Growth and Development Economics

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Take-Home Exam, due March 15 at 1.00pm

Discuss all your results in each and all the following items.

Question 1. Factor Input Misallocation in the Village

1. Individual output, capital and productivity are, respectively, y_i, k_i and s_i. Assume that $\ln s_i$ and $\ln k_i$ follow a joint normal distribution. Assume that the correlation between $\ln s_i$ and $\ln k_i$ is zero, the variance of $\ln s_i$ is equal to 1.416, the variance of $\ln k_i$ is equal to 0.749, and that average s and k is equal to one. Then simulate 10,000 observations and plot the joint density in logs and in levels. We are going to assume that these 10,000 observations are your administrative data that capture the entire population/universe in a given village.

2. Compute firm output y_i for each of your observations as:

$$y_i = s_i k_i^{\gamma}$$

   where $\gamma = 0.5$. From now on consider your simulated s_i, k_i and y_i as actual data.

3. Solve the following maximization problem:

$$Y^e = \max_{k_i} \sum_i s_i k_i^{\gamma}$$

   subject to $K = \sum_i k_i$. To solve this problem use s_i from the actual data that you created in item 1.

4. Compare the optimal allocations k^e against the data.

5. Compute the ouptut gains from reallocation, i.e., $\frac{Y^e}{Y}$ where $Y = \sum_i y_i$ from the data that you created in item 1.

6. Redo items (2)-(5) assuming that the correlation between $\ln s_i$ and $\ln k_i$ is 0.25.

Question 2. Higher Span of Control

1. Redo the previous Question 1 for $\gamma = 0.8$. Discuss your results.

Question 3. From Administrative to Random Sampling in the Village

1. Random sample 10 observations (that is, a ratio 1/1000 with respect to actual population). Do this 100 times. This implies 100 random samples.
2. Redo items (2) to (5) in previous Question 1 for each of these 100 random samples.
3. Discuss your results. In particular discuss the distribution of output gains from reallocation from your 100 random samples and compare it to the administrative data results.

Question 4. Endogenous Productivity

Use the 10,000 observations from Question 1.1 to do the following items:

1. Compute firm output y_i for each of your observations as:

$$y_i = s_i(a_i, k_i)k_i^{\gamma}$$

where $\gamma = 0.5$. We assume that managerial ability now depends on the level of capital with come degree of complementerity between true ability (a_i) and capital (k_i),

$$s_i(a_i, k_i) = \left[\alpha a_i^{\frac{\sigma-1}{\sigma}} + (1-\alpha)k_i^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

with $\alpha = 0.5$ and $\sigma = 1.0$. From now on consider your simulated s_i, k_i and y_i as actual data.

2. Solve the following maximization problem:

$$Y^e = \max_{k_i} \sum_i s_i(a_i, k_i)k_i^{\gamma}$$

subject to $K = \sum_i k_i$. To solve this problem use s_i from the actual data that you created in item 1.

3. Compare your the optimal allocations k_i^e against the data.
4. Compute the ouptut gains from reallocation, i.e., $\frac{Y^e}{Y}$ where $Y = \sum_i y_i$ from the data that you created in item 1.
5. Redo items (2)-(5) for $\sigma = 0.5$ and $\sigma = 2.0$. Discuss your results.
6. Compare your results to Question in this question to those in Question 1.

Question 5. Dynamic Endogenous Productivity [Bonus]

- Write down a dynamic version of the problem in Question 4. How would you solve the problem? How will the solution of this problem compare to its static version in Quesiton 4?