Asset Pricing and Re-sale in Networks

Gabriela Stockler*

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Abstract

I study asset pricing when re-trade can take place in co-existing and interconnected markets. In my framework, there is a divisible asset and a finite set of traders with identical single-peaked preferences. They are distributed over a trading network. Traders can acquire shares at a common price, and then they may buy more shares from their connections at possibly different prices. I develop a novel network metric, trading centrality, that is a sufficient statistic for the equilibrium. A trader's asset acquisition is proportional to his centrality, and the asset common price is defined by aggregating centrality globally. For the re-trades in the network, a trader demands the gap between his optimal level of asset and his centrality; while each price is defined by aggregating centrality locally in the seller's network. Welfare is also determined by traders' centrality. I investigate what market outcomes arise at different trading networks. Implications for asset issuance and interdealer markets are examined.

KEYWORDS: networks; decentralized markets; interdealer trade; primary market; inventory; network games

JEL CLASSIFICATION: D4, D53, D85, G11, G12

*Universitat Autonoma de Barcelona and Barcelona School of Economics. stockler.gab@gmail.com.
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1 Introduction

For a variety of financial assets, traders first acquire shares in the primary market of asset issuance and then go on to trade those shares in co-existing and interconnected secondary markets. This is the case for fixed-income securities such as corporate and government bonds, for example. In the United States alone, issuance and trading values of fixed-income markets have surmounted $7,189.9 billion and $931.1 billion, respectively, in 2022 (SIFMA). Secondary markets typify the ubiquitous decentralized nature of modern financial markets in that assets move around by being re-traded among different counterparties at different prices. In contrast, primary markets are centralized in the sense that the traders bid for asset shares and pay a common price, the issuance price.

How asset price is affected by such dichotomy in trading configurations? This paper examines how the primary market is affected by the trading network of secondary markets. I study the decision of traders to acquire asset shares in anticipation of possibly being able to buy more shares later in the trading network. I reveal that traders act strategically in response to the interdependent terms of future re-trades. That's because asset shares are bought at issuance in anticipation of a variety of re-trade markets. But prices and demands of re-trades depend on how many shares traders already hold. As a consequence, I show that asset price in the (centralized) primary market is determined by the trading network structure of the (decentralized) secondary markets.

In the model, a finite set of traders must decide how much of an asset to purchase in two periods. The amount of asset shares available is exogenously-fixed. Traders have identical single-peaked preferences and are distributed over an exogenous *trading network*. At period one, everyone participates in the *primary market* (PM). Afterwards, trade happens in *local markets* (LM) described by the trading network.[1] With some positive probability, at most one trader is selected and forced to re-sell all his shares. This seller determines the *active local market* at period two where his connections are the buyers. All markets operate as a one-sided uniform-price auction.[2] Local markets can be thought of as meeting places

[1]The trading network describes secondary markets which are interliked through traders. It captures the fact that traders can participate in many different types of trading venues for possibly nondisjoint subsets of traders.

[2]As I explain later, traders are price-takers and thus truthful when submitting demand schedules in all markets they can participate. This means that traders ignore the *direct* impact of their bids on prices, as in Swinkels (2001) and Feldman et al. (2015). Price-taking assumption is the main departure from the well-known imperfect competition framework (as in Kyle (1989), Vives (2011) and Malamud and Rostek (2017)). However, it allows me to distill the equilibrium effects coming *only* from the structure of the trading network. And it is enough to guarantee the existence and uniqueness of equilibrium.

where traders *can* trade, and the active local market as *when* exchanges are realized. The probability of being the seller is referred as the re-sell shock, and it captures a sudden need of liquidity.

I develop a network metric, *trading centrality*, that is a sufficient statistic for the unique equilibrium, and the main contribution of this paper. I show that trading centrality neatly characterizes all market outcomes. Each trader's PM demand is given by his centrality and his LM demand is simply the gap between the his optimal holdings and his centrality. PM price is defined by aggregating trading centrality globally, and each LM price is defined by aggregating trading centrality locally in the network. Lastly, welfare is a weighted average of traders' centrality with weights that are functions of the trading network structure.

At the core of the model are the conflicting incentives to acquire asset shares in anticipation of future re-trades. On the one hand, traders can secure a level of asset holdings in the PM. On the other hand, they face higher competition there. I show that this conflict is resolved by an endogenous substitutability of demands. A trader optimally decides to invest in the opposite way of his direct and indirect connections.

As a buyer in the PM, a trader demand less when others demand more in expectation of a lower price in period two. That's because there will be i) more being sold - by his connections; and ii) less being demanded - by his competitors (i.e. they will be closer to their optimal holdings). Thus, traders defer asset consumption from the first to the second period, and the extent they do so depends on the local markets they can participate.

Traders use their network position to conjecture the set of local markets equilibria that could arise and, contingent on that, they strategically decide PM asset acquisition. Network position is the only dimension of ex-ante heterogeneity and it is the unique source delivering difference in demands. The trading network enables trade and, at the same time, constrains and correlates traders' behavior. The environment boils down to a one-shot, simultaneous-move network game of strategic substitutes played in the PM. Traders' best-respond to each others' demand schedule and the Nash equilibrium is given by the trading centrality.

With trading centrality in hand, I am able to investigate what market outcomes to expect across different and arbitrary trading networks. In term of prices, there are three main findings. First, PM price is bounded: it is the highest on symmetric networks, and the lowest on core-periphery structures.[3] Second, changes in the degree distribution,[4]

[3]In fact, for a fixed network size, the complete trading network exhibits the highest PM price; and the star network has the lowest PM price.

[4]In a trading network, degree refers to how many connections (i.e. counterparties) a given trader (node) has.

as stochastic dominant shifts, lead to monotonic changes in trading centrality and, in turn, in the PM price. A first-order stochastic dominant shift, by increasing connectivity, unambiguously increases PM price. A mean preserving spread, by increasing degree inequality, unambiguously decreases PM price. Third, PM price is higher than any LM price. And so a seller incurs capital losses[5] The unique exception is a star trading network which has a relatively small probability of price increase over time. That's because its core trader is the only one who obtains profits from re-selling.

Welfare, in terms of total expected utility, conversely echoes the findings of PM price. Welfare is the highest on core-periphery networks and the lowest on symmetric networks[6] The aforementioned changes in the degree distribution also affect welfare unambiguously. Increasing network connectivity decreases welfare; while increasing degree inequality increases welfare. All together, this reveals that reducing trading asymmetry, either by increasing network connectivity and/or reducing degree inequality, must not necessarily be welfare enhancing because of two opposing effects. On the one hand, more connected and equal trading networks are allocative efficient, as traders have the same (or complete) local market participation; on the other, prices are higher because competition is greater and uniform.

Notice that, at period one, traders make decisions anticipating they can be both a buyer and a seller. However, as I show in the Appendix, the selling motive is negligible in large enough networks - which I focus on.[7] Therefore, what drives traders' behavior in the model - that is, why asset acquisition is postponed - is their role as buyers. The seller exists simply to create supply at period two. Indeed, the results showcase perfectly why the model is all about buying incentives. The highest welfare is on the network structures that exhibit the lowest prices in expectation. And, in equilibrium, re-selling is costly but for the core trader of a star network.

My paper speaks to the behavior of dealers, the financial intermediaries for assets traded off-exchange.[8] Dealers often absorb substantial inventory position at issuance or from their costumers, and then use the interdealer market to offload these positions. I

I define connectivity as the average degree, and degree inequality as degree variance.

[5] Duffie (2010) discusses the large body of research aiming to explain price reaction to supply (or demand) shocks observed in several financial markets. He argues that price concessions are given by those who have limited opportunities to trade with counterparties, in line with findings.

[6] Just as with PM price, I find that welfare is the highest (lowest) in the star (complete) network.

[7] A quick way to see why is to notice that the probability of being a buyer is always greater than being a seller. For large enough networks this means that, on the aggregate, traders expect to be buyers.

[8] Dealers are the backbone of trades for bonds (government, corporate and sovereign ones), derivatives, commodities, and currencies - to name a few. (See He et al. (2017) among others).

attend to two unaddressed questions regarding how the interdealer network structure influences financial markets. I do so by leveraging the rationale that dealers' willingness to take on inventory is affected by their subsequent trading network; and prices and quantities in the interdealer network are influenced by their current inventory holdings.

First, my findings provides a justification for the often observed core-periphery trading network among dealers themselves.[9] Such structure coordinates dealers' inventory positions in a way that ensures the lowest average cost to take on asset shares, making them all better off. This suggests that the core-periphery interdealer network supports dealers' inventory management and guarantees market liquidity in secondary markets. Second, I reveal a novel effect of the interdealer network structure: it determines the issuance price of an asset, thus regulating credit provision for the issuers of securities (such as governments and firms) in the primary markets.

To gain further insights on how my framework maps to the data, I proceed in two distinct ways. I first layout the empirical implications to the interdealer market. Trading centrality helps rationalize the inconclusive evidence on whether central dealers[10] have better or worse terms of trade. I show that the trading centrality can induce both centrality premium and discount. But this ambiguity is resolved once we take into account the entire structure of the interdealer network. Trading centrality also provides a novel liquidity measure that only requires information about the trading network structure, and it is informative of prices and quantities.[11] Additionally, I illustrate the empirical application of the model with data on the US Corporate bonds secondary markets.[12] I document an interdealer network with a core-periphery structure, and then I analyze how it relates to the observed interdealer trades. As my theory predicts, I find central dealers sell more and at a higher price; and buy less and at a lower price.

Related Literature: This paper is related to three areas of research: decentralized markets, network games, and over-the-counter (off-exchange) financial markets.

[9]Recent empirical studies document that the interdealer market can be seen as a stable trading network with a core-periphery structure. Such interdealer network has high trading frequency and volume, with spillover effects on the overall market outcomes. See Section 2.

[10]In finance, essentially all studies so far define centrality by standard network metrics such as degree and eigenvector centrality.

[11]As I show in Section 5, this liquidity measure quantifies the re-sell cost: the difference between PM price and a LM price. It is similar to Kyle's lambda (Kyle (1985)) in that it measures the equilibrium price impact of a sell order flow.

[12]I use a restricted version of the TRACE data used by Friewald and Nagler (2019) which the authors made publicly available for replication purposes.

I share the perspective of Malamud and Rostek (2017) of modelling markets as incomplete and co-existing: traders cannot exchange with one another at all times and they can participate in more than one trading venue. The novelty of my paper is to combine both centralized and decentralized trades in a unified and dynamic framework.[13] Most importantly, decentralized markets are random realizations and only take place *after* the centralized one. Moreover, several other features set my paper apart from theirs. Here traders are price-takers, there is just one asset, and I restrict the pricing protocol to a one-sided uniform-price auction.

My paper allows for any market structure "between" centralized and bilateral trading.[14] A vast literature on decentralized markets makes the later extreme assumption. Here there are two modelling approaches. Search models, with the seminal contribution of Duffie et al. (2005) in finance; and network models, such as Kranton and Minehart (2001), Corominas-Bosch (2004), Manea (2016). While suitable for many markets such as the over-the-counter one between dealers and customers, in reality several assets are not traded as pairwise exchanges. Treasure securities for instance, are sold in auctions to a set of 20-30 dealers. Interdealer broker systems, electronic trading platforms for interdealer trades, also work as auctions.

The multi-lateral aspect of trades is also a natural implication of auction as a trading protocol. In my model, traders play a game by submitting demand schedules in each sub-graph of the network. Allowing (not assuming) strategic behavior is central in my framework. Here I build upon the game-theoretical view of decentralized trading with imperfect competition - as in Kyle (1989), Vives (2011) Rostek and Weretka (2012) and Rostek and Weretka (2015) to name a few. In this line of work, (finite) traders account for their impact on price and, because of that, they strategically "shade" their bids in the demand game.[15] My paper, in contrast, assumes that traders are price-takers and thus truthful.[16] Although a strong departure, price-taking renders great tractability of the model and ensures the equilibrium outcomes are driven solely by the trading network, without

[13]Somewhat related, Rostek and Yoon (2021) have unified imperfect competition and decentralized markets in a framework.

[14]In Appendix H I show that restricting the model to bilateral trades would miss all the interesting forces coming from the trading network that drives equilibrium outcomes.

[15]More specifically, imperfect competition means equilibrium is determined by a uniform-price (double) auction with traders submitting demand schedules taking into account their endogenously-determined price impact.

[16]My model accommodates imperfect competition, which I discuss in Section 9. This is also work in progress and available upon request.

compromising the strategic aspect of trades.[17]

My paper is a novel application of games in networks. As I show later (Section 4), the model is set of network games of global strategic substitutes and I rely on the findings of Bramoullé et al. (2014) and Bramoullé and Kranton (2016) to characterize equilibrium.

As aforementioned, a core motivation of this paper is the interdealer market and so my paper relates to the research on off-exchange markets. Given this particular interest, I devote Section 2 to discuss recent empirical findings of the literature and my contributions

Outline: The rest of the paper is organized as follows. Section 2 contextualizes my framework into the interdealer market for off-exchange securities.Section 3 introduces the model and Section 4 solves it. Section 5 gives the main result. Section 6 analyzes how the network structure affects equilibrium and Section 7 studies welfare. Section 9 discusses different extensions. Section 10 concludes. Details and proofs are found in the Appendix.

2 Interdealer Networks and Off-Exchange Securities

A substantial proportion of financial instruments - the so-called ”off-exchange“ assets such as Treasury and corporate bonds, debt securizations, currencies, etc. - are traded in primary and secondary markets. Primary markets are for asset issuance and serve to raise capital. They are centralized in the sense that dealers bid to acquire shares, usually in uniform-price auctions, and no trade happens among dealers themselves. Secondary markets is where trades actually happen in a decentralized way, and asset prices are dispersed. In the bonds market, for example, a firm or government (the issuer) creates a new bond and allocates it to dealers[18] at a common price; who then take the bond to secondary markets.

Primary and secondary markets are paramount for the well-function of financial markets and the economy. For instance, as of 2021, US fixed-income securities have issuance value of roughly $US\$13.5$ billion, outstanding value of $US\$52.9$ trillion and average daily traded value of $US\$969$ billion (SIFMA). The backbone of such trading activity are deal-

[17]Strategic behavior in my model arises purely from the interaction of re-sale risk and trading frictions imposed by the trading network. Not concerns about price impact as in the imperfect competition models.

[18]Usually investors do not participate in the primary market. The noteworthy example is the US Treasury securities market where a group of more than 20 primary dealers commit to buy large quantities of Treasuries every time the government issues debt, and stand ready to trade them in the otc market (FINRA). The use of primary dealers is common across many countries and it has been in force since 1960 (Arnone and Ugolini (2005)).

ers,[19] the market-makers, who provide immediacy to other traders and ensure market liquidity. There are currently 3,394 dealers registered with Financial Industry Regulatory Authority (FINRA) in the US alone.

Dealers do not operate in a vacuum. Rather, when making markets, they rely on being able to trade with one another. Thus, the existence of a trading network among dealers themselves.[20] Interdealer trades are useful because dealers often absorb substantial inventory position in primary markets or from their costumers, and then use the interdealer market to offload these position and rebalance their inventory.[21]

In reality, most interdealer networks have a stable core-periphery structure.[22] This is the case, for instance, for US corporate bonds (Dick-Nielsen et al. (2020), Goldstein and Hotchkiss (2020), Di Maggio et al. (2017b)); US foreign exchange (Hasbrouck and Levich (2020)); US debt securization (Hollifield et al. (2017)); US municipal bonds (Li and Schürhoff (2019)). This indicates that trading relationships exist and are persistent, and typically, a few large dealers (the core) are responsible for large share of the trading volume.

An extensive and growing empirical literature[23] relates the dealers' position in the network to their trading behavior. The recurring finding is that a dealer's centrality, his "importance" in the network, is a determinant for his: bid-ask spreads; trade volume and frequency; clientele characteristics; and trade execution speed. Moreover, there is heterogeneity at the linkage (relationship) level: a dealer's trading price, frequency and volume depends on the identity of his counterparty: if it is a customer or another dealer, and which customer/dealer is. Recent studies (Eisfeldt et al. (2018) and Di Maggio et al. (2017b)) also show that changes in the interdealer network, by the exit of a dealer, has significant impact in market outcomes.

[19] A dealer, or broker-dealer, is a financial institution in the business of buying or selling securities on behalf of its customers or its own account or both. Dealers must be registered with a specific regulatory authority, such as Financial Industry Regulatory Authority (FINRA).

[20] By forming trading relationships, dealers can reduce trading frictions, such as different forms of trading costs, funding constraints, search, and informational frictions. See Vayanos and Wang (2012).

[21] That's because inventory is risky and costly, and interdealer trades are the main tool to manage it.

[22] In fact, the observed majority of financial networks are core-periphery. See Bech and Atalay (2010) for evidence on US federal funds market;Boss et al. (2004), Craig and von Peter (2014), in 't Veld and van Lelyveld (2014) for interbank market in other countries. This is also true for the dealer-client networks. See Hasbrouck and Levich (2020), Hendershott et al. (2020), Hollifield et al. (2017), Li and Schürhoff (2019), Di Maggio et al. (2017b), Kondor and Pintér (2022) Czech et al. (2021) for otc markets; and Di Maggio et al. (2017a) for the stock market.

[23] This is possible due to the structure of financial data and the decentralized nature of secondary markets that allows us to identify different counterparties and construct trading networks.

However, evidence on whether centrality makes a dealer to have better or worse terms of trade, in terms of bid-ask spread, is still inconclusive. Li and Schürhoff (2019) and Di Maggio et al. (2017b) find a centrality premium: core (central) dealers charge a wider spread than peripheral dealers. Di Maggio et al. (2017b) also document that more central dealers pay lower spread. Meanwhile, Hollifield et al. (2017), Goldstein and Hotchkiss (2020) and Dick-Nielsen et al. (2020) find centrality discount: core dealers charge a narrower spread. Goldstein and Hotchkiss (2020) and Dick-Nielsen et al. (2020) are two exceptions who also look at interdealer trades, and they both find centrality premium.

The next figures show examples of real core-periphery interdealer networks. The more central dealers form the core, while the less central ones the periphery. Centrality is measured by the standard network metrics of degree and/or eigenvector centrality.[24] In the US municipal bonds markets, Li and Schürhoff (2019) infer the interdealer network with 2,238 dealers and a core size of 10 to 30 highly interconnected dealers (Figure 1). In the securization markets, Hollifield et al. (2017) document an interdealer network with 658 dealers (Figure 2). Both find that the most central dealers are more active and account for the vast majority of trades - what can be seen in the right-hand side network in those figures.

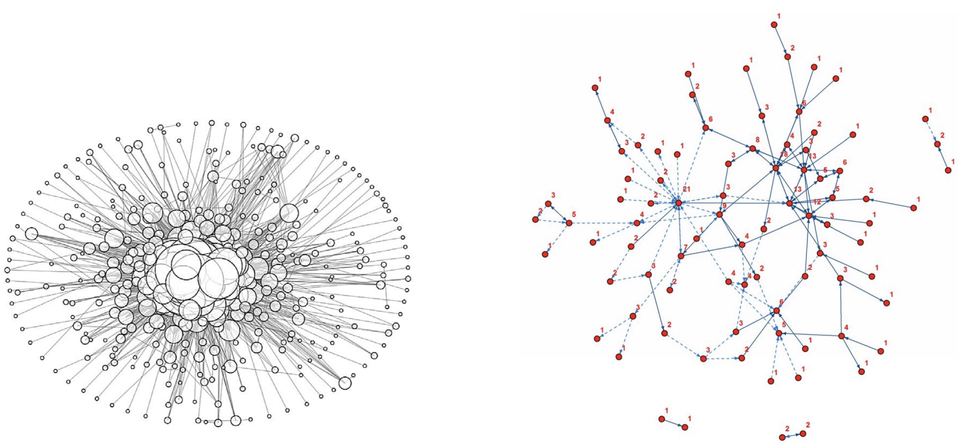

Figure 1: Hollifield et al. (2014)
On the left is the interdealer network: each node is a dealer and each arrow represents a directed order flow between a pair of dealers. On the right is the same network but only with the most active dealers. The active network only keep links with at least 50 trades and worth at least $\$10$ million. Dealers are labeled by their degree; links with trades worth more than $\$100$ million are shown in solid lines.

[24]Degree centrality means that a dealer is more central the more connection he has. Eigenvector centrality also accounts for indirect connections: a dealer is more central as he and his connections are more central.

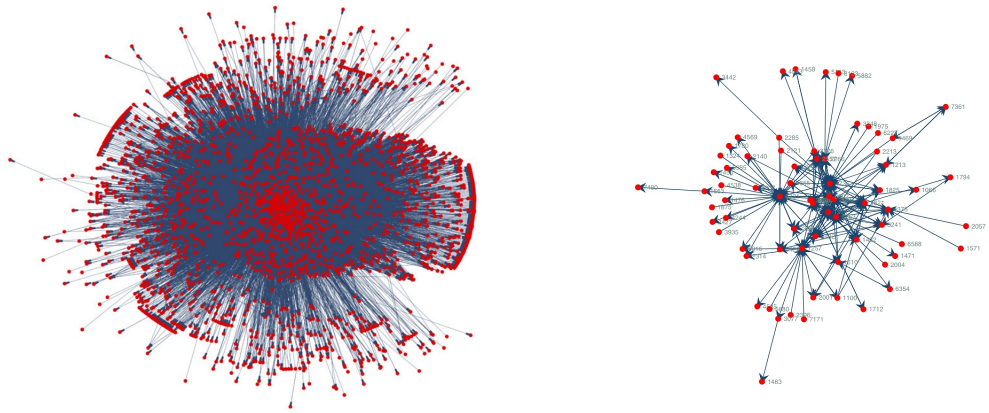

Figure 2: Li and Schürhoff (2019)
On the left is the interdealer network: each node is a dealer and each arrow represents a directed order flow between a pair of dealers. On the right is the same network but only with the most active dealers. The active network only keep links that exceeds 10,000 transactions over the sample period.

Figure 3 and Figure 4 display different interdealer networks in the US Corporate bonds markets inferred by Dick-Nielsen et al. (2020) and Di Maggio et al. (2017b), respectively. Dick-Nielsen et al. (2020) investigate immediacy provision from 2002 to 2013 in a network with 3,499 dealers. In the right-hand side, the (inverse) distribution of eigenvector centrality clearly reveals that most dealers are less central (the periphery) and only a few (the core) are very central. The same is true for the interdealer network reported by Di Maggio et al. (2017b) who look at the entire universe of trades from 2005 to 2011. In the right-hand side, the cumulative distribution of trades as a function of a dealer's centrality shows that the top 50 dealers account for roughly 80% of all transactions.

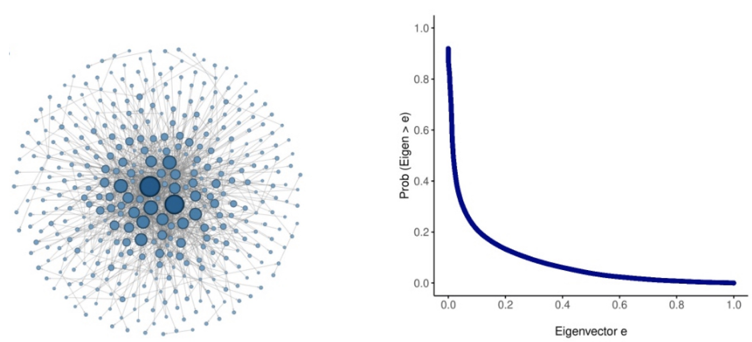

Figure 3: Dick-Nielsen et al. (2020)
Interdealer network where the size of a node (dealer) reflects his eigenvector centrality. The inverse distribution of eigenvector centrality is plotted on the right-hand side.

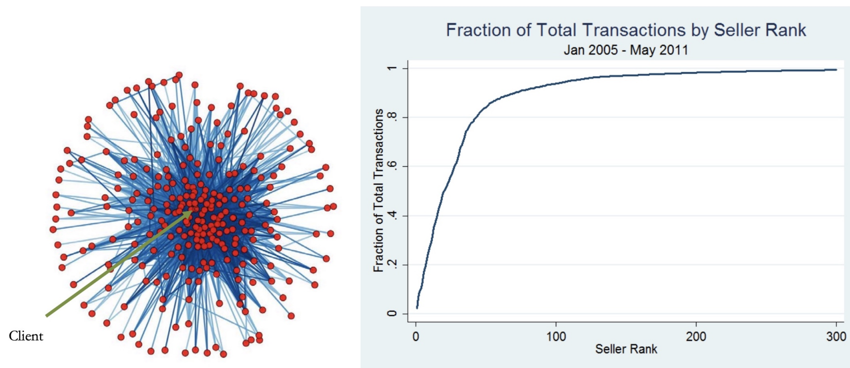

Figure 4: Di Maggio et al. (2017b)
On the left, the interdealer network; darker lines indicate higher number of transactions between a pair of dealers. On the right, the cumulative distribution of dealer's eigenvector centrality, measured by selling trades.

Even within the same market, the interdealer network and dealers' behavior vary greatly in the cross-section of assets. Hollifield et al. (2014) analyze various segments of the securitization market during eight months in 2011-2012. Figure 5 depicts the interdealer networks, dealer-customer trades, interdealer trades and prices for two securization instruments in their sample. The networks have notable different structures. The left one has roughly a monotonic evolution of prices, but the right one doesn't. Also, bid,ask and interdealer prices are similar in the left network; while in the right one interdealer prices can be above and below costumer-trade prices. It also seems that the volume in each dealer-customer and interdaler segments behave similarly: the right network with more customer trades also have more interdealer trades.

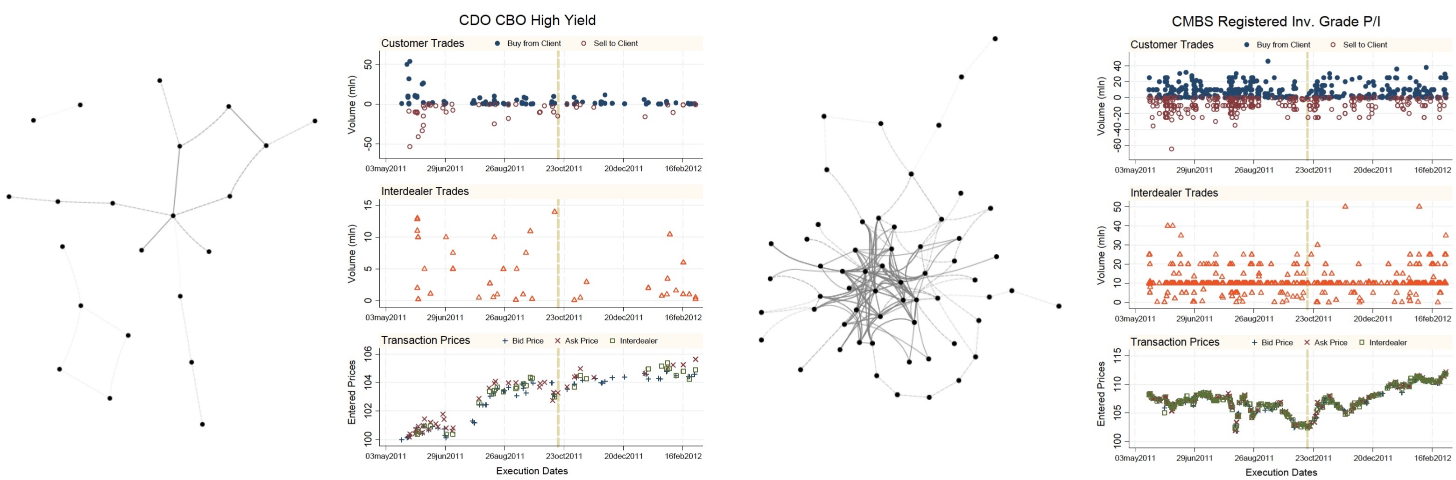

Figure 5: Hollifield et al. (2014)
At each panel, on the left is the interdealer networks: darker lines indicate higher number of transactions between a pair of dealers; on the right, the evolution of dealer-customer trade volume, interdealer trade volume and transaction prices.

The take-away so far is that the structure of the interdealer network shapes, to some extent, the market behavior of several financial assets. Perhaps surprisingly, the relationship between primary markets and interdealer networks has received scant attention.[25] After all, dealers are the link between primary and secondary markets. The extent that a dealer can successfully re-sell shares of the asset acquired at issuance relies on him being able to manage his inventory in the interdealer market. At the same time, how much inventory is accumulated at issuance influences liquidity need and provision in the interdealer trades. Hence, there is a two-way feedback effect between primary markets and the interdealer market.

Despite the vast theoretical and empirical literature,[26] two questions remain unaddressed regarding the interdealer network. The first is its relation with the issuance price of an asset. The second is how to explain the mixed evidence on dealers' centrality effects on market outcomes. My paper fills these gaps, and I show that both are related to how the structure of the interdealer network conveys information about *expected and correlated* terms of trades among dealers themselves and, thus, determines dealers' willingness to take on inventory and asset issuance price.

3 The Model

Markets and the Trading Network

There are two periods $t = \{1, 2\}$ and $N > 2$ traders. There exists a divisible asset in exogenous and fixed supply $Q > 0$[27] All traders can acquire asset shares at $t = 1$ in the *primary market* (PM) . Afterwards shares can be re-traded among traders themselves in *local markets*.

There exists a *trading network*[28] in which nodes are the traders and connections determine which traders have access to a particular local market jointly but not separately. A link between i and j means that trade between i and j is *possible*. Formally, the trading network is characterized by the adjacency matrix G such that $[G]_{ij} \equiv g_{ij} = 1$ if $i \in N$ and

[25] I am not aware of any study, either theoretical or empirical, that explores it.

[26] Over the last two decades, the center of attention has been on the otc markets, with the seminal contribution of Duffie et al. (2005). Research has made great progress in advance our understanding of the bilateral trading behavior and its implication for prices and liquidity, both at the dealer-to-client and dealer-to-dealer segments.

[27] This specification may, for example, capture cases in which dealers face a common outside opportunity for customer sell order, or when they allocate a new security at issuance.

[28] The trading network is unweighted, undirected, fixed, exogenous and known.

$j \in N$ are connected, and $g_{ij} = 0$ otherwise. By convention, $g_{ii} = 0$. The set of linkages of trader $i \in N$ is given by his neighborhood $N_i = \{j \in N : g_{ij} = 1\}$, and i's degree is the number of connections he has: $d_i = |N_i| = \sum_{j \in N} g_{ij} > 0$.[29]

Thus, the trading network summarizes the set of local markets at $t = 2$. There are N of them. Each is defined by a trader's neighborhood. I refer to the local market of trader $i \in N$ as when i sells his shares to his linked traders, his buyers, at an endogenously-determined uniform price (more details below).

Market participation at $t = 2$ is random. With a probability $\phi > 0$, which I refer as the *re-sell shock*, only trader $i \in N$ is selected and is forced to re-sell in his local market. This happens at the same probability ϕ for each trader. The seller establishes the *active local market* at $t = 2$. I make two assumptions:

Assumption 1. *At most one trader experiences the re-sell shock: $\phi < \frac{1}{N}$.*

Assumption 2. *Supply in any local market is inelastic: the seller does not choose his supply.*

By Assumption 1, there is only *one* active local market or *none*.[30] Local markets can be thought of as meeting places where traders *can* trade, and the active local market as *when* exchanges are realized.[31]

Assumption 2 means that all markets are one-sided. The decision of a trader is how many shares to purchase in each market he has access to. The re-sell shock ϕ is interpreted as a sudden need to unload shares to exit the market. Re-trade in the local markets is for immediacy provision among traders themselves and, in turn, it enables the increase or decrease of asset holdings (i.e inventory management - more details below).

Traders

Traders have initial wealth $w > 0$, and no one is endowed with asset shares. Traders'

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[29] I focus my analysis on trading networks with a minimum degree of one, i.e. every trader has at least one connection. However, this is by no means a restrictive assumption, it is just the most interesting case. It also does not mean that the trading network must be connected. In the Appendix, I provide my main results for when there are "isolated" traders in the network. All the analysis and intuition presented in the main paper hold.

[30] In principle, there are 2^N possible states of the world: $\Omega = \varnothing \cup \{\omega_s : s \in \{1, 2..., 2^{N-1}\}\}$. I assign probability $\phi > 0$ to each state $s \in [1, N]$, which represents the identity of the single trader $i \in N$ who experiences the re-sell shock; probability $1 - N\phi$ to the empty state, in which no trader experiences the shock; and probability zero to the remaining $s \in [N + 1, 2^{N-1}]$ states, which represents the possible combinations of more than one trader getting shocked.

[31] In other words, the re-sell shock "activates" a local network which is a subgraph of the entire network.

goal is to build up asset inventory q_i.[32] Let $q_{i,1} \geq 0$ denote how many shares trader $i \in N$ acquires in the PM, and $q_{i,s} \geq 0$ the amount bought in the local market of seller $s \in N_i$. By assumption, if i and s are not connected they cannot trade and so $q_{i,s} = 0 \; \forall s \notin N_i$. Also, if i is the seller he must liquidate his position and so $q_{i,i} = -q_{i,1}$. At the end of period two, a trader's inventory q_i is the sum of the shares purchased at each period, $q_i = q_{i,1} + q_{i,s}$.

Each trader receives the net payoff of his trading activity. It is defined as the total utility derived from inventory q_i minus the total payment. Purchasing quantities $(q_{i,1}, q_{i,s})$, he enjoys a utility of

$$U(q_{i,1}, q_{i,s}) = (q_{i,1} + q_{i,s}) - \frac{1}{2}\left(q_{i,1} + q_{i,s}\right)^2 \tag{1}$$

and pays $P_1 q_{i,1} - P_s q_{i,s}$ where P_1 is the PM price and P_s the price of seller s.

Preferences are single-peaked and traders have an optimal inventory of 1[33] Intuitively, a trader buys shares at each period to reduce the gap between the optimal inventory and his current one. But holding inventory entails a cost of $\frac{1}{2}q_i^2$. For dealers in financial markets, the costly inventory can be due to several reasons (e.g. regulatory capital or collateral requirements). Here it represents the expected cost of being forced to re-sell to raise liquidity by quickly disposing inventory into a restricted, and possibly illiquid, local market (Duffie (2010),Duffie and Zhu (2017)).

The role of having two consecutive markets is best understood by analyzing the partial utility of the quantity traded in either period:

$$\frac{\partial U(q_{i,1}, q_{i,s})}{\partial q_{i,m}} = 1 - q_{i,1} - q_{i,s} \qquad m = \{1, s\} \tag{2}$$

This partial utility is the trader's marginal willingness to pay for the asset in market m given that he obtains $q_{i,-m}$ shares in the other market. It decreases in both quantities traded. Demands are then substitutes across markets. That's because more of the asset is preferred rather than less only up to the optimal inventory, as inventory is costly[34]

Pricing Mechanism

[32] For dealers in off-exchange markets, inventory is used to facilitate future trades with their customers in the otc market.

[33] This is a normalization. The more general setup with individual asset valuation α_i, possibly heterogeneous, is presented in Section 9.

[34] In financial markets, a dealer's inventory is used in his intermediary activity with customers in the otc markets. Inventory allows dealers to provide immediacy and liquidity to costumers. I abstain from the discussion of dealer-customer trades as my focus is on inter-dealer trades.

Traders are price-takers[35] and every market (local or otherwise) operates as a one-sided uniform-price auction. Each trader $i \in N$ submits a demand schedule $q_{i,m}(\cdot; P_m)$ in every market $m = \{1, \{s\}_{s \in N_i}\}$ he can participate. Equilibrium price in a market is determined by equating aggregate demand of the participant buyers with the asset inelastic supply.

The primary market features complete participation and a global market clearing condition holds. The PM price P_1, common to all traders, is given by

$$\sum_{i \in N} q_{i,1}(\cdot; P_1) = Q \tag{3}$$

The price of a local market is seller-specific. For a seller $s \in N$, his local market price P_s is given by the local market clearing condition

$$\sum_{i \in N_s} q_{i,s}(\cdot; P_s) = q_{s,1}(\cdot; P_1) \tag{4}$$

The pricing mechanism indicates that a feedback effect across traders' demands emerge. There are two reasons for that. First, even though the asset supply is exogenous in the PM, it is endogenous in every local market: the shocked trader re-sell his PM holdings. Second, the buyers in a local market may already have acquired shares in the PM what influences their willingness to pay for the seller's supply.

Figure 6 depicts the timeline of the model.

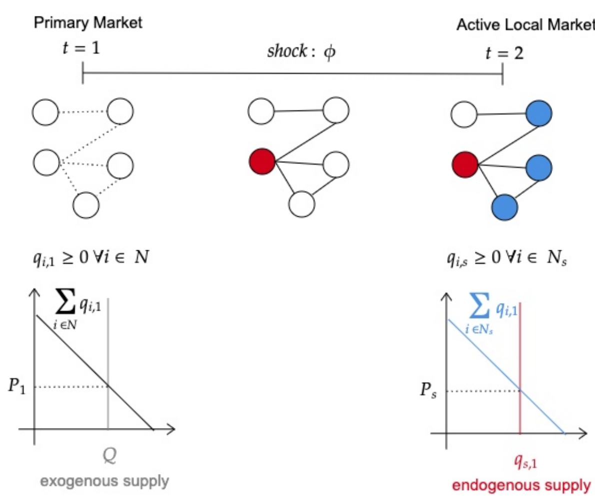

Figure 6: Timeline

One attractive feature of the baseline model is that it can be solved in closed form

[35]That is, traders are truthfull and ignore their direct price impact.

and is thus a parsimonious workhorse with which to develop intuition (Section 4). The tractability relies on the core assumptions I make. Although not very general, they capture realistic features of the interdealer market, which I discuss in Appendix C. In Section 9 I present extensions of my framework that allows for: heterogeneity in asset valuation and risk preference; expected fundamental asset returns; and imperfect competition.

4 Equilibrium Analysis

Notation:

Bold lowercase letters refer to N-dimensional vectors and bold uppercase letter to N-square matrices. All Primary Market variables $(\{q_{i,1}(\cdot)\}_{i\in N}, P_1(\cdot))$ are conditioned on the primitives of the model: the trading network $\boldsymbol{G}$ and the shock parameter ϕ. I omit such notation for the sake of clarity

The model is solved backwards. I first characterize the local market equilibrium given a shock realization. Then, the equilibrium in the primary market is determined. In Appendix D I solve in detail traders' optimization problem.

Traders are rational and forward-looking. They decide their optimal demand schedules in anticipation of the re-sell shock and the different local markets that can take place at period two. Trader $i \in N$ chooses $(q_{i,1}(\cdot), q_{i,s}(\cdot))$ to maximize his expected net payoff,

$$\max_{q_{i,1}(\cdot),q_{i,s}(\cdot)} \quad E\left[(q_{i,1}+q_{i,s}) - \frac{1}{2}\left(q_{i,1}+q_{i,s}\right)^2 - \left(P_1 q_{i,1} + P_s q_{i,s}\right) + w \;\middle|\; \boldsymbol{G}, \phi\right] \tag{5}$$

Each trader faces a trade-off: how many shares to purchase at each period. On the one hand, he can acquire the asset with certainty in the PM but the price is likely to be higher - due to higher competition - and he faces the risk of re-selling. On the other hand, buying at period two is probably cheaper but he does not know if he will need to sell nor if he will participate in the active local market.

Jointly, ϕ and $\boldsymbol{G}$ imply that each trader has three levels of uncertainty: i) if he will participate in one or two markets; conditional on trading at $t = 2$, ii) if he will buy or sell asset shares; and iii) if he is a buyer, with whom will he trade.

As it will be clear from the results (Section 6), although buyers are not forced to provide liquidity to the seller they optimally decide to do so. For two reasons. First, no trader reaches his optimal inventory at $t = 1$. Second, buyers guarantee a price concession to absorb seller's supply in any local market, in any trading network.[36] Thus, gains from trade arise because local markets provides more and cheaper shares to the participants buyers, and liquidity for the seller.

4.1 The Active Local Market

At $t = 2$, both the PM and the re-sell shock have realized. The seller identity $s \in N$ is common knowledge and so it is the fixed asset supply $q_{s,1} \leq Q$ (recall Assumption 2). Notice that the PM at $t = 1$ can be seen as endogenously determining individual asset endowment in the static market of a seller s.

Conditional on a level of current holdings $q_{i,1}$, each trader i as a buyer in period-two chooses his demand schedule $q_{i,s}(\cdot)$ for a local market price P_s. i's demand is given by[37]

$$q_{i,s}(P_s; q_{i,1}) = (1 - q_{i,1}) - P_s \tag{6}$$

Since traders are price-takers, i's demand eq. (6) is the same in every local market he can participate. It is simply given by his willingness to pay more for the asset, i.e. his current marginal utility eq. (2).

Using eq. (6) and local market clearing conditions eq. (4), the set of equilibria at $t = 2$ can be found, one for each possible seller $s \in N$.

Lemma 1. *Local Market Equilibrium*

Consider a Primary Market asset allocation $\{q_{i,1}\}_{i \in N}$. The equilibrium in the local market of seller $s \in N$ with his network-induced set of buyers $i \neq s : i \in N_s$ is given by the selling price P_s^*,

$$P_s(\boldsymbol{q}_{N_s,1}) = 1 - \frac{\left(\sum_{i \in N_s} q_{i,1} + q_{s,1}\right)}{d_s} \tag{7}$$

and buyers' asset allocation,

$$q_{i,s}(\boldsymbol{q}_{N_s,1}) = \frac{1}{d_s}\left(q_{s,1} + \sum_{k \neq i, k \in N_s} q_{k,1}\right) - \frac{d_s - 1}{d_s} q_{i,1} \qquad \forall i \in N_s \tag{8}$$

where $\boldsymbol{q}_{N_s,1} \equiv (q_{s,1}, \{q_{i,1}\}_{i \in N_s})$.

[36]The unique exception is the star network, which is in itself an interesting finding that I discuss later on (Section 6).

[37]This is simply the the first-order condition of (1) with respect to $q_{i,s}$ keeping $q_{i,1}$ and P_1 fixed.

Assumption 1 ensures there is a unique equilibrium at period two as there can be at most one shocked trader.

Corollary 0.1. *Active Local Market Equilibrium*

For a given shock realization, equilibrium at period two is unique. The interior equilibrium - that is, if trader $s \in N$ is the one shocked - is given by Equation (7) *and Equation* (8)*.*

If no trader is shocked, then no local market is active: $\boldsymbol{q}_{i,s} = 0 \forall i, s \in N$ and $\boldsymbol{q}_{i,1} > 0$.

The trading network directly affects the local market prices in two opposite ways. First, trough the positive "participation effect": the seller's degree d_s determines how large his market is. The higher d_s, the higher is $P_s(\boldsymbol{q}_{N_s,1})$. Second, through the negative "inventory effect": PM holdings in the seller's neighborhood, $q_{s,1}$ and $\{q_{i,1}\}_{i \in N_s}$, determine his supply and buyers' willingness to pay. The higher are PM holdings, the lower is P_s^*.

But that's not the whole story because these effects are interdependent what results in additional indirect effects. A larger pool of buyers (high d_s) may imply higher aggregate PM asset holdings just because there are more terms in the sum ($\sum_{i \in N_s} q_{i,1}$), what could drive $P_s(\boldsymbol{q}_{N_s,1})$ down. At the same time, a high d_s may imply lower aggregate PM asset holdings because the buyers anticipate the high competition and so ensured asset holdings in the PM, what could drive $P_s(\boldsymbol{q}_{N_s,1})$ up.

At the buyer level, equilibrium asset allocation $q_{i,s}(\boldsymbol{q}_{N_s,1})$ is decreasing in the seller's degree, as price increases in the later. And it is increasing in other buyers' (competitors) PM holdings $\left(\sum_{k \neq i, k \in N_s} q_{k,1}\right)$, since the higher these are the higher is i's residual supply and the lower is seller's price. Moreover, local markets allocate asset shares in accordance to who values it the the most at $t = 2$, to those who acquired the least shares in the PM. Thus, the buyers who provide more liquidity are those who would need less liquidity if they were to be hit by the re-sell shock.[38]

As I show next, traders understand that PM outcome determines liquidity supply and demand in local markets, what in turn influences traders' asset acquisition, and thus price, in the PM. And that's precisely why the trading network plays a crucial role in the PM. It incorporates the two-way feedback effects across markets and traders.

[38] This apparent "coordination" between liquidity demand and supply of a trader reflects the strategic nature of the environment. This is going to be clear when in the upcoming analysis of traders' behavior in the PM (Subsection 4.2)

4.2 Primary Market as a Trading Game

Recall that the only source of ex-ante heterogeneity among traders is on network position. Different network positions mean that traders have different expected trades (i.e. local market participation) and, consequently, their asset acquisition decision will differ. More importantly, this decision depends on all other traders' decision as well since they determine the terms of trade in local markets.

In turn, the expected payoff (5) of each trader $i \in N$ in the PM, before any trade takes place, is only function of his and others PM holdings choice, $q_{i,1}$ and $q_{-i,1}$,[39] parameterized by the PM price P_1, the trading network $\boldsymbol{G}$, and the re-sell shock ϕ[40]:

$$\pi_i\big(q_{i,1}, \boldsymbol{q_{-i,1}}; P_1, \boldsymbol{G}, \phi\big) = w + (1 - P_1) \cdot q_{i,1} - \frac{1}{2v_i(\boldsymbol{G},\phi)} q_{i,1}^2 - \phi \sum_j \tilde{g}_{ij}(\boldsymbol{G}) \cdot q_{j,1} q_{i,1} + \phi \sum_j \bar{g}_{ij}(\boldsymbol{G}) \cdot q_{j,1} \quad (9)$$

Perhaps surprisingly, equation (9) coincides with the payoff function of a network game of global strategic substitutes (notice that $\frac{\partial q_{i,1}}{\partial q_{j,1}} \leq 0 \quad \forall i \neq j$). The "network coefficients" $\{v_i(\boldsymbol{G},\phi), \tilde{g}_{ij}(\boldsymbol{G}), \bar{g}_{ij}(\boldsymbol{G})\}_{\forall i,j \in N}$ are endogenous, non-negative, and each is a function of the trading network structure. They encapsulate the local market effects on $q_{i,1}$ given i's network position (more details below).

Others' PM holdings $q_{-i,1}$ negatively impact trader i's optimal choice while also having positive externality. This is best understood if we put ourselves in the shoes of trader i conjecturing his local market trades. Consider first the PM demand of i's neighbors (i.e. his direct connections), $q_{N_i} \equiv \{q_{k,1}\}_{k \in N_i}$. As a seller, q_{N_i} pushes i's price down (the "inventory effect"). So i demands less if he expects his buyers to demand more in the PM. As a buyer, q_{N_i} pushes down the price i faces (also the "inventory effect"). So i also demand less (to afford more shares in local markets) if he expects his sellers and competitors to have high PM demand.

However these are just *first-order* effects. The connections of i's connections are also i's competitors and they offer alternative markets for i's neighbors, thus influencing i's PM decision. This is also true for the connections of the connections of i's neighbors, and so on. In all cases, the same reasoning holds: i takes into account the effect of every other trader when he acts a buyer and as a seller in local markets.

[39] As usual, the notation $q_{-i,1}$ represents the demand of all traders but i, i.e. $\{q_{j,1}\}_{j \neq i, j \in N}$

[40] A quick way to see this result is just to replace local market variables in the expected payoff (5) with the equilibrium result of local markets (Lemma 1). With a little algebra, the "clean" representation in (9) is obtained. Full details are found in Appendix D.

Bottom line is that the asset acquisition of each trader is negatively influenced by the same decision of each and every other trader, irrespectively if they are connected or not. This is the key insight of this paper because it reveals that the dynamic framework boils down to a one-shot, simultaneous-move network game played in the PM.

Since traders are price-takers, each PM price P_1 induces a game. In each game, the strategy for trader $i \in N$ is his asset acquisition decision in the PM. It is a mapping $q_{i,1} : q_{-i,1} \times P_1 \to \mathcal{R}$ where $q_{-i,1}$ is the strategy of all other traders different than i. Traders simultaneously choose their demand schedules by best-responding to the demand schedules of others. The equilibrium concept is pure-strategy Nash Equilibrium.[41]

Lemma 2. *Primary Market Trading Game*

For each PM game with price P_1, a trader i's asset demand schedule (best-response) is

$$q_{i,1}(\boldsymbol{q}_{-i,1}; P_1\phi, \boldsymbol{G}) = v_i(\boldsymbol{G}, \phi)\left[(1 - P_1) - \phi \sum_j \tilde{g}_{ij}(\boldsymbol{G}) \cdot q_{j,1}\right] \tag{10}$$

where

$$v_i(\boldsymbol{G}, \phi) \equiv \left[\frac{2\phi}{d_i} + 1 - \phi(d_i + 1) + \phi \sum_j g_{ij} \cdot \frac{(2d_j - 1)}{d_j^2}\right]^{-1} \tag{11}$$

and

$$\tilde{g}_{ij}(\boldsymbol{G}) = g_{ij} \cdot \left[\frac{1}{d_i} + \frac{(d_j - 1)}{d_j^2}\right] + \left[\sum_{\substack{z \neq j \\ z}} g_{iz} g_{jz} \frac{(d_z - 1)}{d_z^2}\right] \geq 0 \tag{12}$$

The trading network determines the influence among traders' strategies, while the re-sell shock ϕ regulates the global degree of substituability among PM demands: the higher ϕ, the greater is the chance of local market trading and thus the greater is the feedback effect between traders' demands.

The individual network effect, $v_i(\phi) > 0$, summarizes trader i's interactions in local markets, and it can be seen as the marginal benefit of acquiring shares in the PM.

Each global network coefficient $\bar{g}_{ij} \geq 0$ captures bilateral influences: it gives how

[41] An important step in the paper is to formulate the model as a game. Network games of global strategic substitutes have been extensively studied - see Bramoullé et al. (2014) and Galeotti et al. (2010). I rely on the advances of this literature to characterize the equilibrium in every possible game and for *any* network graph $\boldsymbol{G}$.

influential is trader j on i's demand. Its value depends on how far apart i and j are[42] Notice that it is increasing in the number of overlapping connections i and j have. Implying that indirect connections can be more influential to i's decisions than i's neighbors. And that neighbors with the same degree can have different effects.

Lastly, the first term in the demand schedule, $(1 - P_1) \geq 0$, is common across all agents. It represents the optimal action absent network interactions: $(1 - P_1)$ is the individual demand in a Walrasian (competitive, static) market of size N.

In the Appendix D I derive the results above, and I discuss more deeply the functional forms of the payoff function and demand schedule. I also prove the existence of a unique interior equilibrium for each network game.[43]

It is useful to look at two traders' optimal PM demand schedules to understand how the Nash Equilibrium of each network game is found. The left-hand side of Figure 7 shows that, for a given P_1, the Nash Equilibrium is given by the intersection of traders' i and j best-responses. The right-hand side of Figure 7 depicts how the primary market equilibrium is one of the Nash equilibria such that equilibrium aggregate demand meets the exogenous asset supply $\bar{Q}$.

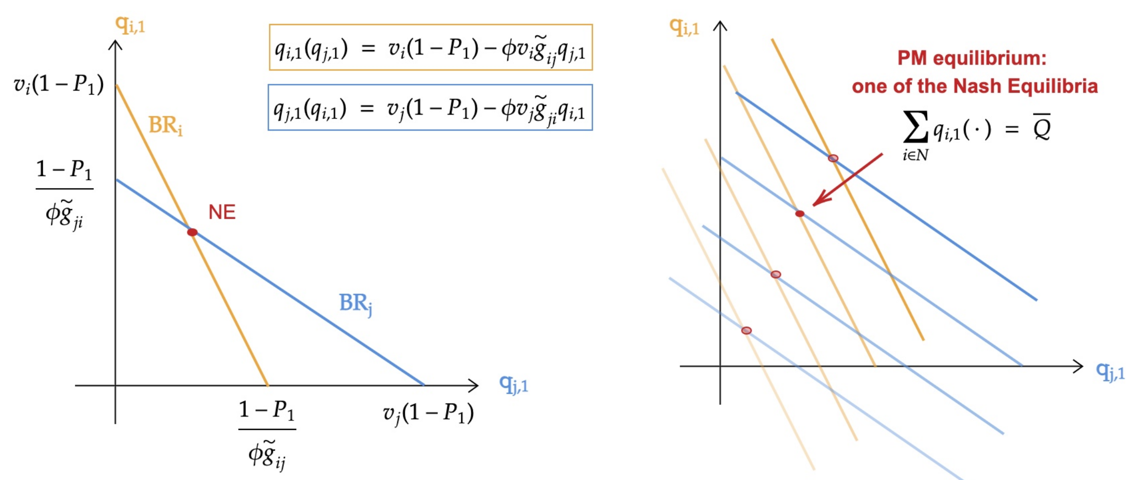

Figure 7: The demand schedules of two traders

[42]That's because, as I discussed before,there are three channels through which $q_{j,1}$ impacts $q_{i,1}$: as a buyer from i or as a seller to i, if they are connected; and as a competing buyer to i if they share a common linkage to another agent z (i.e. if i, j have overlapping connections).

[43]The existence and uniqueness is guaranteed by Assumption 1, i.e. as long as the shock probability $\phi < 1/N$ (Proposition 11). Intuitively, $\phi > 1/N$ implies that traders expect more than one seller in the local market. The anticipation of "too much" local market trading may lead traders to either demand too much ($q_{i,1} \to 1$) or too little ($q_{i,1} \to 0$) in the PM. In the former case, local markets would collapse as buyers' willingness to trade would be virtually small. In the latter case, the PM would collapse as the market would not clear.

Relative to the related literature of imperfectly competitive trading models (i.e. demand games, which Rostek and Yoon (2020a) provide an excellent review), the way I find the equilibrium in the model is different. The key feature of imperfect competition is that traders conjecture their endogenous (and unknown) price impact and have to do the same for others' price impact and demands, due to private information. As Rostek and Yoon (2020a) show, the equilibrium is characterized by two conditions: market clearing and correct price impacts. That is, each trader optimally chooses his demand schedule given his price impact such that his price impact equals the slope of his residual inverse supply function.[44] With price-taker traders (as in my model), finding equilibrium is simpler because traders only respond to each other demands and thus only one condition - market clearing - characterizes equilibrium. Although this is a strong assumption, I view it as plausible given the main goal of this paper: to distill the network effects on equilibrium. I obtain formal and closed formed solutions and I can study the implications of the structure of the trading network in isolation[45]

5 Trading Centrality, a sufficient statistic for Equilibrium

The analysis so far has two main conclusions. First, the presence of local markets leads to a substitution effect across PM demands $q_{i,1}(\cdot)$ (eq. (10)). Second, for each trader, how he reacts to others' behavior is determined by his network position in complicated ways.

The main contribution of this paper is that, by solving the model, I show that the unique equilibrium can be described as a function of a simple measure, *trading centrality*. Trading centrality, $\boldsymbol{c}(\boldsymbol{G}, \phi)_{N\times 1} : c_i(\boldsymbol{G}, \phi) \; \forall i \in N$, is a recursive network metric that produces a "score" for each trader. In Appendix F I provide the formal definition of trading centrality and its analytical expression. However, the reason why trading centrality is a sufficient statistic is best understood by simply using economic intuition for what it does.

The trading centrality score measures the trader's endogenous valuation for the asset

[44] More specifically, with imperfect competition, each trader chooses his demand as optimal pointwise for each price realization against a family of the residual supply (all other trader's demand schedule) with a deterministic slope (price impact) and random intercept (due to other traders private information).

[45] However, my main methodological contribution, which is to derive a sufficient network metric for the equilibrium (Theorem 1), is not limited by the price-taking assumption. In the Appendix, I derive and show the equilibrium in a setting where markets are imperfectly competitive (see Section 9 for further discussion). As I will explain later on, if traders were strategic with respect to price, the endogenous price impact would depend on the network structure and would also influence the equilibrium. Then, the PM price would be determined by two related but distinct forces: price impact and network structure.

in the PM. The higher is c_i, the higher is i's marginal utility for asset holdings and thus the higher is he willingness to pay for shares in the PM. Moreover, it also implies that a trader is *more* central as his direct and indirect connections are *less* central.

That is, the PM demand schedule $q_{i,1}(\cdot)$ of each trader i (Equation 10) can be expressed in terms of his centrality c_i,

$$q_{i,1}(\boldsymbol{q}_{-i,1}; P_1, \boldsymbol{G}, \phi) = (1 - P_1)c_i(\boldsymbol{G}, \phi) \tag{13}$$

And c_i can be expressed in terms of other traders' centralities $\{c_j\}_{j\neq i}$,

$$c_i(\boldsymbol{G}, \phi) = v_i(\boldsymbol{G}, \phi)\left(1 - \phi\sum_j \tilde{g}_{ij} \cdot c_j(\boldsymbol{G}, \phi)\right) \tag{14}$$

The recursive form of trading centrality is illustrated in Figure 7 by noticing that the best-responses (i.e. demand schedules) depicted there are essentially Equation (14) replacing $c_i(\cdot), c_j(\cdot)$ with $q_{i,1}(\cdot), q_{j,1}(\cdot)$ and setting $P_1 = 0$.

The next figure depicts the demands of two traders with different centralities. The trader with higher centrality (in green) has a more elastic demand curve than the one with lower centrality (in orange). In turn, for any price P_1, the more central trader will acquire more shares than the less central one.

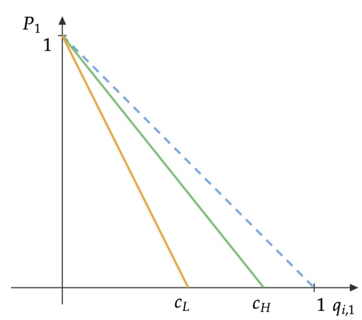

Figure 8: Demand schedules are determined by traders' centrality
The orange demand is of a trader with lower trading centrality; and the green demand if of a trader with higher trading centrality.
In blue is the demand schedule if the trading network did not exist.

Throughout the paper, with abuse of notation, I omit the functional arguments of trading centrality and write $\boldsymbol{c} = \boldsymbol{c}(\boldsymbol{G}, \phi, Q)$.

Since trading centrality determines traders' willingness to acquire the asset in the PM, it defines the equilibrium demands and, in turn, the equilibrium PM price. Thus, the unique equilibrium of the model is determined solely by the structure of the trading network and the shock parameter ϕ.

Theorem 1. *Primary Market Equilibrium and Trading Centrality*

Given an asset supply Q, the unique and interior equilibrium in the PM is determined by trading centrality $\boldsymbol{c}(\boldsymbol{G}, \phi)$.

Equilibrium PM price is given by

$$P_1^*(\boldsymbol{c}; Q) = 1 - \frac{Q}{c_A} \tag{15}$$

where $c_A = \sum_{i \in N} c_i$ is the aggregate trading centrality.

Each trader's equilibrium PM asset allocation is

$$q_{i,1}^*(\boldsymbol{c}; Q) = Q \frac{c_i}{c_A} \tag{16}$$

Or in matrix notation, $\boldsymbol{q}_1^* = \frac{Q}{c_A} \boldsymbol{c}$.

Trading centrality neatly characterizes equilibrium, and in an intuitive way. PM price is simply defined by aggregating trading centrality globally, as all traders can participate in that market. And each trader's asset acquisition is proportional to his centrality.

In the same spirit, for each local market, equilibrium price is defined by aggregate trading centrality locally in the seller's neighborhood. And each trader's demand schedule is simply determined by the gap between his optimal holdings and his centrality.

Corollary 1.1. *Local Market Equilibrium and Trading Centrality*

Given an asset supply Q, trading centrality $\boldsymbol{c}(\boldsymbol{G}, \phi)$ uniquely determines the equilibrium in a local market.

A trader's demand schedule is

$$q_{i,s}(\boldsymbol{c}; P_s) = \left(1 - \frac{Q}{c_A} c_i\right) - P_s \quad \forall i \in N \tag{17}$$

Each LM price is

$$P_s(\boldsymbol{c}) = 1 - \frac{Q}{c_A} \left(\frac{c_s + \sum_{i \in N_s} c_i}{d_s} \right) \quad \forall s \in N \tag{18}$$

The results above show how trading centrality is a sufficient statistic for market outcomes. Given an arbitrary trading network and a shock parameter ϕ, trading centrality can be computed. In turn, prices and demands in every possible market are found.

In equilibrium, PM price is increasing in the aggregate centrality of the network and a trader's PM holdings is increasing in his centrality.[46] Also, traders *always* have a strictly positive demand in all markets they can participate at. That's due to three reasons. First, as the PM has more competing buyers, no trader is able to reach his optimal inventory at $t = 1$. Second, the anticipated yet uncertain re-sell shock also lowers PM demand to mitigate the risk of being a seller. Lastly, since the likelihood of being a buyer is weakly greater than of being a seller, traders command lower prices in local markets to compensate the shift from securing asset holdings in the PM at $t = 1$ to the risky asset allocation at $t = 2$.

All this leads to PM price being greater than any local market price in any network. The unique exception is for star network (Corollary 1.1).

Proposition 1. *Price Dynamics*

A trading network with a star structure is the unique one that can exhibit price increase over time. For any other network structure, asset price drops from $t = 1$ to $t = 2$.

Thus, liquidity (re-selling) is costly for the seller. Analysing price dynamics then means measuring traders' liquidity cost. Just as with equilibrium prices, liquidity cost is determined by the gap between the aggregate centrality in the seller's neighborhood and the price he would have in absence of PM holdings among his buyers (i.e. if his buyers had no asset endowment when trading with him):

$$P_i - P_1 = \frac{Q}{c_A} \left| 1 - \frac{c_i + \sum_{j \in N_i} c_j}{d_i} \right| \neq 0 \quad \forall i \in N \tag{19}$$

The next corollary of Proposition 1 reveals why price can increase in a star network. Its core trader is the only one who make profits from re-selling. That's because his price is higher than the price in any other market within the star network. In fact, the core's price is higher than the prices in any trading network structure of the same size.

Corollary 1.1. *Re-sell Cost*

For a trader $i \in N$, re-selling is profitable, i.e. $P_i - P_1 > 0$, if and only if i is the core of a star network.

Looking at the aggregate centrality in a trader's local market is advantageous because it has a straightforward and monotonic relation with liquidity cost. The next natural question is a trader's selling price P_i and liquidity cost $(P_1 - P_i)$ relate to his trading centrality c_i

[46] I use "trading centrality" and "centrality" interchangeably.

alone. It turns out that the answer is non-trivial because the relation between d_i with P_i and c_i ambiguous.[47] I find that in certain trading networks, c_i and d_i are positively correlated and, thus, so it is c_i and P_i. But other networks exhibit the opposite relation..[48]

The non-monotonic relation between trading centrality and degree has two crucial implications. First, in my model, central traders are not necessarily those highly connected. Second, central traders do not always have better terms of trades. Although these implications might be counter-intuitive, in reality they are observed in interdealer markets. The growing empirical literature on interdealer networks find mixed evidence on whether "central" dealers have higher or lower liquidity costs (Section 2). My results suggest that a reason for this inconclusiveness could be that all these studies use standard network metrics such as degree and eigenvector centrality that are positively correlated to degree. Consequently,"centrality" only reflects the extensive margin of trades and imply that the centrality of a dealer is weakly increasing in the centrality of his connections (and connections' connections, etc.).[49] My novel trading centrality measure reveals that this might be misleading, particularly so when analyzing markets with endogenous and correlated terms of trade.

In my model, central traders emerge as market makers only for particular trading networks in which trading centrality and degree have a positive relationship. This is the case, for instance, for "nicely behaved" networks such as symmetric and core-periphery networks. In those cases, central dealers buy more in the PM to sell at a higher price in the local market, compared to others.

The fact that liquidity is costly is reminiscent of the long-standing theoretical literature of inventory behavior in the interdealer market, pioneered by Ho and Stoll (1981) and Ho and Stoll (1983). In these models, while interdealer trading enables inventory risk sharing, the initiating dealer must give up some portion of the spread to his counterparty.[50] It is also a common phenomena across a variety of securities and markets (Duffie (2010)).

The rest of the paper explores how the structure of the trading network itself affects PM price. I also study welfare. But in order to understand all my findings - including Theorem 1

[47] In the appendix, Lemma 8 provides a formal statement of this result.

[48] I illustrate this on a simple example on appendix A. See also Appendix F for more details.

[49] This is how we usually think of centrality, as a proxy for "prestige" and "influential power".

[50] In these models, using interdealer trades to unwind inventory is a choice. However, as long as dealers are farther from their optimal inventory, the benefits of risk sharing via the interdealer market outweigh the trading costs. So dealers (almost) always choose to sell the inventory in the interdealer market if needed. This is the main conceptual difference with my paper since I assume that the seller (the initialing dealer) *must* sell. Interdealer trading in my model is a choice just for the buyers because they can demand zero.

- it is crucial to distill what information trading centrality encapsulates. Before proceeding, the next simple example distils what information trading centrality encapsulates. In sequence, I discuss the information content and mathematical form of trading centrality (Subsection 5.2).

5.1 Example

The trading network structure is described by the $\boldsymbol{G}$ such that $g_{ij} = \{0, 1\}, g_{ij} = g_{ji} \forall i, j \in N$. It is essential to keep in mind the substituability of demands that arises because of the feedback between the PM and local markets. This implies that the trading network induces endogenous "trading costs" for holdings asset shares that reflect local market trades.

The *implied* trading network is given by the modified adjacency matrix $\tilde{\boldsymbol{G}}$ such that $\tilde{g}_{ij} \geq 0, \tilde{g}_{ij} \neq \tilde{g}_{ji} \forall i, j \in N$ (eq. (12)). $\tilde{\boldsymbol{G}}$ is a weighted and asymmetric matrix and a modified version of $\boldsymbol{G}$.[51] Take a trader $i \in N$. Each $\tilde{g}_{ij}$ is the marginal cost imposed by the holdings of a trade counterparty $j \neq i, j \in N$. It accounts for when i and j trade as a seller and a buyer to one another, and as competitors for a common linkage. On top of that, i incurs a marginal cost from his own holdings given his possible trade participations (i.e. every local market where he is a buyer, a seller or out of it), which is given by the individual coefficient $\frac{1}{v_i}$.

All these network-induced "trading costs" define i's marginal benefit of holding asset shares. Then, to optimally decide his PM demand, i equates it to the marginal cost of acquiring the asset, i.e. the PM price P_1. In matrix notation this gives,

$$\underbrace{\boldsymbol{1}_N - \left(\boldsymbol{V} + \phi\tilde{\boldsymbol{G}}\right) \cdot \boldsymbol{q}_1}_{\text{asset holdings marginal benefit}} = \underbrace{\boldsymbol{P}_{\boldsymbol{1}N}}_{\text{asset holdings marginal cost}}$$

where $\boldsymbol{V}_{N\times N} \equiv diag(\frac{1}{v})$ is the diagonal matrix with entries $1/v_i$.

Notice that the above is simply the system of first-order conditions of traders' optimization problem in the PM (eq. (9)). Since all traders have the same optimal holdings of 1, the lower is the row-wise sum of $\left(\boldsymbol{V} + \phi\tilde{\boldsymbol{G}}\right)$ (i.e. trading costs), the higher is the trader's marginal utility and thus the higher is his willingness to pay. Then, in equilibrium, the trader's demand - and share allocation - is higher.

[51] An interesting aspect of the forces in my model is that they turn the "plain" trading network, which is undirected and unweighted, into a weighted and directed network graph. Moreover, a graph that is weakly more connected than the trading network itself. See Figure 9

This operation is exactly what trading centrality does. But it gives the information in terms of marginal benefit instead of marginal costs:

$$\boldsymbol{c} \equiv \left(\boldsymbol{V} + \phi\tilde{\boldsymbol{G}}\right)^{-1} \mathbf{1}_N \tag{20}$$

Thus, the score of each trader is precisely his marginal utility of holdings asset shares. Or, in other words, the marginal benefit of acquiring shares in the PM.

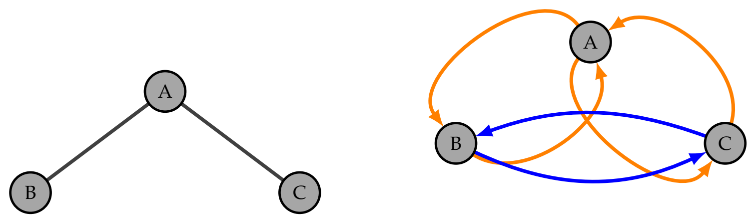

Figure 9: Orange links imply two trades (nodes) are buyers and sellers to one another (direct counterparties) in local markets. Blue links imply they are competitors (indirect counterparties).

Now let's look at a simple example. Consider $N = 3$ traders in the star trading network with structure described by the adjacency matrix $\boldsymbol{G}$ below (left graph of Figure 9). The implied star trading network is given by the modified adjacency matrix $\tilde{\boldsymbol{G}}$ (right graph of Figure 9):

$$\boldsymbol{G} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \tilde{\boldsymbol{G}} = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 1.25 & 0 & 0.25 \\ 1.25 & 0.25 & 0 \end{pmatrix}$$

The core trader A has the same interaction with both B and C, and that's why $\tilde{g}_{AB} = \tilde{g}_{AC} = 0.5$. However, B and C interact differently with one another and with A. Not only that, but they trade directly with A and indirectly with each other through A. That's why $\tilde{g}_{BA} = 1.25 > \tilde{g}_{BC} = 0.25$ and $\tilde{g}_{CA} = 1.25 > \tilde{g}_{CB} = 0.25$. The individual coefficients are $\frac{\mathbf{1}}{\boldsymbol{v}} = (1, 1.1875, 1.1875)'$. A has a lower marginal cost of holding shares because he has greater local market participation.

Traders' marginal benefit of asset holdings is then[52]

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 & 0.125 & 0.125 \\ 0.3125 & 1.1875 & 0.0625 \\ 0.3125 & 0.0625 & 1.1875 \end{pmatrix} \cdot \begin{pmatrix} q_{A,1} \\ q_{B,1} \\ q_{C,1} \end{pmatrix} \boldsymbol{q}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot P_1$$

[52]Notice that $\left(\boldsymbol{V} + \phi\tilde{\boldsymbol{G}}\right) \cdot \mathbf{1}_N = (1.25, 1.5625, 1.5625)$. That is, the core has lower trading cost compared to the peripheries.

Thus, trading centrality is

$$\boldsymbol{c} = \begin{pmatrix} 1.067 & -0.1067 & -0.1067 \\ -0.267 & 0.871 & -0.0178 \\ -0.267 & -0.0178 & 0.871 \end{pmatrix} \cdot \mathbf{1}_N = \begin{pmatrix} 0.853 \\ 0.587 \\ 0.587 \end{pmatrix}$$

Letting the asset supply be one, the equilibrium asset allocation is $\boldsymbol{q}_1^* = (0.421, 0.289, 0.289)'$, which is proportional to trading centrality $\boldsymbol{c}$. The core trader is the most central and has the highest asset holdings. The equilibrium PM price is given by the aggregate trading centrality $c_A = 2.027$, such that $P_1^* = 1 - \frac{1}{c_A} = 0.5066$.

5.2 The Information Content of Trading Centrality

Now it is clear that trading centrality process information about each and every local market interaction, and maps it to the asset acquisition decision in the PM. Hence, trading centrality defines traders' behavior before trade and it implies that the centrality of a trader is decreasing in all other traders' centralities.[53]

The recursive representation in eq. (20) is reminiscent of the *negative* Bonacich centrality[54] with weights that are not a simple geometric series (as in the Bonacich measure), but instead are endogenous and capture trading incentives. They are the network-induced trading costs $\left(\{\frac{1}{v_i}\}\ \forall i \in N, \{\tilde{g}_{ij}\}\ \forall i,j \in N\right)$ that defines traders' payoff function eq. (9).

The strength of effect between two traders' centralities varies with how far apart they are. On one hand, a trader i respond *more* negatively to the demand of traders at most two links apart (i.e direct connections or common connections). On the other, i respond *less* negatively to traders further away.

6 Trading Network Structure and Equilibrium

The overall takeaway so far is that the trading network induces a complex relationship between traders and markets what is encapsulated in the trading centrality, the sufficient

[53]This is makes sense as the framework is fundamentally a game of global strategic substitutes.

[54]The Bonacich centrality is a well-known network measure of node importance. Its common adoption in economics has been with a *positive* scalar as it means an agent is more powerful (central) the more powerful are his connections. This interpretation is meaningful in many economic scenarios that exhibits local complementarities and it was first invoke by Ballester et al. (2006). My paper contributes to the less explored network models in which Bonacich centrality with negative scalar is the appropriate measure of node influence.

statistic for equilibrium and my main contribution. Now I investigate how changes in the trading network structure affect equilibrium outcomes.

The relationship between the trading network and PM price can be broken down into three effects: market size N, connectivity and degree inequality.[55] I define connectivity as the average degree of the trading network. And degree inequality as the variance of traders' degrees. Notice that such effects are tightly interrelated. For instance, increasing the number of traders can affect how traders are connected. Or changing (re-arranging, deleting or adding) linkages can reduce or increase differences in traders' degree.

In Appendix A I illustrate the main findings of this section with a relatively simple example.

A first natural question is which network structure, if any, delivers a maximum or minimum value for PM price. I find that the complete trading network delivers the highest PM price while the star trading network the lowest possible PM price.

Proposition 2. *Bounds on Primary Market Price*

Consider an arbitrary trading network of size N. The equilibrium primary market price is bounded by its level on two specific networks of the same size: above by the complete network, and below by the star network.

There are no trading frictions in a complete network. All traders are connected with one another and everyone trade in both markets, either as a buyer or a seller. In equilibrium, demand schedules and asset allocation are homogeneous across traders in every markets, and so are local market prices. From a buyer's perspective, trading in a local market is as competitive as in the primary market. For this same reason, the seller's price is likely to be higher. Both buyer and seller's effects combined induce traders to demand more in the primary market because i) as a buyer, higher PM demand lowers a seller's price; and ii) as a seller, a higher price will be obtained. Higher willingness to trade in the primary market pushes price up, even though the equilibrium allocation $q^*_{i,1} = \frac{\bar{Q}}{N} \forall i \in N$ is the same as in a frictionless market.

The bounds on PM price might indicate that price is monotonically affected by the two network effects since the complete (star) network has the highest (lowest) connectivity and lowest (highest) degree inequality. However, this is not true.

Lemma 3. *PM price is non-monotonic in connectivity and degree inequality.*

[55] One way to see why these effects emerge is to look at PM demand schedule $q_{i,1}$ (10). That depends on the first and second moments of the degree distribution: individual degrees and their square appear in the demand function.

This results raises the question on how to compare equilibrium outcomes in the cross-section of trading networks of the same size. I do this by investigating how PM equilibrium is affected by specific changes in the degree distribution of the trading network.

Consider, in particular, a change in the probability distribution over the degrees of traders that reflects an unambiguous increase in connectivity, as given by the criterion of First Order Stochastic Dominance (FOSD). Denote the degree distribution of two trading networks as P and P'. If P first-order stochastically dominates P', then the average degree under P is higher than under P', the reverse not true.

Proposition 3. *Changes in connectivity: FOSD*

Suppose that P' FOSD P. Then the PM price under P' is unambiguously higher than under P. All traders' demand are higher after the change.

Now consider an unambiguous increase in degree inequality, while keeping average degree unchanged. This change is capture by a mean-preserving in the degree distribution. If P' is a mean-preserving spread of P, then the variance under P' is higher than in P, reverse not true.

Proposition 4. *Changes in degree inequality: Mean-preserving spread*

Now suppose P' is a mean-preserving spread of P. Than PM price is lower under P' than under P.

Traders' demand can increase or decrease depending on their network position. More (less) connected traders have an increase (decrease) in demand.

In other words, if we weakly increase all traders' degree - implying greater connectivity - PM price also increases (Proposition 3). And if degree inequality increases, keeping connectivity constant, PM price decreases (Proposition 4). It is worth pointing out that the reverse does not necessarily hold (i.e., the above propositions are not if and only if statements). As argued before, information on the number of traders' connections (i.e. degree distribution) is not enough to characterize equilibrium. Trading centrality - the key statistic for equilibrium - reveals that one must look at the pattern of connectivity.

Lastly, straightforward changes in the trading network are to add/remove a link or a trader. Both changes, when made alone and keeping everything else the same, lead to increase (decrease) in PM price. Moreover, changing the number of traders affects every local market price in the same direction as the PM one. Both facts are useful for the coming discussions.

Lemma 4. *Removal and addition of a link*

Everything else the same, adding (removing) a link from the trading network increases (decreases) PM price unambiguously.

Lemma 5. *Trading Network size*

Everything else the same, PM price and local market prices are increasing in N.

In the rest of this section, I study the equilibrium for particular trading network structures: symmetric and core-periphery. Doing so renders great tractability of the model because one just needs to keep track of one or two trading centrality scores respectively. In turn, I show that such networks contribute to understating the relationship between PM price and the trading network.

6.1 Symmetric Networks

In symmetric networks (or regular graphs), all traders have the same number of links and, thus, same network position. This means that a symmetric network is solely characterized by the number N of traders in it and their degree d. Examples are the complete network in which $d = N - 1$ and the ring network in which $d = 2$.

It immediate follows from Theorem 1 that prices and demands across traders and markets are homogeneous[56]

Proposition 5. *Equilibrium in Symmetric Networks*

Consider a symmetric network with N traders and degree d. Then, Primary Market equilibrium is

$$P_1^* = 1 - \frac{\bar{Q}}{N}\left(\frac{\phi + d(1+\phi)}{d}\right) \tag{21}$$

$$q_{i,1}^* \equiv q_1^* = \frac{Q}{N} \quad \forall i \in N \tag{22}$$

and the equilibrium in any Local Market is

[56]Notice that traders have the same demand and selling price if and only if they have the same network position. These two facts make it easier to study symmetric networks. There is just one unknown, a single trading centrality score; and four equilibrium values - the PM price, the single PM demand, the single LM price, and the single LM demand.

$$P_s^* = 1 - \frac{(d+1)}{d}\frac{Q}{N} \tag{23}$$

$$q_{i,s}^* \equiv q_2^* = \frac{Q}{Nd} \quad \forall i, s \in N \tag{24}$$

In any symmetric network, asset supply is divided equally among buyers and it is the as if no trading network exists, i.e in the perfectly (Walrasian) market. Although traders have the same willingness to pay, their demand schedule[57] $q_1 = (1 - P_1)\frac{d}{\phi + d(1+\phi)}$ is not the same as in perfect competition, $q_{ce} = (1 - P_1)$. Traders respond to the trading network and the re-sell shocked by submitting a more inelastic demand.

In contrast, PM price varies considerably across symmetric networks and it is never equal to the price of a perfectly competitive market. Thus, the distribution of asset shares alone is not informative about PM price nor about the network structure (since all symmetric networks have the same allocation). For financial market, this suggests that by looking only at dealers' inventories we could miss an important consideration for the cost of credit for the issuers.

Comparing price across periods, while PM price is increasing in degree the LM is not. In turn, more connected symmetric networks exhibits higher price drop.

Recall that the degree distribution of the trading network, although not fully informative about the equilibrium, is useful to compare market outcomes across network structures. It turns out that combining what the degree distribution tells us with the notion of symmetric brings further insights on PM price.

Fixing N, we already know that the complete network has the highest PM price (Proposition 2). Further fixing the number of linkages in the trading network, I find that the PM price in every symmetric network is higher than in any other structure.

Proposition 6. *Network Symmetry and Primary Market Price*

Suppose there are N traders and a fix number of connections among them. Then, the PM price in a symmetric network, if it exists, is higher than in any other network.

Benchmark cases:

The complete network is a special case of a regular network where no trading frictions exist since all traders are connected with one another. Equilibrium prices are $P_1^* = 1 - \frac{\bar{Q}}{N(N-1)}(N(1+\phi) - 1)$ and demand schedules are $q_1 = \frac{N-1}{N(1+\phi)-1}$.

[57]This follows because trading centrality is $c_i \equiv c = \frac{d}{\phi + d(1+\phi)} < 1$.

The other two extremes cases of symmetry are i) the empty network (i.e. without any trading relationships but with $\phi > 0$); and ii) the static, competitive market (i.e. without a trading network and/or $\phi = 0$).

The competitive market PM price equilibrium is $P_1^{ce} = 1 - \frac{Q}{N}$ and demand schedules are $q_1 = 1 - P_1$. In an empty network traders could still face the re-sell shock even though no local market actually exist. This risk makes demand schedules less elastic and drives PM price down. In equilibrium, PM price and demand schedules are, respectively, $P_1^* = (1-\phi)\left(1 - \frac{\bar{Q}}{N}\right)$ and $q_1(P_1) = 1 - \frac{1}{1-\phi} P_1$.

6.2 Core-Periphery Networks

Empirical evidence suggests that several financial markets exhibit a core-periphery structure (Section 2). We already know that the star network, a particular core-periphery structure, has the lowest PM price (Proposition 2). A core-periphery network consists of a well-connected set of nodes, the core, and the remainder nodes, the periphery, well connected to the core but sparsely connected internally (Figure 10). Motivated by the above, I now restrict my study of equilibrium in core-periphery trading networks. First, I look at the star network (left in Figure 10). Then, I analyze how price changes as the size of the core (right in Figure 10) and periphery grows.

I find that when the core consists of a single trader, he is the unique trader obtaining capital gains (Proposition 7). And that, even though PM price is increasing in the size of the core, the price drop monotonically decreases as the network grows (Proposition 8).

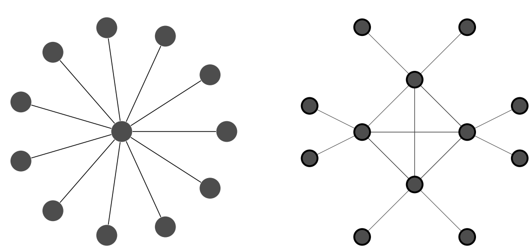

Figure 10: The figure depicts two core-periphery networks of the same size but with different cores

The star network is the simplest case of a core-periphery structure with a single core. An arbitrary core-periphery network can be understood as adding more traders to the core of the star network and keeping core's connectivity the same[58] The size of a core-periphery

[58]Core traders are fully connected among themselves, they have the same number of connections to peripheral nodes, and each periphery is linked to a single core node.

network is altered in two ways: by changing the number of core nodes and peripheral nodes.

The core traders have incentives to increase their demand. They expect to sell at high price and not be able to buy a lot in any local market, even though their buying prices are likely to be low. The reverse logic holds for peripheral trader. The peripheries anticipate a very low selling price and low marginal utility for their supply since they would only be trading with a core trader. This drives their demand down. In equilibrium, the traders in the core (periphery) have the highest (lowest) asset holdings.

What drives apart equilibrium proprieties of the star and other core-periphery networks are two facts. First, deviating from a single core decreases core traders' trading centrality. Second, cores' centrality is more affected than peripheries' one. The reason being traders in the core also trade with each other, who themselves are relatively more central than the peripheries. Lower trading centrality drives core's demand down and, thus, their selling price decreases. At the same time, demand inequality decreases since the peripheries' demand is greater.

First, as the size of the core increases, core traders' trading centrality decreases. Second, cores' centrality is more affected than peripheries' one. The reason being they also trade with each other, who themselves are relatively more central than the peripheries. Lower trading centrality drives core's demand down and, thus, their selling price decreases.

The next proposition shows the asset pricing implication of such behavior.

Proposition 7. *Prices and the size of the Core*

Compared to any trading network with N traders, only the single core trader of the star network has a selling price higher than the PM price.

For any core-periphery network different than the star, selling prices are lower than the PM price for every traders. Although all prices are increasing in the size of the core.

The core trader of the star network, and only him, obtains capital gains from selling his shares. His price is the highest across all markets with N traders, including the PM. The deviation from a single core trader leads to higher PM price and lower local market prices, including for the core. Consequently, price drops over time - in stark difference with the star network.

Behind this result is the fact that cores' centrality is more affected than peripheries' once the network changes from a star to a core-periphery structure. Core traders' trading centrality decreases since they also trade with each other. Lower trading centrality drives their demand down and, thus, selling price decreases.

As the size of the core increases, competition among core traders intensify, reducing even further their centrality and demand. However, the market size effect dominates and all prices increases. Interestingly, the difference between core-periphery demands becomes smaller. The next proposition summarizes how PM equilibrium changes as the core-periphery network grows.

Proposition 8. *Equilibrium in Core-Periphery Networks*

As N and /or the number of core traders increase, the core-periphery network exhibits:

i) lower demand inequality ($q_{core,1} - q_{periphery,1}$), and

ii) smaller price change over periods $|P_1 - P_i|\ \forall i \in N$.

If there is one core trader, there is lower price raise. For any other core size, there is lower price drop.

Summarizing, the star network has three interesting features: lowest PM price, highest demand inequality and possibility of capital gains. Such properties do not hold for any other core-periphery structure. Increasing the size of the core reduces demand inequality and no seller can obtain capital gain.

In Appendix J, I provide analytical solutions and further details on the equilibrium for core-periphery networks.

7 Welfare

I study welfare in terms of traders' expected indirect utility. Similarly to equilibrium prices (Theorem 1, Corollary 1.1), welfare is determined by aggregating trading centrality in a particular way (as I show below).

Even so, welfare comparison across different trading networks is challenging. I find that it is not necessarily true that welfare is enhanced by i) reducing the disparities between traders' number of connections and/or PM demand; or ii) increasing connectivity. That's because, since welfare depends on trading centrality, it inherits the non-trivial relation with network connectivity and degree inequality.

Formally, the expected indirect utility EU_i^* of a trader $i \in N$ can be written in terms of his and others' centrality,[59]

[59]Recall that i's expected indirect utility is given by $EU_i^* = w + (1 - P_1^*)q_{i,1}^* - \frac{1}{2v_i}(q_{i,1}^*)^2 - \phi \sum_j \tilde{g}_{ij} q_{j,1}^* q_{i,1}^* + \sum_j \bar{g}_{ij} q_{j,1}^*$.

$$EU_i^*\left(\boldsymbol{c};\boldsymbol{G},\phi,\bar{Q}\right)=w+\left(\frac{Q}{Nc_A}\right)^2\left[c_i-\frac{1}{2v_i}c_i^2-\phi c_i\sum_j\tilde{g}_{ij}c_j\right]+\phi\left(\frac{Q}{Nc_A}\right)\left[\sum_j\bar{g}_{ij}c_j\right] \tag{25}$$

and so EU_i^* is only a function of the trading network structure $\boldsymbol{G}$ and the parameters of the model (ϕ, Q).

Welfare of a trading network $\boldsymbol{G}$ is defined as the sum of traders' expected utility, $EU^* \equiv \sum_i EU_i^*\left(\boldsymbol{G},\phi,\bar{Q}\right)$. Not surprisingly, it is also determined by traders' centrality and given by

$$EU^*\left(\boldsymbol{c};\boldsymbol{G},\phi,\bar{Q}\right)=Nw+\frac{Q^2}{2c_A}-\phi\frac{Q^2}{c_A^2}\sum_i\left[c_i\left(\frac{3\phi}{2}+\sum_{j\neq i}\bar{g}_{ji}\right)-\frac{3\phi}{2}\frac{c_i^2}{v_i}\right] \tag{26}$$

To gain some insights on welfare, I now focus on the extreme cases of symmetric and core-periphery networks[60] - i.e. the complete, the ring and the star structure - plus the line network. All depicted in fig. 11.

Welfare study in these network structures is facilitated because they all share the same property of trading centrality being monotonically increasing in degree.[61] I provide two welfare rankings: traders' expected utility within a trading network, and aggregate expected utility across trading networks of the same size. Thus, it holds that central dealers are those with i) higher PM demand (i.e. asset holdings); ii) lower liquidity costs (i.e. higher re-selling prices); and iii) who face lower local market prices as buyers.

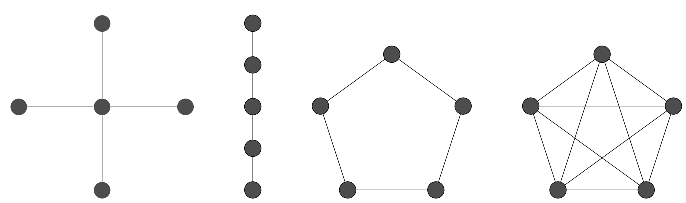

Figure 11: The star, the line, the ring and the complete trading network with five traders.

At the individual level, I find that a trader's expected utility EU_i^* is increasing in his centrality c_i.[62] Thus welfare ranking of traders is given by trading centrality: more central traders achieve a higher expected utility.

[60]In Appendix L I explicit discuss the ring and the star networks. Their simplicity provides insights on how network size and degree distribution interaction with one another, and how it affects the equilibrium.
[61]Recall Lemma 8 and **??** that this is not always the case.
[62]This results from the fact that trading centrality is increasing in degree See Appendix M and Lemma 10.

Proposition 9. *For $N > 3$ and if the trading network is either a complete, ring, line or star graph, then a trader's expected utility increases in his trading centrality.*

At the aggregate level, higher connectivity leads to higher aggregate trading centrality and, thus, higher PM price (eq. (15)). Not only that, but also higher local market prices since they are increasing in degree. As a consequence, I find that welfare decreases with aggregate trading centrality - and connectivity.[63]

Proposition 10. *For $N > 3$, welfare ranking across the following networks is:*

$$\textit{Star} > \textit{Line} > \textit{Ring} > \textit{Complete}.$$

Thus welfare ranking of trading networks is according to the aggregate trading centrality, and it is the reverse as the PM price rank. At a first glance this result seems odd. Welfare increases with degree (and asset allocation) inequality, and decreases with connectivity.

However, keep in mind that traders in the model are natural buyers of the asset by assumption. And each trader has a (weakly) greater probability of being a buyer of the asset in both periods. In turn, the opposite effect of the degree distribution on prices pushes them at such a greater level that is detrimental to traders' utility. On top of that, the likelihood of high local market price is higher in more connected and less unequal networks, what also drives utility down. Ultimately, greater welfare is determined by traders *expecting* lower prices in all markets - the PM and local markets.

It is worth emphasizing that asset allocation inequality is an equilibrium outcome. Traders, by taking into account their own and others' position in the trading network, optimally decide their holdings. That's why demand inequality is not necessarily detrimental for welfare, but a reflection of the strategic response between traders themselves.

My welfare analysis concludes that the trading network delivering the highest (lowest) PM price, the complete (star) structure, is the exact one delivering the lowest (highest) welfare. I stress that Proposition 10 only compares the four network structures illustrated in fig. 11. A more interesting exercise is to consider any arbitrary trading network and investigate which trader(s) or linkages should be removed or added to increase welfare. I leave this for future research.

In the literature of decentralized markets, a typical result argues that the absence of frictions would correspond to maximal welfare. This is not the case in my model and it's similar to the result obtained in Malamud and Rostek (2017), Wittwer (2021) andGlode

[63] See Appendix M. There I show that this result follows from the previous ones regarding PM price bounds (Proposition 2) and the price effect of the degree distribution (Proposition 3 and Proposition 4).

and Opp (2020), who demonstrate that decentralized markets might be more efficient than centralized markets. In contrast to these previous work, my framework is the first so show that allocative efficiency in decentralized markets (i.e. local markets) does not lead to greater (or maximal) welfare. Moreover, intertemporally, welfare is maximized by having the two most extreme trading schemes: a centralized (PM) market followed by a market in which one trader intermediate all trading flows (star network).

8 Empirical Exercise: Real-world Interdealer Network

An attractive feature of my model is that it has a straightforward empirical application to off-exchange assets, and it generates generates a rich set of empirical predictions. In this section, I give guidelines for future empirical work by an illustrative example on the US Corporate Bonds market. Namely, conditioning on the inferred interdealer network, I use trading centrality to compute dealers' inventory, and interdealer trade prices and quantities. Then I explore the sensitivity of various observable variables of interest with respect to trading centrality.

I use a sample of the TRACE data provided by Friewald and Nagler (2019)[64] with information on secondary markets of US Corporate bonds.[65] The TRACE database contains detailed transaction information on the prices and volumes both between dealers and customers (d2c trades)[66] and among dealers themselves (d2d trades). Crucially, the information is at the dealer-level. This allows me to recover one main ingredient needed for the empirical exercise: the interdealer network.

I document an interdealer network with 201 dealers and 452 links. The distribution of dealers' number of trading relationships and trading centrality are highly skewed. There is also great pair-wise heterogeneity of trading frequency and volume traded. I also document that dealers sell more often to the customer, instead of buying from him; and that the more central dealers have a greater volume of sell trades with the customer.

I find that, in the interdealer trades in the network, trading centrality has a positive effect in sell volume, and a negative effect in buy volume. Thus, as the model predicts,

[64]The authors' sample captures all trades executed by more than 2,600 dealers over the 2003 to 2013 period. In this empirical exercise, I use a restricted version of it which the authors made available for replication purposes.

[65]The ideal empirical exercise would combine data on primary markets, from the Mergent/FISD database in the case of US Corporate bonds, and the secondary markets, from the TRACE data. This is work in progress.

[66]Customers cannot be identified and d2c trades are all assigned to a representative "Customer".

central dealers sell more and buy less in the interdealer market. For prices, trading centrality has a positive effect on sell price and a negative effect on buy price. That is, central dealers sell at a higher price and buy at a lower price, as implied by the model as well.

Section 8.1 depicts the inferred dealer network; and Section 8.2 discusses regressions outcomes. All details and regression outputs tables are found in the Appendix P.

8.1 The dealer network

There are 201 dealers in the sample, who trade 5 different bonds over 42 trading days (what corresponds to two months, May and June 2009).[67] I infer the interdealer network from the realized trades between any pair of dealers (d2d trades). I link two dealers if they trade with one another at least once during the sample period. I restrict the set of dealers to those who trade both at the d2d and d2c markets.

I document an interdealer network with 201 dealers and 452 links, depicted below (Figure 12).

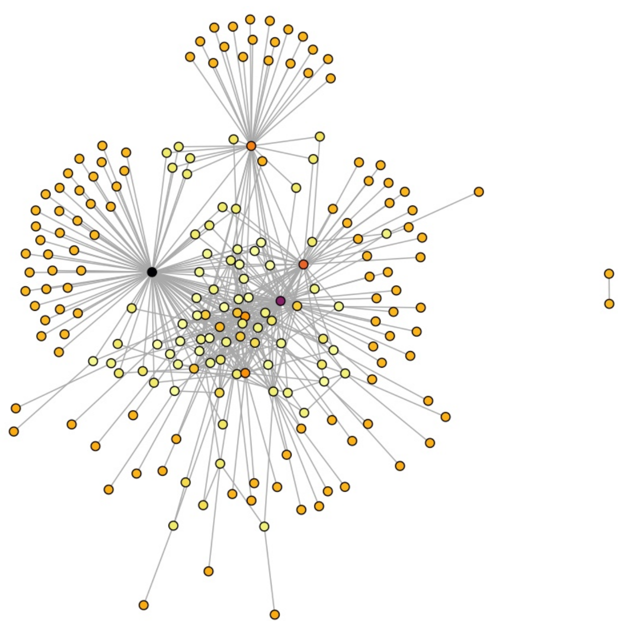

Figure 12: Dealer network: trading centrality increases from darker to lighter colored nodes

The table 1 shows some network characteristics. Both degree and trading centrality distribution skewed: the former is skewed to the right, and the latter to the left - as depicted

[67] My main analysis focuses on the trade data and model predictions using the full sample with all bonds.

in fig. 13. There is also high asymmetry in pair-wise trading frequency and volume traded. On average, two dealers trade less than six times. But there are few pairs that trade quite frequently. This relates to the core-periphery structure of the network, which suggests that just a few dealers intermediate most of the trades.

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
degree	201	4.498	9.473	1	1	4	93
trading cent.	201	0.873	0.097	0.000	0.836	0.944	1.000
#path-two connections	201	105.000	101.900	0	38	130	483
pair frequency	452	5.872	21.480	1	1	3	320

Table 1: Interdealer network statistics

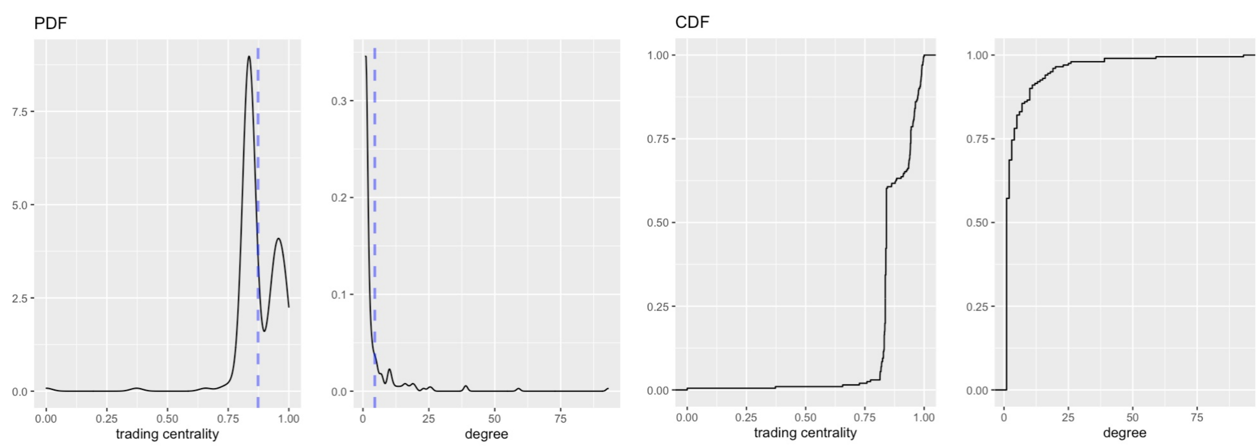

Figure 13: Trading centrality distribution, and degree distribution

Trading centrality and degree are negatively related. And so is my centrality with eigenvector centrality (fig. 14). This is not surprising as, for trading centrality, a dealer's centrality is deceasing in the centrality of his direct and indirect connections.

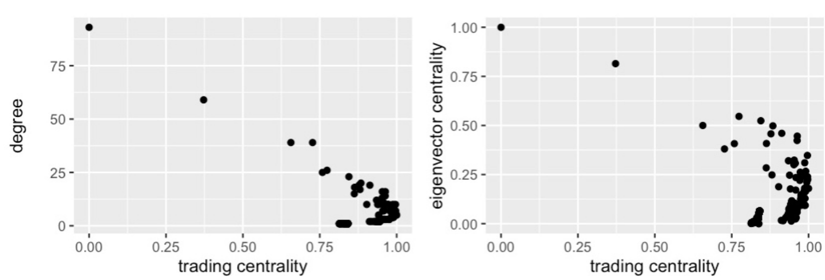

Figure 14: Trading centrality and degree relationship (left); and Trading centrality and eigenvector centrality relationship (right)

8.2 Trading Centrality and Interdealer Trades

Through the lens of my model, trading centrality determines and prices and quantities in the interdealer market, the empirical counterpart of period-two local markets. Is this the case in the data as well? I investigate the relationship between the trades in the interdealer network and trading centrality. According to the model, more central dealers sell more in the interdealer market. The relationship of trading centrality and selling price is ambiguous.

My analysis is at the daily level (i.e. a panel data). I estimate the following regressions for trade volume and price, respectively:

$$vol_{i,t} = \alpha_{i,t} + \beta_{ci} tc_i + \boldsymbol{\beta}_i' \boldsymbol{F}_i + \epsilon_{i,t} \tag{27}$$

$$pr_{i,t} = \alpha_{i,t} + \beta_{ci} tc_i + \boldsymbol{\beta}_i' \boldsymbol{F}_i + \epsilon_{i,t} \tag{28}$$

where $\boldsymbol{F}_i$ is the vector of dealer i's time-invariant observable characteristics such as degree deg, net total d2c qi (i.e. inventory), average price of inventory pi.[68] I also control for the transaction price pr in the volume regression.

Regressing trade volume and trade price on trading centrality delivers significant results. For trade volume, the sign of trading centrality coefficient varies depending on the set of controls. Trading centrality alone has a positive effect on volume, and so it does when controlling for inventory and trade price. However, adding degree and as control, turns the centrality coefficient negative (Section P.3). This implies that central dealers trade more volume in the network. Through the lens of my model, the change in sign when controlling for degree can be explained as follows. In the inferred interdealer network,degree and trading centrality are negatively related. Thus, since central dealers have less connections they unavoidably have less participation in the interdealer trade and thus less trade volume.

For trade price though trading centrality is only significant when controlling for degree. In this case, trading centrality has a positive effect on price: more central dealers face higher interdealer prices (Section P.3).

In my framework, an important distinction in local markets regards the side at which traders are, i.e., when traders are buyers and sellers. Because of that, I re-estimate eq. (27)

[68] In Appendix P I discuss dealer-customer trades and how I calculate dealers' inventory.

controlling for the transaction side: if the dealer is buying or selling.

Trading centrality has a negative, significant coefficient in all buy volume regression specification. Thus, as the model predicts, central dealers buy less in the interdealer market. However, the sign of the centrality coefficient varies in the sell volume regressions. The coefficient is positive if degree is not added as a control. Thus, in general, central dealers sell more in the interdealer market. Again, as implied by the model. Recall that in the model, selling price decreases in centrality if one controls for the seller's degree. This suggests that a more central trader would want to sell less, just as the regression results with degree suggest (Section P.3).

Looking at prices, again centrality is only significant when degree is not added as control. In this case, trading centrality has a positive coefficient on sell price and a negative coefficient on buy price. Thus, central dealers sell at a higher price and buy at a lower price, as implied by the model as well.

Moreover, for both buy and sell trades, inventory price (from customer trades, i.e. primary market price) has a positive effect on prices. Through the lens of the model this makes sense. As a seller, higher inventory price means it is more costly to sell. And as a buyer, higher inventory price means it is relatively cheaper to buy in local markets, what increases the demand from the seller and so pushes his price up (Table P.4).

9 Discussion

With the intuition for my results in place, I now establish that the model accommodates pertinent extensions. In any of them, even though the model becomes less tractable, the main finding still prevails: primary market price is characterized by the trading network. The general formulation for trader's optimal demand schedule (10) is given by

$$q_{i,1}\left(q_{-i,1}; P_1, \phi, \mathrm{G}, \Psi\right) = \beta_i \left[a_i - bP_1 - \phi \sum_j \beta_{ij} q_{j,1}\right] \tag{29}$$

where Ψ is the set of model parameters apart from the shock ϕ. Coefficients $(\{\beta_i, a_i\}_{i\in N}, b, \{\beta_{ij}\}_{\forall i,j\in N})$ are endogenous and functions of one or all arguments (ϕ, G, Ψ).

What differs across model specifications is, apart from *parameters*,[69] how the patterns of trading linkages map into trading centrality and, thus, prices and demands. I point out that the study of equilibrium outcomes and the trading network structure in Section 6 and

[69]Namely, i) the different individual valuations and risk aversion, and ii) the different beliefs.

Section 7 do not hold in general. I leave this exploration for a companion paper.

All details and full analytical solution for each extension are found in Appendix N.

Heterogeneity in Preferences:

Consider the more general set up in which traders have different individual valuation $\alpha_i > 0$ and risk-aversion γ_i in the quasilinear-quadratic utility,

$$U_i(Q_i) = \alpha_i q_i - \frac{\gamma_i}{2} q_i^2$$

In financial markets, heterogeneity in α_i captures the different and persistent close relationships traders tend to form with their clients in OTC markets (Di Maggio et al. (2017b)). The different cost γ_i may be related to fund outside investments, regulatory capital or collateral requirements, which may vary across traders.

I find that the equilibrium is determined by a modified trading centrality that incorporates the different levels of risk-aversion and asset valuation. While risk-aversion affects the network effect across traders' PM demand, the individual valuations only affects the level of demand of each trader.

Expected Fundamental Returns

In reality, traders care about the fundamental return of an asset. They hold an asset not just for the sake of holding it (i.e. to enjoy utility flow) but because they expect that the asset itself is a good financial investment, with high intrinsic value. Suppose then the asset has uncertain return f which is normally distributed with mean μ and variance σ^2, and it is realized after all trading activities take place[70] traders have initial wealth w_0 and choose asset inventory q_i to maximize the expected CARA utility of final wealth $E\left[-\exp\left(\gamma W\right)\right]$ given by $W = f(q_{i,1} + q_{i,s}) - (P_1 q_{i,1} + P_s q_{i,s}) + w_0$.

I find that, in equilibrium, asset price reflects both the traders' beliefs on returns and the trading network. Importantly, the way the former is incorporated into price depends on the later. That's because, a trader i's PM demand depends on market price P_1, his information *and* the information and demand of *all other traders*, including those he is not directly connected to but who are connected with his connections. This is in stark difference with the canonical linear asset pricing model where individual demands depend on all agents' information set *but not directly* on other demands. That's because in such setting equilibrium price aggregate all useful information and so it is not necessary to

[70]The normality assumption is standard in this literature. See, for instance, Kyle (1989), Vives (2011), Rostek and Weretka (2012), (Duffie and Zhu (2016)) and others.

know other demands. In my model, however, even an anticipated shock and the fact the it leads to different trading possibilities make agents to conditional on others demands, since this is informative about the market structure.

Price Impact

In my model, traders are price-takers and strategic in terms of demands in the sense that they understand that their PM demands directly affects prices in the trading network. In other words, they do take into account the effect of their first-period choices on the second period price, and vice-versa. In turn, traders' do have price impact in the PM which arises endogenously precisely because of intertemporal demand dependence.

Notice that this is different than saying that traders try to manipulate prices by 'shading their bids' in the PM, as in the models with imperfect competition such as Kyle (1989) and Rostek and Weretka (2015). Nonetheless, my framework can be used to study the economy with strategic traders that take into account their price impact, as Rostek and Yoon (2020b). I find that the equilibrium with imperfect competition is determined by a modified trading centrality that is only a function of the network structure - just as in my model.

10 Conclusion

This paper shows why and how future re-sale market structures affect asset pricing before trade, in what I call the primary market (PM). I develop a dynamic trading model where re-sale of a divisible asset takes place in local markets of limited and random participation, captured by a trading network. I show that to find the equilibrium is enough to look at the structure of the trading network.

Trading centrality, a novel network metric, is a sufficient statistic for equilibrium. Behind this result is the interdependency of demands across traders and markets due to the interaction of re-sell risk and the interconnected local markets. The key insight is that my network measure processes all the information driving traders' behavior.

My results are of interest to regulators, scholars and participants of financial markets alike. I argue that the interdealer network not just guarantee the well-functioning of over-the-counter markets but determines the cost of credit in the economy. Also, trading centrality offers a new measures of liquidity and "importance" in the interdealer market that only require information about the interdealer network structure. Both can be useful in empirical applications.

This paper allows for several and interesting extensions. Straightforward ones are having multiple sellers, and to allow for the choice of trading venues or contemporaneous access to all markets. One present limitation is that, due the two-period environment, I do not study intermediation chains which have been pointed outed as important in otc markets. However, having multiple periods natural extension of the model. Apart from financial market applications, the model I develop is suitable for any environment in which a good is initially priced by a large set of agents but subsequently it is only valued or can only be traded by a restricted subset of them.

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Appendices

Appendix A Illustrative Example of the Main Results

Consider five traders A, B, C, D and E, and six different trading networks depicted in Figure A.1. The networks are arranged in descending order of connectivity from left to right, as the first row of Table A.1 shows.

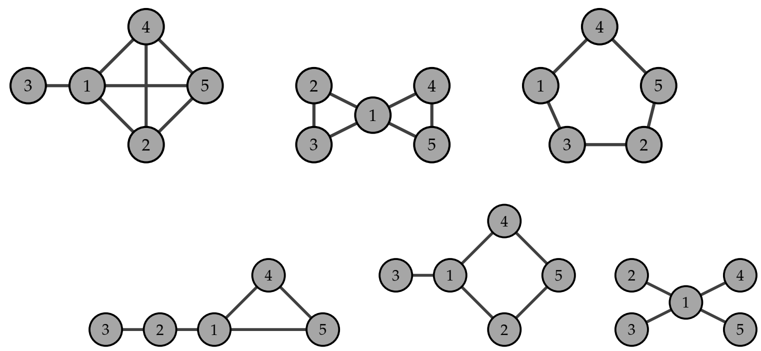


Figure A.1: Trading networks 1, 2 3, 4, 5 and 6 - respectively

	Net 1	Net 2	Net 3	Net 4	Net 5	Net 6
connectivity	2.8	2.4	2	2	2	1.6
inequality	1.2	0.8	0	0.5	0.5	1.8
PM price (P)	$P_2 > P_1 > P_3 > P_4 > P_5 > P_6$					
liquidity cost (PD)	$PD_6 > PD_5 > PD_4 > PD_3 > PD_1 > PD_2$					
welfare (EU)	$EU_6 > EU_3 > EU_5 > EU_4 > EU_2 > EU_1$					

Table A.1: Characteristics and equilibrium outcomes of Figure A.1. Connectivity is the average degree and inequality degree variance. Liquidity cost is the average difference between PM price and local market prices. Welfare is the aggregate expected utility.

We can first compare the structural proprieties of the trading networks - i.e. connectivity and degree inequality - and PM price. Network 2, the second most connected one, has the highest PM price. This reveals the non-monotonicity of PM price with respect to connectivity (Lemma 3). PM price is the lowest on network 6, the star network, what turns out to be the lower bound of PM price across all networks of the same size $N = 5$ (Proposition 2). Not depicted in fig. A.1 is the complete network, which has the highest

PM price. Indeed, as Proposition 2 shows, the complete network imposes the upper bound on PM price level.

Comparison of networks 2 and 3 depicts that (weakly) increasing all traders' degree results in a higher price: every trader in network 2 is at least as connected as in network 3 (despite the former having higher degree inequality) and, consequently, $P_2 > P_3$ (Proposition 3).[71] Comparison of networks 3 and 4 (and/or 5) shows that, keeping connectivity fixed, the higher is degree inequality (network 4), the lower is PM price (Proposition 4).[72] Finally, notice that networks 4 and 5 have the same connectivity and degree inequality (degree distribution) but different PM prices.

We've also seen that asset price almost always drop over time (Proposition 1). Just as with PM price, such price dynamics is non-monotonic in connectivity and degree inequality. Interestingly, the rank of the trading networks with respect to PM price is the reverse to their rank with respect to price drop ($3rd$ and $4th$ rows of table A.1). This means that a higher price for acquiring asset shares does not lead to higher liquidity cost on average.

Welfare comparison offers additional insights (Section 7). Network 6, the star network, delivers the highest welfare. That's because its lowest PM price compensates its highest liquidity cost. Network 1, the most connected one, exhibits the lowest welfare. Actually, the complete network delivers the lowest welfare. Moreover, just as with trading centrality and PM price, connectivity and inequality are not sufficient information to analyze welfare - for example, look at networks 3, 4 and 5.

Lastly, we can compare traders within a network based on trading centrality and degree (Theorem 1, Lemma 8) .[73] Network 2 is an example of when centrality and degree are negatively related: $d_1 > d_2 = d_4 = d_5 = d_3 \leftrightarrow c_1 < c_2 = c_4 = c_5 = c_3$. In turn, central dealers in network 2 have higher asset holdings and higher liquidity cost (lower re-selling price). In the other networks of fig. A.1, central dealers are market makers - they acquire more shares in the PM and have lower liquidity cost - because centrality and degree are positively related. Even so, this relationship is not the same across networks. For instance, networks 1 and 5 are the only cases when centrality is monotonically increasing in degree.

[71] Formally, the degree distribution of 2 FOSD the distribution of 3 and thus price in the former is greater.

[72] Formally, the degree distribution of network 4 is a mean-preserving spread of the degree distribution of network 3 and thus price in the former is lower.

[73] Recall that (as discussed when solving for the equilibrium) the relationship between trading centrality and degree plays an important role in shaping individual outcomes (i.e. demands and re-selling prices), although it is non-trivial. That's because certain network structures deliver a positive relationship between trading centrality and degree, while other structures deliver the opposite.

.[74]

In sum, this example illustrates the non-trivial relationship between welfare, connectivity and degree inequality.

Appendix B Buyer and Seller Effects

I focus in the environment in which the seller effect is minor. This holds for all networks of at least size four. As the network grows, the probability of being a seller vanishes - and it does so at a higher rate than the probability of being a buyer (Figure B.1). Because of that, traders expect to be buyers.

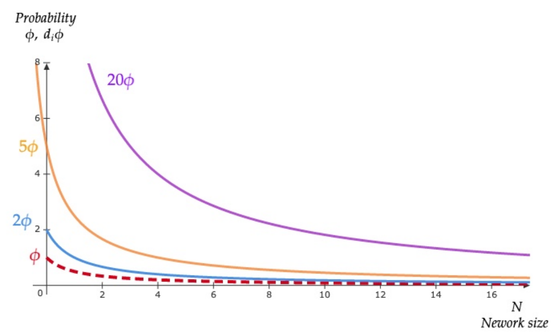


Figure B.1: Probability of being a seller ϕ versus being a buyer $d_i\phi$ as the network size N grows. I plot $\phi = \frac{1}{N+1}$.

In small networks (with two or three traders) the seller effect is not dominated by the buyer effect, an equilibrium might not exists. That's because the probability of being a seller is just as large as being a buyer for all traders. For example, in networks depicted in fig. B.2. With two traders these probabilities are literally the same. With three traders, they can either be equal for all traders or be equal for most traders.

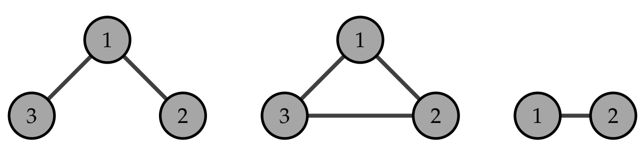


Figure B.2: Networks A, B and C (respectively) where the probability of being a seller ϕ and a buyer $d\phi$ are similar.

[74] That is, in network 1 we have $d_1 > d_2 = d_4 = d_5 = d_3 \leftrightarrow c_1 > c_2 = c_4 = c_5 = c_3$; and in network 5 $d_1 > d_2 = d_4 = d_5 = d_3 \leftrightarrow c_1 > c_2 = c_4 = c_5 = c_3$. However, in network 3, centrality is positively related to degree, but non-monotonically: $d_1 > d_2 = d_4 = d_5 > d_3 \leftrightarrow c_2 > c_1 > c_4 = c_5 > c_3$.

$$A: \phi = \frac{1}{4}, \quad d\phi = \{\frac{1}{4}, \frac{1}{2}\}$$
$$B: \phi = \frac{1}{4}, \quad d\phi = \frac{1}{2}$$
$$A: \phi = d\phi = \frac{1}{3}$$

However, this does not mean that the re-sell risk does not give rise to interesting price dynamics.

Appendix C The Model the Real-World Interdealer Market

In the context of off-exchange markets, the timing of the model captures the idea that dealers often absorb substantial inventory position in primary markets of asset issuance or from their costumers, and then use the interdealer market to offload these positions. Interdealer trades is how dealers provide liquidity to one another.

The re-sell shock[75] is interpreted as the risk of selling under pressure, or the risk of having inventory imbalances resulting from unexpected and large customer orders. In either scenario, a dealer is forced to raise liquidity by quickly disposing his inventory in a possibly illiquid market (Duffie and Zhu (2017)), the interdealer market. This rationalizes why the shocked trader *must sell all* his holdings. And why I do not allow for the choice of being a seller, nor that two traders can be shocked at the same time. Such views of the re-sell shock have been empirically documented, and it turns out that they are not uncommon events.[76]

The quadratic utility function in the quantity traded,[77] apart from being the standard in the literature, also contributes to the tractability of the model. Specifically, it leads linear equilibria that have proved to be useful as a basis for empirical analysis and are supported in the empirical literature on single-sided multi-unit auction - which is the main mechanism of asset issuance in primary markets and electronic interdealer trades.[78]

[75]The use of a random shock to generate trade is common both in the finance (Duffie et al. (2005), Vayanos and Weill (2008)) and network literature (Gale and Kariv (2007), Condorelli et al. (2021)).

[76]See Di Maggio et al. (2017b), Balasubramaniam et al. (2020) among others.

[77]See, for instance, Kyle (1989) Vives (2011), Rostek and Weretka (2012), Rostek and Yoon (2020a) among others.

[78]For example, Hortacsu (2199), using data from Turkish Treasury auctions, shows that linear demands fit actual bidding behavior quite closely

Moreover, this utility representations reflects the fact that dealers are risk-averse with respect to inventory (Ho and Stoll (1983)). Because of that, they often have a desired but costly inventory position, and they trade such to avoid large deviations from this target. Finally, the quadratic utility is microfounded in the mean-variance trade-off of a trader (CAPM theory), and it is equivalent for trading behavior to the the classic CARA-Normal setting.

The trading network captures trading frictions that have been extensively explored in the OTC markets literature, both theoretical and empirically. The premise is that different traders must be sufficiently close on some dimension to be able to trade. This could be justified by having lower trading costs, or similar clientele so that both value the asset being issued. One possible interpretation is that linkages are due to previous investments in relationships: traders may invest time and resources to contact other traders and to know them better. An alternative view is that trading is costly, and linkages capture parties with an easiness to trade. In any case, I'm agnostic on how the network linkages have aroused and I assume it boils down to a pre-determined and fixed set of trading relationships, the trading network.

Appendix D Solving the Model

D.1 Pricing Mechanism

It is convenient to visualize inter-trader trading, i.e. the local market stage, as involving three steps. First, the shocked trader - the seller - $s \in N$ hands over his entire holding of the asset, $q_{s,1}$, to an auctioneer (the inter-trader broker). Then, the auctioneer solicits bids from all *available* traders[79] - the buyers - in the form of demand schedules: combinations of price and quantity. A typical buyer i's trading strategy, as a function of the equilibrium seller's price and and his own pre-trading position, is the quantity that is awarded to him by the auctioneer. The equilibrium single price of the seller, P_s, is determined by equating demand and supply. After the auctioneer collects payment from all buyers, the total proceeds are returned to the seller and are his to keep. Note that, at the conclusion of the inter-trader trading, seller's holding is zero, while buyers total position is $q_{i,1} + q_{i,s}$. In other words, the seller exits the market while buyers increase their asset holdings.

[79]This means that the seller is inactive at the local market stage. In the Appendix I consider allowing the seller to choose his supply as well.

The assumption of a unique active local market ensures there is a single equilibrium price at period two, even though multiple prices are ex-ante possible. This is a distinct feature of the dealership market nowadays which are conducted through an inter-trader broker. The brokerage system guarantees anonymity and that transactions take place at a single price.

D.2 Optimal Demand Decisions

At $t = 1$, trader i submits demand schedule $q_{i,1}(\cdot)$ considering the probability of future local markets he could participate. His optimization problem is

$$\max_{q_{i,1}(\cdot):\mathcal{R}^N \to \mathcal{R}} \quad \phi \left\{ w + q_{i,1} \left(P_i - P_1 \right) \right\} + \phi \sum_{s \in N_i} \left\{ \left(q_{i,1} + q_{i,s} \right) - \frac{1}{2} \left(q_{i,1} + q_{i,s} \right)^2 + w - \left(P_1 q_{i,1} + P_s q_{i,s} \right) \right\}$$
$$+ \left(1 - \phi(d_i + 1)\right) \cdot \left\{ q_{i,1} - \frac{1}{2} q_{i,1}^2 + w - P_1 q_{i,1} \right\} \tag{30}$$

The first component of (30) is the what i gets when she is the seller in the local market. The second term accounts for every possible payoff i gets from trading as a buyer with each of her network-implied sellers. The last term is the payoff of just trading in the primary market, when i is not shocked nor connected to the shocked agent.

All traders face the same idiosyncratic shock ϕ and so they all have the same probability of being a seller. However, the likelihood of being a buyer is determined by how many connections i has. The network also dictates trader i's willingness to trade at $t = 2$, since buying in the PM brings i's closer to his target portifolio. Because of that, the choice of $q_{i,1}$ affects and is affected by local market demands $\{q_{i,s}\}_{i \cup s \in N_i}$ and prices $\{P_s\}_{i \cup s \in N_i}$.

The first order condition of (30) is

$$\begin{aligned} &\phi \cdot \left\{ P_i - P_1 + q_{i,1} \frac{\partial P_i}{\partial q_{i,1}} \right\} \\ &+ \phi \cdot \sum_{s \in N_i} \left\{ 1 + 1 \frac{\partial q_{i,s}}{\partial q_{i,1}} - \left(q_{i,1} + q_{i,s} \right) \left(1 - \frac{\partial q_{i,s}}{\partial q_{i,1}} \right) - P_1 - P_s \frac{\partial q_{i,s}}{\partial q_{i,1}} - q_{i,s} \frac{\partial P_s}{\partial q_{i,1}} \right\} \\ &+ \left(1 - \phi(d_i + 1)\right) \cdot \left\{ 1 - q_{i,1} - P_1 \right\} = 0 \end{aligned} \tag{31}$$

At $t = 2$, both the PM and the re-sell shock have realized. The seller identity $s \in N$

is common knowledge and, by assumption, he does not make any decision in the local market: he supplies all his PM shares, $q_{s,1} \leq Q$. Each buyer $i \in N_s$ chooses how many shares to buy from the seller s, $q_{i,s}$, taking into account his PM holdings:

$$\max_{q_{i,s}(\cdot):\mathcal{R}\to\mathcal{R}} \quad (q_{i,1}+q_{i,s}) - \frac{1}{2}\left(q_{i,1}+q_{i,s}\right)^2 + w - (P_1 q_{i,1} + P_s q_{i,s}) \tag{32}$$

Notice that $(w - P_1 q_{i,1})$ is the capital available to invest after trading at $t=1$. First-order condition delivers buyer i's demand schedule for seller $s \in N_i$:

$$q_{i,s}(P_s;\boldsymbol{q}) = (1-q_{i,1}) - P_s \tag{6}$$

Buyer i's downward-slopping demand $q_{i,s}$ does not directly depend on the network, and it is negatively related to his PM holdings. That's because a buyer's willingness pay for q_s (i.e. his marginal utility from trading with the seller) is given only by how far he is from the target inventory, $(1-q_{i,1})$ (See Section 3). The higher is $q_{i,1}$, the more satisfied i is with his current amount of asset shares and so he will be less willing to trade with the seller for any possible price.

Demand (6) confirms that the local market is *perfectly competitive with heterogeneous valuations*: buyers' demands differ only by their asset holdings coming into $t=2$.[80] As so, the local market can be interpreted as a static Walrasian market with heterogeneous asset endowment.

D.3 Active Local Market

The immediate corollary of Proposition regards positive price.

Corollary 10.1. *Seller's Price and Excess Demand*

Any seller $s \in N$ has positive equilibrium price if and only if there exists an excessive average demand in his neighborhood:

$$P_s > 0 \leftrightarrow \sum_{i\in N_s} \frac{(1-q_{i,1})}{d_s} > \frac{q_{s,1}}{d_s}$$

In principle, P_s could be negative. To ensure weakly positive prices $P_s \geq 0$, there must be an *excessive average demand* in the seller's neighborhood, i.e the neighborhood's valuation

[80]Two different buyers demand the same amount *if and only if* their first-period demand are them same.

of the asset is greater than the supply : $\sum_{i\in N_s}\frac{(1-q_{i,1})}{d_s}\geq\frac{q_{s,1}}{d_s}$. This constraint ensures that buyers are willing to trade with the seller since their asset holdings at $t=2$, $\{q_{i,1}\}_{i\in N_s}$, are low enough. To secure their optimal inventory - that is, to reduce $1-(q_{i,1}+q_{i,s})$ - a buyer's demand schedule is positive at the seller's equilibrium price.

From pricing equation (7) we can draw three conclusions. First, P_s^* is determined by the total average PM asset holdings, including the selling quantity. Equivalently, seller's price is given by the difference between the average of target inventories of his buyers and his per-link supply: $P_s=\frac{1}{d_s}\sum_{i\in N_s}(1-q_{i,1})-\frac{1}{d_s}q_{s,1}$. Second, seller's price is constrained by the buyers' total amount of PM shares: $P_s^*\in[(1-q_{s,1})-q^{\max}_{N_s,1},(1-q_{s,1})-q^{\min}_{N_s,1}]$, where $q^{\max}_{N_s,1}$ is the highest first-period consumption of a buyer and $q^{\min}_{N_s,1}$ is the lowest PM consumption of a buyer. Thus, the more heterogeneous buyers' asset holdings, the wider the range of possible prices for a given seller s. Finally, two cases can make P_s to be zero (negative): if buyers have high too much bond holdings (i.e., too high PM demand); and if the supply is much greater than seller's aggregate demand (i.e. excessive supply is large enough). In both cases, P_s has to be low enough to induce buyers to demand from the seller.

Appendix E The Trading Network Game

It turns out that the model is in essence a network game of strategic substitute. This is the key insight of this paper as it allows the equilibrium characterization in the Primary and all Local markets. Moreover, the game belongs to a particular class of games: those with quadratic payoff function and linear best replies. The networks literature has extensively studied this type of games.[81] In this Appendix section, I derive in details the characterization of the game as a result of the model and prove the equilibrium outcomes. I start by describing the model as a network game, comprised of the degree distribution and each agent's action and payoffs. I then discuss the equilibrium concept, Nash Equilibrium, and provide equilibrium results for the game.

E.1 Actions, Links and Payoffs

Traders $i=\{1,2,...,N\}$ simultaneously choose actions: each trader i chooses his PM demand $q_{i,1}\geq 0$. Traders are embedded in the fixed Trading Network represented by the matrix $\boldsymbol{G}\in\{0,1\}^{N\times N}$ with $g_{ij}=1$ implying a link between agents i,j.

[81] See Bramoullé et al. (2014) and Galeotti et al. (2010).

Each trader's payoff is a function of own action, $q_{i,1}$, others' actions, $\boldsymbol{q_{-i,1}}$, the network $\boldsymbol{G}$, and the shock parameter $\phi \in (0, \frac{1}{N})$:

$$\pi_i\big(q_{i,1}, \boldsymbol{q_{-i,1}}; \boldsymbol{G}, \phi\big) = w + (1 - P_1) \cdot q_{i,1} - \frac{1}{2v_i(\phi)} q_{i,1}^2 - \phi \sum_j \tilde{g}_{ij} q_{j,1} q_{i,1} + \phi \sum_j \bar{g}_{ij} q_{j,1} \qquad \textbf{(??)}$$

where

$$v_i(\phi) \equiv \left[\frac{2\phi}{d_i} + 1 - \phi(d_i + 1) + \phi \sum_j g_{ij} \cdot \frac{(2d_j - 1)}{d_j^2}\right]^{-1} \qquad \textbf{(??)}$$

$$\tilde{g}_{ij} = g_{ij} \cdot \left[\frac{1}{d_i} + \frac{(d_j - 1)}{d_j^2}\right] + \left[\sum_{\substack{z \neq j \\ z}} g_{iz} g_{jz} \frac{(d_z - 1)}{d_z^2}\right] \qquad \textbf{(??)}$$

$$\bar{g}_{ij} = g_{ij} \cdot \frac{1}{2d_j^2} + \sum_{z \neq \{i,j\}} g_{iz} g_{jz} \cdot \frac{1}{d_z^2} \qquad (33)$$

The endogenous coefficients $\{v_i(\phi), \tilde{g}_{ij}, \bar{g}_{ij}\}_{\forall i,j \in N}$, all non-negative, are determined by network graph $\boldsymbol{G}$ such that

$$\begin{aligned}
&\frac{\partial \tilde{g}_{ij}}{\partial d_i} \leq 0, \qquad \frac{\partial \bar{g}_{ij}}{\partial d_i} = 0 \\
&\frac{\partial \tilde{g}_{ij}}{\partial d_j} \leq 0, \qquad \frac{\partial \bar{g}_{ij}}{\partial d_j} \leq 0 \quad for \quad d_j \geq 2 \\
&\frac{\partial \tilde{g}_{ij}}{\partial d_z} \leq 0, \qquad \frac{\partial \bar{g}_{ij}}{\partial d_z} \leq 0 \quad for \quad d_z \geq 2, \quad z \neq \{i,j\} : g_{iz} = g_{jz} = 1
\end{aligned}$$

Individual payoff function (9) is strictly concave in own-action $\frac{\partial^2 \pi}{\partial^2 q_{i,1}^2} = (2v_i(\phi))^{-1} > 0$ for all $i \in N$. The re-sell shock probability $\phi > 0$ regulates the global interaction effect among agents. As ϕ increases, the payoff externalities of agents' action become globally stronger. That's because, the higher ϕ, the greater is the chance of being both a buyer and a seller in the local market. And so, the greater is the feedback effect between markets.

The term multiplying individual action $q_{i,1}$, $(1 - P_1) \geq 0$ is common across all agents and represents an agent's optimal action absent network interactions. That is, $(1 - P_1)$ corresponds to the individual demand in a competitive market of size N, when future trading does not opportunity exists.

The individual network effect $v_i(\phi)$ is increasing in both i and his connections' degree, $d_i, \{d_j\}_{j \in N_i}$. The higher d_i, the higher is i's selling price and thus the more he consumes in the PM. The higher d_j, the higher the prices i will face in a local market, thus also inducing more demand in the PM.

The global network coefficients $\tilde{g}_{ij} \geq 0$ captures bilateral influences. Traders' actions are strategic substitutes since $\frac{\partial^2 \pi_i}{\partial q_{i,1} \partial q_{j,1}} = -\tilde{g}_{ij} \leq 0$.

The individual payoff equation (**??**) is exactly the individual expected utility (**??**) in the baseline model. Hence, payoff maximization is equivalent to trader's expected utility maximization problem.

E.2 Best Replies and Nash Equilibrium

The solution concept considered is pure-strategy Nash Equilibrium. Bramoullé et al. (2014) and Bramoullé and Kranton (2016) give the conditions that guarantees a unique interior equilibrium of a quadratic-linear network game of strategic substitutes. As I show, such conditions are met in the model and thus market outcomes are unique and interior.

Given the quadratic linear payoff function (**??**), an interior Nash equilibrium in pure-strategies $q^*_{i,1} > 0$ is such that $\partial \pi_i / \partial a_i(\boldsymbol{a}^*) = 0$ and $q^*_{i,1} > 0$ for all $i \in N$.

Lemma 6. *For a given price level P_1, each trader i's best-response to others' demands is given by the first order condition of* (**??**)*:*

$$q_{i,1}(q_{-i,1}; P_1, \phi, \boldsymbol{G}) = \max\left(0, v_i(\phi)\left[(1 - P_1) - \phi \sum_j \tilde{g}_{ij} q_{j,1}\right]\right) \qquad \textbf{(??)}$$

Optimal demand $q_{i,1}(q_{-i,1}; P_1, \phi, \boldsymbol{G})$ is linear in others' demands and $q_{i,1}(\boldsymbol{q_1}) \in [0, 1 - P_1]$ Define the following matrices: $\boldsymbol{V}$ is the N-diagonal matrix with entries $\{v_i\}_i$; $\tilde{\boldsymbol{G}}$ is te N-square, not symmetric matrix with entries $\{\tilde{g}_{ij}\}_{i,j}$; $\mathbf{1}_N$ is the N-vector of ones.

The existence of a unique interior equilibrium for each game is guaranteed as long as the shock probability ϕ is no greater than $1/N$.

Proposition 11. *Primary Market Equilibrium Demand*

Denote $\boldsymbol{V}\tilde{\boldsymbol{G}}_S$ the symmetric part of the N-square matrix $\boldsymbol{V}\tilde{\boldsymbol{G}}$. For each price P_1, a unique and interior Nash equilibrium exists if and only if $\phi < -1/\lambda_{min}\left(\boldsymbol{V}\tilde{\boldsymbol{G}}_S\right)$. Then, the vector of optimal PM demands $\boldsymbol{q}_1$ is

$$\boldsymbol{q}_1 = (1 - P_1)\left(\boldsymbol{I} + \phi \boldsymbol{V}\tilde{\boldsymbol{G}}\right)^{-1} \boldsymbol{V}\mathbf{1} \tag{34}$$

Traders' demands $\boldsymbol{q}_1$ is a linear function of the weighted adjacency matrix of global network effects $\tilde{\boldsymbol{G}}$ and the vector of individual effects $\boldsymbol{v}$. In **??**, I show how the Nash equilibrium looks like for a simple economy with four traders and six different trading networks.

The equilibrium PM price is determined by the market clearing condition (3). It is a particular Nash Equilibrium of the set of equilibria characterized by (10) such that aggregate demand meets exogenous asset supply $\bar{Q}$. Thus, equilibrium PM price follows directly from **??**.

Theorem 2. *Assume conditions of* **??** *hold. Then, given asset supply $\bar{Q} > 0$, the unique and positive equilibrium primary market price is*

$$P_1^* = 1 - \bar{Q}\left(\boldsymbol{v}'\left(\left(\boldsymbol{I} + \phi \boldsymbol{V}\tilde{\boldsymbol{G}}\right)^{-1}\right)'\mathbf{1}_{\boldsymbol{N}}\right)^{-1} \tag{35}$$

where $\boldsymbol{v}'\left(\left(\boldsymbol{I} + \phi \boldsymbol{V}\tilde{\boldsymbol{G}}\right)^{-1}\right)'\mathbf{1}_{\boldsymbol{N}}$ is a strictly positive scalar.

As argued, the crucial demand decision is the primary market one and it allows us to determine i) traders' inventory at the end of period 2; ii) the PM (issuance) price of the bond, and so the cost of credit for the issuer; iii) all possible market outcomes in the local market.This last point is a corollary of Theorem 2.

Corollary 11.1. *Secondary Market Outcomes*

If conditions of Theorem 2 hold, equilibrium outcomes in the secondary market are determined by the network graph $\boldsymbol{G}$, the shock ϕ and issuer's asset supply $\bar{Q}$. They are characterized by the price vector $\boldsymbol{p}_{\boldsymbol{d}N\times 1}$ and square matrix of demands $\boldsymbol{Q}_{N\times N}$:

$$\boldsymbol{p}_{\boldsymbol{d}} = \mathbf{1}_N - \boldsymbol{D}^{-1}\left(\boldsymbol{G} + I\right)\boldsymbol{q}_{\mathbf{1}}^{*} \tag{36}$$

$$\boldsymbol{Q}_d = diag\left(\mathbf{1}_N - \boldsymbol{q}_1^*\right)\boldsymbol{G} - \boldsymbol{G}diag\left(\boldsymbol{p}_{\boldsymbol{d}}\right) \tag{37}$$

where $\boldsymbol{D} = diag\left(\boldsymbol{d}\right)$ is the diagonal matrix of individual degrees

The demand matrix $\boldsymbol{Q}_d$ gives the amount of asset shares traded between two agents at each possible market in period 2: rows indicate the buyer and columns the seller. For example, $[\boldsymbol{Q}_d]_{ij} = q_{i,j}$ is agent i's demand when j is the seller. Clearly, if i and j don't share a connection then $q_{i,j} = 0$. $\boldsymbol{Q}_d$ has a zero-diagonal as it does not include asset supply.

Appendix F Trading Centrality

Two crucial features sets my trading centrality measure apart from the typical graph-theoretic measures of network centrality that are not suitable for the model. First, it captures not only first and higher-order inter-connectivity (friends, friends of friends, etc..) but also it encodes how such connections interact in local markets (as buyers, sellers, and competitors). Second, the relationship between centrality and individual degree depends on the structure of the trading network. Some networks - like symmetric and core-periphery graphs, and lines - have trading centrality increasing in individual degree. For arbitrary networks the reverse can hold. For example, if the network exhibits high-degree nodes connected to one another, than low-degree traders are the central ones.

Definition 1. *Trading centrality is N-dimensional vector $\boldsymbol{c}$ defined as*[82]

$$\boldsymbol{c}\left(\boldsymbol{G}, \phi\right) = \left(\boldsymbol{V}(\boldsymbol{G}, \phi) + \phi \tilde{\boldsymbol{G}}(\boldsymbol{G})\right)^{-1} \mathbf{1}_N \tag{20}$$

where $\boldsymbol{V}(\boldsymbol{G}, \phi)$ is the N-diagonal matrix with entries $\{1/v_i\}_i$; $\tilde{\boldsymbol{G}}(\boldsymbol{G})$ is the N-square, not symmetric matrix with entries $\{\tilde{g}_{ij}\}_{i,j}$; and 1_N is the N-vector of ones.

Equivalently, trader i's trading centrality c_i is

$$c_i(\boldsymbol{G}, \phi) = v_i \left((1 - \phi \sum_j \tilde{g}_{ij} \cdot c_j(\boldsymbol{G}, \phi) \right) \tag{14}$$

Trading centrality fixed point representation (20) and recursive representation (14) are related as follows:

[82] More precisely, trading centrality is a measure $\boldsymbol{c} : \boldsymbol{G} \to \mathcal{R}^N$, where $c_i(\boldsymbol{G}, \phi)$ is the trading centrality of trader (node) i in the trading network $\boldsymbol{G}$.

$$
\begin{aligned}
\boldsymbol{c} = \left(\boldsymbol{V} + \phi\tilde{\boldsymbol{G}}\right)^{-1}\mathbf{1}_N &= \left(I + \phi\boldsymbol{V}\tilde{\boldsymbol{G}}\right)^{-1}\boldsymbol{V}\mathbf{1}_N \\
&\mathit{or} \\
\boldsymbol{c} &= \boldsymbol{V}\mathbf{1}_N - \phi\boldsymbol{V}\tilde{\boldsymbol{G}}\boldsymbol{c} \\
&= \boldsymbol{V}\mathbf{1}_N - \phi\boldsymbol{V}\tilde{\boldsymbol{G}}\left(\boldsymbol{V}\mathbf{1}_N - \phi\boldsymbol{V}\tilde{\boldsymbol{G}}\boldsymbol{c}\right) \\
&= \boldsymbol{V}\mathbf{1}_N - \phi\boldsymbol{V}\tilde{\boldsymbol{G}}\boldsymbol{V}\mathbf{1}_N + \phi^2\left(\boldsymbol{V}\tilde{\boldsymbol{G}}\right)^2\boldsymbol{c} \\
&= \boldsymbol{V}\mathbf{1}_N - \phi\boldsymbol{V}\tilde{\boldsymbol{G}}\boldsymbol{V}\mathbf{1}_N + \phi^2\left(\boldsymbol{V}\tilde{\boldsymbol{G}}\right)^2\left(\boldsymbol{V}\mathbf{1}_N - \phi\boldsymbol{V}\tilde{\boldsymbol{G}}\boldsymbol{c}\right) \\
&= \boldsymbol{V}\mathbf{1}_N - \phi\boldsymbol{V}\tilde{\boldsymbol{G}}\boldsymbol{V}\mathbf{1}_N + \phi^2\left(\boldsymbol{V}\tilde{\boldsymbol{G}}\right)^2\boldsymbol{V}\mathbf{1}_N - \phi^3\left(\boldsymbol{V}\tilde{\boldsymbol{G}}\right)^3\boldsymbol{c}... \\
&... \\
&= \left[\sum_{t=0}^{\infty}(-\phi)^t\left(\boldsymbol{V}\tilde{\boldsymbol{G}}\right)^t\right]\boldsymbol{V}\mathbf{1}_N
\end{aligned}
$$

That is

$$
c_i = v_i\left[1 + \sum_{t=0}^{\infty}(-\phi)^t\left(v_i\sum_j \tilde{g}_{ij}\right)^{t+1}\right]
$$

The trading network adjacency matrix $\boldsymbol{G}$ describes all linkages among traders: it is symmetric and unweighted. But the crucial matrix for the model is the global network effect matrix $\tilde{\boldsymbol{G}}$ such that $\tilde{\boldsymbol{g}}_{ij} \geq 0$: it is asymmetric and weighted and it describes traders' interaction in any the Secondary Market. $\tilde{\boldsymbol{G}}$ itself induces a graph. Denote the trading network graph by $\mathcal{G}$ and the global network graph by $\tilde{\mathcal{G}}$, both undirected. It holds that $\mathcal{G} \subseteq \tilde{\mathcal{G}}$.

Matrix $\tilde{\boldsymbol{G}}$ has entries in the $[0, 1]$ interval except when $d_i = 1$ and $d_j > 1$. The centrality matrix $(I + \phi\boldsymbol{V}\tilde{\boldsymbol{G}})^{-1}$ has positive diagonal and negative off-diagonal. The diagonal is increasing in one's connectivity and neighbor's connectivity.

Lemma 7. *Centrality Matrix*

The centrality matrix $(I + \boldsymbol{V}\tilde{\boldsymbol{G}})$ has diagonal equals to 1 and off diagonal elements in $[0, 1)$. Off-diagonal entry is zero iff (i, j) are not connected nor share a path-two link.

The inverse centrality matrix $(I + \boldsymbol{V}\tilde{\boldsymbol{G}})^{-1}$ is a N-square matrix with positive diagonal. Off-diagonal elements can be a) negative, if (i, j) are connected or share a path-two link; b) positive, if (i, j) are not connected nor share a path-two link

To grasp the intuition behind Lemma 7, take trader $i \in N$. Traders j, k at most two link apart from i directly influence i's local market trading. If i is a buyer, he demands less in the PM when j, k demand more. In this way, i can buy more shares at a lower price in local markets. The same logic holds when i responds to j (or k) as a competitor for k (or j). As a seller, i also lowers his PM demand in response to higher PM demand from j, k. In this way, i sells less at a lower price. For a trader z further apart, i still responds negatively to z's demand. But this response is not as strong since z does not influence directly i's local market trading. Trader z only affects i because z's PM demand determines the terms of trade in other local markets (i has no access to) and, consequently, the PM demand of other traders. What ultimately determines the equilibrium PM price and asset allocation.

Trading centrality translates the above discussion into the matrix $\left(\boldsymbol{V} + \phi\tilde{\boldsymbol{G}}\right)^{-1}$. The *sign* of $\left(\boldsymbol{V} + \phi\tilde{\boldsymbol{G}}\right)^{-1}$ varies with how far apart traders are. If traders i, j are directly connected or have one common connection, then the (i, j) entry is *negative*. However, if i, j are more than two links apart, then (i, j) entry is *positive*.

F.1 Trading Centrality and Degree

As discussed in Section 5, the relationship between trading centrality and individual degrees is non-trivial. The centrality measure encapsulates information above and beyond connectivity. And connectivity by itself is not enough to understand the feedback effect between the primary market and subsequent local markets. Meanwhile, as the network structure defines trading centrality, it also defines te relationship between centrality and degree. In some networks, being more central means being more connected. In others, the opposite holds.

Lemma 8. *Trading Centrality and Degree*

The relationship between trading centrality $\boldsymbol{c}$ and individual degree $\boldsymbol{d}$ depends on the structure of the trading network.

Consider an arbitrary trading network. If $corr(\boldsymbol{c}, \boldsymbol{d}) > 0$, then local markets (re-selling) prices $\{P_i\}_{\forall i \in N}$ are increasing in both $(\boldsymbol{c}, \boldsymbol{d})$.

If $(\boldsymbol{c}, \boldsymbol{d}) < 0$, the opposite holds.

F.2 Centrality and the Adjacency Matrix

Trading centrality is a function of the adjacency matrix $\boldsymbol{G}$ defining the trading network. To express trading centrality in terms of $\boldsymbol{G}$ alone, it is useful to introduce some notation. The vector of degrees is $\boldsymbol{d} = \boldsymbol{G}\mathbf{1}$. Define the following Nvectors which are functions of individual degrees:

$$\begin{aligned}
\boldsymbol{d}^{\mathbf{1}} &= (\boldsymbol{G}\mathbf{1})^{-1} \\
\boldsymbol{d}^{\mathbf{2}} &= (\boldsymbol{G}\mathbf{1})^{-1} - (\boldsymbol{G}\mathbf{1})^{-2} \\
\boldsymbol{d}^{\mathbf{3}} &= 2(\boldsymbol{G}\mathbf{1})^{-1} - (\boldsymbol{G}\mathbf{1})^{-2}
\end{aligned}$$

so that the entries of each vector are $d_i^1 \equiv \frac{1}{d_i}$, $d_i^2 \equiv \frac{d_i - 1}{d_i^2}$ and $d_i^3 \equiv \frac{2d_i - 1}{d_i^2} = \frac{d_i}{d_i^2} + \frac{(d_i-1)}{d_i^2} = d_i^1 + d_i^2$.

Let $\boldsymbol{D}^{\mathbf{1}}, \boldsymbol{D}^{\mathbf{2}}, \boldsymbol{D}^{\mathbf{3}}$ denote the diagonal matrix with entries $\boldsymbol{d}^{\mathbf{1}}, \boldsymbol{d}^{\mathbf{2}}, \boldsymbol{d}^{\mathbf{3}}$, respectively:

$$\begin{aligned}
\boldsymbol{D}^{\mathbf{1}} &= diag(\boldsymbol{d}^{\mathbf{1}}) = diag\left((\boldsymbol{G}\mathbf{1})^{-1}\right) = \left((\boldsymbol{G}\mathbf{1})^{-1}\right)\mathbf{1}^T\boldsymbol{I} \\
\boldsymbol{D}^{\mathbf{2}} &= diag(\boldsymbol{d}^{\mathbf{2}}) = diag\left(\boldsymbol{G}\mathbf{1})^{-1} - (\boldsymbol{G}\mathbf{1})^{-2}\right) = \left((\boldsymbol{G}\mathbf{1})^{-1} - (\boldsymbol{G}\mathbf{1})^{-2}\right)\mathbf{1}^T\boldsymbol{I} \\
\boldsymbol{D}^{\mathbf{3}} &= diag(\boldsymbol{d}^{\mathbf{3}}) = diag\left(2(\boldsymbol{G}\mathbf{1})^{-1} - (\boldsymbol{G}\mathbf{1})^{-2}\right) = \left(2(\boldsymbol{G}\mathbf{1})^{-1} - (\boldsymbol{G}\mathbf{1})^{-2}\right)\mathbf{1}^T\boldsymbol{I}
\end{aligned}$$

The global network matrix $\tilde{\boldsymbol{G}}$ has zero-diagonal and non-negative entries everywhere else. It is useful to decompose it into direct and indirect effects:

$$\tilde{\boldsymbol{G}} = \tilde{\boldsymbol{G}}_1 + \tilde{\boldsymbol{G}}_2$$

where

$$\begin{aligned}
\tilde{\boldsymbol{G}}_1 &\equiv \boldsymbol{D}^{\mathbf{1}}\boldsymbol{G} + \boldsymbol{G}\boldsymbol{D}^{\mathbf{2}} \\
\tilde{\boldsymbol{G}}_2 &\equiv \boldsymbol{G}\boldsymbol{D}^{\mathbf{2}}\boldsymbol{G} - diag\left(\boldsymbol{G}\boldsymbol{D}^{\mathbf{2}}\boldsymbol{G}\right)
\end{aligned}$$

Then

$$
\begin{aligned}
\tilde{\boldsymbol{G}} &= \boldsymbol{D^1 G} + \boldsymbol{G D^2}\left(I + \boldsymbol{G}\right) - diag\left(\boldsymbol{G D^2 G}\right) \\
&= (\boldsymbol{G1})^{-1}\boldsymbol{1}^T\boldsymbol{I}\boldsymbol{G} + \boldsymbol{G}\left((\boldsymbol{G1})^{-1} - (\boldsymbol{G1})^{-2}\right)\boldsymbol{1}^T\boldsymbol{I}(\boldsymbol{I} + \boldsymbol{G}) \\
&- \left(\boldsymbol{G}\left((\boldsymbol{G1})^{-1} - (\boldsymbol{G1})^{-2}\right)\boldsymbol{1}^T\boldsymbol{I}\boldsymbol{G}\right)\boldsymbol{1}^T\boldsymbol{I}
\end{aligned}
$$

The matrix of indirect effects $\tilde{\boldsymbol{G}}_2$[83] is a weighted count of the paths of length two in the network. This path plays an important role in determining a trader trading centrality because it captures competition in the local market.

Now I turn into the individual effect vector $\boldsymbol{v}(\phi)^{-1}$,

$$
\boldsymbol{v}(\phi)^{-1} = (1 - \phi)\boldsymbol{1}_N + 2\phi\boldsymbol{d^1} - \phi\boldsymbol{d} + \phi\boldsymbol{G d^3}
$$

The diagonal matrix $\boldsymbol{V}$ has entries $\boldsymbol{v}(\phi)^{-1}$, and so in terms of $\boldsymbol{G}$ and ϕ,

$$
\begin{aligned}
\boldsymbol{V} &= diag\left(\boldsymbol{v}(\phi)^{-1}\right) \\
&= \left((1 - \phi)\boldsymbol{1} + 2\phi(\boldsymbol{G1})^{-1} - \phi(\boldsymbol{G1}) + \phi\boldsymbol{G}\left(2(\boldsymbol{G1})^{-1} - (\boldsymbol{G1})^{-2}\right)\right)\boldsymbol{1}^T\boldsymbol{I}
\end{aligned}
$$

Hence, trading centrality $\boldsymbol{c}(\boldsymbol{G}, \phi)$ (eq. (20)) is given by

$$
\begin{aligned}
\boldsymbol{c} = \Bigg(&\left((1 - \phi)\boldsymbol{1} + 2\phi(\boldsymbol{G1})^{-1} - \phi(\boldsymbol{G1}) + \phi\boldsymbol{G}\left(2(\boldsymbol{G1})^{-1} - (\boldsymbol{G1})^{-2}\right)\right)\boldsymbol{1}^T\boldsymbol{I} \\
&+ \phi\Big((\boldsymbol{G1})^{-1}\boldsymbol{1}^T\boldsymbol{I}\boldsymbol{G} + \boldsymbol{G}\left((\boldsymbol{G1})^{-1} - (\boldsymbol{G1})^{-2}\right)\boldsymbol{1}^T\boldsymbol{I}(\boldsymbol{I} + \boldsymbol{G}) \\
&- \left(\boldsymbol{G}\left((\boldsymbol{G1})^{-1} - (\boldsymbol{G1})^{-2}\right)\boldsymbol{1}^T\boldsymbol{I}\boldsymbol{G}\right)\boldsymbol{1}^T\boldsymbol{I}\Big)\Bigg)^{-1}\boldsymbol{1}
\end{aligned} \tag{38}
$$

F.3 Local Trading Centrality

From Lemma 1 and Theorem 1, the selling price of trade i is given by his and his buyers' centralities, $c_i, \{c_j\}_{j \in N_i}$ respectively:

[83]The matrix of path-two is given by $\boldsymbol{P}_2 = \boldsymbol{G^2} - diag(\boldsymbol{G}^2)$.

$$P_i^* = 1 - \frac{\bar{Q}}{Nc_A} \frac{\left(c_i + \sum_{j \in N_i} c_j\right)}{d_i} \tag{39}$$

Then, i's trading cost is

$$\begin{aligned} P_1 - P_i &= \left(1 - \frac{\bar{Q}}{Nc_A}\right) - \left(1 - \frac{\bar{Q}}{Nc_A} \frac{\left(c_i + \sum_{j \in N_i} c_j\right)}{d_i}\right) \\ &= \frac{\bar{Q}}{Nc_A} \left(\frac{c_i + \sum_{j \in N_i} c_j}{d_i} - 1\right) \end{aligned} \tag{40}$$

It is useful to denote $\tilde{c}_i \equiv \frac{c_i + \sum_{j \in N_i} c_j}{d_i}$ as the local centrality of trader i, that is, the sum of his local market participants' centrality, including himself, controlled for his degree. With that, there is a straightforward relation between trading cost and local centrality.

Proposition 12. *Trading Cost and Centrality*

The trading cost of trader $i \in N$ is increasing in his local centrality $\tilde{c}_i$:

$$P_1 - P_i = \frac{Q}{Nc_A} (\tilde{c}_i - 1) > 0 \tag{41}$$

Moreover, $P_1 - P_i < 0$ if and only if i is the core of a star network of size $N \geq 3$.

Local centrality captures both participation and inventory effects. And that's the reason it's a sufficient statistic for liquidity cost.

Appendix G Comparative Statics

Best-replies:

Trader i's response to changes in behavior of others is independent of PM price. In the best-replies space, varying price P_1 corresponds to parallel shifts of the demand schedules. As a trader $j \neq i$ changes his PM demand $q_{j,1}(\cdot)$, i reacts by changing his demand $q_{i,1}$ according to

$$\frac{\partial q_{i,1}(\cdot)}{\partial q_{j,1}(\cdot)} = -\phi v_i(\phi) \tilde{g}_{ij} \tag{42}$$

As expected, the partial derivative (42) is negative as the game is one of strategic substitutes. It is also asymmetric: the way j affects i is not the same as i affects j. That's because i and j can have different positions in the network and so they will face different sets of potential sellers, buyers and competitors. This is precisely the reason that just looking at how many connections (degree) of a trader is not enough to understand his behavior. Lastly, it only depends on the trading network and shock parameter since, as shown in Theorem 2, these are necessary and sufficient information to find PM equilibrium (along with the exogenous supply level $\bar{Q}$). Mathematically, the matrix $\tilde{\boldsymbol{G}}$ is asymmetric and individual coefficients $\{v_i(\phi)\}_i$ are heterogeneous.

The shock effect:

P_1 only affects how demands change in response to the shock. To see this, first notice that ϕ has a direct and indirect effect on $q_{i,1}$:

$$\frac{\partial q_{i,1}(\cdot)}{\partial \phi} = \frac{\partial v_i}{\partial \phi}\left[(1-P_1) - \phi \sum_j \tilde{g}_{ij} q_{j,1}\right] - v_i \sum_j \tilde{g}_{ij} q_{j,1}$$

The indirect effect comes trough $v_i(\phi)$:

$$\begin{aligned} \frac{\partial v_i}{\partial \phi} &= -v_i^2 \cdot \psi_i > 0 \qquad if \;\; \psi_i < 0 \Longleftrightarrow \underbrace{\frac{(d_i^2 + d_i - 2)}{d_i}}_{>0} > \sum g_{ij} \cdot \underbrace{\left(\frac{2d_j - 1}{d_j^2}\right)}_{>0} \\ &< 0 \qquad if \;\; \psi_i > 0 \Longleftrightarrow \underbrace{\frac{(d_i^2 + d_i - 2)}{d_i}}_{>0} < \sum g_{ij} \cdot \underbrace{\left(\frac{2d_j - 1}{d_j^2}\right)}_{>0} \end{aligned} \tag{43}$$

where, to simplify notation, I let

$$\psi_i \equiv \left[\frac{2}{d_i} - (d_i + 1) + \sum_j g_{ij} \cdot \left(\frac{2d_j - 1}{d_j^2}\right)\right]$$

ψ_i is a measure of relative connectivity between i and his neighbors. When i is, loosely, more (less) well-connected than his friends then $\psi_i < 0$ ($\psi_i > 0$). In turn, the total effect of ϕ on $q_{i,1}$ is positive (negative). Thus an increase in the shock probability makes a trader i to increase his demand only if he is more connected than his neighbors:

$$\frac{\partial q_{i,1}(\cdot)}{\partial \phi} = -v_i^2 \cdot \psi_i > 0 \qquad if \;\; \psi_i < 0$$

As the secondary market becomes more likely, i expects to sell relatively more asset shares at a high price if he is shocked, and to buy relatively few shares at low price is he is connected to the seller.

The individual degree effect:

Individual degrees $\{d_i\}_{i \in N}$ appear in all PM demand's network components $(v_i(\phi), \{\tilde{g}_{ij}\}_{j \neq i}$. Their effect is can be broken down by a trader's own degree, his neighbors' degrees, and his neighbors' connections degrees.

First, individual degree d_i has a positive effect on v_i and a negative effect on $\{\tilde{g}_{ij}\}_{j \neq i}$

$$\begin{aligned} \frac{\partial v_i}{\partial d_i} &= -\left(-\frac{2\phi}{d_i^2} - \phi\right) \cdot v_i^2 = \left(\frac{2\phi}{d_i^2} + \phi\right) \cdot v_i^2 > 0 \\ \frac{\partial \tilde{g}_{ij}}{\partial d_i} &= -\frac{1}{d_i^2} < 0 \qquad \forall j \;\; s.t. \;\; g_{ij} \geq 1 \end{aligned} \tag{44}$$

Both effects combined imply that $q_{i,1}$ is increasing in individual degree d_i.

Second, each direct connection's degree d_j has a direct and indirect effects, the latter coming from common friends:

$$\begin{aligned} \frac{\partial v_i}{\partial d_j} &= -\phi \left(-\frac{2}{d_j^2} + \frac{2}{d_j^3}\right) \cdot vi^2 = \phi \left(\frac{2(d_j - 1)}{d_j^3}\right) \cdot v_i^2 \geq 0 \\ \frac{\partial \tilde{g}_{ij}}{\partial d_j} &= -\frac{(d_j - 2)}{d_j^3} \left(1 + \sum_{z \neq \{i,j\}} g_{ij} g_{jz}\right) \begin{cases} \leq 0 & d_j \geq 2 \\ > 0 & d_j = 1 \end{cases} \end{aligned} \tag{45}$$

Lastly, purely indirect connections' degrees, i.e. those who are connected to i only trough a common friend, have an effect on i's demand that depends only on he common friend's degree. That is, the effect of trader k's degree d_k such that $g_{ij} = 1, g_{jk} = 1, g_{ik} = 0$ for all $j, k \in N$ is

$$\frac{\partial \tilde{g}_{ik}}{\partial d_k} = - \sum_{j \neq \{k,i\}} g_{jk} g_{ij} \frac{(d_j - 2)}{d_j^3} \leq 0 \qquad since \quad d_k \geq 2 \tag{46}$$

The price effect:

The elasticity of demand of each trader is given by his individual network effect $v_i(\phi)$:

$$\frac{\partial q_{i,1}}{\partial P_1} = -v_i(\phi) \tag{47}$$

This has two implications. First, the higher $v_i(\phi)$, the more elastic is a trader's PM demand. Traders respond negatively but in different magnitude to changes in the PM price. Second, the slope of a trader's demand schedule changes as P_1 varies. This is precisely because each P_1 induces a different network game and traders' best-replies are game-specific.

Figure G.1 depicts the price effect. In the left-hand graph, each colored line is a trader's demand curve for a given price P_1. For example, the orange line is the schedule when $P_1 = 0.9$. Clearly, as P_1 increases the demand curve becomes steeper. The right-hand panel compares demand curves to two traders i (orange) and j (blue) such that $v_i(\phi) > v_j(\phi)$.

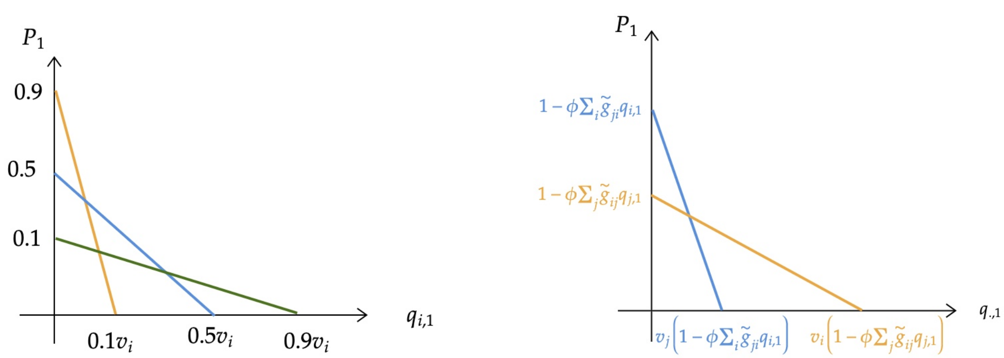


Figure G.1: Best-replies and Primary market price

Appendix H Bilateral Trading Comparison

In this section, I discuss the equilibrium outcomes if only bilateral trade was allowed. Maintaining the assumptions of the model, this would be the case of a regular network with degree two, that is, a network with regular components of size two. Using the results for regular networks, PM equilibrium price and asset consumption are $P_1^* = 1 - \frac{\bar{Q}}{N}(1+2\phi)$ and $q_1^* = \frac{\bar{Q}}{N} \;\; \forall i \in N$, respectively. Any local market equilibrium is given by price and asset consumption, $P_2^* = 1 - 2\frac{\bar{Q}}{N}$ and $q_2^* = q_1^* \;\; \forall i \in N$.

Bilateral trade delivers the *lowest* possible prices in the primary market and all local markets. In equilibrium, asset consumption is the same in both periods even though their demands are different. A trader's demand schedule in the PM is $q_1 = \frac{1-P_1}{1+2\phi}$ and in the local market is $q_2 = \frac{P_1}{1+2\phi} - P_s$.

Traders' expected total asset holdings is simply the average supply $\frac{\bar{Q}}{N}$. Also, traders' expected utility is given by

$$\begin{aligned} EU &= w + (1 - P_1^*)q_1^* - \frac{1+2\phi}{2}(q_1^*)^2 \\ &= w + \frac{(1+2\phi)}{2}\left(\frac{\bar{Q}}{N}\right)^2 \end{aligned}$$

With bilateral trade, if the restriction in the shock $\phi < \frac{1}{N}$ is discarded, local market price can be greater than the primary price.

Proposition 13. *Bilateral Trade and Difference in Prices*

If the trading network only allows for bilateral trades, equilibrium prices in the PM and any local market are $P_1^* = 1 - \frac{\bar{Q}}{N}(1+2\phi)$ and $P_2^* = 1 - 2\frac{\bar{Q}}{N}$, respectively. Traders consume the same amount of asset shares in both periods.

The negative demand shock ϕ controls the capital gain/loss of the seller. Selling bilaterally delivers capital loss (gain) if ϕ is less (greater) than $\frac{1}{2}$. If $\phi = \frac{1}{2}$, prices are equal.

It is useful to see the equilibrium in matrix notation for a direct comparison with my main result. First, demand schedule coefficients are $v_i(\phi) = (1+\phi)^{-1} \;\; \forall i$ and $\tilde{g}_{ij} = 1 \;\; \forall i, j : g_{ij} = 1$ and zero otherwise. This means that the global network matrix $\tilde{\boldsymbol{G}}$ equals the adjacency matrix $\boldsymbol{G}$ and that $\boldsymbol{V}\tilde{\boldsymbol{G}}$ is simply $(1+\phi)^{-1}\boldsymbol{G}$. Trading centrality simplifies to $\boldsymbol{C} = \frac{1}{1+\phi}\left(\mathbf{1} + \frac{\phi}{1+\phi}\boldsymbol{G}\right)^{-1}\mathbf{1}_N$.

Appendix I Results

Degree Distribution

I study changes in the trading network structure by changing its degree distribution: that is, the number of connections of each trader. I do two exercises based on stochastic dominance: varying the average degree and the degree variance. Specifically, I compare two degree distributions such that i) one is a first-order stochastic shift of the other; and ii) one is a mean-preserving spread of the other.

Recall the stochastic dominance definition: a cumulative distribution F first-order stochastically dominates (FOSD) another distribution G iff $F(x) \leq G()$ for all x. Note that if F FOSD G, then F necessarily has a strictly larger expected value than G, $EF(x) > EG(x)$ (the reverse not true). Recall also the mean-preserving spread definition: F is a mean-preserving spread of G if and only if $EF(x) = EG(x)$ and $\int F(x)dx \leq \int_{\infty}^{c} G(x)dx$ for all c. Note that if F is a mean-preserving spread of G than F necessarily has a larger variance than G.

Denote the degree distribution of the trading network as P. Consider a change in the probability distribution over the degrees to P' that reflects an unambiguous increase in connectivity. In particular, suppose that P' FOSD P. Then, the average degree under P' is higher than under P. Moreover, each trader's degree under P' is at least as large as his degree under P'. I am interested in how traders' demands, and hence price, changes as the trading network shifts from P to P'.

Looking at network coefficients in (11) and (12), we know that $v_i(\phi)$ weakly increases while $\tilde{g}_{ij}$ weakly decreases for all $i, j \in N$ (see Appendix G). Then, from the demand function (10), each trader demands more at each possible price level, i.e. his demand schedule becomes flatter. Since PM price is increasing in traders' demands, PM price increases as well.

Network Symmetry

A symmetric network, or regular graph, is such that degree of each node is equal. A graph is called k-regular if degree of each node is k. Regular graphs have useful properties. First, the necessary and sufficient conditions for a k-regular graph with N nodes to exist are that $N \geq k + 1$ and that Nk is even. Second, the number of links E is given by $\frac{Nk}{2}$. It is also known that for any undirected graph E also relates the the sum of individual degrees such that $\sum_i d_i = 2E$. These four facts, combined to the findings regarding the degree distribution, lead to Proposition 6.

Take a k-regular network with N traders and, thus, $\frac{Nk}{2}$ links. Now consider any other network structure with the same number of traders and links. It must hold that the average degree in this network is such that $\frac{1}{N}\sum_i d_i = 2\frac{k}{2} = k$, i.e. the same as the in k-regular graph. And that its degree variance is positive, otherwise it would be a regular network (i.e. with zero variance). Thus, from Proposition 4, it holds the PM price in this network is lower than the one in the k-regular network.

Appendix J Core-Periphery Networks

A core-periphery network structure typically consists of a well-connected set of nodes, the core, and the remainder nodes, the periphery, well connected to the core but sparsely connected internally. The most common example is the star network in which one node is fully connected to all other nodes, who themselves are only connected to the core.

?? shows that the star network is the unique structure delivering capital gains for a seller while exhibiting the lowest primary market price. At the same time, empirical evidence has documented a core-periphery structure for different inter-trader markets. Motivated by these two facts, in this section of the Appendix I provide detailed results and proof for the class of core-periphery networks. In particular, I focus on the star graph, regular core-periphery networks, and the most extreme cases of a fully connected core (the complete case) and the sparsely connected core (the ring case).

J.1 Star Network

In this part of the appendix, I prove the following lemmas.

Lemma 9. *Across all markets and across networks of the same size N, the core's price of the star network is the highest.*

Another interesting feature of the star network is that it is the most unequal: it delivers the highest dispersion in asset allocation. Traders located in the periphery shift their asset consumption relatively more to the local market even though the seller's price is high. That's because, if a periphery is shocked, his selling price is so low that his capital loss would be larger than the difference in prices between the two markets he can act as a buyer. The next proposition state this result.

Proposition 14. *Inequality in a Star Trading Network*

Across trading networks of the same size N, the star structure delivers the highest dispersion in asset allocation. The core (periphery) has the highest (lowest) possible primary market demand.

Proposition 15. *Demand inequality is decreasing in the size of the star network N.*

Core's network coefficients are

$$\begin{aligned} v_c^{-1} &= \frac{2\phi}{N-1} + N(1-\phi) = \frac{2\phi + (N^2 - N)()1 - \phi}{N-1} \\ \tilde{g}_{cp} &= 1 \end{aligned}$$

and his demand function is then

$$q_c = v_c \left(1 - P_1 - \phi(N-1)q_p\right) \tag{48}$$

Peripheries' network coefficients are

$$\begin{aligned} v_p^{-1} &= 1 + \phi\frac{(2N-3)}{(N-1)^2} \\ \tilde{g}_{pc} &= 1 + \frac{(N-2)}{(N-1)^2} = \frac{N^2 - N - 1}{(N-1)^2} \\ \tilde{g}_{pc} &= \frac{N-2}{(N-1)^2} \end{aligned}$$

and their demand function is then

$$q_p = v_p \left(1 - P_1 - \phi\tilde{g}_{pc}q_p - \phi(N-2)\tilde{g}_{pp}q_p\right) \tag{49}$$

Now I can write the system of demands in 2-by-2 matrix format in which the first row/column refers to the core and the second row/column to a periphery. Matrices $\mathrm{V}, \tilde{\boldsymbol{G}}$ are given by

$$\left(\mathrm{V}^{-1} + \phi\tilde{\boldsymbol{G}}\right) = \begin{pmatrix} \frac{1}{v_c} & \phi(N-1) \\ \phi\tilde{g}_{pc} & \frac{1}{v_p} + \phi(N-1)\tilde{g}_{pp} \end{pmatrix} \tag{50}$$

and trading centrality is then

$$\boldsymbol{TC} \equiv \begin{pmatrix} C_c \\ C_p \end{pmatrix} = \left(\mathrm{V}^{-1} + \phi\tilde{\boldsymbol{G}}\right)^{-1} 1 = \frac{1}{\Delta}\begin{pmatrix} \frac{1}{v_p} + \phi(N-1)\tilde{g}_{pp} - \phi(N-1) \\ \frac{1}{v_c} - \phi\tilde{g}_{pc} \end{pmatrix} \tag{51}$$

where $\Delta \equiv det\left(\left(\mathrm{V}^{-1} + \phi\tilde{\boldsymbol{G}}\right)^{-1}\right)$ and it holds that $\frac{1}{v_p} + \phi(N-1)\tilde{g}_{pp} - \phi(N-1) > \frac{1}{v} - \phi\tilde{g}_{pc}$ Equilibrium is determined by the weighted sum of centralities,

$$C_T \equiv \frac{1}{\Delta}\begin{pmatrix} \frac{1}{v_p} + \phi(N-1)\tilde{g}_{pp} - \phi(N-1) & \frac{1}{v} - \phi\tilde{g}_{pc} \end{pmatrix}\begin{pmatrix} 1 \\ N-1 \end{pmatrix} \tag{52}$$

Thus primary market price and asset allocation are, respectively

$$P_1^* = 1\frac{\bar{Q}}{C_T}$$
$$q_i^* = \frac{\bar{Q}}{C_T}C_i \quad i = \{c, p\}$$

J.2 Core-Periphery Networks

Growing the size of the core:

The next figures depicts Proposition 8. Figure J.1 shows the equilibrium of the model for growing core-periphery networks by increasing the number of N traders. The number of core traders, with the same connectivity to 2 peripheries, increases.

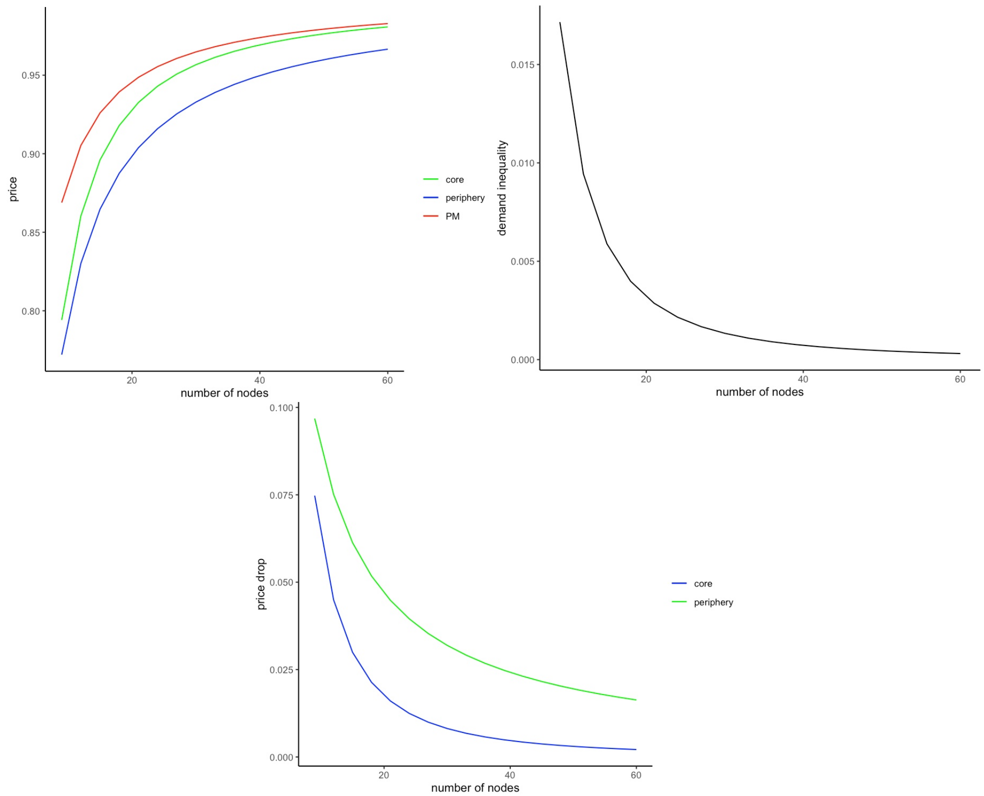


Figure J.1: Growing a Core-Periphery network: increasing N by adding more core traders with the same number of peripheral connections.

Similarly, Figure J.2 shows the equilibrium for different core-periphery network with the same number of $N = 12$ traders and different core sizes. The dashed lines are equilibrium outcomes for the star network of the same size $N = 12$.

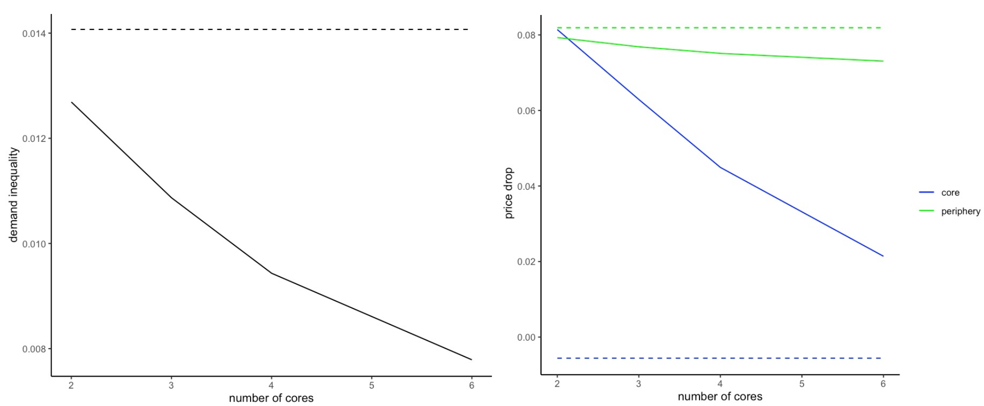


Figure J.2: Growing a Core-Periphery network: for a fixed $N = 12$ traders, the core size increases by moving a peripheral node to the core while keeping cores' connectivity to the periphery homogeneous

To make it clear the difference in the two exercises above, the first is moving from network B to C below. The second is moving from network A to B.

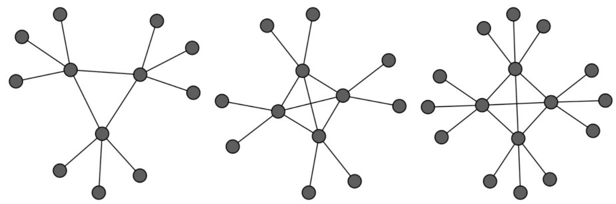


Figure J.3: Growing a Core-Periphery network: from left to right - network A, B and C

J.3 Regular Core-Periphery Network

A regular core-periphery network structure is quite tractable since cores' share the same demand, and so do peripheries. Thus I just need to keep track of two variables, a core's demand $q_{,1c}$ and a periphery's demand $q_{,1p}$.

The primitives for an arbitrary core-periphery structure are i) the number of cores n_c; ii) the number of peripheries *per core* n_p. So there are $(n_c n_p)$ peripheries and $N = n_c(1+n_p)$ nodes. Notice that core's degree is then $d_c = n_p + (n_c - 1) = N - (n_c - 1)n_p$.

Suppose $n_c \geq 2$. Let $\tilde{g} \equiv \frac{(d_c-1)}{d_c^2}, \quad \tilde{g}_c = \tilde{g}d_{cc} + \frac{1}{d_c}$. Then, cores and peripheries' demand schedules (10) are, respectively

$$\begin{aligned} q_c &= v_c \left[1 - P_1 - \phi\tilde{g}_c \left(d_{cc}q_c + n_p q_p\right)\right] \\ q_p &= v_p \left[1 - P_1 - \phi\tilde{g}_p \left((d_{cc}+1)q_c + (n_p-1)q_p\right) - \phi q_c\right] \end{aligned} \tag{53}$$

The network-induced coefficients in (10), $\{(v_i, \tilde{g}_{ij})\}_{i,j\in N}$, become for a core

$$\begin{aligned} v_c &= \left[\frac{2\phi}{d_c} + 1 - \phi(d_c+1) + \phi d_{cc}\left(\tilde{g} + \frac{1}{d_c}\right) + \phi n_p\right]^{-1} \\ \tilde{g}_{cc} &= \tilde{g}_c d_{cc} \\ \tilde{g}_{cp} &= \tilde{g} n_p \end{aligned}$$

and for a periphery

$$v_p = \left[1 + \phi\left(\tilde{g} + \frac{1}{d_c}\right)\right]^{-1}$$
$$\tilde{g}_{pc} = \tilde{g}(d_{cc} + 1) + 1$$
$$\tilde{g}_{pp} = \tilde{g}(n_p - 1)$$

That is, demands are $q_c = v_c\left[1 - P_1 - \phi\tilde{g}_{cc}q_c - \tilde{g}_{cp}q_p\right] \quad \forall c \in N$ and and $q_p = v_p\left[1 - P_1 - \phi\tilde{g}_{pc}q_c - \tilde{g}_{pp}q_p\right] \quad \forall p \in N$.

Now I can write the system of demands in matrix format. Define 2-by-2 matrices Ψ_1, Ψ_2 in which the first row/column refers to a core and the second row/column to a periphery,

$$\Psi_1 = \begin{pmatrix} \psi_{1c} & 0 \\ 0 & \psi_{1p} \end{pmatrix} \quad \Psi_2 = \begin{pmatrix} 0 & \psi_{2c} \\ \psi_{2p} & 0 \end{pmatrix} \tag{54}$$

such that $\psi_{1c} \equiv \left[\frac{1}{v_c} + \phi\tilde{g}_c d_{cc}\right]$, $\quad \psi_{1p} \equiv \left[\frac{1}{v_p} + \phi\tilde{g}(n_p - 1)\right]$, and $\psi_{2c} \equiv \tilde{g}_c n_p$, $\quad \psi_{2p} \equiv \tilde{g}(d_{cc} + 1) + 1$.

Then the system of demands is

$$\begin{aligned} \Psi_1\boldsymbol{q} &= (1 - P_1)\mathbf{1}_2 - \phi\Psi_2\boldsymbol{q} \\ (\Psi_1 + \phi\Psi_2)\,\boldsymbol{q} &= (1 - P_1)\mathbf{1}_2 \\ \boldsymbol{q} &= (\Psi_1 + \phi\Psi_2)^{-1}\,\mathbf{1}_2(1 - P_1) \end{aligned} \tag{55}$$

(55) is the counterpart of (34). Since Ψ_1, Ψ_2 are 2-by-b2 taking the inverse is easy:

$$(\Psi_1 + \phi\Psi_2)^{-1} = \frac{1}{\psi_{1c}\psi_{1p} - \psi_{2c}\psi_{2p}}\begin{pmatrix} \psi_{1p} & -\phi\psi_{2c} \\ -\phi\psi_{2p} & \psi_{1c} \end{pmatrix}$$

Trading centrality (20) is now given by

$$\begin{aligned} \boldsymbol{c} &= (\Psi_1 + \phi\Psi_2)^{-1}\,\mathbf{1}_2 \\ &= \frac{1}{\psi_{1c}\psi_{1p} - \psi_{2c}\psi_{2p}}\begin{pmatrix} \psi_{1p} - \phi\psi_{2c} \\ \psi_{1c} - \phi\psi_{2p} \end{pmatrix} \end{aligned} \tag{56}$$

where it holds that core's centrality is higher than periphery' centrality:

$$\psi_{1p} - \phi\psi_{2c} > \psi_{1c} - \phi\psi_{2p}$$

The next proposition gives the equilibrium for a regular core-periphery network with n_c cores, $d_c n_p$ peripheries and connectivity among cores of d_{cc}.

Proposition 16. *Equilibrium in a Regular Core-Periphery Network*

Consider regular core-periphery network with n_c cores, $d_c n_p$ peripheries and connectivity among cores of d_{cc}. Suppose $n_c \geq 3$ and $d_{cc} \geq 2$. Then, trading centrality is given by

$$\boldsymbol{C} = (\Psi_1 + \phi\Psi_2)^{-1}\,\mathbf{1}_2 \tag{57}$$

Primary Market equilibrium price and demands are, respectively

$$P_1^* = 1 - \bar{Q}\frac{1}{C_c n_c + C_p n_p} \tag{58}$$

$$q_{i,1} = \bar{Q}\frac{C_i}{C_T} \quad i = \{c, p\} \tag{59}$$

where $C_T = C_c n_c + C_p n_p$.

To understand how equilibrium change as we change the structure of the core-periphery graph one must look at the trading centrality. One can show that: i) cores' (peripheries') centrality is decreasing (increasing) in cores' connectivity; ii) both centralities are increasing in the number of cores and/or peripheries.

Focusing on the extremes structures - complete and ring cores, cores' (peripheries') centrality is decreasing (increasing) in in the number of the cores and/or peripheries.

Complete and Ring Cases

The two most extreme cases of regular core-periphery networks are i) when the core is fully connected (complete); and ii) when each core is connected to other two (ring). I now compare primary market equilibrium in these two core-peripheries structure. In particular, I show that:

- Price is higher in the complete core than in the ring core, for any number of cores and peripheries (from the main result)

- Cores' (peripheries') centrality is lower (higher) in the complete structure
- Cores' (peripheries') demand is lower (higher) in the complete structure
- Demand dispersion (inequality) is lower (higher) in the complete (ring) structure

In addition, I study how the comparison changes as the number of peripheries changes. Increasing the number of peripheries results in

- Lower price difference between the complete and ring structures - even though price increases in both
- Lower difference in demand dispersion - inequality decreases in both

I obtain these results using Proposition 16 that shows that characterizing the equilibrium in any core-periphery structure is quite straightforward. One only needs to determine two variables: the demand schedules for a core and a periphery. For the complete case, notice that $d_{cc} = n_c - 1$ and $\tilde{g}_c = (n_c - 1)\tilde{g} + \frac{1}{d_c}$. Then, cores and peripheries' demands are, respectively

$$\begin{aligned} q_c &= v_c \left[1 - P_1 - \phi \tilde{g}_c \left((n_c - 1) q_c + n_p q_p\right)\right] \\ q_p &= v_p \left[1 - P_1 - \phi \left(\tilde{g} n_c + 1\right) q_c - \phi \tilde{g} (n_p - 1) q_p\right] \end{aligned} \tag{60}$$

For the ring case, $d_{cc} = 2$ $\tilde{g}_c = 2\tilde{g} + \frac{1}{d_c}$. Then, cores and peripheries' demand schedules are, respectively

$$\begin{aligned} q_c &= v_c \left[1 - P_1 - \phi \tilde{g}_c \left(2 q_c + n_p q_p\right)\right] \\ q_p &= v_p \left[1 - P_1 - \phi \tilde{g}_p \left(3 q_c + (n_p - 1) q_p\right) - \phi q_c\right] \end{aligned} \tag{61}$$

The next proposition shows that as the number of cores become too large, price in these structure converge to the same level.

Proposition 17. *As $n_c \to \infty$, then $P_1^{complete} - P_1^{ring} \to 0$ irrespective of the number of peripheries.*

Appendix K Symmetric Networks

In regular networks, all nodes have the same number of links and position in the network. It immediate follows from Theorem 1 that traders have the same demand and selling price if and only if they have the same network position. These two facts make it easier to study regular networks. Equilibrium asset allocation is independent of the network structure, and it is the same as in the Walrasian market. PM and local market demands are, respectively, $q^*_{i,1} \equiv q^*_1 = \frac{Q}{N}, q^*_{i,s} \equiv q^*_2 = \frac{\bar{Q}}{Nd}$. The local market price is exclusively determined by traders' degree d, $P^*_s = 1 - \frac{(d+1)}{d}\frac{\bar{Q}}{N}$.

Proposition 18. *Primary Market in Regular Networks*

In regular networks, PM price is increasing in the size and degree of the network.

Differently, primary market equilibrium allocation is independent of the network. It is the same as in the Walrasian market: all traders demand $\frac{\bar{Q}}{N}$ in the primary market, and $\frac{\bar{Q}}{Nd}$ in any local market.

Proposition 18 shows that, for a fixed number of N traders, the higher is traders' degree the higher is the PM price. Similarly, for a fixed degree d, increasing the size of the network increases PM price.

The demand schedule in every market is homogeneous across traders: any asset supply is divided equally among buyers, as they have the same willingness to pay. In other words, asset allocation is the same as in a perfectly competitive market. As I show next, PM price varies considerably across regular networks and it is never equal to the price of a perfectly competitive market. The results in this section highlights that if we ignore asset issuance price and only look at traders' inventories in OTC we are missing important considerations of funding costs.

Using market clearing conditions, equilibrium asset allocation in the PM and local market are, respectively: $q^*_{i,1} \equiv q^*_1 = \frac{\bar{Q}}{N}$ and $q^*_{i,s} \equiv q^*_2 = \frac{\bar{Q}}{Nd}$. The local market price is exclusively determined by traders' degree as d: $P^*_s = 1 - \frac{\bar{Q}}{N}\left(\frac{d+1}{d}\right)$

Turning to primary market equilibrium, first notice that individual network coefficient v_i becomes $v_i^{-1} = v^{-1} = \frac{\phi + d(1+\phi-\phi d)}{d}$ and, in turn, trading centrality for every trader is $c_i = \frac{d}{\phi + d(1+\phi)} \forall i \in N$. Then, the traders' demand schedule is $q_1 = v(1 - P_1 - \phi d q_1) = \frac{d}{\phi + d(1+\phi)}$. And equilibrium PM is thus $P^*_1 = 1 - \frac{\bar{Q}}{N}\left(\frac{\phi + d(1+\phi)}{d}\right)$. As P^*_1 is increasing degree d, it is easy to compare different regular networks.

K.1 Complete Graph

The complete network is a special case of a regular network with degree $d = N - 1$. No trading frictions exist since all traders are connected with one another. Everyone trade in both markets, either as a buyer or a seller. Using the result from Proposition 18, primary market price is $P_1^* = 1 - \frac{\bar{Q}}{N(N-1)}(N(1+\phi) - 1)$, and any local market equilibrium price is $P_s^* = 1 - \frac{\bar{Q}}{N-1}$. Demands in each market are, respectively,$q_1 = \frac{\bar{Q}}{N}$ and $q_2 = \frac{\bar{Q}}{N(N-1)}$.

It is easier to see that the complete network delivers the highest primary market price.

Proposition 19. *Complete Network*

For a fixed network size N and in comparison with other regular networks, the Complete Network exhibits the highest primary market price.

It is worth pointing out that the static, competitive market is different than an economy with an empty network, i.e. without any trading relationships Even though there is no local market in both scenarios, in an empty network agents still face the negative demands shock. This risk makes demand schedules less elastic and drives PM price down. In equilibrium, PM demand and price are, respectively, $q_{i,1}(P_1) = q_1(P_1) = 1 - \frac{1}{1-\phi}P_1 \;\; \forall i \in N$ and $P_1^* = (1-\phi)\left(1 - \frac{\bar{Q}}{N}\right)$.

Appendix L Ring versus Star Trading Networks

The conclusion from the study of core-periphery networks (Section 6.2) is that prices and demands are tightly related to the number of traders in the core of core-periphery networks. Meanwhile, PM price is increasing the number of traders and it is determined by how they are connected among themselves. This suggest that the effect of growing the trading network on equilibrium outcomes depends on the way new traders and their linkages are added in the network. That is, there exists a size effect - increasing N, and a network effect - changing the degree distribution.

One way to differentiate between the size effect and network effect on PM price is to look on how price changes as we grow a ring network and a star network. The former is a regular network.[84] It exhibits no inequality in terms of degree and asset allocation: all traders have the same demands and selling price as they have the same degree and network position. The star network is the most unequal one with respect to both degree and demand.

[84] Appendix K provides equilibrium outcomes for the general class of regular networks.

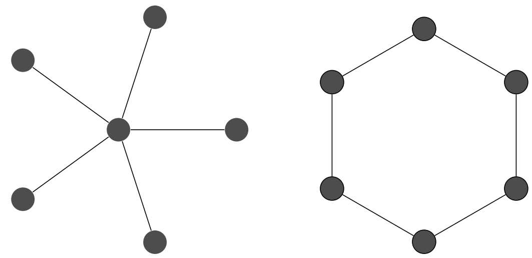


Figure L.1: A ring and a star network of the same size $N = 6$ traders.

The unique effect of growing the ring network is about market size. The growth of the star network, apart from capturing the size effect, carries network effect because the degree distribution changes. Adding one trader in either case means adding just one more link. However, degree inequality and connectivity increase in the star network, while the degree distribution remains unchanged in the ring.[85]

It is useful to first compare market outcomes as the networks grow. By Lemma 5, we know that PM and local market prices in both networks increase. The difference is that in the ring price drops while in the star price can either increase or decrease: price increase if the core is selling and drops if one of the peripheries is the seller.

Even though PM price increases as the network becomes larger, it does so as diminishing rates. More importantly, it grows faster in the star network.

The results above are depicted in the next figure. It shows PM price drop and PM price growth rate as each structure grows.

[85]Notice that the number of linkages in the ring network is N and in the star network $N - 1$, and connectivity is higher in the ring network.

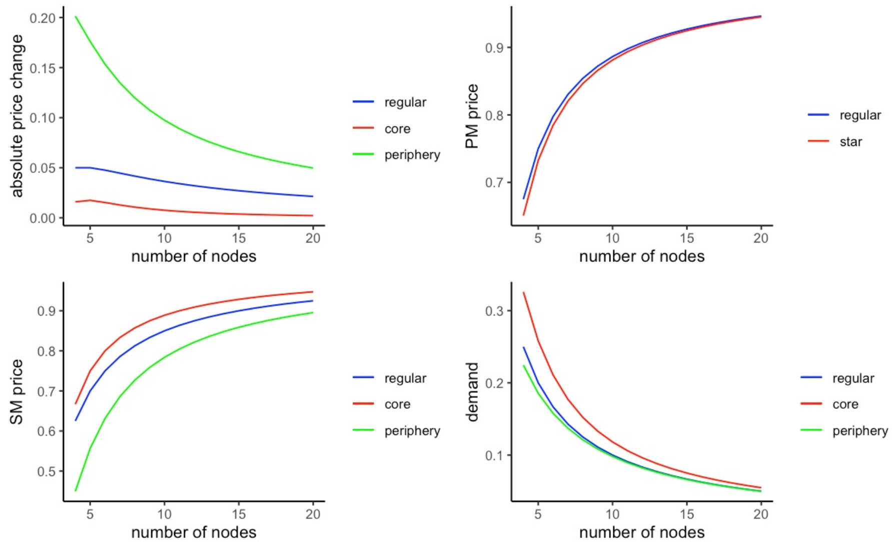


Figure L.2: Growing Networks: Ring versus Star structures

If we divide the star growth rate by the ring growth rate we isolate the network effect. I find that the network effect is positive any finite N, and that it is greater the smaller the size of the trading network.

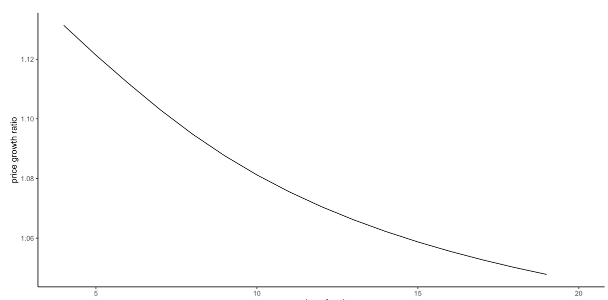


Figure L.3: PM price growth ratio: Star/Ring

This finding is important. The network effect in the star network makes its price diverge from the one in the ring. The importance of this effect though diminishes as the network grows.

More details

Even though PM price increases as both network structures becomes larger, its growth rate differs. To see this, look at how price grows as each structure grows:

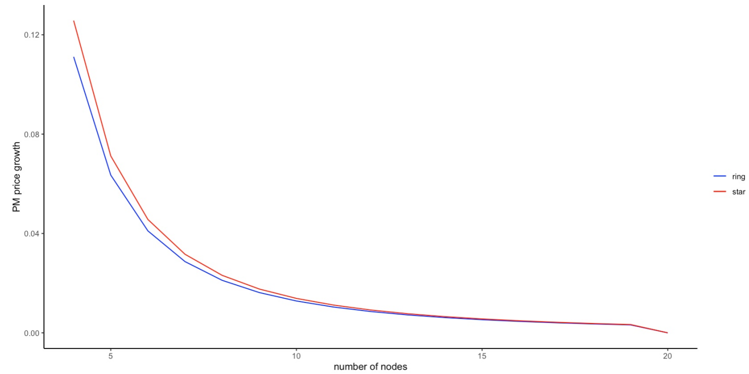


Figure L.4: PM Price growth as the network becomes larger: Ring versus Star structures

Another way to see this is to look at the price ratio of the PM price in star growth by the one in the ring:

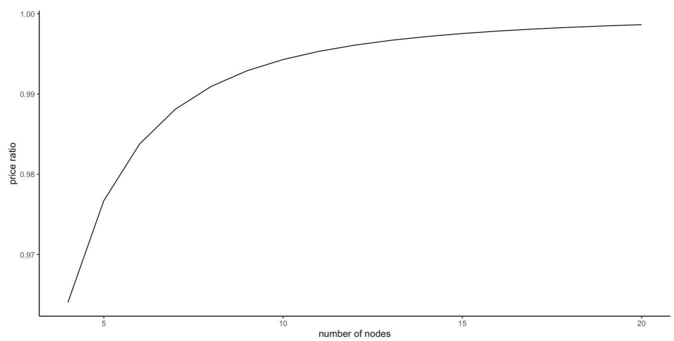


Figure L.5: PM price ratio: Star/Ring

Mathematically, price growth ratio is

$$\left(\frac{P^{star}_{1,N+1} - P^{star}_{1,N}}{P^{star}_{1,N}}\right) \Big/ \left(\frac{P^{ring}_{1,N+1} - P^{ring}_{1,N}}{P^{ring}_{1,N}}\right) = \frac{P^{star}_{1,N+1} - P^{star}_{1,N}}{P^{ring}_{1,N+1} - P^{ring}_{1,N}} \cdot \underbrace{\frac{P^{ring}_{1,N}}{P^{star}_{1,N}}}_{>1}$$

We already know that $P^{ring}_{1,N} > P^{star}_{1,N}$. We now find that, due to the change in the network degree distribution, price increases more in the star network than in the ring network as the structure grows. Notice that, in both structures, the price growth is positive but decreasing: price grows with the network at a diminishing rate.

L.1 Growing the Star Network

Even though growing the star leads to a more unequal network in terms of degree, the opposite occurs for demand inequality. The peripheral traders buy relatively more, i.e. become relatively more important in the PM, when this group is large. Difference in demands become smaller. Thus, core's capital gain decreases.

Proposition 20. *Growing a Star Trading Network*

For $N > 3$, core's capital gain and demand inequality decreases as the star network grows.

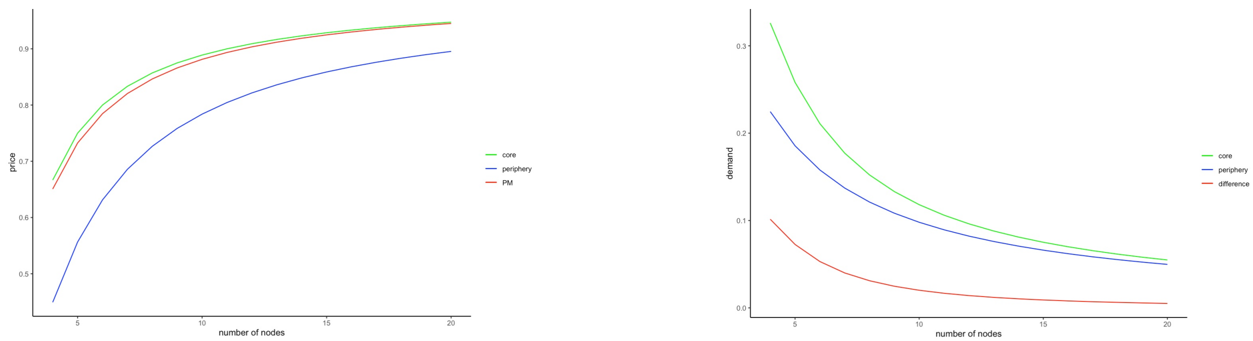


Figure L.6: Growing a Star Network

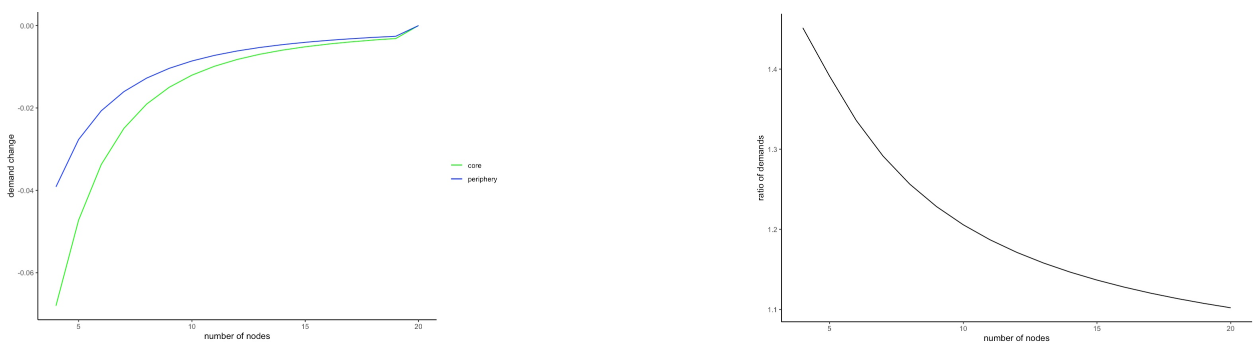


Figure L.7: Demand change and the size of the star network

L.2 Comparing with the Complete network

I find that:

- Complete local market price and the core's price are the same. That's because, the increase in core's demand and the reduction in peripheries' demand exactly compensate each other.

  That is, for a network with N traders,

$$|q_{1,regular} - q_{1,core}| = (N-1) \cdot |q_{1,regular} - q_{1,periphery}|$$

- Complete and ring's demands are the same. And both less than the core's and higher than the peripheries'.

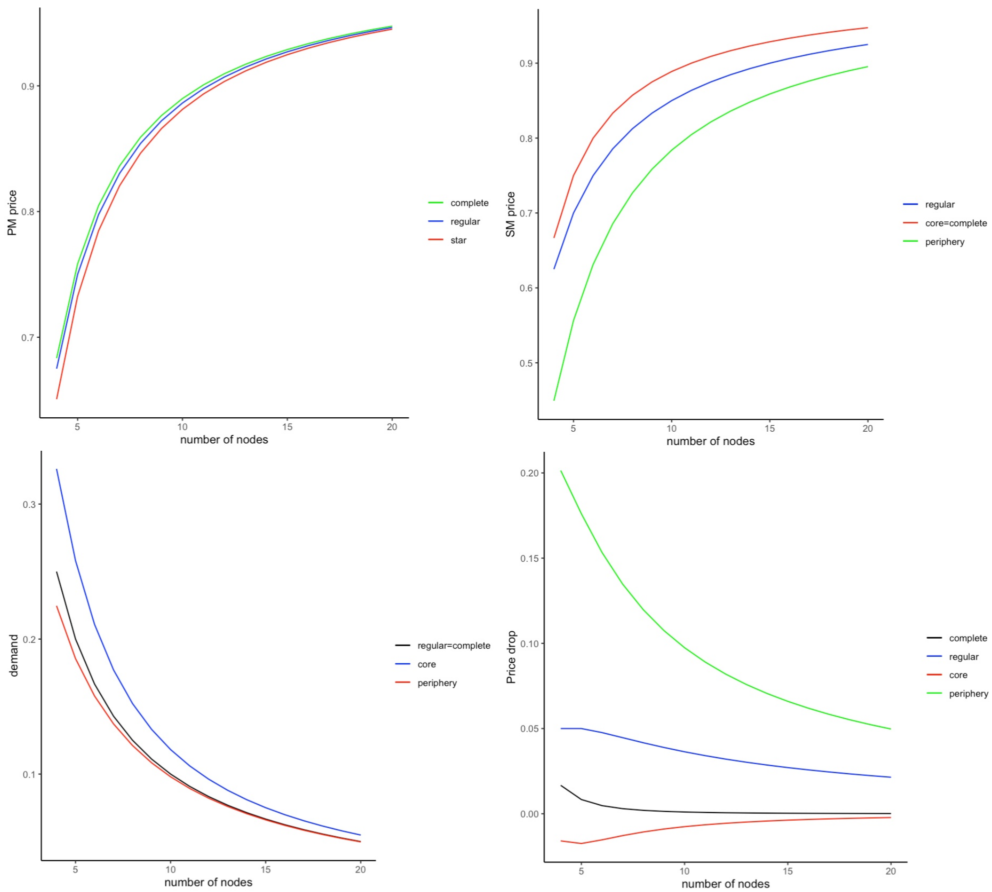


Figure L.8: Growing Networks: Ring versus Star structures

Appendix M Welfare

The expected indirect utility EU_i^* of each trader $i \in N$ is given by eq. (25)

$$EU_i^*\left(\boldsymbol{c}; \boldsymbol{G}, \phi, \bar{Q}\right) = w + \left(\frac{Q}{c_A}\right)^2 \left[c_i - \frac{1}{2v_i}c_i^2 - \phi c_i \sum_j \tilde{g}_{ij} c_j\right] + \phi \left(\frac{Q}{c_A}\right) \left[\sum_j \bar{g}_{ij} c_j\right] \qquad (25)$$

I analyze welfare as the sum of traders' indirect expected utility, $EU^* \equiv \sum_i EU_i^*\left(\boldsymbol{c}; \boldsymbol{G}, \phi, \bar{Q}\right)$:

$$EU^{*} = Nw + \left(\frac{Q}{c_A}\right)^2 \sum_i c_i - \frac{1}{2}\left(\frac{Q}{c_A}\right)^2 \sum_i \frac{c_i^2}{v_i} - \phi\left(\frac{Q}{c_A}\right)^2 \sum_i \left(c_i \sum_j \tilde{g}_{ij} c_j\right) + \phi\left(\frac{Q}{c_A}\right) \sum_i \left[\sum_j \bar{g}_{ij} c_j\right]$$

The above can be simplified to

$$\begin{aligned} EU^{*} &= Nw + \frac{Q^2}{2c_A} - \frac{Q^2}{c_A^2} \sum_i \sum_{j\neq i} \bar{g}_{ij} c_j - \frac{3}{2}\frac{Q^2}{c_A^2}\phi \sum_i \left(c_i - \frac{c_i^2}{v_i}\right) \\ &= Nw + \frac{Q^2}{2c_A} - \frac{Q^2}{c_A^2} \sum_i c_i \sum_{j\neq i} \bar{g}_{ji} - \frac{3\phi}{2}\frac{Q^2}{c_A^2}\phi \sum_i c_i + \left(\frac{3\phi}{2}\frac{Q^2}{c_A^2}\right) \sum_i \frac{c_i^2}{v_i} \\ &= Nw + \frac{Q^2}{2c_A} - \frac{Q^2}{c_A^2} \sum_i c_i \left(\frac{3\phi}{2} + \sum_{j\neq i} \bar{g}_{ji}\right) + \left(\frac{3\phi}{2}\frac{Q^2}{c_A^2}\right) \sum_i \frac{c_i^2}{v_i} \\ &= Nw + \frac{Q^2}{2c_A} - \phi\frac{Q^2}{c_A^2} \sum_i \left[c_i \left(\frac{3\phi}{2} + \sum_{j\neq i} \bar{g}_{ji}\right) - \frac{3\phi}{2}\frac{c_i^2}{v_i}\right] \end{aligned} \tag{62}$$

Hence, welfare $EU\left(\boldsymbol{c}; \boldsymbol{G}, \phi, \bar{Q}\right)$ in a trading network $\boldsymbol{G}$, given a shock and supply parameters $\phi, \bar{Q}$, is given by

$$EU\left(\boldsymbol{c}; \boldsymbol{G}, \phi, \bar{Q}\right) = Nw + \frac{Q^2}{2c_A} - \phi\frac{Q^2}{c_A^2} \sum_i \left[c_i \left(\frac{3\phi}{2} + \sum_{j\neq i} \bar{g}_{ji}\right) - \frac{3\phi}{2}\frac{c_i^2}{v_i}\right] \tag{25}$$

The welfare analysis is focused on four network structures depicted in fig. 11: the complete graph, the line, the star and ring. The following results are used to proof the main results in Section 7.

Welfare comparison across these network structures is easier because we can invoke Proposition 3 and Proposition 4. Notice that the degree distribution of the complete network FOSD all the other ones; the degree distribution of the ring network FOSD the line and the star ones; and the degree distribution of the line FOSD the star one. Moreover, degree distribution of the star is mean-preserving spread of the line one. It is straightforward then that PM price rank is: $complete > ring > line > star$.

When trading centrality is increasing in degree, a trader's expected utility is increasing in his centrality. In turn, we can rank traders' welfare within a network: more central traders achieve higher expected utility.

Lemma 10. *Individual Welfare and Degree*

If trading centrality c_i of trader $i \in N$ is monotonically increasing in his degree d_i, then i's indirect expected utility EU_i^* is monotonically increasing in his centrality c_i and, thus, his degree d_i.

We can compare the complete network and the ring (or any other regular structure). The complete network has higher (average) trading centrality and thus prices. In turn, PM price in the complete network is higher than in the ring. And so is welfare.

For the structural effect, take the complete and the star. The trader in core of the star is fully connected in both networks. But in the star his connections are poorly connected, which pushes his centrality up and others' centrality down. Consequently, a trader as the core has higher expected utility in the complete network.

Lemma 11. *Welfare and Trading centrality*

Aggregate expected utility is decreasing in the aggregate trading centrality c_A.

High aggregate trading centrality implies high PM price. This is the main reason behind Proposition 10.

Another interesting question is to find the welfare maximizing trading network $\boldsymbol{G}^*$ delivering the highest aggregate expected utility. $\boldsymbol{G}^*$ is the solution to the following maximization problem,

$$\boldsymbol{G}^* = \arg\max_{\boldsymbol{G}} \quad EU^* \left(\boldsymbol{G}, \phi, Q\right) \tag{63}$$

Solving (63) is hard. Due to its dependence on trading centrality, welfare shares the property of non-monotonicity with respect to connectivity and degree inequality. Moreover, since trading centrality is a recursive measure (eq. (14)), it is affected by a trader's own degree and also the centrality and degree of other traders. It holds that, for all $i, j \in N$, $\frac{\partial c_i}{\partial c_j} < 0$ but $\frac{\partial c_i}{\partial d_j}$ can be positive or negative depending on all traders' degree, since trading centrality is determined by the distribution of individual degrees $\{d_i\}_{\forall i \in N}$. I leave this inspection for future research.

Appendix N Extensions

N.1 Heterogeneous Preferences

The baseline model is the homogeneous version of the general setup of each trader $i \in N$ having quasilinear-quadratic utility over inventory with parameters of individual valuation $\alpha_i > 0$ and risk-aversion γ_i

$$U_i(Q_i) = \alpha_i q_i - \frac{\gamma_i}{2} q_i^2 \tag{64}$$

so that traders are ex-ante heterogeneous. From building up inventory q_i a trader obtains a marginal value α_i and has a marginal cost of $\frac{\gamma_i}{2} q_i$. Heterogeneity in α_i captures the different and persistent close relationships traders tend to form with their clients in OTC markets (Di Maggio et al. (2017b)). The different cost γ_i may be related to fund outside investments, regulatory capital or collateral requirements, which may vary across traders.

The unique and interior Nash Equilibrium is characterized by the demand schedule

$$q_{i,1} = \beta_i(\boldsymbol{\gamma}; \phi, \mathrm{G}) \times \left[\underbrace{\left(m_i(\boldsymbol{\gamma}; \phi, \mathrm{G})\, \alpha_i + \phi \sum_j m_{ij}(\boldsymbol{\gamma}; \mathrm{G})\, \alpha_j \right)}_{a_i} - P_1 - \phi \sum_j \beta_{ij}(\boldsymbol{\gamma}; \mathrm{G}) \cdot q_{j,1} \right]$$

where now the endogenous network-induced coefficients also depend on the risk aversion of traders, and $b = 1$. In Subsection N.1 give the full specification of coefficients above. An important observation is that individual valuations α_i only affects the level of demand.

The key feature of this extension is that it permits heterogeneous interdependencies among values $\{\alpha_i\}_{i \in N}$ that arises endogenously, as described next.[86]

Lemma 12. *local market Equilibrium*

Equilibrium price of seller s is

[86] This setup follows Rostek and Weretka (2012). The difference is that in that paper the valuations are unknown and traders have private signals on their own and others valuations. Here, however, agents can infer others valuation in the PM through the network

$$P_s^* = \left(\sum_{i\in N_s}\frac{1}{\gamma_i}\right)^{-1}\cdot\left[\left(\sum_{i\in N_s}\frac{\alpha_i}{\gamma_i}\right)-\left(\sum_{i\in N_s}q_{i,1}+q_{s,1}\right)\right] \tag{65}$$

and equilibrium allocation of each buyer $i \in N_s$ is

$$q_{i,s}^* = \left(\frac{\alpha_i}{\gamma_i}-q_{i,1}\right)-\frac{1}{\gamma_i}\cdot\left\{\left(\sum_{i\in N_s}\frac{1}{\gamma_i}\right)^{-1}\cdot\left[\left(\sum_{i\in N_s}\frac{\alpha_i}{\gamma_i}\right)-\left(\sum_{i\in N_s}q_{i,1}+q_{s,1}\right)\right]\right\} \tag{66}$$

Denote $\Gamma_s \equiv \sum_{i\in N_s}\frac{1}{\gamma_i}$. Then network coefficients become

$$v_i(\phi) \equiv \left\{\gamma_i(1-\phi-\phi d_i)+\phi\frac{2}{\Gamma_i}+\phi\sum_j g_{ij}\cdot\frac{1}{\gamma_i\Gamma_j^2}(2\gamma_i\Gamma_j-1)\right\}^{-1}$$

$$m_i(\phi) \equiv \left((1-\phi-\phi d_i)+\frac{1}{\gamma_i}\cdot\phi\sum_j g_{ij}\cdot\left[\frac{1}{\Gamma_j^2}+\frac{2(\gamma_i\Gamma_j-1)}{\gamma_i\Gamma_j^2}\right]\right)$$

$$\tilde{g}_{ij}^1 \equiv \left[g_{ij}\cdot\left(\frac{1}{\Gamma_i}+\frac{1}{\Gamma_j}-\frac{1}{\gamma_i\Gamma_j^2}\right)+\sum_{k\neq i,j}g_{ik}g_{jk}\cdot\left(\frac{1}{\Gamma_k}-\frac{1}{\gamma_i\Gamma_k^2}\right)\right]$$

$$\tilde{g}_{ij}^2 \equiv \left[g_{ij}\cdot\frac{1}{\gamma_j}\frac{1}{\Gamma_i}+\sum_{k\neq i,j}g_{ik}g_{jk}\cdot\frac{1}{\gamma_j}\left(\frac{1}{\Gamma_k}-\frac{1}{\gamma_i\Gamma_k^2}\right)\right]$$

The next lemma characterized PM equilibrium demands.

Lemma 13. *PM Equilibrium Demand*

$$q_{i,1} = v_i(\boldsymbol{\gamma};\phi,\boldsymbol{G})\times\left[m_i(\boldsymbol{\gamma};\phi,\boldsymbol{G})\,\alpha_i-P_1-\phi\sum_j\tilde{g}_{ij}^1(\boldsymbol{\gamma};\boldsymbol{G})\cdot q_{j,1}+\phi\sum_j\tilde{g}_{ij}^2(\boldsymbol{\gamma};\boldsymbol{G})\,\alpha_j\right]$$

Assuming further that traders have the same level of risk-aversion, $\gamma_i = \gamma \forall i \in N$, the equilibrium turns out to be quite similar to the baseline model. PM demand is given by

$$q_{i,1} = \frac{1}{\gamma} v_i(\phi) \times \left[m_i(\gamma, \phi)\alpha_i - P_1 - \gamma\phi \sum_j \tilde{g}^1_{ij} \cdot q_{j,1} + \phi \sum_j \tilde{g}^2_{ij} \alpha_j \right] \tag{67}$$

where $v_i(\phi), \tilde{g}_{ij}$ are the same as in the baseline model, and $m_i(\phi, \gamma = 1 - \phi - \phi d_i + \phi \sum g_{ij} \frac{1}{d_j^2} [\gamma + (2d_j - 2)]$ and $\tilde{g}^2_{ij} = g_{ij} \cdot \frac{1}{d_i} + \sum_k g_{ik} g_{jk} \cdot \left(\frac{1}{d_k} - \frac{1}{d_k^2} \right)$.

N.2 Expected Fundamental Returns

In reality, traders care about the fundamental return of an asset. They hold an asset not just for the sake of holding it (i.e. to enjoy utility flow) but because they expect that the asset itself is a good financial investment, with high intrinsic value. My framework accommodates *asset-related information* and with that, as I show next, asset price reflects both the traders' beliefs on returns and the trading network. Importantly, the way the former is incorporated into price depends on the later.

To understand this results a brief description of this extension is enough (See appendix for all the details). Suppose the asset has uncertain return f which is normally distributed with mean μ and variance σ^2, and it is realized after all trading activities take place[87] traders have initial wealth w_0 and choose asset inventory q_i to maximize the expected CARA utility of final wealth $E\left[-\exp\left(\gamma W\right)\right]$ given by

$$W = f(q_{i,1} + q_{i,s}) - (P_1 q_{i,1} + P_s q_{i,s}) + w_0 \tag{68}$$

The counterpart Nash Equilibrium demand of Equation 10 is

$$q_{i,1} = \beta_i(\phi) \left[\underbrace{\mu \cdot \frac{(1 + \phi m_i(\mathrm{G}))}{\gamma\sigma^2}}_{a_i} - \underbrace{\frac{1}{\gamma\sigma^2}}_{b} P_1 - \phi \sum_j \beta_{ij} q_{j,1} \right]$$

where the coefficients only depend on the trading network G. As before, see Subsection N.2 for a detail the full analytical solution.

The equilibrium implies that trader i's PM demand depends on market price P_1, his information *and* the information and demand of *all other traders*, including those he is not directly connected to but who are connected with his connections. This is in stark

[87] The normality assumption is standard in this literature. See, for instance, Kyle (1989), Vives (2011), Rostek and Weretka (2012), (Duffie and Zhu (2016)) and others.

difference with the canonical linear asset pricing model where individual demands depend on all agents' information set *but not directly* on other demands. That's because in such setting equilibrium price aggregate all useful information and so it is not necessary to know other demands. In my model, however, even an anticipated shock and the fact the it leads to different trading possibilities make agents to conditional on others demands, since this is informative about the market structure.

Buyer i's demand from seller s is $q_{i,s} = \frac{\mu - P_s}{\gamma\sigma^2} - q_{i,1}$.

Lemma 14. *Local market Equilibrium*

$$P_s^* = \mu - \frac{\gamma\sigma^2}{d_s}\left(q_{s,1} + \sum_{i\in N_s} q_{i,1}\right)$$
$$q_{i,s}^* = \frac{1}{d_s}\left(q_{s,1} + \sum_{k\neq i, k\in N_s} q_{k,1}\right) - \frac{(d_s - 1)}{d_s} q_{i,1}$$

Network coefficients become

$$v_i(\phi) \equiv \left[(1-\phi) - \phi\frac{(d_i - 2)}{d_i} + 2\phi \cdot \sum_j g_{ij}\frac{1}{d_j}\right]^{-1}$$
$$\tilde{g}_{ij} \equiv g_{ij}\left(\frac{1}{d_i} + \frac{1}{d_j}\right) + \sum_{k\neq i,j} g_{ik} g_{jk}\frac{1}{d_k}$$
$$\tilde{g}_i \equiv -d_i + \sum_j g_{ij}\frac{1}{d_j} + \sum_j\sum_k g_{ik} g_{jk}\frac{1}{d_k}$$

Define vectors $\boldsymbol{v} = [v_i(\phi)]\tilde{\boldsymbol{g}}_{N\times 1} : [v_i(\phi)\cdot\tilde{g}_i]$ and matrices $\mathrm{V}_{N\times N} = diag(\boldsymbol{v}), \tilde{\boldsymbol{G}}_{N\times N} : [\tilde{g}_{ij}]$. Then the system of PM demands can be written in matrix form.

Lemma 15. *PM Equilibrium Demands*

$$\boldsymbol{q}_1^* = \left(\boldsymbol{I} + \phi\boldsymbol{V}\tilde{\boldsymbol{G}}\right)^{-1}\cdot\left(\frac{\mu}{\gamma\sigma^2}\left(\boldsymbol{v} + \phi\tilde{\boldsymbol{g}}\right) - \frac{1}{\gamma\sigma^2}P_1\boldsymbol{v}\right)$$

N.3 Price Impact

My framework is essentially a static demand game because a trader's PM demand depends on expected local market trades, not on realized trade. This is the crucial feature of the model. And it is the reason local market trading induces a set of games in the PM. In equilibrium, prices and demands are not independent across markets. If they were, the markets would operate as independent venues and this is clearly not the case here.

Price impact in the PM arises endogenously in the model precisely because of intertemporal demand dependence. That is, it comes from *all the possible* local market exchanges in the trading network. Moreover, PM asset marginal utility is dictated by participation in the SM, and vice-versa.

Following the imperfect competition literature, the framework can be used to study the economy with strategic traders, as Rostek and Yoon (2020b).

Suppose $d_i \geq 3 \forall i \in N$. In every local market, a buyer $i \in N_s$ trades taking into account his price impact $\lambda_s \equiv \frac{\partial P_s}{\partial q_i}$. In equilibrium, I show that $\lambda_s = \frac{1}{d_s - 2}$ and so price impact is equal across buyers in a given local market. Buyer i's demand is $q_{i,s} = \frac{1}{\lambda_s + 1}(1 - q_{i,1} - P_s) = \frac{d_s - 2}{d_s - 1}(1 - q_{i,1} - P_s)$.

Lemma 16. *Local Market Equilibrium*

The local market or seller $s \in N$ has equilibrium price,

$$\begin{aligned} P_s^* &= 1 - \frac{1}{d_s}\sum_i a_i - \frac{1+\lambda_s}{d_s} a_s \\ &= 1 - \frac{1}{d_s}\sum_i a_i - \frac{d_s - 1}{d_s(d_s - 2)} a_s \end{aligned} \tag{69}$$

and asset allocation

$$q_{i,s}^* = \frac{1}{d_s(1+\lambda_s)} q_{N_s - i,1} + \frac{1}{d_s} q_{s,1} - \left(\frac{d_s - 1}{d_s(\lambda_s + 1)}\right) q_{i,1}$$

Lemma 17. *Primary Market Equilibrium*

$$q_{i,1} = \psi_i \left[(1 - P_1) - \phi \sum_j \psi_{ij} q_{j,1}\right] \tag{70}$$

where

$$\psi_i = \left[1 + \phi(d_i + 1) + 2\phi\frac{(d_i - 1)}{d_i(d_i - 2)} + \phi\sum_j g_{ij}\frac{2}{d_j}\right]^{-1}$$

$$\psi_{ij} = g_{ij} \cdot \left[\frac{1}{d_i} + \frac{1}{d_j}\right] + \sum_{z \neq \{i,j,k\}} g_{iz}g_{jz} \cdot \frac{(d_z - 2)}{d_z(d_z - 1)}$$

Appendix O Local Markets with Outside Traders

I assume that all traders participate in the primary market. However, this is not a restrictive assumption. The results are robust to incorporating outside traders who may only participate in a local market. In this section, I introduce a representative outside trader with demand q_t for each local market. This trader has the same quadratic-quasilinear preference but only one asset demand. The outside trader is interpreted as investors who only learn about the asset after the first trading round, or that due to financial constraints do not participate in the primary market.

The outsider demand schedule is $q_t = 1 - P_s$. By market clearing, seller s' price is $P_s^* = 1 - \frac{1}{d_s+1}\left(q_{N_s,1} + q_s\right)$.

From the local market equilibrium, one can already see that the only increasing price denominator by 1. In other words, the seller's effective degree is $d_s + 1$. Since this holds for every local market, all results remain unchanged.

The more realistic approach would be to include outside traders just in some local markets, or to allow for preference heterogeneity. The model accommodates all these extensions. Even though it remains tractable, it becomes harder to disentangle the network effects from preference and outside traders' effects. As I mentioned in the Introduction, the goal of this paper is to provide a benchmark framework in which the network effects are the *only driver* of equilibrium outcomes. The extensions presented in Section 9 are pertinent are interesting. I leave them for future work.

Appendix P Empirical Exercise

P.1 Bond-level Dealer Network

In Section 8 I explore the sample of trades of all bonds. But the same analysis can be done at the bond-level. As I show next, each bond in the sample has a different trading network. That's because the main take-away of my paper is that different assets have different prices because each has a different underlying trading network structure.

The next figure depicts the dealer network for each bond. Notice that, according to my definition, 2 bonds do not have a dealer network because only one dealer trade with the customer.

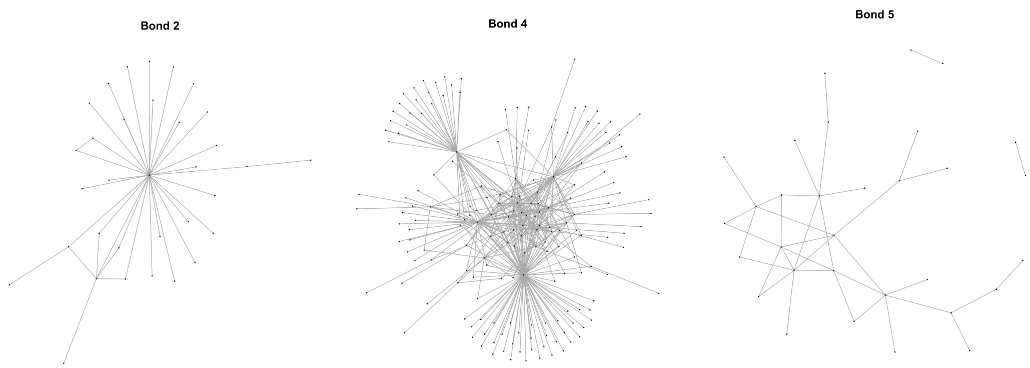


Figure P.1: Dealer networks for different bonds

Each bond in the data exhibits a different inventory price distribution, as the next figure shows.

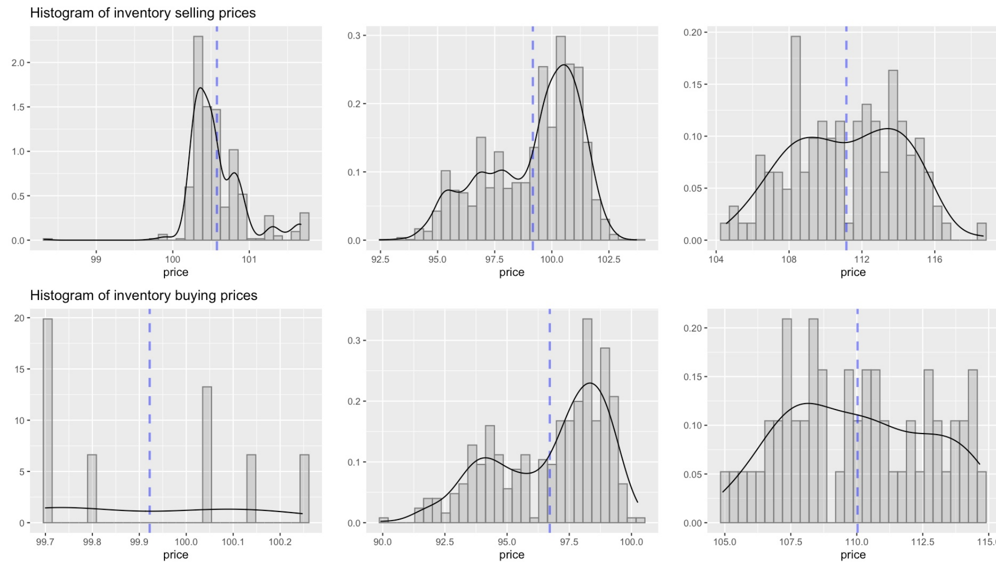


Figure P.2: Distribution of prices of dealer-customer trades

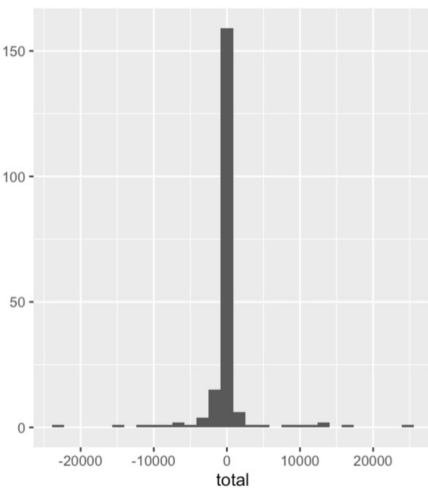
P.2 Dealers' Inventory

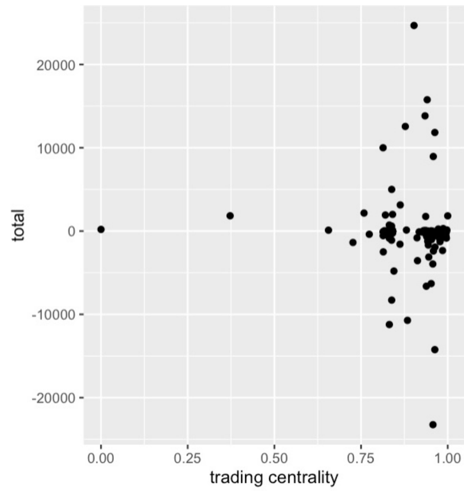
By trading with the customer, dealers accumulate inventory (bond holdings) over time. So, in my analysis, inventory is the empirical counterpart of PM demand. I compute inventory from net d2c trades. A positive (negative) inventory means that a dealer is a net buyer from (seller to) the customer. Through the lens of my model, a positive inventory would lead the dealer to trade in the interdealer network.

I calculate a dealer's (net) inventory at the end-of sample, referred as net total inventory (NTI), and its nominal and absolute values (qi, aqi).[88] NTI gives me a dataset with one observation per dealer with inventory as bond holdings at the end of the 42 trading days. As the figure below shows, inventory is concentrated around zero, indicative that most dealers trade in a way to off-set portfolio imbalances.

The relationship between dealers' inventory and trading centrality is depicted next. Since the attribution of trading centrality is highly skewed, at a first glance it does not have a clear relation with net bond holdings. Even so the graph reveals that the least central dealers have roughly zero net inventory - while the reverse is not true.

[88] I also calculate inventory at the daily level: net daily inventory (NDI) nominal and absolute values $(qid, aqid)$. This gives me an unbalanced panel data with multiple observations per dealer over 42 trading days. See appendix.





P.3 Regressions: Interdealer trades and Trading Centrality

Regressing trade volume or trade price on trading centrality delivers significant results. For trade volume, the sign of trading centrality coefficient varies depending on the set of controls. Trading centrality alone has a positive effect on volume, and so it does when controlling for nominal net total inventory (from customer trades) and trade price. However, adding degree and as control, turns the centrality coefficient negative.

Table P.1: D2D trade volume

	Dependent variable:					
	vol					
	(1)	(2)	(3)	(4)	(5)	(6)
tcn	38.6***		−296.6***	87.2***	84.9***	−150.2**
	(11.7)		(59.6)	(12.4)	(12.4)	(61.0)
deg		−0.6***	−3.9***			−2.7***
		(0.1)	(0.7)			(0.7)
qi				0.01***	0.01***	0.01***
				(0.001)	(0.001)	(0.001)
pr					4.0**	4.1**
					(1.6)	(1.6)
Constant	38.3***	84.1***	395.1***	23.7***	−370.0**	−133.2
	(8.7)	(6.4)	(62.8)	(8.7)	(162.6)	(173.2)
Observations	5,308	5,308	5,308	5,308	5,308	5,308
Adjusted R^2	0.002	0.003	0.01	0.02	0.02	0.03
Note:						*p<0.1; **p<0.05; ***p<0.01

For trade price though trading centrality is only significant when controlling for degree. In this case, trading centrality has a positive effect on price: more central dealers face higher interdealer prices.

Table P.2: D2D trade price

	Dependent variable:					
	pr					
	(1)	(2)	(3)	(4)	(5)	(6)
tcn	0.5^{***}		1.1^{**}	0.1	0.6^{***}	0.5
	(0.1)		(0.5)	(0.1)	(0.1)	(0.5)
deg		-0.01^{***}	0.01			0.01
		(0.001)	(0.01)			(0.01)
pi				0.2^{***}		0.2^{***}
				(0.02)		(0.02)
qi					0.000	
					(0.000)	
Constant	98.8^{***}	99.4^{***}	98.3^{***}	84.2^{***}	98.8^{***}	83.8^{***}
	(0.1)	(0.1)	(0.5)	(1.7)	(0.1)	(1.7)
Observations	5,308	5,308	5,308	5,308	5,308	5,308
Adjusted R^2	0.01	0.01	0.01	0.02	0.01	0.02
Note:						*p<0.1; **p<0.05; ***p<0.01

Conditioning on side

Table P.3: D2D trade volume conditional on side

	Dependent variable:					
	buy volume					
	(1)	(2)	(3)	(4)	(5)	(6)
tcn	-136.8^{***}		-194.2^{***}	-118.4^{***}	-118.9^{***}	-153.5^{**}
	(29.7)		(67.1)	(31.4)	(31.4)	(72.1)
deg		1.2^{***}	-0.7			-0.4
		(0.3)	(0.7)			(0.8)
qi				0.002^{*}	0.002^{*}	0.002^{*}
				(0.001)	(0.001)	(0.001)
pr					4.8^{**}	4.9^{**}
					(2.3)	(2.3)
Constant	182.2^{***}	43.9^{***}	243.5^{***}	176.2^{***}	-303.1	-268.4
	(26.5)	(8.0)	(69.5)	(26.7)	(232.8)	(241.8)
Observations	2,654	2,654	2,654	2,654	2,654	2,654
Adjusted R^2	0.01	0.005	0.01	0.01	0.01	0.01
	Dependent variable:					
	sell volume					
	(1)	(2)	(3)	(4)	(5)	(6)
tcn	120.5^{***}		$-1,939.0^{***}$	107.6^{***}	104.8^{***}	$-1,542.0^{***}$
	(16.5)		(154.8)	(16.0)	(16.2)	(157.4)
deg		-1.6^{***}	-24.0^{***}			-19.3^{***}
		(0.2)	(1.8)			(1.8)
qi				0.02^{***}	0.02^{***}	0.01^{***}
				(0.001)	(0.001)	(0.001)
pr					2.7	2.5
					(2.3)	(2.2)
Constant	11.0	154.2^{***}	$2,234.0^{***}$	1.2	-265.1	$1,537.0^{***}$
	(9.3)	(12.1)	(166.5)	(9.1)	(225.2)	(279.4)
Observations	2,654	2,654	2,654	2,654	2,654	2,654
Adjusted R^2	0.02	0.03	0.1	0.1	0.1	0.1
Note:						*p<0.1; **p<0.05; ***p<0.01

Table P.4: D2D trade prices conditional on side

	Dependent variable:					
	sell price					
	(1)	(2)	(3)	(4)	(5)	(6)
tcn	1.0***		0.9	0.7***	1.0***	2.1
	(0.1)		(1.3)	(0.1)	(0.1)	(1.3)
deg		−0.01***	−0.002			0.02
		(0.002)	(0.02)			(0.02)
pi				0.2***		0.2***
				(0.02)		(0.02)
qi					−0.000	
					(0.000)	
Constant	98.7***	99.9***	98.9***	84.1***	98.8***	82.1***
	(0.1)	(0.1)	(1.4)	(2.2)	(0.1)	(2.8)
Observations	2,654	2,654	2,654	2,654	2,654	2,654
Adjusted R^2	0.02	0.02	0.02	0.04	0.02	0.04
	Dependent variable:					
	buy price					
	(1)	(2)	(3)	(4)	(5)	(6)
tcn	0.2		0.7	−0.7**	0.1	−0.7
	(0.2)		(0.6)	(0.3)	(0.3)	(0.6)
deg		−0.001	0.01			−0.000
		(0.003)	(0.01)			(0.01)
pi				0.2***		0.2***
				(0.03)		(0.03)
qi					−0.000	
					(0.000)	
Constant	99.0***	99.2***	98.5***	80.1***	99.0***	80.1***
	(0.2)	(0.1)	(0.6)	(2.7)	(0.2)	(2.7)
Observations	2,654	2,654	2,654	2,654	2,654	2,654
Adjusted R^2	-0.000	-0.000	-0.000	0.02	0.001	0.02
Note:						*p<0.1; **p<0.05; ***p<0.01

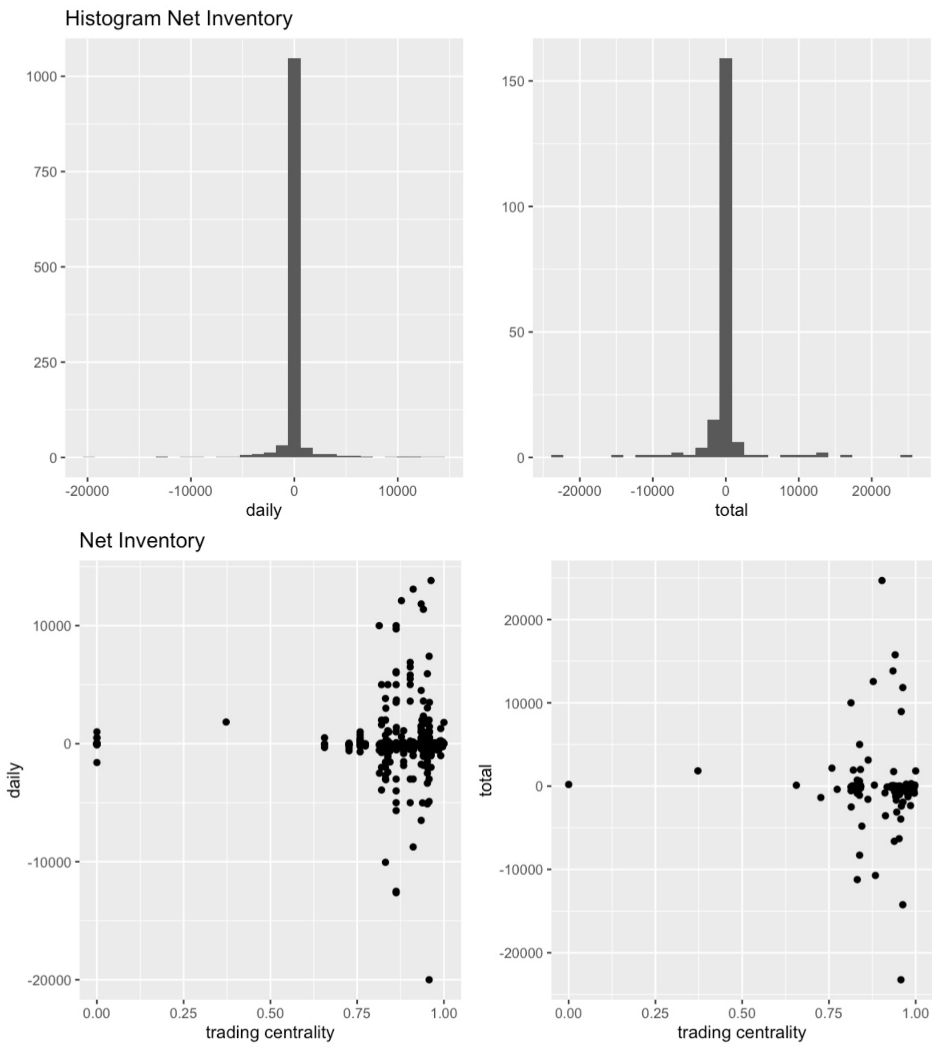
P.4 The Sample

Primary Market: D2C data

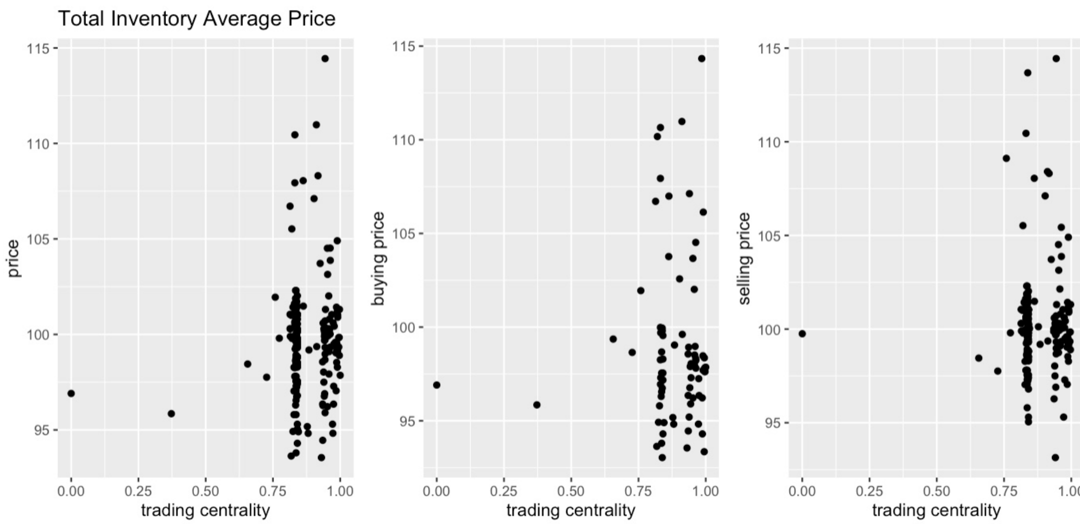
I calculate a dealer's (net) inventory at two different periods, daily and at the end-of sample, and their nominal and absolute values. That is, I compute:

- net daily inventory (NDI) nominal and absolute values ($qid, aqid$), which gives me an unbalanced panel data with multiple observations per dealer over 42 trading days;
- net total inventory (NTI) nominal and absolute values (qi, aqi), which gives me a dataset with one observation per dealer with inventory as bond holdings at the end of the 42 trading days.

As the figure below shows, both inventory measures are concentrated around zero, indicative that most dealers trade in a way to off-set portfolio imbalances.



The concentration of trading centrality values is also reflected in the graph relating it with the price of inventory. The lest central dealers have lower prices, although the reverse is not true.



Interdealer Market: D2D data

The following table summarizing trading frequency and market outcomes (prices and quantities) in the observed D2D trades. There are $2,654$ trades.

Table P.5: $N = 5,308$

Statistic	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
trd_dpair	5.619	6.180	1	1	8	30
trd_mpair	49.950	58.080	1	4	106	182
trd_pair	84.350	104.600	1	6	113	320
trd_ddealer	19.210	20.650	1	4	26	92
trd_mdealer	259.500	234.000	1	57	501	673
trd_dealer	466.400	387.600	1	135	849	1,006
vol	63.520	310.600	1	10	30	5,985
pr	99.190	2.571	91.230	98.300	100.300	116.600

Dealer-level information on transactions:

Table P.6: Buy and sell trades per dealer

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
qs	201	0.000	2,957.000	−24,843	10	151	14,884
qs.mean.all	201	188.300	611.400	3.000	16.250	75.000	5,000.000
ps.mean.all	201	98.990	2.232	93.550	98.020	100.000	112.400

Table P.7: Sell trades per dealer

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
qs.mean	85	−329.900	866.700	−5,000	−177.5	−22.5	−3
ps.mean	85	99.490	3.962	93.360	97.640	100.000	114.400
trd_side	85	31.220	122.800	1	1	12	870

Table P.8: Buy trades per dealer

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
qs.mean	176	114.300	435.000	4.000	15.000	63.540	5,000.000
ps.mean	176	98.930	1.875	94.300	98.020	100.100	107.000
trd_side	176	15.080	56.060	1	1	7	638