Demo#4-Kriging

Omnidirectional Kriging

Jeffrey Yarus

30 juin, 2025

Contents

1	Demo Summary	1
2	Terminology and key functions from this Demo	2
3	Loading Packages	2
4	Initializing the Demo 4.1 Code chucks that apply to lognormal data only	2 4 5
5	Working in R S4 - Preparation for Geostatistical Analysis using RGeostats 5.1 Creating the Databases and Basemap	5
6	creating databases with and without the outlier	5
7	Variogram Construction and modeling 7.1 Omnidirectional Experimental Semivariogram	8 8 9
8	Selecting the variogram model(s) 8.0.1 Omnidirectional Variogram Model	9 10
9	Unique neighborhood	12
10	10.1 Creating the Grid for Kriging	

1 Demo Summary

In this demonstration, the objective is to create a Kriged map and it's associated error variance map. Here, the code begins as usual by reading in the data set, creating the assignment variables, identifying outliers, and removing the outliers. We are now ready to create variograms. Please be sure to read through the text I have included before each code chunk.

Run the entire code chunk by chunk. The variable P_PHI_pct is already in place and ready to run. The variogram model and kriging parameters are already set. Erase your Global Environment before beginning

Perform Omnidirectional kriging of P_PHI_pct For variogram modeling the best models to use are the: - Exponential - Spherical - Cubic - Stable - Be sure the data base you select has omitted any outliers

Note which variogram model types generated the "best results" for omnidirectional kriging (just through visual inspection, you don't have to do bootstrapping)

2 Terminology and key functions from this Demo

1. Kriging

• The geostatistical interpolation method

2. Unique Neighborhood

• Kriging requires knowledge of the neighbors it will use for estimating the values at each grid cell. The default parameter is called a, "Unique" neighborhood which instructs kriging to use all the data in the study area. In this case, the number of samples is considered small, so we can use all the data. In other cases, you will want to establish a moving neighborhood which restricts the number of samples to use in the estimation around a grid cell.

3. Kriging Estimate

• The value determined from the kriging equation at a given location. The estimage is optimized and considered the, "Best Linear, Unbiased Estimate.

4. Error Estimate

• The value of the variance or standard deviation determined from the kriging equation at a given location. The Error Variance is usually reported in terms of the standard deviation as the standard deviation is in the same units as the original variable.

3 Loading Packages

Run the chunk below...

4 Initializing the Demo

As in the previous demos, we:

- Read in our data
- Create the tibble
- Ensure our categorical variables are designated as factors
- Create a backup data frame
- Remove the unnecessary variables from the tibble
- Create our assigned symbolic names
- Identify the outliers. all of this is being done in one code chunk

Run the chunk below...

```
df %<>%
  mutate(across(matches("N_|L_"), factor))
head(df, n = 20)
## # A tibble: 20 x 11
                 N_Well C_X_ft C_Y_ft L_3_FACIES P_Delta_t P_KH_md P_PHI P_PHI_pct
##
     F Top
##
      <chr>
                  <fct>
                          <dbl> <dbl> <fct>
                                                      <dbl>
                                                              <dbl> <dbl>
                                                                              <dbl>
## 1 S_Siltstone 5001
                          11564
                                  5691 2
                                                               3.51 0.094
                                                                                9.4
                                                       60.8
## 2 S_Siltstone 5002
                          10679 13706 2
                                                       59.1
                                                               5.81 0.093
                                                                                9.3
## 3 S Siltstone 5003
                           6311 36307 1
                                                       55.9
                                                               0.77 0.059
                                                                                5.9
## 4 S Siltstone 5004
                           2754 11437 1
                                                       52.5
                                                               0.55 0.044
                                                                                4.4
## 5 S_Siltstone 5005
                           9386 24799 3
                                                       63.3
                                                               6.29 0.099
                                                                                9.9
## 6 S_Siltstone 5006
                          10761 24755 3
                                                       63.1
                                                               2.9 0.099
                                                                                9.9
## 7 S_Siltstone 5007
                                                              12.3 0.113
                          10769
                                 23463 3
                                                       65.9
                                                                                11.3
## 8 S_Siltstone 5008
                          9145 23413 1
                                                       57.9
                                                               0.79 0.074
                                                                                7.4
## 9 S_Siltstone 5009
                           8201 23557 3
                                                       64.4
                                                               8.19 0.104
                                                                                10.4
## 10 S_Siltstone 5010
                           6727
                                23472 1
                                                       55.8
                                                               1.45 0.071
                                                                                7.1
## 11 S_Siltstone 5011
                           3834 23446 1
                                                       53.9
                                                               1.6 0.062
                                                                                6.2
## 12 S_Siltstone 5012
                           2780
                                23494 1
                                                               1.57 0.063
                                                       54.8
                                                                                6.3
## 13 S_Siltstone 5013
                           3994 22098 1
                                                       53.5
                                                               1.51 0.06
                                                                                6
## 14 S_Siltstone 5014
                           5482 22214 1
                                                       53.9
                                                               1.79 0.065
                                                                                6.5
## 15 S Siltstone 5015
                           6692 22138 2
                                                       60.8
                                                               8.21 0.091
                                                                                9.1
                           9324 22199 3
## 16 S_Siltstone 5016
                                                       63.5
                                                               4.48 0.1
                                                                                10
## 17 S Siltstone 5017
                          10817 22305 2
                                                       61.7
                                                               2.73 0.093
                                                                                9.3
                          10761 20774 1
                                                               2.68 0.073
                                                                                7.3
## 18 S_Siltstone 5018
                                                       57.4
## 19 S_Siltstone 5019
                           9350 20761 3
                                                               3.42 0.096
                                                       62.5
                                                                                9.6
                           8237 20748 1
## 20 S Siltstone 5020
                                                       57.2
                                                               3.82 0.079
                                                                                7.9
## # i 2 more variables: P_Thickness <dbl>, P_Top_ft <dbl>
# Creating a duplicate tibble for later use, but eliminate unnecessary variables
df2 <- df %>%
 dplyr::select(L_3_FACIES:P_Top_ft)
# Creating Symbolic Inputs
xlon <- "C_X_ft"</pre>
vlat <- "C Y ft"
out_analysis <- "Raw (outlier analysis performed on raw data)"</pre>
property <- "P_PHI_pct" # The property you are selecting for analysis
# Idnetifying the outliers
df <-
 df %>%
  mutate(
   iqr_val = IQR(!!sym(property)),
    # calculates the IQR value
   igr\ val\ adj = ((igr\ val) * 1.5),
   third_q = quantile(!!sym(property), prob = 0.75, na.rm = TRUE),
   first_q = quantile(!!sym(property), prob = .25, na.rm = TRUE),
    # Creating a column of True and False, True = an outlier, False = not an
    #outlier
   outlier =
```

```
(!!sym(property)) > (third_q + iqr_val_adj)
      (!!sym(property) < (first_q - iqr_val_adj))</pre>
  )
df
## # A tibble: 262 x 16
##
      F_Top
                  N_Well C_X_ft C_Y_ft L_3_FACIES P_Delta_t P_KH_md P_PHI P_PHI_pct
##
      <chr>
                   <fct>
                           <dbl>
                                  <dbl> <fct>
                                                         <dbl>
                                                                 <dbl> <dbl>
                                                                                  <dbl>
##
   1 S Siltstone 5001
                           11564
                                   5691 2
                                                         60.8
                                                                  3.51 0.094
                                                                                    9.4
    2 S Siltstone 5002
                           10679
                                  13706 2
                                                         59.1
                                                                  5.81 0.093
                                                                                    9.3
##
    3 S Siltstone 5003
                                  36307 1
                                                         55.9
                                                                  0.77 0.059
##
                            6311
                                                                                    5.9
##
   4 S Siltstone 5004
                            2754
                                  11437 1
                                                         52.5
                                                                  0.55 0.044
                                                                                    4.4
##
   5 S Siltstone 5005
                            9386
                                  24799 3
                                                         63.3
                                                                  6.29 0.099
                                                                                    9.9
   6 S_Siltstone 5006
                           10761
                                                         63.1
                                                                  2.9
                                                                       0.099
                                                                                    9.9
##
                                  24755 3
##
   7 S_Siltstone 5007
                           10769
                                  23463 3
                                                         65.9
                                                                 12.3
                                                                       0.113
                                                                                   11.3
##
   8 S_Siltstone 5008
                                                                  0.79 0.074
                                                                                   7.4
                            9145
                                  23413 1
                                                         57.9
   9 S_Siltstone 5009
                            8201
                                  23557 3
                                                         64.4
                                                                  8.19 0.104
                                                                                   10.4
## 10 S_Siltstone 5010
                            6727
                                  23472 1
                                                         55.8
                                                                  1.45 0.071
                                                                                    7.1
## # i 252 more rows
## # i 7 more variables: P_Thickness <dbl>, P_Top_ft <dbl>, iqr_val <dbl>,
       iqr_val_adj <dbl>, third_q <dbl>, first_q <dbl>, outlier <lgl>
```

4.1 Code chucks that apply to lognormal data only

The variable, P_PHI_pct is roughly symetrically distributed. If you decide to try a different variable, you need to check to see if it is symetrically distributed. If it is not, and the variable data are symmetrically distributed YOU MAY SKIP THE NEXT TWO CHUNKS!!!

Adding the log of a right-skewed (~normally distributed) variable This property will be added to the data frame for the purpose of identifying the outlier. However, it's only useful if the variable being evaluated is log-normally distributed. should you decide to change the variable under investigation from P_PHI_pct to P_KH_md, then this section becomes relevant.

ADJUSUTING FOR THE MIN AND MAX WITHOUT THE OUTLIER Once the outlier is found, in order to identify the proper interquartile range (without the outlier), we need to redefine the max and min values. Note, again, this is operating on the Log_property in case the variable under investigation is log-normally distributed.

Run the code chunk below if the variable you have selected is log-nomrally distributed, uncomment the code chunk below and run it. This will take the log of the variable temporarily just to correctly identify outliers.

```
# df <-
# df %>%
# mutate(Log_property = log(!!sym(property)))
#
# df <-
# df %>%
# mutate(
# maxval_trans = max(Log_property),
# # not necessary to calculate max
# minval_trans = min(Log_property),
```

```
# not necessary to calculate min
#
      iqr_val_trans = IQR(Log_property),
      # calculates the IQR value
#
#
      iqr_val_adj_trans = ((iqr_val_trans) * 1.5),
      third_q_trans = quantile(Log_property, prob = 0.75, na.rm = TRUE),
#
      first_q_trans = quantile(Log_property, prob = .25, na.rm = TRUE),
#
      outlier =
        (Log_property) > (third_q_trans + iqr_val_adj_trans) /
#
        (Log_property) < (first_q_trans - iqr_val_adj_trans)
#
   dplyr::select(-maxval_trans:-first_q_trans)
```

4.2 End of code chunks for log-normally distributed data data

5 Working in R S4 - Preparation for Geostatistical Analysis using RGeostats

5.1 Creating the Databases and Basemap

The following chunk creates two databases, one with outliers, and one without outliers from a data frame and assigns the location of the coordinates and the selected mapping variable ("property"). Then, the basemap with no outlier is plotted for reference.

Run the code chunk below...

6 creating databases with and without the outlier

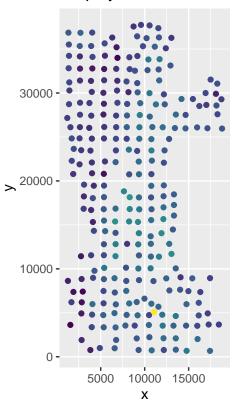
```
# creating a database with the outlier
df1 = df %>% dplyr::select(-F_Top)
# creating a database with the outlier from the Data Frame
db = Db_fromTL(df1)
# Define the different variables
err = db$setLocators(c(xlon, ylat), ELoc_X())
err = db$setLocator(property, ELoc_Z())
db
##
## Data Base Characteristics
## -----
##
## Data Base Summary
## -----
## File is organized as a set of isolated points
## Space dimension
## Number of Columns
                            = 15
## Total number of samples = 262
##
```

```
## Variables
## -----
## Column = 0 - Name = N Well - Locator = NA
## Column = 1 - Name = C_X_ft - Locator = x1
## Column = 2 - Name = C_Y_ft - Locator = x2
## Column = 3 - Name = L_3_FACIES - Locator = NA
## Column = 4 - Name = P Delta t - Locator = NA
## Column = 5 - Name = P_KH_md - Locator = NA
## Column = 6 - Name = P_PHI - Locator = NA
## Column = 7 - Name = P_PHI_pct - Locator = z1
## Column = 8 - Name = P_Thickness - Locator = NA
## Column = 9 - Name = P_Top_ft - Locator = NA
## Column = 10 - Name = iqr_val - Locator = NA
## Column = 11 - Name = iqr_val_adj - Locator = NA
## Column = 12 - Name = third_q - Locator = NA
## Column = 13 - Name = first_q - Locator = NA
## Column = 14 - Name = outlier - Locator = NA
# Creating databases with no outliers
# Remove the first variable
df1 = df %>% dplyr::select(-F_Top) %>% filter(outlier == FALSE)
# creating a database with the outlier from the Data Frame
db_noout.db = Db_fromTL(df1)
# Define the different variables
err = db_noout.db$setLocators(c(xlon, ylat), ELoc_X())
err = db_noout.db$setLocator(property, ELoc_Z())
db noout.db
##
## Data Base Characteristics
## -----
##
## Data Base Summary
## -----
## File is organized as a set of isolated points
## Space dimension
## Number of Columns
                               = 15
## Total number of samples
                             = 261
##
## Variables
## -----
## Column = 0 - Name = N_Well - Locator = NA
## Column = 1 - Name = C_X_ft - Locator = x1
## Column = 2 - Name = C_Y_ft - Locator = x2
## Column = 3 - Name = L_3_FACIES - Locator = NA
## Column = 4 - Name = P_Delta_t - Locator = NA
## Column = 5 - Name = P_KH_md - Locator = NA
## Column = 6 - Name = P_PHI - Locator = NA
## Column = 7 - Name = P_PHI_pct - Locator = z1
## Column = 8 - Name = P Thickness - Locator = NA
## Column = 9 - Name = P_Top_ft - Locator = NA
## Column = 10 - Name = iqr_val - Locator = NA
## Column = 11 - Name = iqr_val_adj - Locator = NA
```

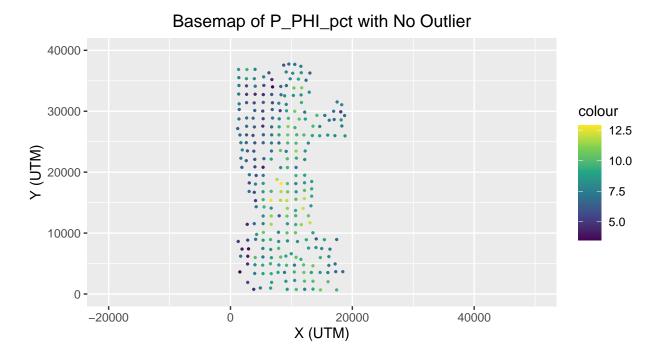
```
## Column = 12 - Name = third_q - Locator = NA
## Column = 13 - Name = first_q - Locator = NA
## Column = 14 - Name = outlier - Locator = NA

p = plot.init(asp=1)
p = p + plot.symbol(db, nameColor=property)
p = p + plot.decoration(title="Display of Grid Nodes")
plot.end(p)
```

Display of Grid Nodes



```
p = plot.init(asp=1)
p = p + plot.symbol(db_noout.db, nameColor = property, pch = 19, cex = 0.5, flagLegend = TRUE)
p = p + plot.geometry(xlim = c(-20000, 50000), ylim = c(0, 40000))
p = p + plot.decoration(title=paste("Basemap of", property, "with No Outlier"), xlab = "X (UTM)", ylab = plot.end(p)
```



To determine the ceiling of the grid (or the coordinates of the upper-right corner), I wrote a roundUp() function that take the largest value in both selected C_X and C_Y column and find the nearest 10, 100, 1000, etc. that's larger than the largest value. The purpose of this code is to establish the extents of the data so we can build a grid.

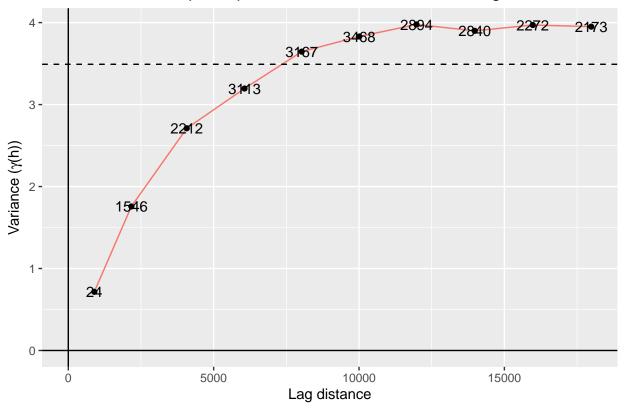
7 Variogram Construction and modeling

7.1 Omnidirectional Experimental Semivariogram

The code chunks below calculate and plot the variograms. Initially, they do not correct for outliers. Replace the database (db) with the database that removes the outliers (db_noout.db). Observe the impact.

Run the chunk below...

P_PHI_pct Experimental Ominidirectional Variogram



7.2 Modeling the Experimental Semivariogram

8 Selecting the variogram model(s)

Run the chunk below...

ECov_printAll()

```
-2 -
              UNKNOWN: Unknown covariance
##
     -1 -
             FUNCTION : External covariance function
##
##
      0 -
               NUGGET : Nugget effect
##
      1 - EXPONENTIAL : Exponential
            SPHERICAL : Spherical
##
      2 -
      3 -
             GAUSSIAN : Gaussian
##
##
      4 -
                CUBIC : Cubic
      5 -
              SINCARD : Sine Cardinal
##
      6 -
              BESSELJ : Bessel J
##
      7 -
               MATERN : Matern
##
      8 -
                GAMMA : Gamma
##
      9 -
               CAUCHY: Cauchy
##
     10 -
               STABLE : Stable
##
     11 -
               LINEAR : Linear
##
     12 -
                POWER : Power
     13 -
            ORDER1_GC : First Order Generalized covariance
##
            SPLINE GC : Spline Generalized covariance
##
     14 -
##
     15 -
            ORDER3_GC : Third Order Generalized covariance
     16 -
##
            ORDER5_GC : Fifth Order Generalized covariance
```

```
##
     17 -
              COSINUS : Cosine
     18 -
##
             TRIANGLE : Triangle
               COSEXP : Cosine Exponential
##
     19 -
                REG1D : 1-D Regular
##
     20 -
##
     21 -
                PENTA: Pentamodel
##
     22 -
           SPLINE2 GC: Order-2 Spline
              STORKEY: Storkey covariance in 1-D
##
     23 -
     24 -
            WENDLANDO: Wendland covariance (2,0)
##
##
     25 -
            WENDLAND1: Wendland covariance (3,1)
            WENDLAND2: Wendland covariance (4,2)
##
     26 -
##
     27 -
               MARKOV : Markovian covariances
            GEOMETRIC : Geometric (Sphere only)
##
     28 -
              POISSON : Poisson (Sphere only)
##
     29 -
            LINEARSPH : Linear (Sphere only)
##
     30 -
## NULL
```

Run the chunk below... Note the assignment variable, "struct" is then seen in the Values section of the Global Environment.

```
struct <- c(2)
```

Note, if you included many basic structures into the auto-fitting algorithm of RGeostats, model.auto() will try to fit every included structure and pick out the best fit to plot with the experimental variogram.

8.0.1 Omnidirectional Variogram Model

Be sure to look at the information printed out on the omnidirectional variogram.

Run the chunk below...

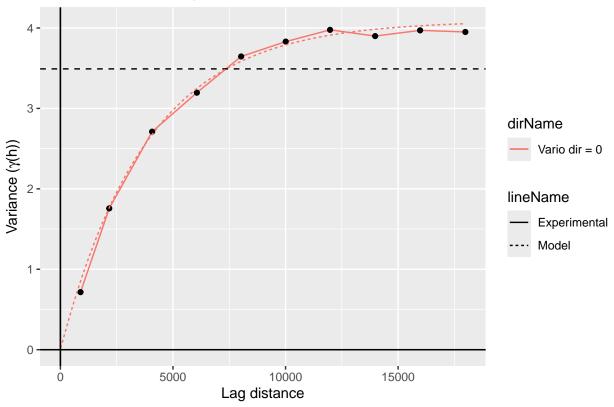
```
data.model.omni <- Model()
types = ECov_fromKeys(c("EXPONENTIAL"))
err = data.model.omni$fit(vario_omni, types=types)

data.model.omni
##</pre>
```

```
## Model characteristics
## ========
## Space dimension
## Number of variable(s)
## Number of basic structure(s) = 1
## Number of drift function(s) = 0
## Number of drift equation(s) = 0
##
## Covariance Part
## -----
## Exponential
## - Sill
                       4.091
## - Range
                    11500.465
## - Theo. Range =
                     3838.950
## Total Sill
                       4.091
## Known Mean(s)
                    0.000
```

We can more fully annotate the variograms; change the curve colors, line type and width, subtitles, enhanced legends, etc. Below is an example.

P_PHI_pct Model Omnidirectional



```
# Step 3: Extract lag distances
gamma_exp = vario_omni$getAllGg()

# Step 4: Evaluate model values at those lags
lags = vario_omni$getAllHh()
mode = CovCalcMode()
mode$setAsVario(TRUE)

## NULL
model_vals = data.model.omni$evalIvarNlag(lags,mode=mode)

# Step 5: Compute sum of squared differences
```

squared_diff <- sum((gamma_exp - model_vals)^2)</pre>

print(paste0("Sum of squared difference = ", round(squared_diff,4)))

```
## [1] "Sum of squared difference = 0.0528"
```

9 Unique neighborhood

In the chunk below, we're going to create a unique neighborhood using function neigh.create() with type "0" for Unique Neighborhood and nidm = 2 as we are in two dimensions. Recall, a unique neighborhood uses all the data.

Run the chunk below...

```
neigh.unique <- NeighUnique_create()</pre>
```

At the same time, if you want to experiment with creating a moving neighborhood for further purposes you may. Because there's not a need for moving neighborhood in this script, it's best to comment it out to decrease the running time of the program.

Moving Neighborhood The inactive code below can be used to construct a moving neighborhood

```
#neigh.moving <- NeighMoving_create(nmaxi=10, radius=10, nsect=8, nsmax=2)</pre>
```

10 Kriging

Kriging is the geostatistical interpolation method that uses the variogram to weight neighboring data by distance and azimuth. (azimuth is an angular measurement in a spherical coordinate system).

The different about Kriging comes from the fact that the weights come from the spatial (variogram) model using the following equation: Z(0) = SUM(i=1 to n)(lambda(i)*Z(i))

10.1 Creating the Grid for Kriging

Run the chunk below...

10.2 Kriging the variable of interest

The chunk of code below performs kriging of the selected property on a grid. It takes about a minute to run. Because of the small number of data, the kriging in this script will be performed in Unique Neighborhood. The use of the **radix** enables the creation of a second set of variables avoiding the confusion with those that already exist. Note below, radix = "Omni" creates a set of variables with the prefix, "Omni.P_PHI_pct." Look at the results of dbgrid3 once the code is run. You can see the two variables it created; Omni.P_PHI_pct.estim and Omni.P_PHI_pct.stdev. Now, all we need to do is map them.

Run the chunk below...

```
err <- kriging(
   dbin = db_noout.db,
   dbout = dbgrid2,
   model = data.model.omni,
   neigh = neigh.unique,
   namconv = NamingConvention("Omni"))
dbgrid2</pre>
```

##

Data Base Grid Characteristics

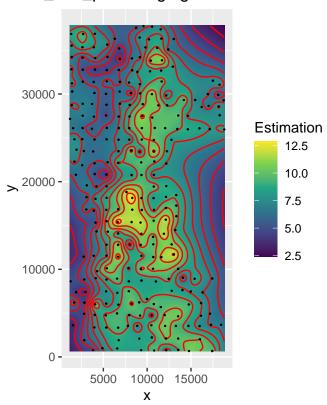
```
##
## Data Base Summary
## -----
## File is organized as a regular grid
## Space dimension
## Number of Columns
                             = 11
                            = 65844
## Total number of samples
##
## Grid characteristics:
## -----
## Origin: 1198.000 670.000
## Mesh : 100.000
                       100.000
## Number :
               177
                           372
##
## Variables
## Column = 0 - Name = x1 - Locator = x1
## Column = 1 - \text{Name} = x2 - \text{Locator} = x2
## Column = 2 - Name = L_3_FACIES - Locator = NA
## Column = 3 - Name = P_Delta_t - Locator = NA
## Column = 4 - Name = P_KH_md - Locator = NA
## Column = 5 - Name = P_PHI - Locator = NA
## Column = 6 - Name = P_PHI_pct - Locator = NA
## Column = 7 - Name = P_Thickness - Locator = NA
## Column = 8 - Name = P_Top_ft - Locator = NA
## Column = 9 - Name = Omni.P_PHI_pct.estim - Locator = z1
## Column = 10 - Name = Omni.P_PHI_pct.stdev - Locator = NA
```

From that we can plot the kriged map

Run the chunk below...

```
p = plot.init(asp=1)
p = p + plot.raster(dbgrid2, name = "*.estim", flagLegend=TRUE, legendName="Estimation")
p = p + plot.contour(dbgrid2, name = "*.estim", color = 'red')
p = p + plot.symbol(db_noout.db, nameSize = property, pch = 19, cex = 0.2)
p = p + plot.decoration(title = paste(property, "- Kriging Estimate"))
plot.end(p)
```

P_PHI_pct - Kriging Estimate



```
p = plot.init(asp=1)
p = p + plot.raster(dbgrid2, name = "*.stdev", flagLegend=TRUE, legendName = "St. Dev.")
p = p + plot.contour(dbgrid2, name = "*.stdev", color = 'black')
p = p + plot.symbol(db_noout.db, nameSize = property, pch = 19, cex = 0.2, color= 'white')
p = p + plot.decoration(title = paste(property, "- Kriging Error St. Deviation"))
plot.end(p)
```

P_PHI_pct – Kriging Error St. Deviation

