Growth & Development - Take Home Exam

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Abstract

This document presents my solution to the Take Home Exam. The computational part was done on Matlab and the code is available under *TakeHome_GGSB.m*. [1].

[1] Any errors are exclusively my responsibility.

Contents

Considerations

A couple of remarks are worth making before presenting the results.

Firstly, and as specified below, I generated a sample assuming $\ln(s_i)$ and $\ln(k_i)$ have mean equal to one. In the Take Home Exam Document, however, it is specified as s_i, k_i having mean one, what consequently would imply $\ln(s_i)$ and $\ln(k_i)$ having mean 0. This was an honest mistake and by the time I caught it all the results had been generated and I was about to submit my solutions. Despite this, results should be qualitatively the same. Only the level of variables would be a little lower[2]. Hence, I do not believe this is an relevant depart from what is asked and I stand by the results I present.

Also, regarding the histograms included in this document, their x-axis label is wrong: it should not be the number of individuals since I am plotting the log level of variables.

Finally, I abstain from the bonus question due to time-constraint.

Question 1. Factor Input Misallocation in the Village

Case when $\rho_{sk} = 0.25$

1 Data Generating Process

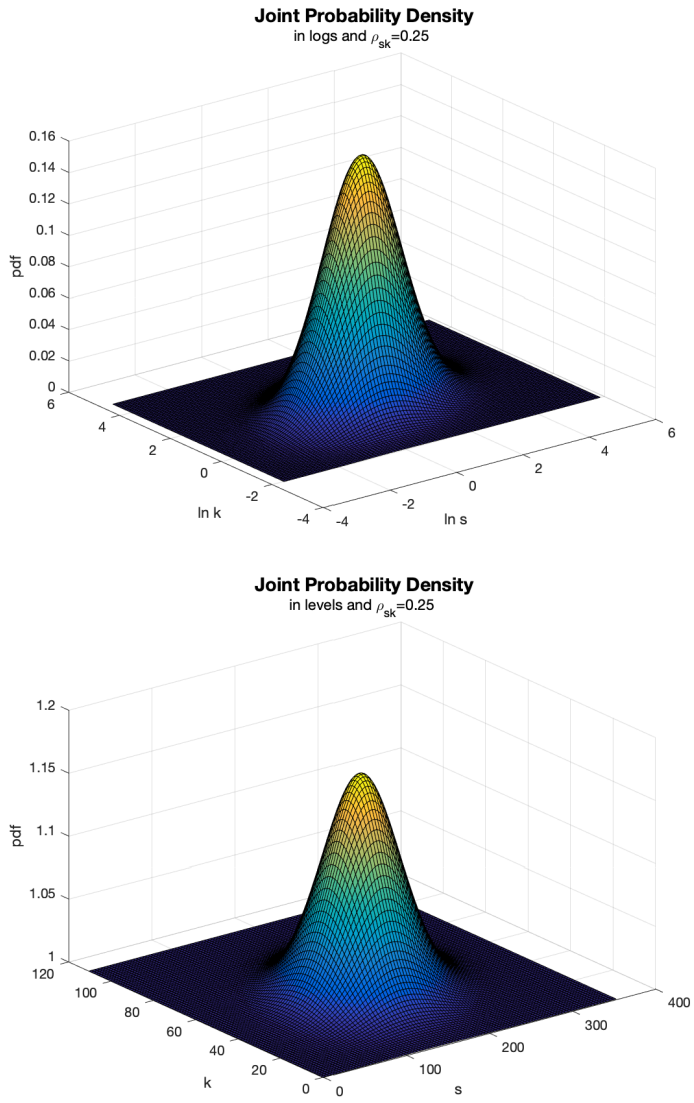
Given the specifications on individual capital (k_i) and productivity (s_i) processes, I use the following as the baseline distributions for the data generating process:

$$\begin{pmatrix} \ln s_i \\ \ln k_i \end{pmatrix} \sim N\left[\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1.416 & 0.2575 \\ 0.2575 & 0.749 \end{pmatrix} \right] \tag{1}$$

I simulate 10,000 observations and their summary statistics as well as the joint density are given below.

	mean	variance	min	1st-q	median	3rd-q	max
s_i	5.434	90.771	0.0257	1.228	2.749	6.122	353.34
k_i	3.946	18.501	0.133	1.5058	2.680	4.782	110.48

[2] I did check if this is the case and it is indeed.



Clearly, capital and productivity vary greatly among agents, the latter having an even higher variance.

2 Output Data

With the generated data, I also calculate firm's individual output y_i given by:

$$y_i = s_i k_i^{\gamma} \tag{2}$$

where $\gamma = 0.5$.

Below is a summary of the generated y_i's.

	mean	variance	min	1st-q	median	3rd-q	max
y_i	11.062	555.902	0.0216	1.797	4.485	11.353	763.78

As expected, individual output substantially differ across individuals. Its average is considerably higher than its means, and it is located approximately in the third quantile of individual output distribution. These implies that most agents have lower than average output, and a few group of firms exhibits very large production.

Hence, I treat the above generated data $\{s_i, k_i, y_i\}_{i=1}^{10,000}$ as the actual administrative data for a given village.

3 Efficient allocation

To solve for the Planner's problem, I follow the approach we saw on class[3] and recast the problem in terms of an arbitrary individual - without loss of generality, I take $i = 1$.

That is, I rewrite the Planner's optimization problem as (where $I = 10,000$):

$$\max_{k_1,...k_I} s_1 \underbrace{\left(K - \sum_{i\neq 1}^{I} k_i\right)^{\gamma}}_{=k_1} + \sum_{i\neq 1}^{I} s_i k_i^{\gamma} \tag{3}$$

by using the resource allocation.

Then, FOCs give the optimality condition for each individual, defined by:

$$s_1 k_1^{\gamma-1} = s_i k_i^{\gamma-1} \forall i \tag{4}$$

That is, marginal product across all firms are equal in the efficient allocation.

On the next question I provide a detailed comparison between the efficient allocations and the data.

When computing the individual efficient capital k_i^e, I checked if the above optimal condition was met. As expected, all marginal products are equalized and equal to 5.5212.

[3] Available on the class notes *"Resource Misallocation and Policy Distortions"*

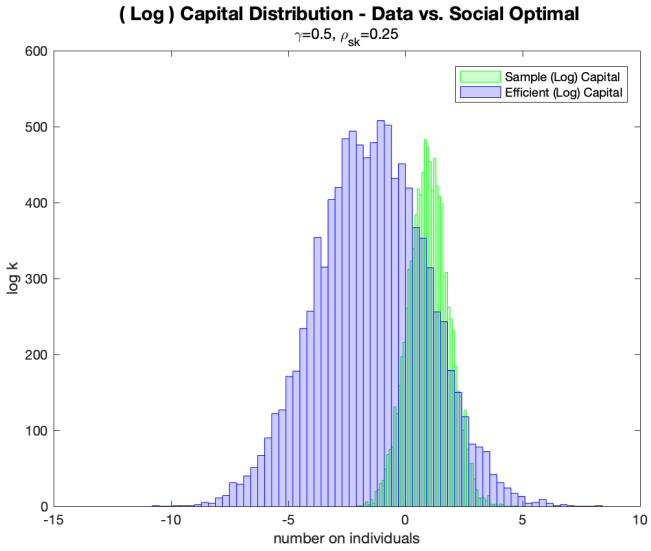
4 Data vs. Efficient Individual Capital Allocations

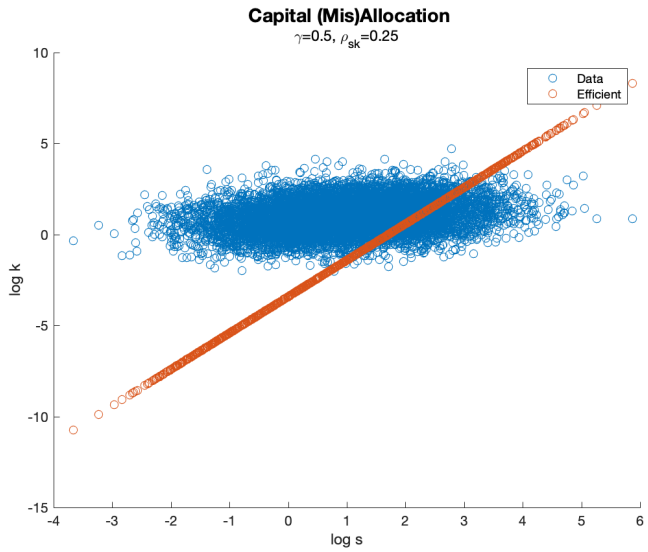
To compare capital data and its efficient allocation, I check their distribution, summary statistics and the difference between these allocations.

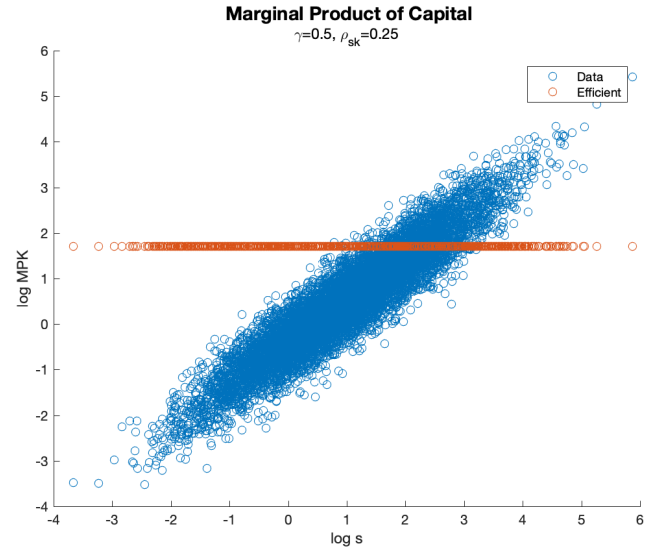
	mean	variance	min	1st-q	median	3rd-q	max
k_i	3.946	18.501	0.133	1.506	2.680	4.782	110.48
k_i^e	3.946	2.2944e+03	2.1743e-05	0.0495	0.248	1.229	4095.6

When taking their difference $k_i - k_i^e \forall i$, I get the following:

	mean	variance	min	1st-q	median	3rd-q	max
$k_i - k_i^e$	-2.8944e-15	2.2996e+03	-4093.2	0.70785	1.866	3.7788	101.79







Discussion:

Data and efficient allocations of capital are substantially different. Notice that, as expected, their mean are the same since the efficient allocation simply implies reallocation of resources. However, efficient capital holdings exhibit a much greater variance and range.

When comparing their (log) distribution, it is clear the reshuffle of allocations "done by the Planner". This reallocation results in a higher individual capital for the majority of agents. On the other hand, there is a greater proportion of agents with lower k_i, when comparing to the data.

Looking at the relationship between productivity and capital, the data does not exhibit any clear relation among these variables. Meanwhile, efficient allocation implies a positive relationship: individual

capital is increasing in individual productivity. Hence, this suggests a misallocation of capital on the data: those who are more productive do not benefit from more capital, and consequently greater production (all else equal).

This results is made clear when looking at marginal products of capital (MPK). On data, (log) individual MPK is linearly increasing in (log) individual productivity. However, as discussed above, the efficient allocation ensures equalized MPKs.

5 Output Gains from Reallocation

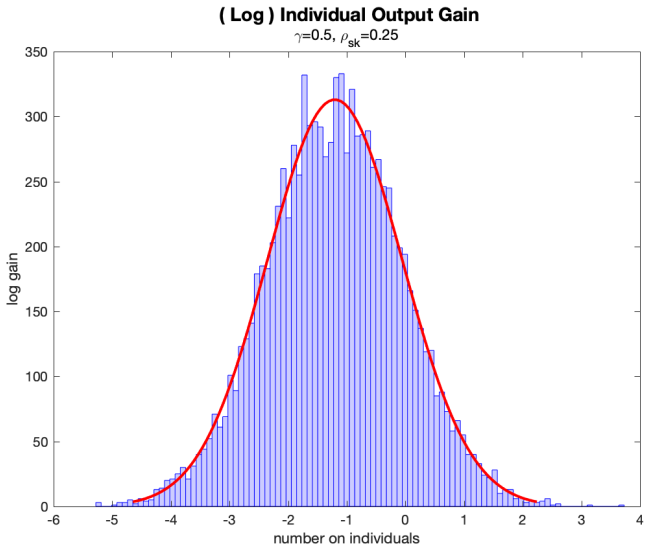
The output gains from reallocation are

$$\frac{Y^e}{Y} = 1.9697 \leftrightarrow \log \frac{Y^e}{Y} = 0.6779 \tag{5}$$

This means that efficient factor reallocation would increase aggregate output by a factor of approximately 2. Or a 67.8% increase in output.

I also look at individual gains.

	mean	variance	min	1st-q	median	3rd-q	max
gain	0.5813	1.0021	0.00534	0.138	0.303	0.654	41.334



The mean individual gain is 0.5813 and its median is 0.303. This is indicative of the aforementioned reallocation of capital resources in the efficient solution. Given individual productivity levels, optimal allocation of capital among agents results in increase on production for all agents.

The individual gains distribution fairly approximates a normal distribution, with variance 1.0021.

Case when $\rho_{sk} = 0$

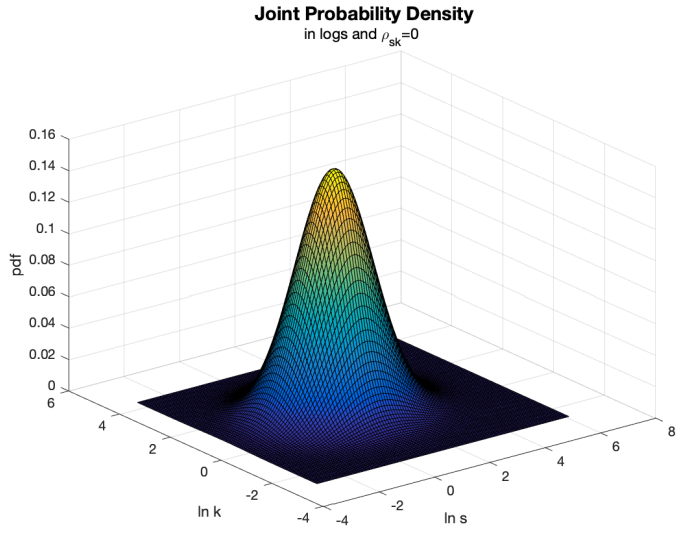
1 Data Generating Process

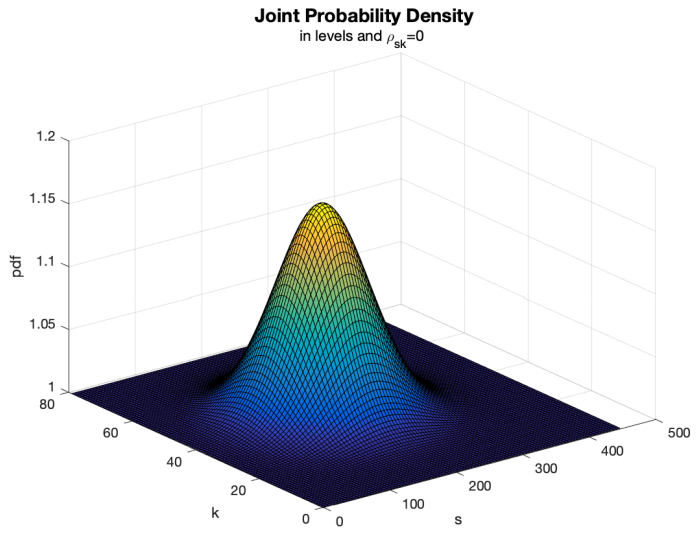
Given the specifications on individual capital (k_i) and productivity (s_i) processes, I use the following as the baseline distributions for the data generating process:

$$\begin{pmatrix} \ln s_i \\ \ln k_i \end{pmatrix} \sim N\left[\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1.416 & 0 \\ 0 & 0.749 \end{pmatrix} \right] \tag{1}$$

I simulate 10,000 observations and their summary statistics as well as the joint density are given below.

	mean	variance	min	1st-q	median	3rd-q	max
s_i	5.3958	94.0277	0.049316	1.2382	2.7335	6.0843	447.59
k_i	3.9725	18.9101	0.068772	1.4853	2.6983	4.8163	79.061





Comparing this new data set with the case of positive correlation, the main difference is on the range of s_i, k_i: with no correlation among these variables, both have a lower minimum and higher maximum. Their variance and mean are (not much) higher than in the previous case.

2 Output Data

With the data generated above, I also calculate firm individual output y_i given by:

$$y_i = s_i k_i^{\gamma} \tag{2}$$

where $\gamma = 0.5$.

Below is a summary of the generated y_i's.

	mean	variance	min	1st-q	median	3rd-q	max
y_i	9.7851	397.8181	0.046226	1.9114	4.4897	10.557	827.8

Hence, I treat the above generated data on $\{s_i, k_i, y_i\}_{i=1}^{10,000}$ as the actual administrative data for a given village.

Comparing to the case with correlation of 0.25 between s_i, k_i, one can see that no correlation results in lower average individual output but also in lower variance of this variable. Moreover, it seems that the individual output distribution is concentrated on higher values, since its minimum, median and maximum are all greater without any correlation.

3 Efficient allocation

The procedure is exactly the same as in the previous question, but the values of the efficient allocation are obviously different.

When computing the individual efficient capital k_i^e, I checked if the above optimal condition was met. As expected, all marginal products are equalized and equal to 5.5674.

Notice that the fact of having or not positive correlation between s_i, k_i does not significantly change the efficient capital level. Its difference is just around 0.05.

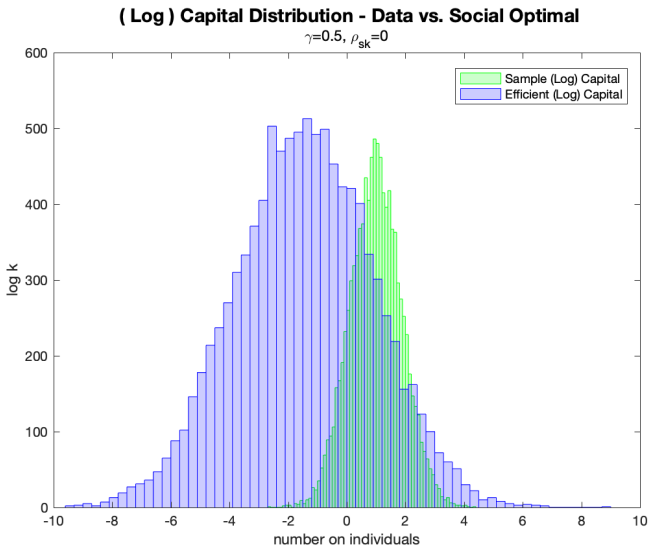
4 Data vs. Efficient Individual Capital Allocations

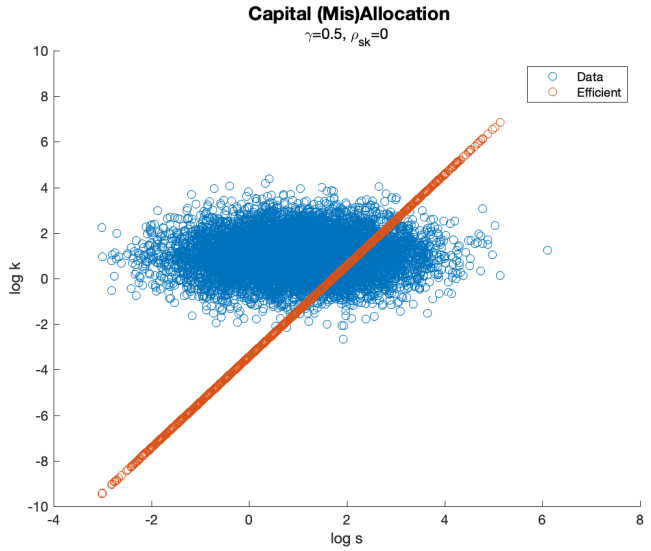
To compare capital data and its efficient allocation, I check their distribution, summary statistics and the difference between these allocations.

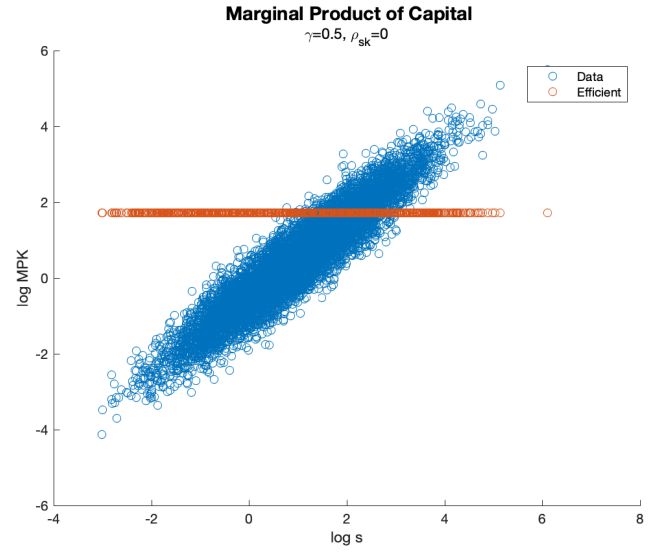
	mean	variance	min	1st-q	median	3rd-q	max
k_i	3.9725	18.9101	0.068772	1.4853	2.6983	4.8163	79.061
k_i^e	3.9725	4.6847e+03	7.8465e-05	0.049463	0.24106	1.1943	6463.2

When taking their difference $k_i - k_i^e \forall i$, I get the following:

	mean	variance	min	1st-q	median	3rd-q	max
$k_i - k_i^e$	-7.0383e-15	4.7033e+03	-6459.8	0.62568	1.9795	4.0644	78.987


(Log) Capital Distribution - Data vs. Social Optimal
γ=0.5, ρsk=0
Sample (Log) Capital
Efficient (Log) Capital
log k
number on individuals


Capital (Mis)Allocation
γ=0.5, ρsk=0
Data
Efficient
log k
log s


Marginal Product of Capital
γ=0.5, ρsk=0
Data
Efficient
log MPK
log s

Discussion:

The results are qualitatively the same as in the case with positive correlation. However, now the efficient (log) capital distribution is more dispersed.

5 Output Gains from Reallocation

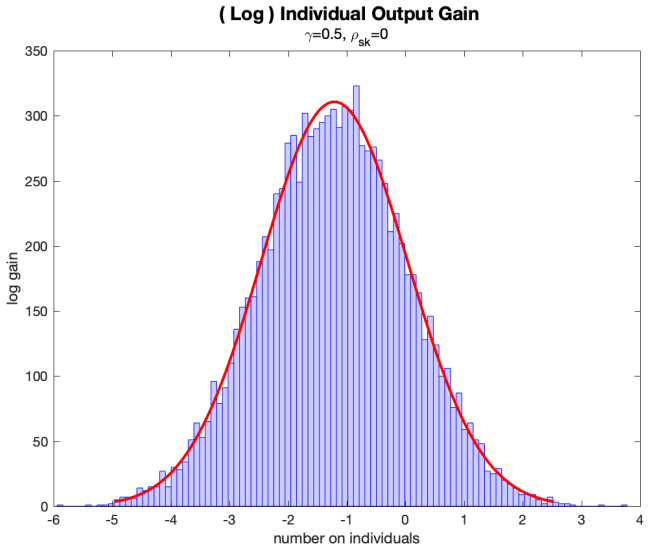
The output gains from reallocation is

$$\frac{Y^e}{Y} = 2.2603 \leftrightarrow \log \frac{Y^e}{Y} = 0.8155 \tag{3}$$

This means that efficient factor reallocation would increase aggregate output by a factor of approximately 2.3. Or a 81.55% increase in output.

I also look at individual gains.

	mean	variance	min	1st-q	median	3rd-q	max
gain	0.6425	1.3986	0.0028776	0.12762	0.2967	0.69192	43.469



Again, individual gains distribution resembles a normal. Its average and variance are greater compared to the case with positive correlation. Also, output gains are greater now.

These results suggest that having independent capital and productivity individual levels enable higher reallocation of (capital) resources and also that this generates higher output gains - both at the individual and aggregate levels.

Question 2. Higher Span of Control

Case when $\gamma = 0.8, \rho_{sk} = 0$

The data used is the one generated previously, on Question 1.1.

2 Output Data

With the data generated, I also calculate firm individual output y_i given by:

$$y_i = s_i k_i^{\gamma} \tag{1}$$

where $\gamma = 0.8$.

Below is a summary of the generated y_i's.

	mean	variance	min	1st-q	median	3rd-q	max
y_i	15.2896	1.2947e+03	0.039522	2.3818	6.0431	15.442	1376.6

Hence, I treat the above generated data on $\{s_i, k_i, y_i\}_{i=1}^{10,000}$ as the actual administrative data for a given village.

With a higher span of control, individual output is overall higher. Its mean and variance are much greater when compared to the lower span of control case. However, qualitatively its distribution resembles the previous one: most firms have lower than average output.

3 Efficient allocation

The procedure is exactly the same as in the previous question, but the values of the efficient allocation are obviously different.

When computing the individual efficient capital k_i^e, I checked if the above optimal condition was met. As expected, all marginal products are equalized and equal to 54.1877.

Interestingly, a higher span of control implies in a much greater efficient marginal product. Hence, the span of control parameter has positive effect on optimal reallocation of individual capital holdings. With a larger span of control, firms gain more from leaving the misallocation state.

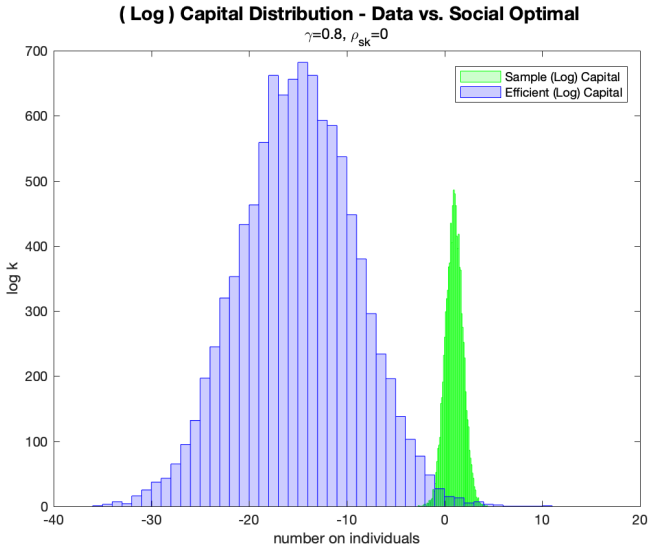
4 Data vs. Efficient Individual Capital Allocations

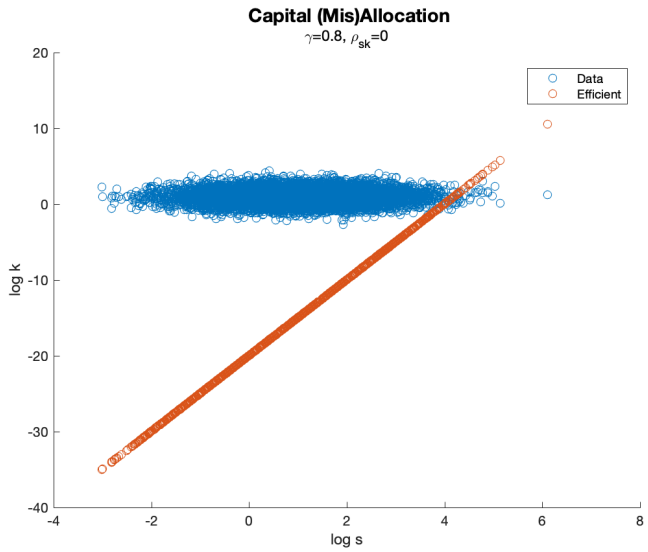
To compare capital data and its efficient allocation, I check their distribution, summary statistics and the difference between these allocations.

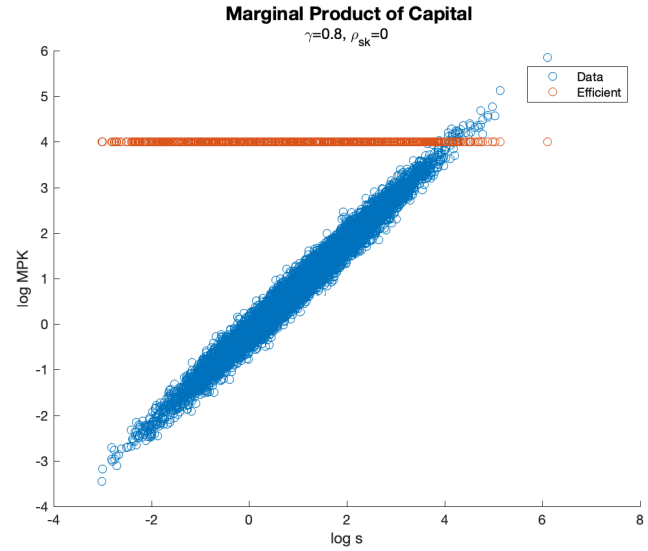
	mean	variance	min	1st-q	median	3rd-q	max
k_i	3.9725	18.9101	0.068772	1.4853	2.6983	4.8163	79.061
k_i^e	3.9725	1.4785e+05	6.2437e-16	6.2296e-09	3.2664e-07	1.7846e-05	38449

When taking their difference $k_i - k_i^e \forall i$, I get the following:

	mean	variance	min	1st-q	median	3rd-q	max
$k_i - k_i^e$	4.9262e-15	1.4787e+05	-38445	1.4661	2.6821	4.7835	79.061







Efficient allocation implies a greater reallocation of resources, when compared to the case of $\gamma = 0.5$. The vast majority of firms profits from higher individual capital under the social optimum. Moreover, the data (log) distribution of capital is more concentrated and exhibits less proportion of agents with high individual capital.

Just as discussed previously and as expected, efficient resource allocation delivers equalized MPKs and a positive linear relationship between (log) individual productivity and (log) individual capital.

5 Output Gains from Reallocation

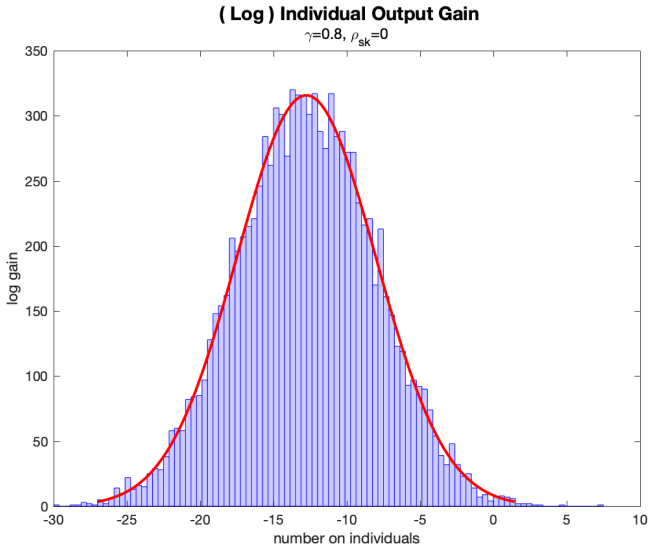
The output gains from reallocation is

$$\frac{Y^e}{Y} = 14.0790 \leftrightarrow \log \frac{Y^e}{Y} = 2.6447 \tag{2}$$

This means that efficient factor reallocation would increase aggregate output by a factor of approximately 14. Or more than 200% increase in output.

I also look at individual gains.

	mean	variance	min	1st-q	median	3rd-q	max
gain	0.2015	303.8310	1.1352e-13	1.1649e-07	2.8581e-06	7.0234e-05	1740.3



As mentioned before, results clearly indicate firm gain more from efficient allocation of resources under higher span of control. Output gains are more than four times larger than with $\gamma = 0.5$

However, at the individual level, output gains distribution is more drastically more dispersed, with a lower mean.

Case when $\gamma = 0.8, \rho = 0.25$

The data used is the one generated previously, on Question 1.6.

2 Output Data

With the data generated above, I also calculate firm individual output y_i given by:

$$y_i = s_i k_i^{\gamma} \tag{1}$$

where $\gamma = 0.8$.

Below is a summary of the generated y_i's.

	mean	variance	min	1st-q	median	3rd-q	max
y_i	18.5224	2.3768e+03	0.019512	2.1662	6.0195	16.942	2007.2

Hence, I treat the above generated data on $\{s_i, k_i, y_i\}_{i=1}^{10,000}$ as the actual administrative data for a given village.

Compared to the above case of no correlation between s_i, k_i and higher span of control, positive correlation decreases (even though not much) individual capital. However, the latter is still substantially higher than with a lower span of control.

3 Efficient allocation

The procedure is exactly the same as in the previous question, but the values of the efficient allocation are obviously different.

When computing the individual efficient capital k_i^e, I checked if the above optimal condition was met. As expected, all marginal products are equalized and equal to 43.6563.

Now, the combination of high span of control and high correlation between s_i, k_i lowers the efficient marginal product level. But still, it is much bigger compared to the lower γ case.

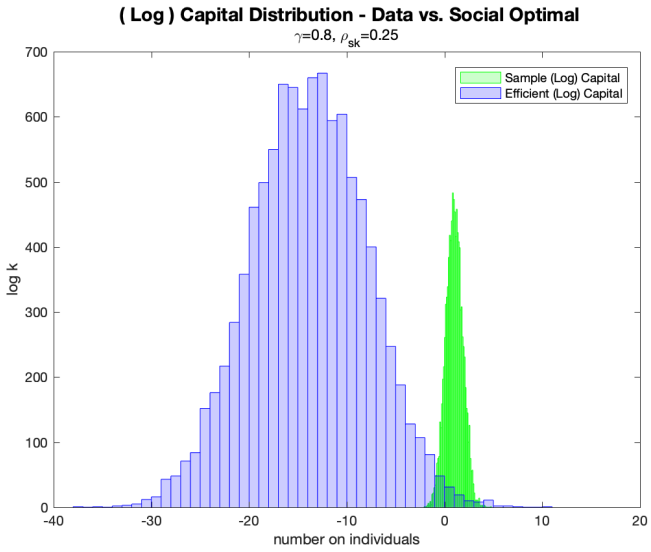
4 Data vs. Efficient Individual Capital Allocations

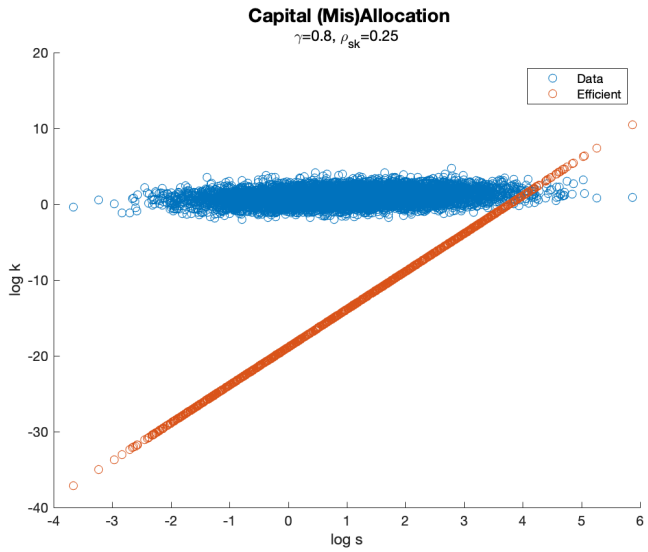
To compare capital data and its efficient allocation, I check their distribution, summary statistics and the difference between these allocations.

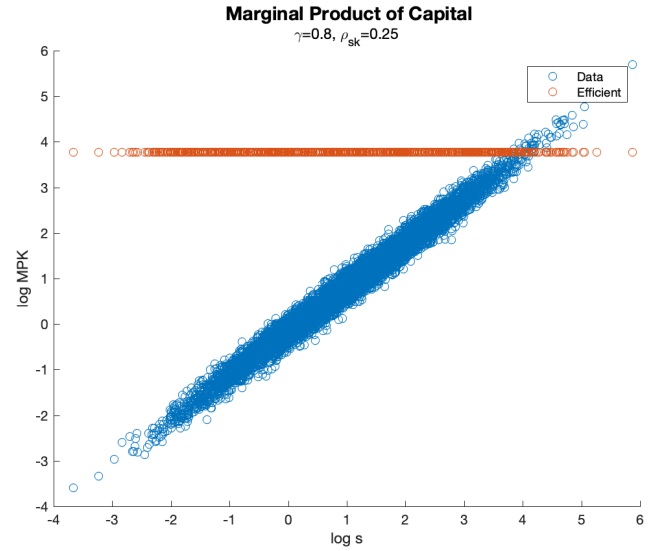
	mean	variance	min	1st-q	median	3rd-q	max
k_i	3.9463	18.501	0.133	1.506	2.680	4.782	110.48
k_i^e	3.9463	1.2099e+05	7.1326e-17	1.7631e-08	9.9023e-07	5.423e-05	34732

When taking their difference $k_i - k_i^e \forall i$, I get the following:

	mean	variance	min	1st-q	median	3rd-q	max
$k_i - k_i^e$	2.3000e-14	1.2101e+05	-34729	1.4839	2.6534	4.7582	110.47







Results are qualitatively similar to the previous case.

5 Output Gains from Reallocation

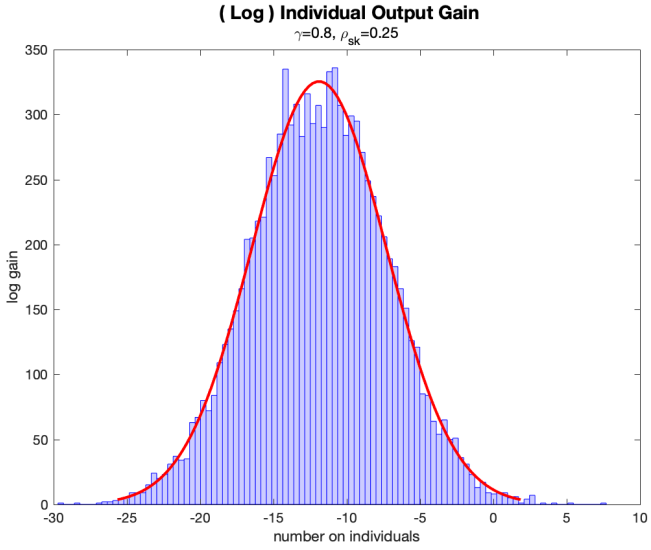
The output gains from reallocation is

$$\frac{Y^e}{Y} = 9.3011 \leftrightarrow \log \frac{Y^e}{Y} = 2.2301 \tag{2}$$

This means that efficient factor reallocation would increase aggregate output by a factor of approximately 9.3. Or more than 200% increase in output.

I also look at individual gains.

	mean	variance	min	1st-q	median	3rd-q	max
gain	0.2688	458.8132	1.5959e-13	3.0331e-07	7.024e-06	0.00015375	2132.2



As mentioned above, output gains are still much bigger than with a lower γ, and lower than the case with no correlation and high span of control.

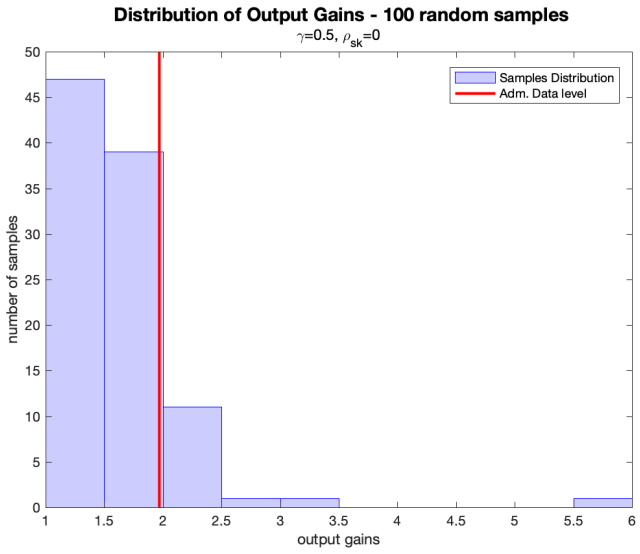
Question 3. From Administrative to Random Sampling in the Village

Throughout this question, I use the data generated on Question 1.1. Thus, I assume $\gamma = 0.5$ and $\rho_{sk} = 0$.

Please refer to the code for the details of exercises 1 and 2.

3 Output Gains from Reallocation

Using the 100 random samples, I get a distribution of output gains from reallocation.



I also look at the summary statistics:

	mean	variance	min	1st-q	median	3rd-q	max
gain	1.6449	0.2823	1.1004	1.3541	1.522	1.7672	5.5309

The 100-random sample average output gains is less than the gain computed with the entire sample (2.2603). Moreover, the greater part of the samples exhibit less output gain than the full sample. There are just a few samples that generate higher gains.

This suggest that using smaller random sample would potentially underestimate the output gains of resources reallocation. One potential explanation is that in order to achieve the efficient allocation, the Planner takes into account all firms, including those who have very high capital levels[4]. Consequently, having very small samples reduces the probability of having as many resources to reallocate optimally.

[4]When running the code, I looked for outliers and indeed they are always present. I found, for the case of no correlation and $\gamma = 0.5$ approximately 300 firms with output greater than three standard deviations from the mean.

Question 4. Endogenous Productivity

Throughout this section, I assume $\rho_{sk} = 0$ and use the data generated on Question 1.1.

In order to solve this Question, I assume the data on s_i is now a_i. That is, my data set consists of observations on true ability a_i and capital k_i, such that:

$$\begin{pmatrix} \ln a_i \\ \ln k_i \end{pmatrix} \sim N\left[\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1.416 & 0 \\ 0 & 0.749 \end{pmatrix} \right] \tag{1}$$

Then, I proceed by calculating the implied managerial ability s_i according to the CES specification:

$$s_i(a_i, k_i) = \left[\alpha a_i^{\frac{\sigma-1}{\sigma}} + (1-\alpha) k_i^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{2}$$

Case when $\sigma = 1.0$

Notice that this is the limiting case of the CES function, and so, $s_i(a_i, k_i)$ has actually a Cobb-Douglas specification:

$$s_i(a_i, k_i) = a_i^{\alpha} k_i^{1-\alpha} \tag{1}$$

1 Output Data

Below is a summary of the generated s_i and y_i.

	mean	variance	min	1st-q	median	3rd-q	max
s_i	3.5450	8.8098	0.18868	1.651	2.7089	4.4852	50.403
y_i	7.7819	118.8752	0.10022	2.1822	4.3898	9.2262	233.2

Hence, I treat the above generated data on $\{s_i, k_i, y_i\}_{i=1}^{10,000}$ as the actual administrative data for a given village.

Comparing this data with the exogenous productivity case, s_i now has lower values, overall. Both its mean, range and variance are considerably lower. This endogeneity also reflects on individual output data, which now is also lower in all means: average, variance and range.

Notice that the individual capital data remains the same.

2 Efficient allocation

To solve for the Planner's problem, I proceed exactly as in the previous questions. That's because I am using the data generated above for s_i.

Thus, I can rewrite Planner's objective function just as before and the optimality conditions implies equalized marginal product of capital among firms.

On the next question I provide a detailed comparison between the efficient allocations and the data.

When computing the individual efficient capital k_i^e, I checked if the above optimal condition was met. As expected, all marginal products are equalized and equal to 2.3197.

Endogenous productivity results in a lower efficient marginal product of capital. This may due to the trade-off between capital and true ability on manegerial productivity. That is, capital now plays two roles: it increase output but also enhances manegerial ability.

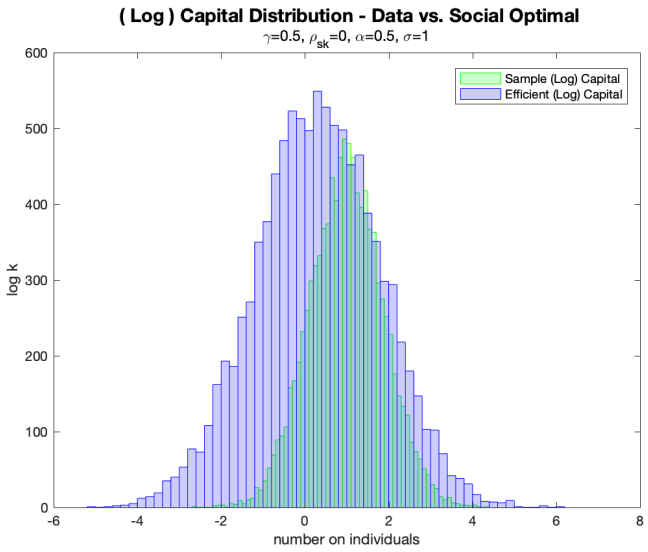
3 Data vs. Efficient Individual Capital Allocations

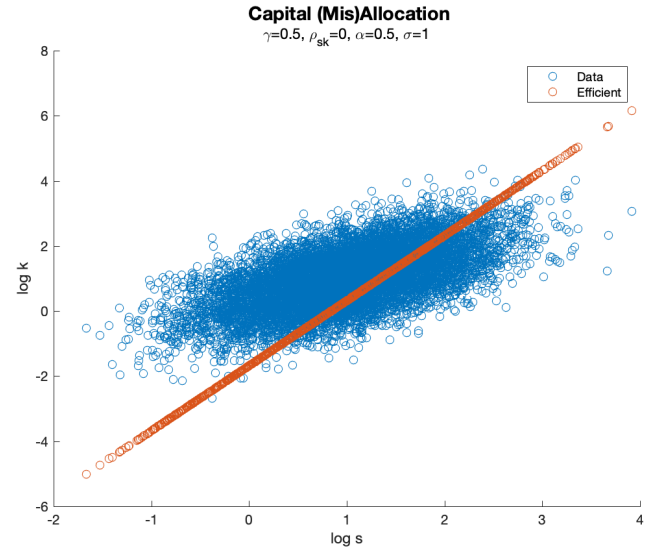
To compare capital data and its efficient allocation, I check their distribution, summary statistics and the difference between these allocations.

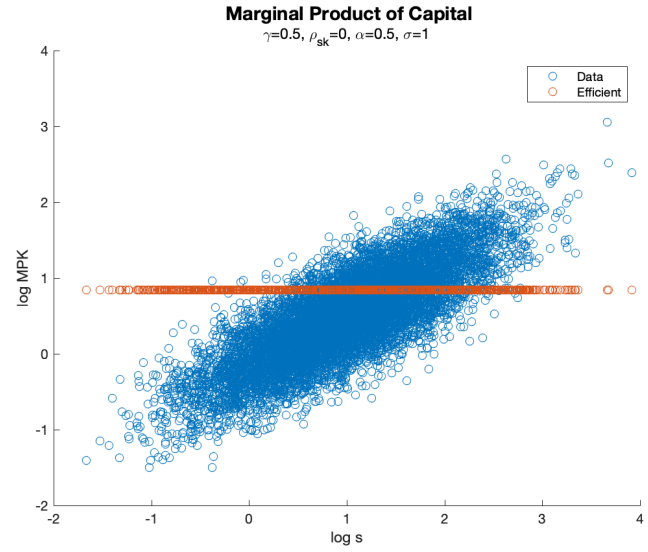
	mean	variance	min	1st-q	median	3rd-q	max
k_i	3.9725	18.9101	0.068772	1.4853	2.6983	4.8163	79.061
k_i^e	3.9725	112.0863	0.0066162	0.50659	1.3637	3.7385	472.13

When taking their difference $k_i - k_i^e \forall i$, I get the following:

	mean	variance	min	1st-q	median	3rd-q	max
$k_i - k_i^e$	6.2434e-15	95.8668	-450.73	-0.25582	0.86261	2.2591	56.83







Capital efficient allocation implies less reallocation of resources, when compared to the case of

exogenous productivity.

One interesting present feature is that the data exhibits a more linear and positive relationship between (log) individual productivity and (log) individual capital. This is expected just by looking at the s_i functional form.

This results might suggest that accounting for endogenous productivity results on having resources allocations closer to their efficient level.

4 Output Gains from Reallocation

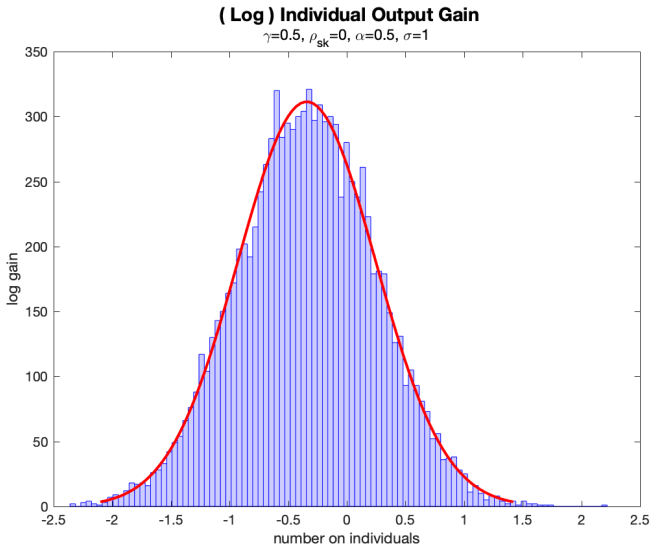
The output gains from reallocation is

$$\frac{Y^e}{Y} = 1.1842 \leftrightarrow \log \frac{Y^e}{Y} = 0.1690 \tag{2}$$

This means that efficient factor reallocation would increase aggregate output by a factor of approximately 1.2. Or a 16.9% increase in output.

I also look at individual gains.

	mean	variance	min	1st-q	median	3rd-q	max
gain	0.8439	0.2906	0.095734	0.4797	0.71274	1.0634	9.1203



Endogenous productivity implies in lower output gains from reallocation. As discussed above, this could be explained by the indirect effect of capital on production through manegerial ability.

Case when $\sigma = 0.5$

1 Output Data

Below is a summary of the generated s_i and y_i.

	mean	variance	min	1st-q	median	3rd-q	max
s_i	2.9063	6.0259	0.098029	1.3045	2.2171	3.7309	36.271
y_i	6.4113	84.3685	0.035713	1.7196	3.5441	7.5596	169.73

Hence, I treat the above generated data on $\{s_i, k_i, y_i\}_{i=1}^{10,000}$ as the actual administrative data for a given village.

Having lower elasticity of substitution between a_i and k_i results on lower levels of both both s_i and y_i.

2 Efficient allocation

When computing the individual efficient capital k_i^e, I checked if the above optimal condition was met. As expected, all marginal products are equalized and equal to 1.9086.

Hence, lower elasticity of substitution results in lower MPK.

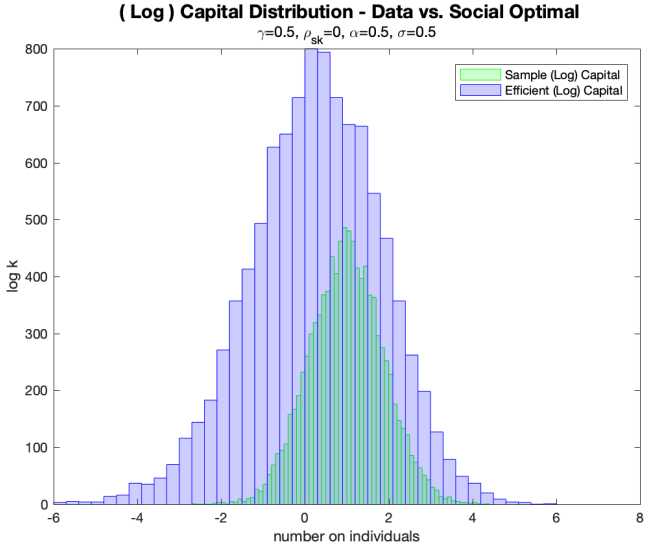
3 Data vs. Efficient Individual Capital Allocations

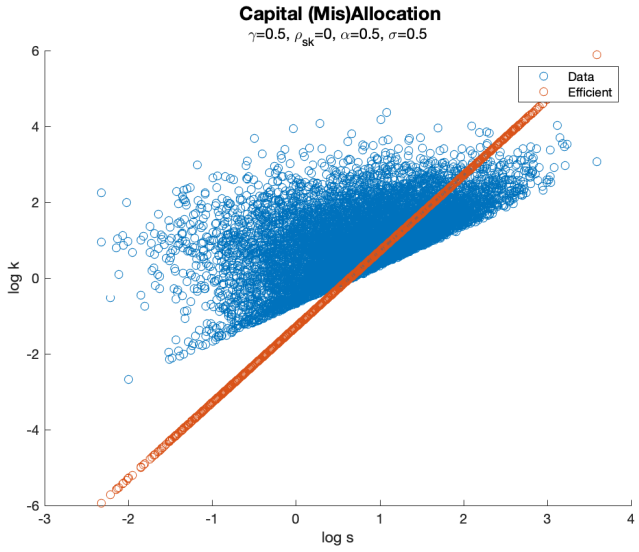
To compare capital data and its efficient allocation, I check their distribution, summary statistics and the difference between these allocations.

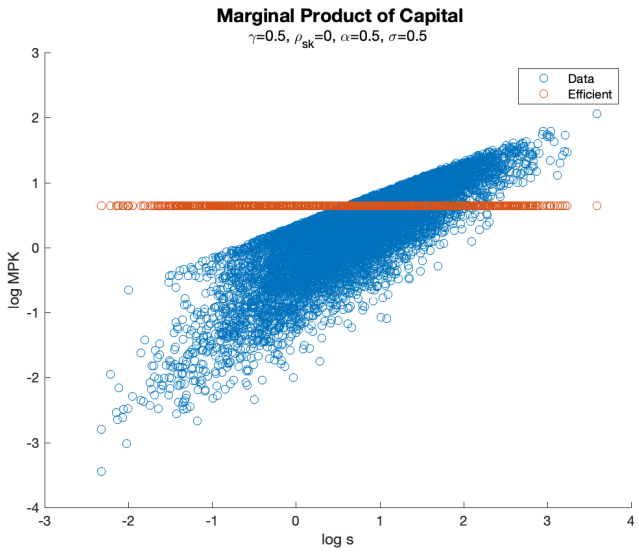
	mean	variance	min	1st-q	median	3rd-q	max
k_i	3.9725	18.9101	0.068772	1.4853	2.6983	4.8163	79.061
k_i^e	3.9725	91.9617	0.0026379	0.46711	1.3494	3.821	361.14

When taking their difference $k_i - k_i^e \forall i$, I get the following:

	mean	variance	min	1st-q	median	3rd-q	max
$k_i - k_i^e$	-4.8583e-15	73.5487	-339.73	-0.37268	0.51102	1.968	76.641







Lower σ results in a closer gap between data and efficient allocations.

4 Output Gains from Reallocation

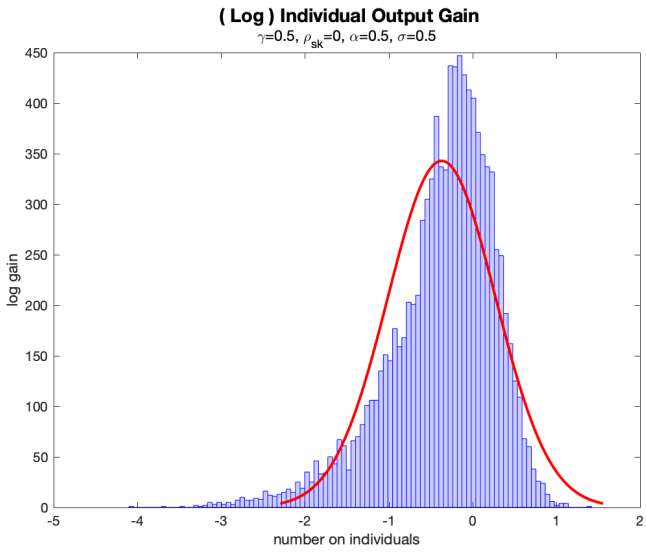
The output gains from reallocation is

$$\frac{Y^e}{Y} = 1.1826 \leftrightarrow \log \frac{Y^e}{Y} = 0.1677 \tag{1}$$

This means that efficient factor reallocation would increase aggregate output by a factor of approximately 1.2. Or a 16.9% increase in output.

I also look at individual gains.

	mean	variance	min	1st-q	median	3rd-q	max
gain	0.8206	0.1892	0.0167	0.50452	0.7748	1.0738	4.1073



As expected, output gains are lower than with a higher σ.

Case when $\sigma = 2.0$

The results presented below are the opposite of what I get by comparing $\sigma = 1$ and $\sigma = 0.5$.

Having higher elasticity of substitution between a_i and k_i results in higher output gains from reallocation. Also, s_i and y_i are considerably lower and higher - in all aspects: mean, variance and range -, respectively.

1 Output Data

Below is a summary of the generated s_i and y_i.

	mean	variance	min	1st-q	median	3rd-q	max
s_i	4.1146	15.8140	0.24926	1.8463	3.0325	5.1028	132.32
y_i	8.9957	177.1828	0.11014	2.4575	4.9882	10.464	278.64

Hence, I treat the above generated data on $\{s_i, k_i, y_i\}_{i=1}^{10,000}$ as the actual administrative data for a given village.

2 Efficient allocation

When computing the individual efficient capital k_i^e, I checked if the above optimal condition was met. As expected, all marginal products are equalized and equal to 2.8709.

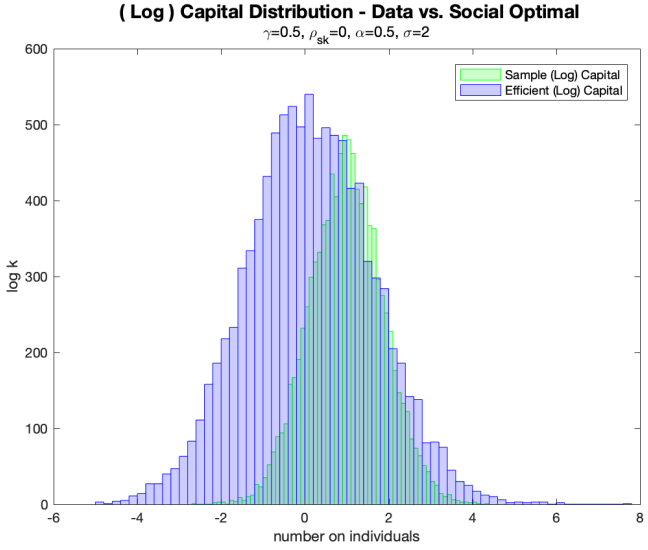
3 Data vs. Efficient Individual Capital Allocations

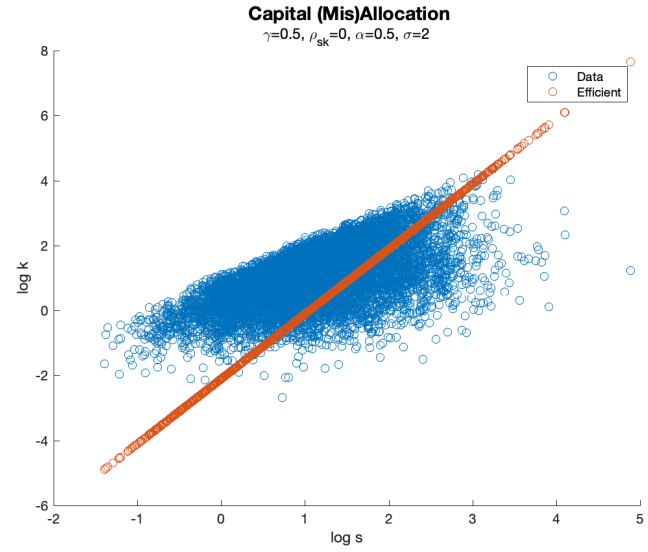
To compare capital data and its efficient allocation, I check their distribution, summary statistics and the difference between these allocations.

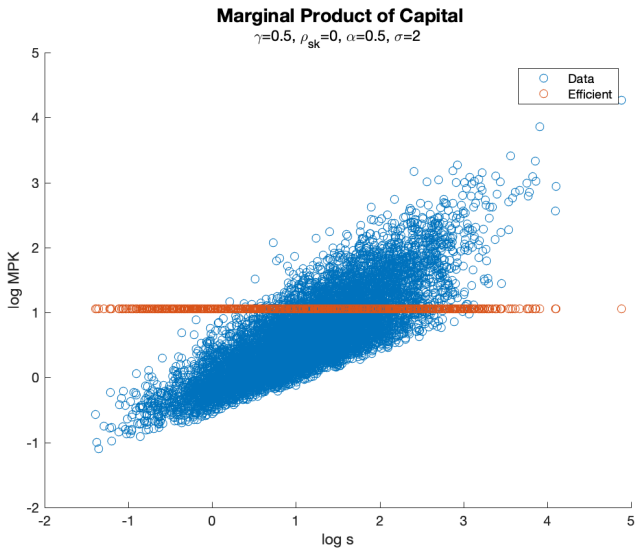
	mean	variance	min	1st-q	median	3rd-q	max
k_i	3.9725	18.9101	0.068772	1.4853	2.6983	4.8163	79.061
k_i^e	3.9725	606.2711	0.0075384	0.41357	1.1157	3.1592	2124.1

When taking their difference $k_i - k_i^e \forall i$, I get the following:

	mean	variance	min	1st-q	median	3rd-q	max
$k_i - k_i^e$	8.8535e-15	594.1138	-2120.7	0.23941	1.2967	2.719	23.015







4 Output Gains from Reallocation

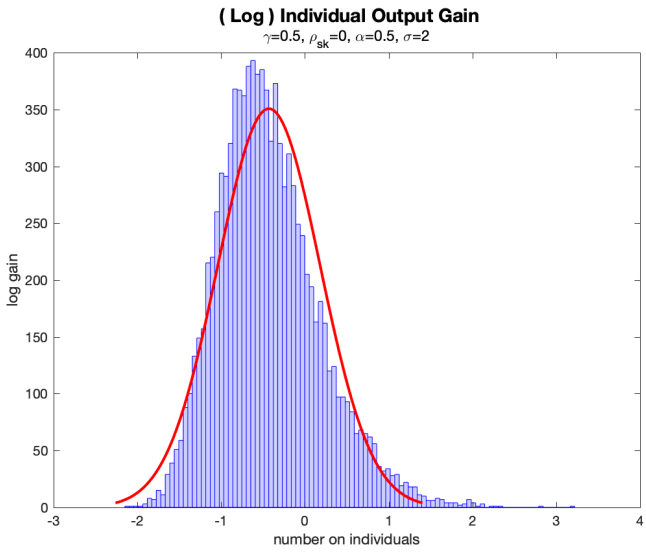
The output gains from reallocation is

$$\frac{Y^e}{Y} = 1.2678 \leftrightarrow \log \frac{Y^e}{Y} = 0.2373 \tag{1}$$

This means that efficient factor reallocation would increase aggregate output by a factor of approximately 1.2. Or a 23.73% increase in output.

I also look at individual gains.

	mean	variance	min	1st-q	median	3rd-q	max
gain	0.8099	0.5704	0.11652	0.42545	0.60594	0.92621	24.92


(Log) Individual Output Gain
γ=0.5, ρsk=0, α=0.5, σ=2
log gain
number on individuals
400
350
300
250
200
150
100
50
0
-3
-2
-1
0
1
2
3
4