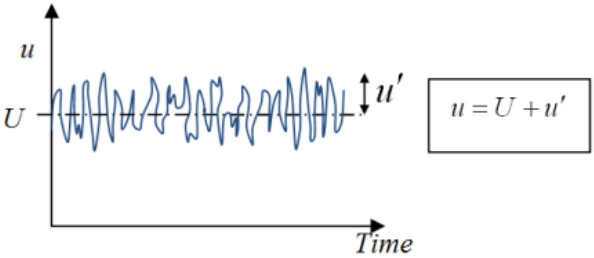
$\Downarrow$

5) Following the above idea, different approaches have been invented so far: Reynolds Averaged Navier Stokes (RANS), Large Eddy Simulation (LES) . . .

RANS: some ingredients

Reynolds decomposition and additional unknowns

- Reynolds decomposition (*fluctuations* of the velocity): $\boldsymbol{v} = \overline{\boldsymbol{v}} + \boldsymbol{v}'$



- Reynolds average is a **time average**: $\overline{\boldsymbol{v}} = \int_{t_1}^{t_2} \boldsymbol{v}\,\mathrm{d}t$
- When we apply the Reynolds average to the NS equations $\int_{t_1}^{t_2} \mathrm{NS}\,\mathrm{d}t$, **extra unknowns** pop up: $\boxed{\tau_{ij} \equiv -\rho\overline{u_i' u_j'}}$ and we call it the "**Reynolds stress tensor**" (know the **Einstein notation**)

Source: http://www.finot.com/ecrits/Damien Lafforgue

The problem of “closure” of the equations

- Concept of “**closure of the equations**”: we need to define $\tau_{ij} \equiv \overline{u_i' u_j'}$ in terms of the **mean flow variables** $\overline{\boldsymbol{v}}$ and not in terms of the fluctuations $\boldsymbol{v}'$
- First closure attempt by Joseph Valentin **Boussinesq** (1842-1929):

$$\boxed{\tau_{ij} \approx \mu_t}$$

in other words, we **approximate** the Reynolds stress tensor $\Rightarrow$ we have an **additional viscosity**, called **eddy viscosity** (or turbulent viscosity) μ_t so that $\boxed{\mu = \mu_L + \mu_t}$

How to “detect” turbulent regions?

- Prandtl (1945) used the **turbulent kinetic energy** (turbulent fluctuations) k as a basis for “detecting” turbulent regions:

$$k = \frac{1}{2}\overline{u_i' u_i'} = \frac{1}{2}\left(\overline{u_i'^2} + \overline{v_i'^2} + \overline{w_i'^2}\right) = -\frac{2}{\rho}\tau_{ii}$$

(mind Einstein notation of repeated index ii, and remember that the Reynolds stress is $\tau_{ij} \equiv -\rho\overline{u_i' u_j'}$)

- Thus, we can get the turbulent viscosity following dimensional analysis:

$$\mu_t = \text{constant} \cdot \rho k^{1/2} \ell$$

and add it in the turbulent cells of the domain as $\mu = \mu_L + \mu_t$

How to compute the turbulent kinetic energy k?

Transport equation for turbulence

Similarly as we did for $\rho, \rho\boldsymbol{v}, \rho E$ when deriving the Navier Stokes equations, we can write a **transport equation** for k trying to represent **physical processes** in turbulence:

$$\underbrace{\underbrace{\frac{\partial k}{\partial t}}_{\substack{\text{Local}\\ \text{derivative}}} + \underbrace{\overline{u}_j \frac{\partial k}{\partial x_j}}_{\text{Advection}}}_{\text{Rate of change of } k} = -\underbrace{\frac{1}{\rho_o}\frac{\partial \overline{u_i' p'}}{\partial x_i}}_{\substack{\text{Pressure}\\ \text{diffusion}}} - \underbrace{\frac{1}{2}\frac{\partial \overline{u_j' u_j' u_i'}}{\partial x_i}}_{\substack{\text{Turbulent}\\ \text{transport}\\ \mathcal{T}}} + \underbrace{\nu \frac{\partial^2 k}{\partial x_j^2}}_{\substack{\text{Molecular}\\ \text{viscous}\\ \text{transport}}}$$

$$\underbrace{-\overline{u_i' u_j'}\frac{\partial \overline{u_i}}{\partial x_j}}_{\substack{\text{Production}\\ \mathcal{P}}} - \underbrace{\nu \overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial u_i'}{\partial x_j}}}_{\substack{\text{Dissipation}\\ \varepsilon_k}} - \underbrace{\frac{g}{\rho_o}\overline{\rho' u_i'}\delta_{i3}}_{\substack{\text{Buoyancy flux}\\ b}}$$

Transport equation for turbulence

The terms in the previous equation have **physical interpretation**, as for example:

- Production $\mathcal{P} = -\overline{u_i' u_j'} \dfrac{\partial \overline{u_i}}{\partial x_j} = + \dfrac{\tau_{ij}}{\rho} \dfrac{\partial \overline{u_i}}{\partial x_j}$ is the **transfer of kinetic energy** from the **mean flow** to turbulence
- Dissipation $\varepsilon_k = -\nu \overline{\dfrac{\partial u_i'}{\partial x_j} \dfrac{\partial u_i'}{\partial x_j}}$ is the rate at which turbulence kinetic energy is **converted** into thermal internal energy

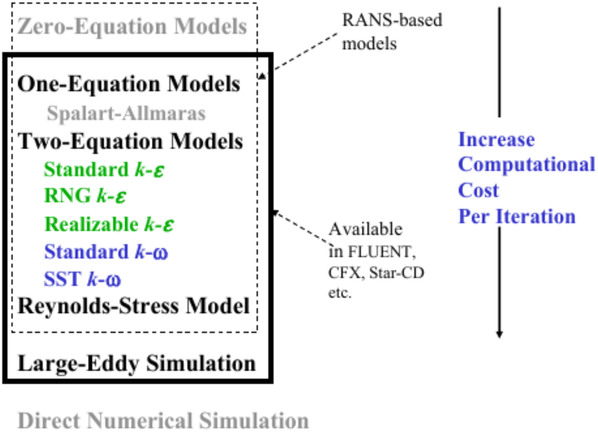
Closure of transport equations

- **Term-by-term** unknown correlations of the exact equation are **replaced** with **closure approximations**
- This process is by **no means rigorous**
- Our hope is that we can find **universally valid** closure approximations that make accurate solutions possible

Completeness of various models

- **One-equation models** (i.e. Spalart-Allmaras): we only use the turbulent transport equation (or similar quantity)
- **Two-equation models** (i.e. k-ϵ, k-ω, SST etc): A transport equation for the **dissipation** ϵ is added, thus these models are **more complete**
- **Beyond Boussinesq approximation - Reynolds stress equation model (RSM)** (second moment closures): the exact Reynolds tensor $\tau_{ij} \equiv -\rho\overline{u_i' u_j'}$ is considered; These models are the **most complete**, they take take into account **directional effects** in turbulence transport

Turbulence modelling: final wrap-up



Thumb rules for different models

One-equation: Spalart-Allmaras

1. **Good for:**
   - Low computational cost
   - Lower need to tune parameters for different flows
   - Attached wall-bounded flows
   - Boundary layers with adverse pressure gradients
2. **Bad for:**
   - Abrupt changes from wall-bounded to free shear flows (i.e. trailing edge of airfoil)
   - Free shear flows, especially plane and round jet flows
   - Separated flows
   - Free-shear flows
   - Decaying turbulence

Two-equation: standard k-ϵ

1. **Good for:**
   - **Free-shear layer flows**
   - Relatively low computational cost
   - Relatively easy to convergence
   - Insensible to free-stream boundary conditions
   - Relatively small pressure gradients
2. **Bad for:**
   - Large adverse pressure gradients
   - Separated flows
   - Wall-bounded and internal flows (large pressure gradients)
   - Inlets and compressors
   - Curved boundary layers
   - Rotating flows
   - Flows in non-circular ducts
   - Valid only for **fully developed** turbulent flows

Two-equation: improved RNG k-ϵ

Shares all the good and bad properties of the standard k-ϵ, but improves the following:

- Model for swirl on turbulence
- High streamline curvature and strain rate
- Transition to turbulence
- **Wall heat and mass transfer**

Two-equation: improved Realizable k-ϵ

Shares all the good and bad properties of the standard k-ϵ, but improves the following:

- Planar and round jets
- **Boundary layers under strong adverse pressure gradients or separation**
- **Rotation & recirculation**
- **Strong streamline curvature**

Two-equation: standard k-ω

Note: ω is the dissipation per unit turbulence kinetic energy $\epsilon \approx \omega k$

1. **Good for:**
   - **Internal and wall-bounded flows**
   - Relatively low computational cost
   - Relatively easy to converge
   - Boundary layers
   - Small/large adverse and favourable pressure gradient
   - **Skin friction coefficient in engineering applications**
2. **Bad for:**
   - **Sensible to free-stream** (inlet) boundary conditions
   - Boundary layers for low-Reynolds flows

Two-equation: Menter SST k-ω

1. **Good for:**
   - **Blend between** k-ω and k-ϵ: switches to a k-ϵ in the free-stream and k-ω formulation in boundary layers
   - Usable at low Reynolds numbers
   - Adverse pressure gradients
   - **Separated flow**
2. **Bad for:**
   - Regions with large normal strain, like stagnation regions and regions with strong acceleration (produces too large turbulence levels)
   - Higher computational cost compared to k-ω and k-ϵ

Recommendations

Recommended models for Formula Student

Considering the above characteristics of various turbulence models, the recommended choices would be:

- Menter SST k-ω
- RNG k-ϵ
- Realizable k-ϵ

Compare the results of the three: it **should not** be observed significant change in the results

Wall functions

You might try to perform simulations with **wall functions** to save computational cost

Turbulence modelling: useful readings

Useful readings

- Book *Turbulence Modeling for CFD*, by David C. Wilcox (first chapters)
- NASA website: https://turbmodels.larc.nasa.gov/
- CFD Online: https://www.cfd-online.com/Wiki/Turbulence_modeling
- Tutorial F1 car CFD (Thank you Imanol :) http://www.simscale-academy.com/p/applications-of-cfd-in-formula-student-formula-sae