Development Economics - PS2: Welfare Analysis

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Abstract

This document reports the results of analyzing the welfare costs of seasons. In particular, we analyze the welfare effects of removing seasonal risk components, both deterministic and stochastic cases. Also, we study the decomposition of welfare effects into consumption and labor compensations separately.
[1].

[1] Any errors are exclusively my responsibility.

Part I

Praying for Rain: The Welfare Cost of Seasons

1 Deterministic Seasonal Component of Consumption

Welfare gains of removing the seasonal component and the nonseasonal idiosyncratic risk

One way to measure welfare effects between two scenarios is by the *consumption equivalent variation(CEV)*; that is, to find a constant cv in untis of a percentage of the consumption good that represents the welfare gain (loss) of going from scenario A to scenario B. In this sense, cv represents consumption compensations for variations in consumption stream across different realities.

Given the specification of the model, we need to find cv s.t.

$$\begin{aligned} W(z_{season+risk}) &= \sum_{t=1}^{40} \beta^{12t} \Bigg\{ \sum_{m=1}^{12} \beta^{m-1} u\Bigg((\mathbf{1}+\mathbf{cv}) c_{m,t} \Bigg) \Bigg\} \\ &= \sum_{t=1}^{40} \beta^{12t} \Bigg\{ \sum_{m=1}^{12} \beta^{m-1} \Bigg[\log\big(z(\mathbf{1}+\mathbf{cv}) \big[e^{g(m)} e^{-\sigma_\epsilon^2/2} \epsilon_t \big] \big) \Bigg] \Bigg\} \end{aligned} \tag{1}$$

$$\begin{aligned} W(z_{noseason+risk}) &= \sum_{t=1}^{40} \beta^{12t} \Bigg\{ \sum_{m=1}^{12} \beta^{m-1} u\Bigg(c_t \Bigg) \Bigg\} \\ &= \sum_{t=1}^{40} \beta^{12t} \Bigg\{ \sum_{m=1}^{12} \beta^{m-1} \Bigg[\log\big(z \big[e^{-\sigma_\epsilon^2/2} \epsilon_t \big] \big) \Bigg] \Bigg\} \end{aligned} \tag{2}$$

$$\downarrow$$

$$\boxed{W(z_{season+risk}, \mathbf{cv}) = W(z_{noseason+risk})} \tag{3}$$

Given the spcefication (CRRA) of the utitlity function and the e;asticity of substitution calibration of $\eta = 1$ - so that $u(.) = log(c)$, the exercise boils down to solve the following equation:

$$\boxed{cv = \exp\Bigg(W_{ns,r} - W_{s,r} \Bigg)^{1/\beta_g} - 1} \tag{4}$$

where $\beta_g = \sum_{t=1}^{40} \sum_{m=1}^{12} \beta^{12t+m-1}$.

If the coefficient of risk aversion is different than one, we find cv by solving:

$$\boxed{cv = \Bigg(\frac{W_{ns,r}}{W_{s,r}} \Bigg)^{1/(1-\eta)} - 1} \tag{5}$$

The same computation is valid when removing the nonseasonal component:

$$\boxed{W(z_{season+risk}, \mathbf{cv}) = W(z_{season+norisk})} \tag{6}$$

Thus, if $cv > 0$ it means that removing the risk source - either seasonality or idyosincratic component - is welfare improving. Notice that since z is individual specific, so it will be the consumption compensation. That is, cv is a vector of length N: each agent is compensated differently.

Below are the results for removing the seasonal component from consumption at different degree of seasonality (low, medium and high), and the nonseasonal risk at each season - as specfied in the PS. The results are grouped by the parametrization of the utlity function.

Please refer to the code *ex11* to solve for the welfare gains of removing seasonality, and to the code *ex12* when removing the idiosyncratic risk.

For the sake of simplicty, the codes are written such that one should choose which season to consider and then run the file.

Moreover, one could compute the follwing using the code:
- histogram: distribution of cvs
- summary statistics: mean, variance, coeffcient of variation, median, maximum, minimum, 90-th percentile.

However, I got the same compensation for each consumer even though they all have different z values. I could not figure out the error in my code. But even with the same cv for every agent, results are reasonable.

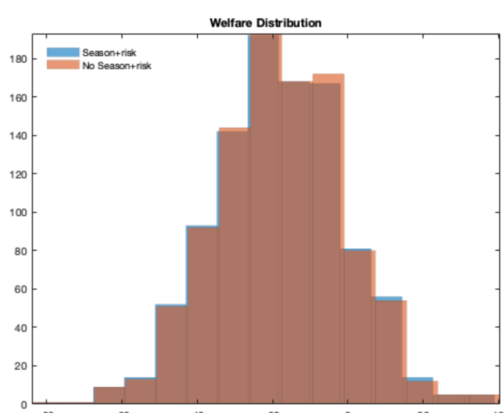
Finally, keep in mind that the permanent level of consumption z was not kept constant throughout the analysis. That is, possibly different values of z were assingned to agents at each season or risk removal. Ideally, it should have been kept constant. Even so, results can indicate that the higher the degree of seasonality, the larger is the cv, thus the large is the welfare gain of removing risk.

A) CRA = 1

Removing the Seasonality Component

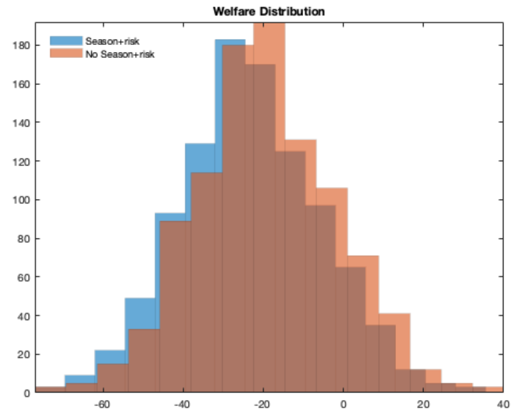
Medium Seasonality

$cv = .0138.$



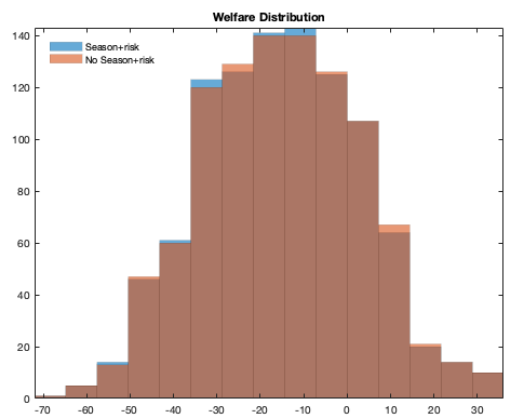
High Seasonality

$cv = .0686.$



Low Seasonality

$cv = .0035.$

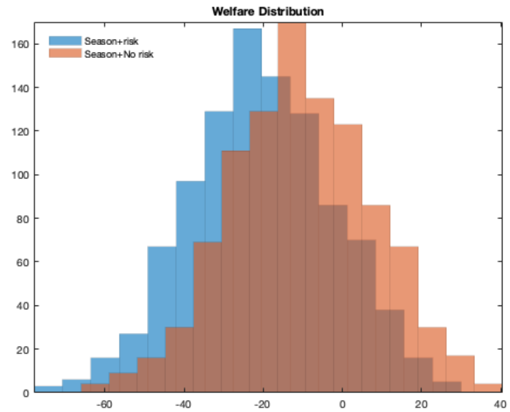


Thus, removing any degree of seasonality is welfare improving.

Removing the NonSeasonality Risk

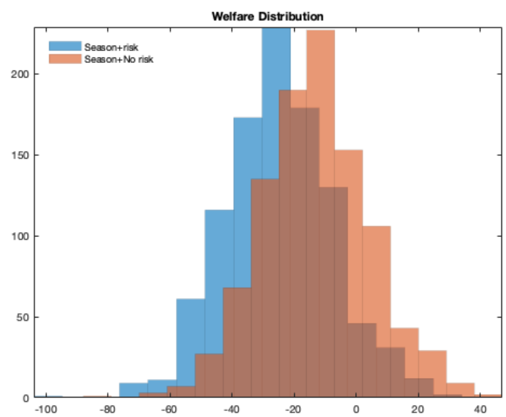
Medium Seasonality

$cv = .1839.$



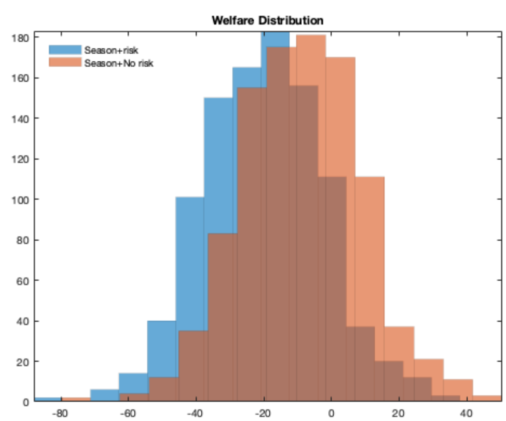
<u>High Seasonality</u>

$cv = .2222.$



<u>Low Seasonality</u>

$cv = .2078$



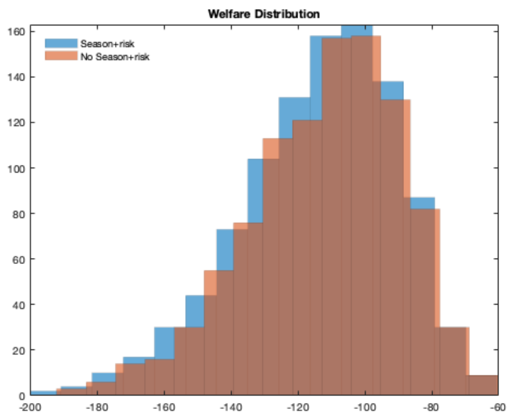
Thus, removing idiosyncratic risk is welfare improving at any level of seasonality.

B) CRA = 2

Removing the Seasonality Component

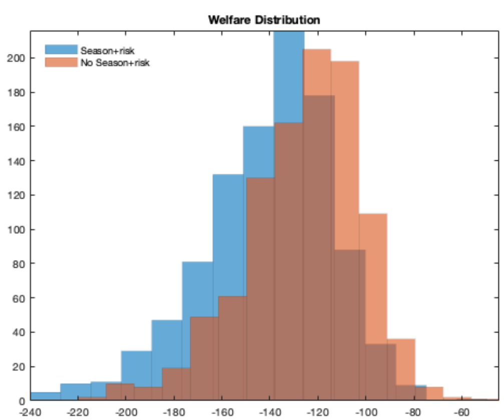
Medium Seasonality

$cv = .0196.$



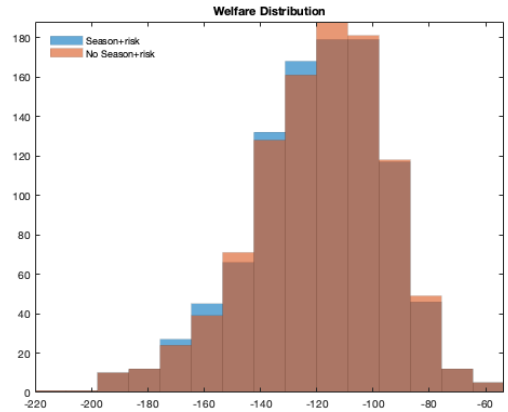
High Seasonality

$cv = .1167.$



Low Seasonality

$cv = .0046.$

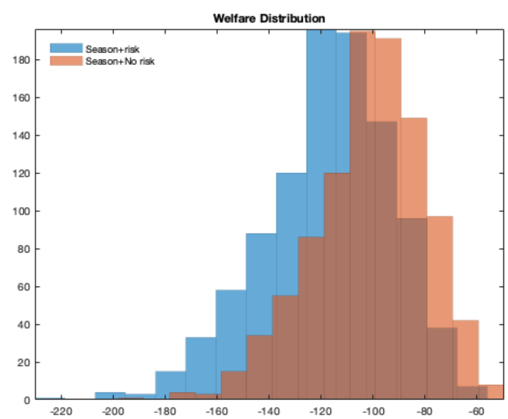


Thus, removing any degree of seasonality is welfare improving.

Removing the NonSeasonality Component

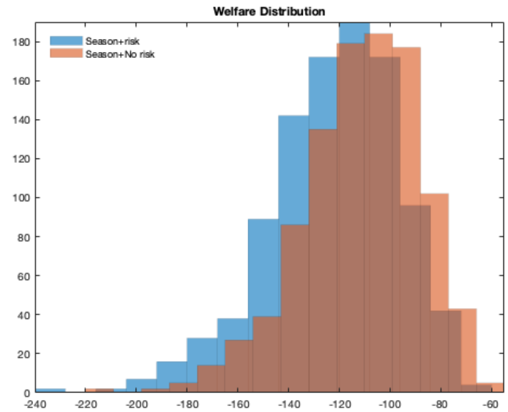
Medium Seasonality

$cv = .1568.$



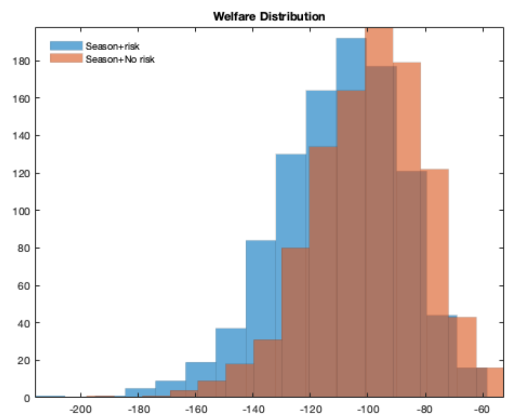
High Seasonality

$cv = .0999.$



<u>Low Seasonality</u>

$cv = .1026.$



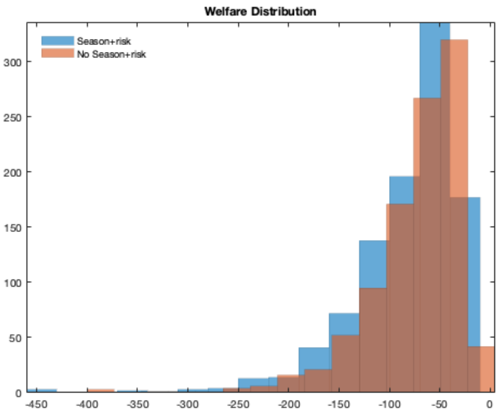
Thus, removing idiosyncratic risk is welfare improving in all degree of seasonality.

C) CRA = 4

Removing the Seasonality Component

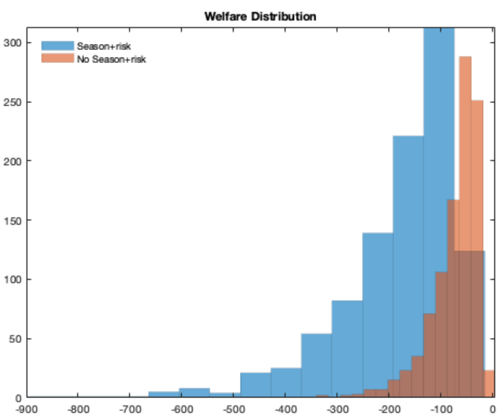
<u>Medium Seasonality</u>

$cv = .0445.$



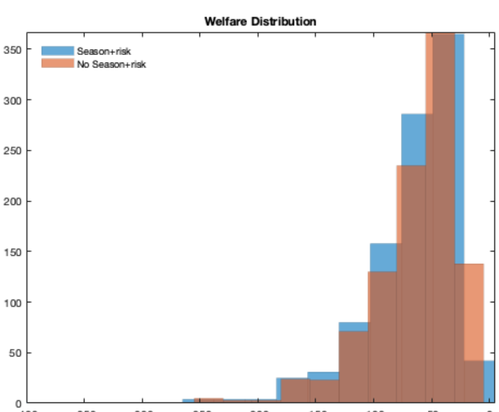
High Seasonality

$cv = .3576.$



Low Seasonality

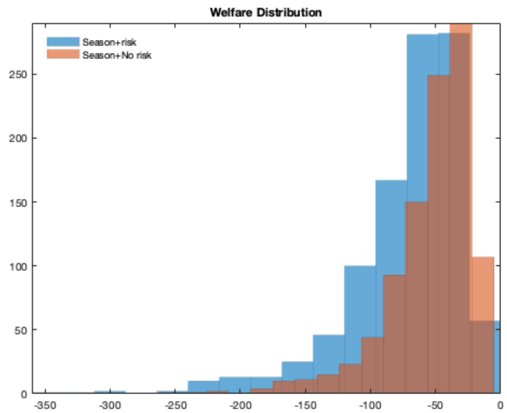
$cv = .0093.$



Removing the NonSeasonality Component

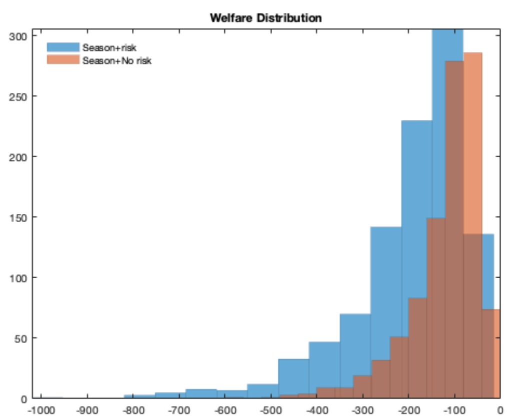
Medium Seasonality

$cv = .1039$.



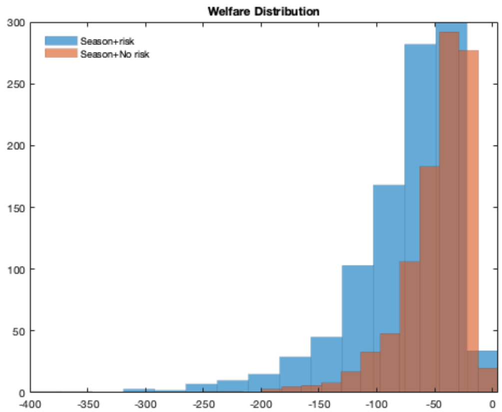
High Seasonality

$cv = .1856$.



Low Seasonality

$cv = .1626$.



Thus, removing idiosyncratic risk is welfare improving in all degree of seasonality.

D) Discussion

Given these results, the higher the degree of seasonality risk, the greater it is the welfare effect of removing it.

On the other hand, they suggest that having seasonality but not idiosyncratic risk delivers a higher positive welfare effect (higher consumption compensations) than keeping the later risk and removing seasonality.

Finally, switiching the parametrization of the utiltity function also impact the results. The above suggests that the more risk averse the agents is - that is, higher η - the greater are the (positive) welfare effects of removing risks.

However, the case of $\eta = 1$ delivers a cv distribution closer to the normal.

2 Additional Stochastic Seasonal Component of Consumption

Now there is an additional source of seasonal risk, which is stochastic and pro-seasonal. The analysis is the same as before.

Notice that now we need to take into account all possible combinations of seasonal risks: 3 deterministics and 3 stochastics. Thus, there are 6 cases to consider.

Please refer to the code *ex21* to solve for the welfare gains of removing seasonality, and to the code *ex22* when removing the idiosyncratic risk.

For the sake of simplicity, I provide the welfare gains (or losses) at each scenario. For a further graphical analysis, please refer to the code.

Also, for studying the removal of the idiosyncratic risk, I report only the results for CRA = 1. Please refer to the code for further details on the other utilities parametrizations.

Each consumer has a different *cv* and so, I provide the summary statistics of the compensations and their distribution.

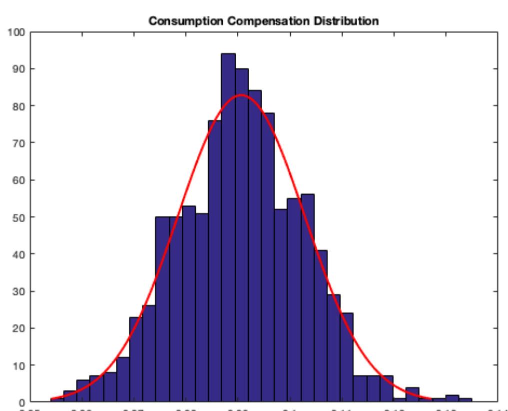
The following titles refer to the deterministic and stochastic seasonal risks, respectively.

A) CRA = 1

Removing the Seasonality Component

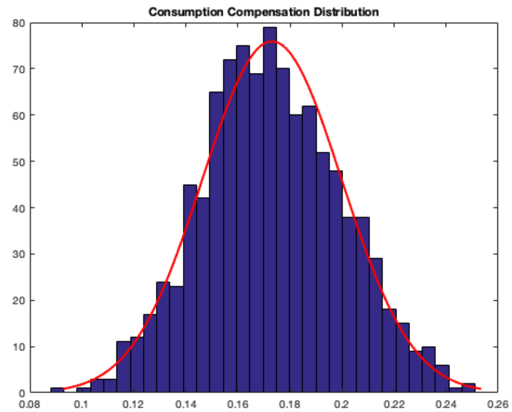
<u>Medium+Medium Seasonality</u>

mean	std	cv	med	max	min	p90
0.09	0.01	13.44	0.09	0.13	0.06	0.11



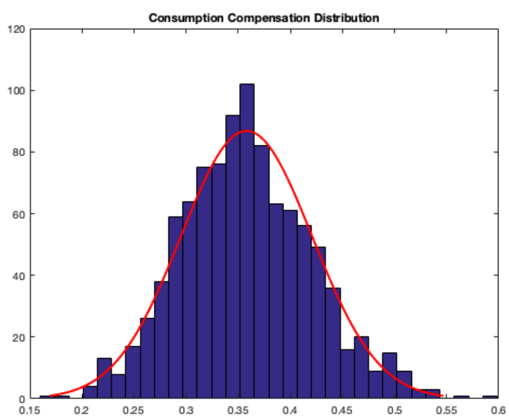
<u>Medium+High Seasonality</u>

mean	std	cv	med	max	min	p90
0.17	0.03	15.48	0.17	0.25	0.09	0.21



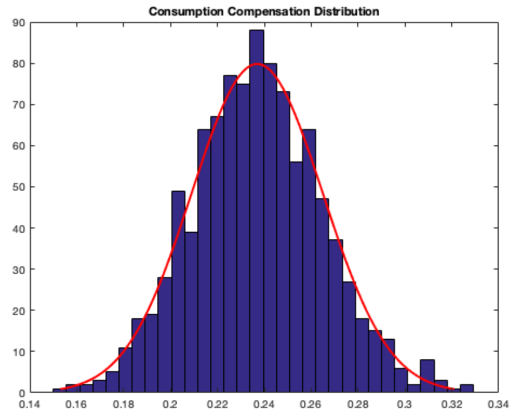
Medium+Low Seasonality

mean	std	cv	med	max	min	p90
0.36	0.06	17.59	0.36	0.60	0.16	0.44



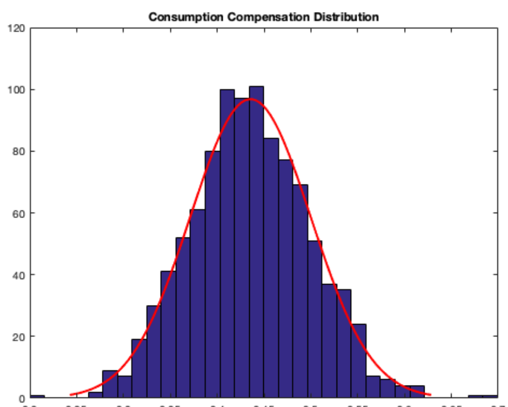
High+High Seasonality

mean	std	cv	med	max	min	p90
0.24	0.03	11.81	0.24	0.33	0.15	0.27



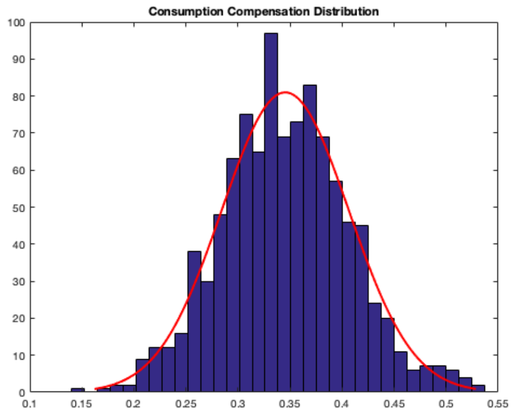
High+Low Seasonality

mean	std	cv	med	max	min	p90
0.44	0.06	14.76	0.43	0.70	0.20	0.52



Low+Low Seasonality

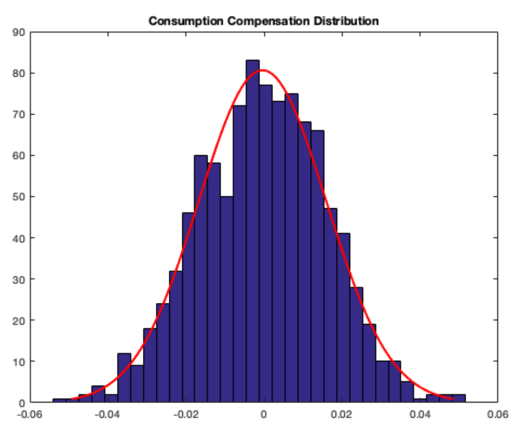
mean	std	cv	med	max	min	p90
0.35	0.06	17.68	0.34	0.54	0.14	0.42



Removing the NonSeasonality Risk Component

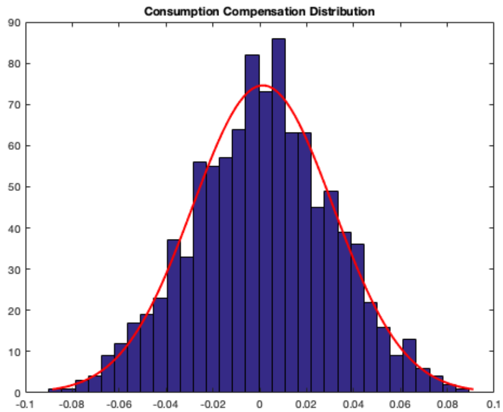
Medium+Medium Seasonality

mean	std	cv	med	max	min	p90
-0.00	0.02	-4107.23	-0.00	0.05	-0.05	0.02



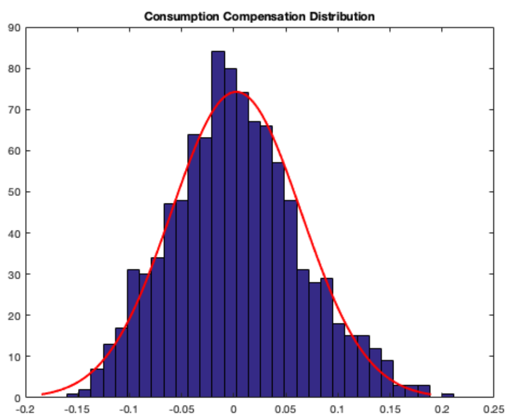
High+High Seasonality

mean	std	cv	med	max	min	p90
0.00	0.03	2268.61	0.00	0.09	-0.09	0.04



Low+Low Seasonality

mean	std	cv	med	max	min	p90
0.09	0.01	13.06	0.09	0.14	0.05	0.11



B) CRA = 2

Removing the Seasonality Component

Medium+Medium Seasonality

mean	std	cv	med	max	min	p90
0.08	0.01	11.16	0.08	0.10	0.04	0.09

Medium+High Seasonality

mean	std	cv	med	max	min	p90
0.15	0.02	12.39	0.15	0.20	0.10	0.17

Medium+Low Seasonality

mean	std	cv	med	max	min	p90
0.35	0.05	13.61	0.35	0.53	0.18	0.42

High+High Seasonality

mean	std	cv	med	max	min	p90
0.24	0.02	6.79	0.24	0.30	0.19	0.26

High+Low Seasonality

mean	std	cv	med	max	min	p90
0.44	0.04	9.14	0.44	0.56	0.31	0.50

Low+Low Seasonality

mean	std	cv	med	max	min	p90
0.35	0.05	12.97	0.35	0.49	0.22	0.40

C) CRA = 4

Removing the Seasonality Component

Medium+Medium Seasonality

mean	std	cv	med	max	min	p90
0.10	0.01	7.99	0.10	0.13	0.08	0.11

Medium+High Seasonality

mean	std	cv	med	max	min	p90
0.21	0.02	10.68	0.20	0.31	0.15	0.23

Medium+Low Seasonality

mean	std	cv	med	max	min	p90
0.65	0.12	18.92	0.63	1.26	0.42	0.81

High+High Seasonality

mean	std	cv	med	max	min	p90
0.48	0.02	3.88	0.48	0.58	0.42	0.50

High+Low Seasonality

mean	std	cv	med	max	min	p90
0.81	0.10	12.50	0.79	1.79	0.61	0.92

Low+Low Seasonality

mean	std	cv	med	max	min	p90
0.67	0.14	20.97	0.65	1.81	0.39	0.84

Part II
Adding Seasonal Labor Supply

When labor supply choice is endogenous implies that welfare effects can be decomposed into the effects of changing consumption and labor allocations separately. Please refer to the codes *ex31, ex32* for the solutions. The first one refers to the welfare effects of removing seasonal risks and the other the effects of removing idiosyncratic nonseasonal risk.

The welfare effects analysis based on consumption compensations is analagous to the case of no labor supply. That is, the welfare effect is given by finding the cv s.t.

$$\boxed{W(z^c_{season+risk}, z^h_{season+risk}, \mathbf{cv}) = W(z^c_{noseason+risk}, z^h_{noseason+risk})} \tag{7}$$

Furthermore, this effect can be decomposed into consumption and labor effects. Let cv_c, cv_h be the consumption and labor decompositions, respectively. Then it follows that:

$$\boxed{W(z^c_{season+risk}, z^h_{season+risk}, \mathbf{cv_c}) = W(z^c_{noseason+risk}, z^h_{season+risk})} \tag{8}$$

$$\boxed{W(z^c_{noseason+risk}, z^h_{season+risk}, \mathbf{cv_h}) = W(z^c_{noseason+risk}, z^h_{noseason+risk})} \tag{9}$$

Given the separable utility form, the above boils down to:

$$\boxed{W(z^c_{season+risk}, \mathbf{cv_c}) = W(z^c_{noseason+risk})} \tag{10}$$

and

$$\boxed{W(z^h_{season+risk}, \mathbf{cv_h}) = W(z^h_{noseason+risk})} \tag{11}$$

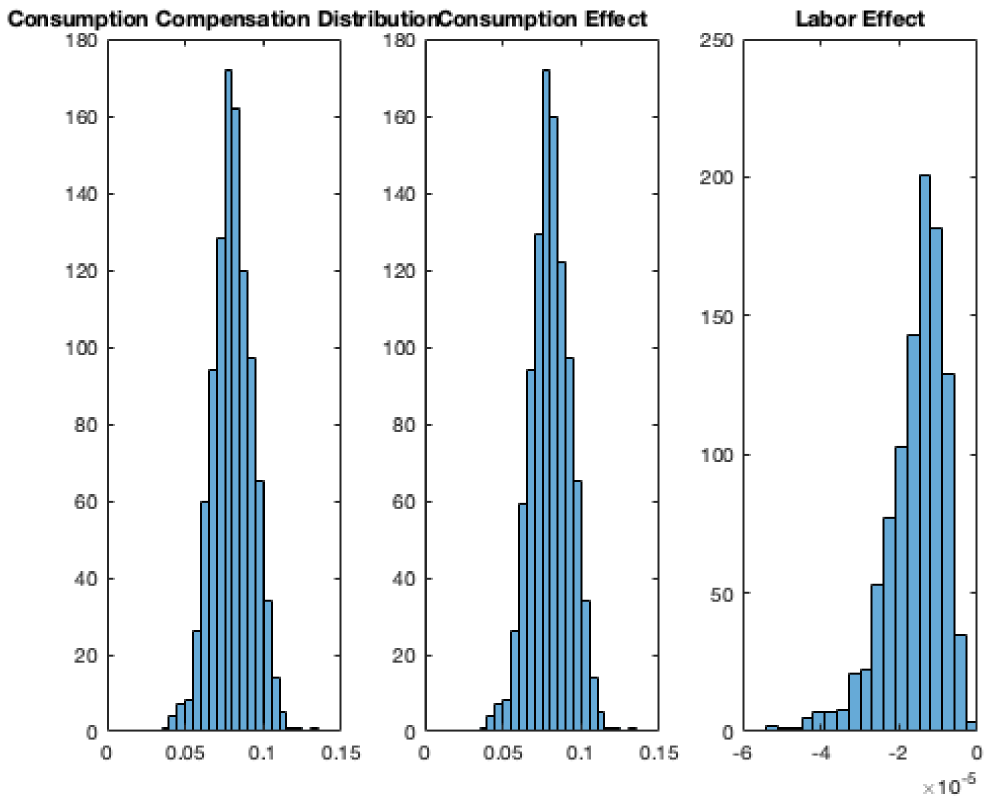
3 Labor and Consumption Positively Correlated Seasonal Components

To get a high positive correlation between labor and consumption components, I assume the deterministic and seasonal components of labor are equal to the consumption ones, and they co-move. That is, whenever the seasonality degree of consumption is low (high), so it is the labor sazonality low (high).

Due to this high positive correlation, basically all the compensation is through the consumption allocation. Also, the major part of the labor effect is negative (even though small), what suggests that agents work less under no seasonality risk.

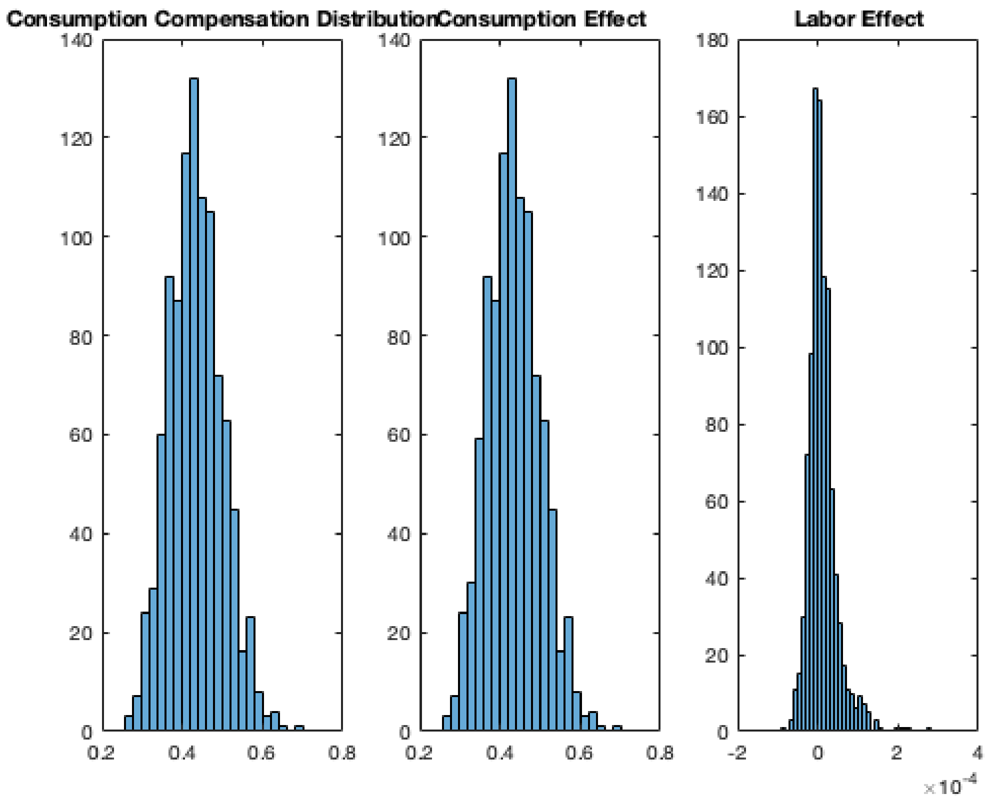
Low Consumption + Low Labor Seasonality Components

mean	std	cv	med	max	min	p90
0.08	0.01	15.56	0.08	0.13	0.04	0.10



High Consumption + High Labor Seasonality Components

mean	std	cv	med	max	min	p90
0.44	0.07	15.08	0.43	0.70	0.26	0.52



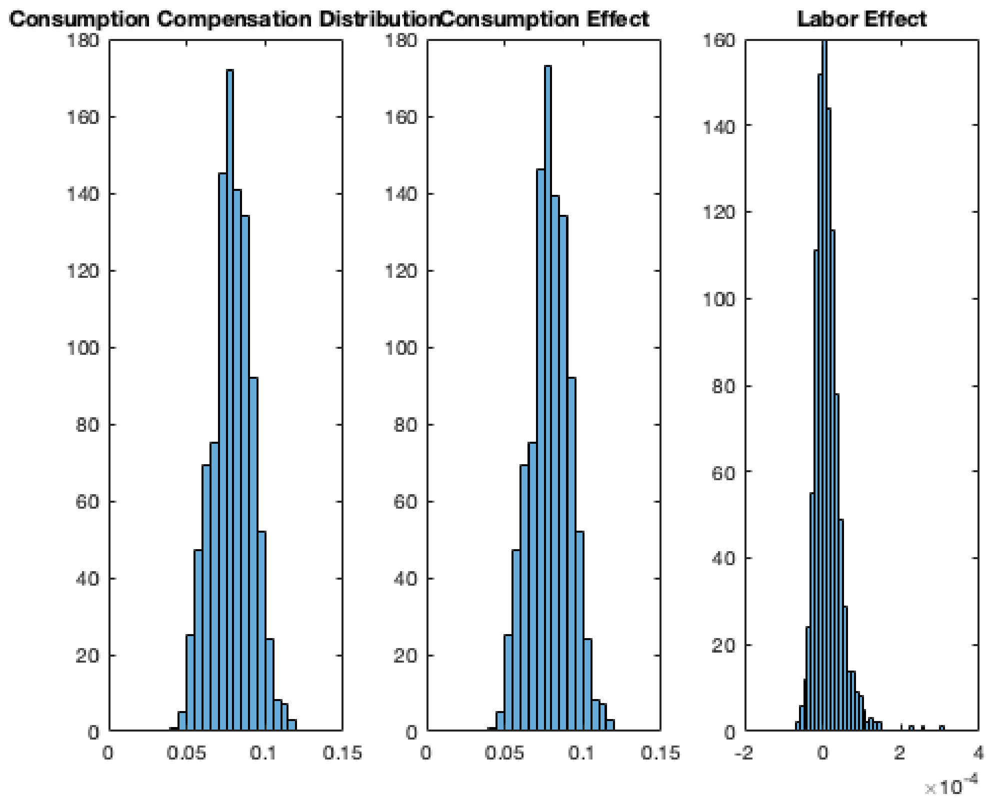
4 Labor and Consumption Negatively Correlated Seasonal Components

To get a high negative correlation between labor and consumption components, I assume the deterministic and seasonal components of labor are equal to the consumption ones, and they move in opposite directions. That is, whenever the seasonality degree of consumption is low (high), its the labor counterpart is high (low).

Results are similar to the positively correlated case.

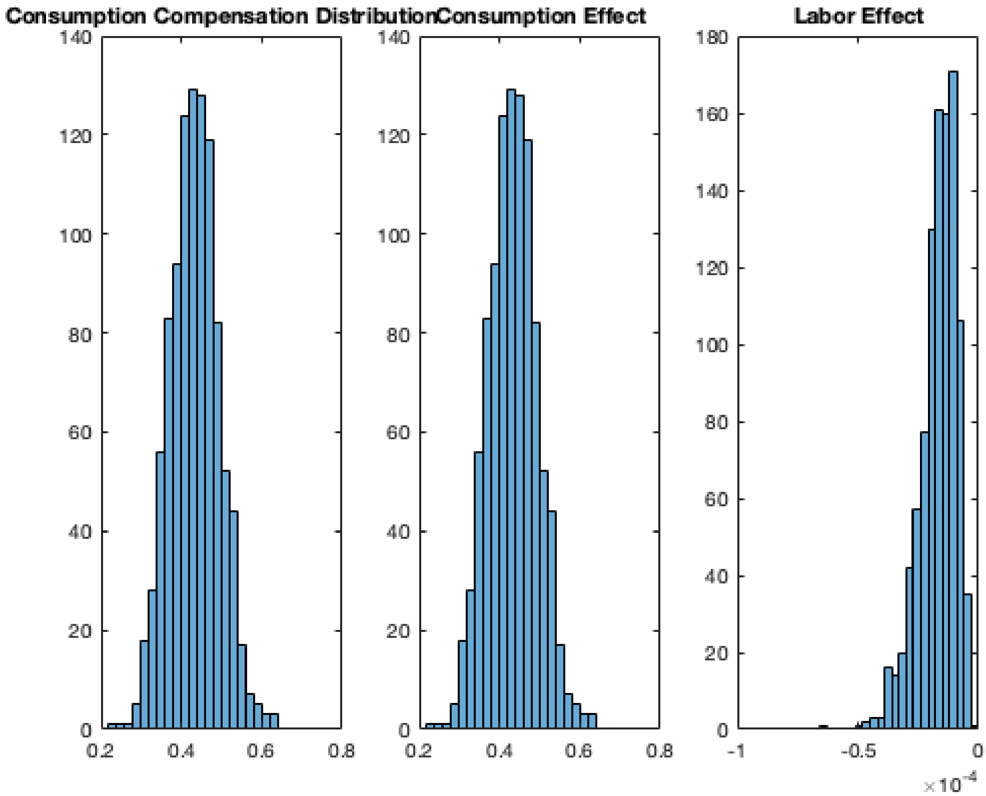
Low Consumption + High Labor Seasonality Components

mean	std	cv	med	max	min	p90
0.08	0.01	15.82	0.08	0.12	0.04	0.09



High Consumption + Low Labor Seasonality Components

mean	std	cv	med	max	min	p90
0.43	0.06	13.82	0.43	0.64	0.23	0.51



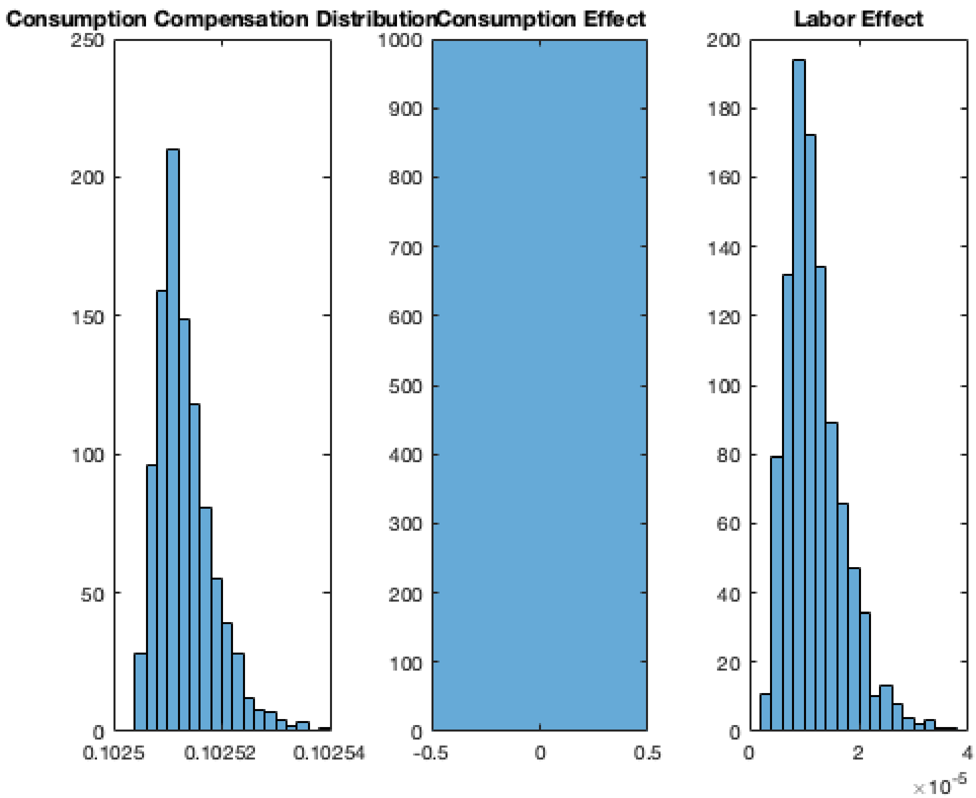
5 Labor and Consumption Correlated NonSeasonal Component

To implement this assumption, I let the permanent consumption and labor components to be the same, and analyze the case of postive and negative correlation between consumption and labor seasonal risks.

Results again indicates that the labor decomposition of the welfare effects is insignificant. However, welfare effects greater than the case of removing the seasonal risks. Moreover, now all agents receive the same compensation.

Low Consumption + Low Labor Seasonality Components: Positive Correlation

	mean	std	cv	med	max	min	p90
Welfare Effects	0.10	0.00	0.01	0.10	0.10	0.10	0.10
Consumption Decompostion	0.10	0.00	0.00	0.10	0.10	0.10	0.10
Labor Decompostion	0.00	0.00	43.46	0.00	0.00	0.00	0.00



Low Consumption + High Labor Seasonality Components: Negative Correlation

Welfare Effects

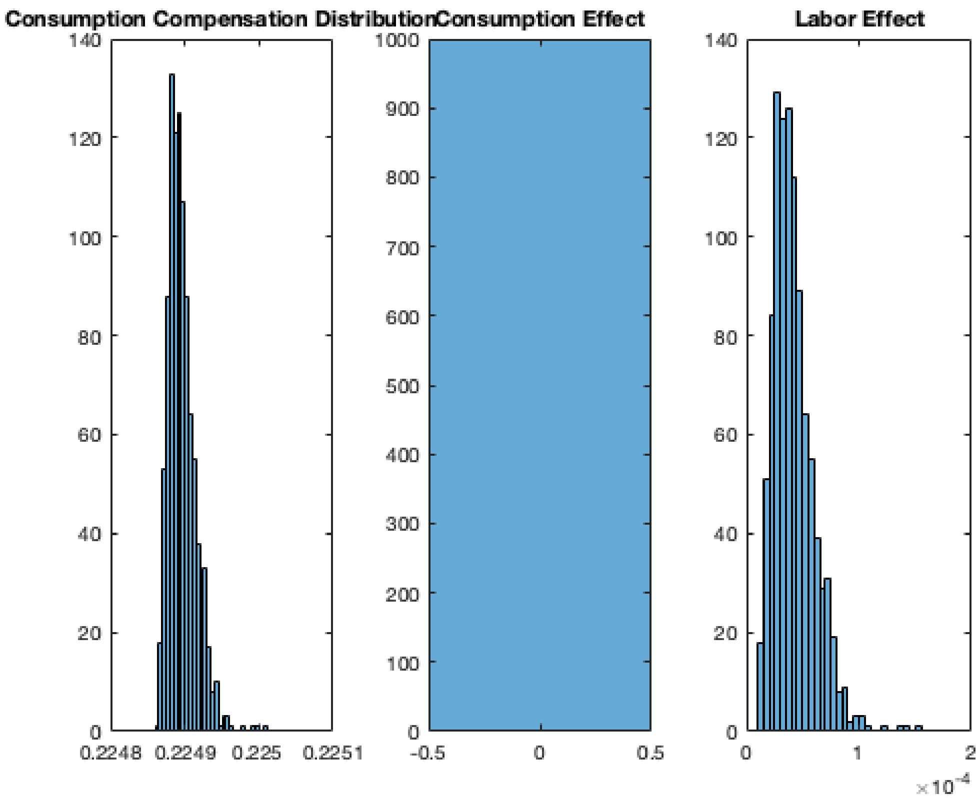
mean	std	cv	med	max	min	p90
0.22	0.00	0.01	0.22	0.23	0.22	0.22

Consumption Decompostion

mean	std	cv	med	max	min	p90
0.22	0.00	0.00	0.22	0.22	0.22	0.22

Labor Decompostion

mean	std	cv	med	max	min	p90
0.00	0.00	43.12	0.00	0.00	0.00	0.00



Consumption Compensation Distribution
Consumption Effect
Labor Effect
140
120
100
80
60
40
20
0
0.2248
0.2249
0.225
0.2251
1000
900
800
700
600
500
400
300
200
100
0
-0.5
0
0.5
140
120
100
80
60
40
20
0
0
1
2
×10^-4