Asset Pricing and Re-sale in Networks

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Abstract

I study asset pricing when there is risk of re-sell in general and interconnected market structures. In my framework, a divisible asset can be acquired at a common price by risk-averse traders, who may then need to re-sell their shares in their local trading network. I develop two novel network metrics that are sufficient statistics for equilibrium variables. Trading centrality defines asset price and demands, while local centrality determines re-selling costs. Both measures map expected re-sale market outcomes into the traders' decision of asset acquisition before re-trade. A trader's demand is proportional to his trading centrality, and his re-selling cost is always positive and increasing in his local centrality. The unique exception is for the core trader in a star network who obtains profits from re-selling. Asset price and welfare are non-monotonic in connectivity and inequality of traders' number of connections. Implications for the interdealer market of off-exchange securities are examined.

KEYWORDS: networks; decentralized markets; interdealer trade; primary market; inventory; network games

JEL CLASSIFICATION: D4, D53, D85, G11, G12, G13, G21

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1 Introduction

Dealers are the backbone of trades for many assets (bonds, securizations, currencies, etc) worth trillion of dollars each year. They often absorb substantial inventory position in primary markets of asset issuance or from their costumers, and then use the interdealer market to offload these positions. Recent empirical studies document that the interdealer market can be seen as a stable trading network with a core-periphery structure.[1] This interdealer network has high trading frequency and volume, with spillover effects on the overall market outcomes.

The interdealer market illustrates the ubiquitous decentralized nature of modern financial markets, often understood as networked markets. Assets move around by being re-sold in contemporaneous and interdependent trading venues. Markets are dominated by large and influential traders, trading relationships are not random, and transactions are not restricted to be bilaterally negotiated but often occur via auctions.

Such features feed into a non-trivial decision of how much of an asset to hold in the first place. Traders acquire asset shares today, anticipating a variety of re-sale markets. But prices and demands of re-sale trades depend on how many shares traders already hold. The purpose of this paper is to analyze how asset price is affected by future re-sale trades in general and interconnected market structures. I show that pre-trade asset acquisition becomes a strategic decision in response to the endogenously correlated terms of re-sell trade. Consequently, asset price is determined by the structure of expected markets.

In the two-period trading model, a finite set of risk-averse and price-taker traders compete for a divisible asset in exogenous inelastic supply. My framework has two key features: re-selling risk and asymmetric market participation. At period one, all traders can acquire shares in the *primary market* (PM). Afterwards, trade can happen in *local markets* that are described by an exogenous *trading network*. Nodes are traders and linkages possible asset flows. A local market is defined by a seller trading with his linked buyers. All markets operate as a one-sided uniform-price auction with traders submitting demand schedules in the markets they can participate[2] With some positive probability,

[1]The core-periphery structure arises because there is great heterogeneity among dealers with respect to trading frequency, volume, and counterparties. See Section 2.

[2]As I explain later, price-taking implies that traders are truthful and ignore the *direct* impact of their bids on prices, as in Swinkels (2001) and Feldman et al. (2015). This assumption is the main departure from the well-known imperfect competition framework (as in Kyle (1989), Vives (2011) and Malamud and Rostek (2017)). However, it allows me to distill the equilibrium effects coming *only* from the structure of the

at most one trader is selected and forced to re-sell all his shares.[3] This determines the *active local market* at period two. The probability of being the seller is referred as the re-sell shock, and it captures a sudden need of liquidity. Local markets can be thought of as meeting places where traders *can* trade, and the active local market as *when* exchanges are realized.

I develop two network metrics that are sufficient statistics for equilibrium variables, and they are the main contributions of this paper. The first, "trading centrality", defines the unique equilibrium in the model. Each trader's demand is proportional to his trading centrality and PM price is increasing in the aggregate trading centrality of the economy. The second, "local centrality", determines liquidity costs (i.e. the difference between PM and local market prices). Liquidity is inverse related to local centrality and a trade's liquidity/re-selling cost is (almost)[4] always positive and monotonically increasing in his local centrality.

At the core of the model is the trade-off the traders face: how many asset shares to acquire at each period. They can buy with certainty in the PM but expecting to pay a higher price and facing re-sell risk. Buying shares from another trader is likely to be cheaper, but traders do not know ex-ante which local market will be active. All this generates interdependency of demands across periods and across traders.

The trading network enables trade and, at the same time, constrains and correlates traders' behavior. Traders use their network position to conjecture the set of local markets equilibria that could arise and, contingent on that, they strategically decide PM asset acquisition. Network position is the only dimension of ex-ante heterogeneity and it is the unique source delivering difference in demands. The environment boils down to a one-shot, simultaneous-move network game of strategic substitutes played in the PM. Traders' best-respond to each others' demand schedule and the Nash equilibrium is given by the trading centrality. This is the critical step for understanding the mechanics behind the model.

The endogenous demand substituability arises because each network linkage carries two opposite forces: a participation effect and an inventory effect. The former is simply the ability to trade, and it guarantees liquidity to the seller and more of the asset for the buyer. The latter is how many shares are carried over from the PM, and it determines

trading network. And it is enough to guarantee the existence and uniqueness of equilibrium.

[3] Alternatively, the selected trader can be seen as the seed for re-selling activity, the seller. Once the seed is specified, the network determines which traders will buy from (trade with) the seed.

[4] The unique exception is for the core trader in a star network, which I discuss below.

the seller's liquidity need (supply) and buyers' willingness to provide liquidity.[5] The participation effect pushes selling price up while the inventory effect pushes selling price down. Before trade though, in the PM, both effects make a trader to demand less shares if he expects others to demand more. If the trader turns out to be the seller, he circumvents his high re-sell cost by having lower liquidity need. If he is a buyer, he can buy more at a lower price. My centrality measures capture these intricate forces driving traders' behavior as they account for trade participation, trading motive and the quantity traded.

I further reveal how the structural properties of the trading network affect market outcomes. I provide two set of results. The first set focuses on the PM price. There are three main findings. First, PM price is bounded by its level in two networks: above by the complete network, and below by the star network. Second, it is non-monotonic in two network features: connectivity and degree inequality.[6] In spite of that, I can show how PM price changes as one of those features vary: PM is increasing in connectivity - due to high competition, and decreasing in degree inequality - due to higher uncertainty. Third, re-sell cost is decreasing in both connectivity and degree inequality.[7] Thus liquidity results from a fine balance between the ability to trade and the correlation of traders' demands. For instance, the core trader of a star network emerges as the unique market market. He acquires the majority of shares in the PM and is the only one having profits from re-selling.

The second set of results assesses welfare, in terms of total expected utility. I find that reducing trading asymmetry, either by increasing network connectivity and/or reducing degree inequality, must not necessarily be welfare enhancing because of two opposing effects. On the one hand, more connected and equal trading networks are allocative efficient, as traders have the same (or complete) local market participation; on the other, traders are more willing to hold asset shares and thereby prices are higher. To gain further insights, I then focus my analysis on networks in which trading centrality and degree are positively correlated.[8] I show that welfare is the highest (lowest) in the star (complete)

[5] In other words, the participation effect is about the extensive margin of trade, while the inventory effect concerns the intensive margin.

[6] In a network, degree refers to how many connections a given node has. I define connectivity as the average degree, and degree inequality as degree variance.

[7] In the AFA Presidential Address, Duffie (2010) discusses the large body of research aiming to explain price reaction to supply (or demand) shocks observed in several financial markets. Typically, price drops considerably and it is followed by a gradual price reversal. Just as in my model, his explanation for such dynamics is that price concessions are given by those who have limited opportunities to trade with counterparties.

[8] Welfare analysis is hard because, just as with PM price, it's determined by trading centrality and so it is

network, the one least (most) connected and most (least) unequal. The reason is that the PM price in the star (complete) is the lowest. I also show that a trader's payoff is increasing in his trading centrality, and that more central traders are hurt the most by increasing network connectivity.

My results highlight that ignoring the patterns of connectivity means ignoring the endogenous correlation of prices and demands, and thus can imply multiplicity in equilibrium. Two trading networks with the same connectivity and/or asset distribution can deliver different prices[9] precisely because each induces a different set of re-trade configuration. However, trading centrality is unique and that's why it uniquely determines price.

I advance our understanding about interdealer networks in financial markets in two ways. First, local centrality - similar to Kyle's lambda (Kyle (1985)) - measures the equilibrium price impact of a sell order flow and it is an inverse proxy of liquidity. The main advantage of my metric is that it only requires the information on the structure of the network although it is informative about quantities and prices. In the same line, trading centrality helps rationalize the ambiguous evidence on whether central dealers, defined by standard network metrics (such as degree and eigenvector centrality) used in the finance literature, have better or worse terms of trade. Second, I reveal a novel effect of the interdealer network structure: it determines the issuance price of an asset, thus regulating credit provision in the primary market.

On a broader level, this paper offers a theoretical benchmark to understand the two-way feedback effect of centralized and decentralized trades for an asset or good. The modelling of a centralized market before re-trade is crucial because it endogenizes willingness to engage in decentralized exchanges, rendering traders' valuation for the asset heterogeneous and correlated.

Related Literature: This paper is related to three areas of research: decentralized markets, network games, and over-the-counter (off-exchange) financial markets.

I share the perspective of Malamud and Rostek (2017) of modelling markets as incomplete and co-existing: traders cannot exchange with one another at all times and they can participate in more than one trading venue. The novelty of my paper is to combine both centralized and decentralized trades in a unified and dynamic framework.[10] Most

non-monotonic in connectivity and degree inequality. I discuss welfare in details in Section 7.

[9]Wittwer (2021) finds a similar result in a framework with strategic agents trading multiple assets in connected or disconnected markets.

[10]Somewhat related, Rostek and Yoon (2021) have unified imperfect competition and decentralized markets

importantly, decentralized markets are random realizations and only take place *after* the centralized one. Moreover, several other features set my paper apart from theirs. Here traders are price-takers, there is just one asset, and I restrict the pricing protocol to a one-sided uniform-price auction.

My paper allows for any market structure "between" centralized and bilateral trading.[11] A vast literature on decentralized markets makes the later extreme assumption. Here there are two modelling approaches. Search models, with the seminal contribution of Duffie et al. (2005) in finance; and network models, such as Kranton and Minehart (2001), Corominas-Bosch (2004), Manea (2016). While suitable for many markets such as the over-the-counter one between dealers and customers, in reality several assets are not traded as pairwise exchanges. Treasure securities for instance, are sold in auctions to a set of 20-30 dealers. Interdealer broker systems, electronic trading platforms for interdealer trades, also work as auctions.

The multi-lateral aspect of trades is also a natural implication of auction as a trading protocol. In my model, traders play a game by submitting demand schedules in each sub-graph of the network. Allowing (not assuming) strategic behavior is central in my framework. Here I build upon the game-theoretical view of decentralized trading with imperfect competition - as in Kyle (1989), Vives (2011) Rostek and Weretka (2012) and Rostek and Weretka (2015) to name a few. In this line of work, (finite) traders account for their impact on price and, because of that, they strategically "shade" their bids in the demand game.[12] My paper, in contrast, assumes that traders are price-takers and thus truthful.[13] Although a strong departure, price-taking renders great tractability of the model and ensures the equilibrium outcomes are driven solely by the trading network, without compromising the strategic aspect of trades.[14]

My paper is a novel application of games in networks. As I show later (Section 4), the model is set of network games of global strategic substitutes and I rely on the findings of Bramoullé et al. (2014) and Bramoullé and Kranton (2016) to characterize equilibrium.

in a framework.

[11] In Appendix F I show that restricting the model to bilateral trades would miss all the interesting forces coming from the trading network that drives equilibrium outcomes.

[12] More specifically, imperfect competition means equilibrium is determined by a uniform-price (double) auction with traders submitting demand schedules taking into account their endogenously-determined price impact.

[13] My model accommodates imperfect competition, which I discuss in Section 8. This is also work in progress and available upon request.

[14] Strategic behavior in my model arises purely from the interaction of re-sale risk and trading frictions imposed by the trading network. Not concerns about price impact as in the imperfect competition models.

As aforementioned, a core motivation of this paper is the interdealer market and so my paper relates to the research on off-exchange markets. Given this particular interest, I devote Section 2 to discuss recent empirical findings of the literature and my contributions

Outline: The rest of the paper is organized as follows. Section 2 contextualizes my framework into the interdealer market for off-exchange securities.Section 3 introduces the model and Section 4 solves it. Section 5 gives the main result. Section 6 analyzes how the network structure affects equilibrium and Section 7 studies welfare. Section 8 discusses different extensions. Section 9 concludes. Details and proofs are found in the Appendix.

2 Interdealer Networks and Off-Exchange Securities

Dealers[15] make trade happens for many assets - such as Treasury and corporate bonds, debt securizations, currencies, etc. - in off-exchange (decentralized) markets. They serve as intermediaries among market participants and provide liquidity. Their activity is sustained by asset inventory, which is risky and costly, and interdealer trades are the main tool for inventory management.

Off-exchange markets then have a two-tier structure: a dealer-customer segment, the over-the-counter (otc) market; and an interdealer market. In many cases, such as for fixed-income assets,[16] off-exchanges are a result of a primary market where the issuer, a firm or the government, raises capital by creating a security and allocating it to dealers, who then take the security to the otc market where it is actually traded.[17]

Interdealer markets exhibit three empirically-documented features, which motivate my theoretical framework. First, market liquidity and prices are sensitive to interdealer

[15] A dealer, or broker-dealer, is a financial institution in the business of buying or selling securities on behalf of its customers or its own account or both. Dealers must be registered with a specific regulatory authority, such as Financial Industry Regulatory Authority (FINRA). There are currently 3,394 dealers registered with FINRA alone.

[16] As of 2021, fixed-income securities have issuance value of roughly $US\$13.5$ billion, outstanding value of $US\$52.9$ trillion and average daily traded value of $US\$969$ billion (SIFMA).

[17] Usually investors do not participate in the primary market. The noteworthy example is the US Treasury securities market where a group of more than 20 primary dealers commit to buy large quantities of Treasuries every time the government issues debt, and stand ready to trade them in the otc market (FINRA). The use of primary dealers is common across many countries and it has been in force since 1960 (Arnone and Ugolini (2005)).

trades[18] Second, there is great heterogeneity among dealers with respect to: trading frequency, volume, and counterparties.[19] Third, the interdealer market exhibits a core-periphery structure which is persistent over time.

A core-periphery interdealer network has been documented in the following markets: Dick-Nielsen et al. (2020), Goldstein and Hotchkiss (2020), Di Maggio et al. (2017) for US corporate bonds; Hasbrouck and Levich (2020) for US foreign exchange; Hollifield et al. (2017) for US debt securization; Li and Schürhoff (2019) for US municipal bonds. Figure 1 depicts some of them.

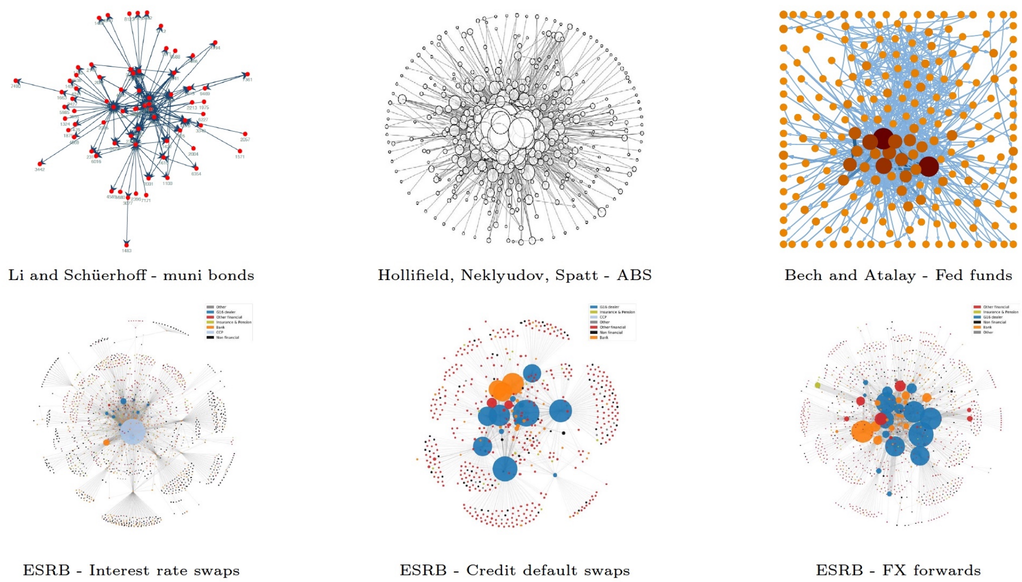

Figure 1: Wang (2016)

Despite the vast theoretical and empirical literature,[20] two questions remain unaddressed regarding the interdealer market. The first is its relation with the issuance price of an asset. The second is how to explain the ambiguous evidence on dealers' centrality

[18]See Schultz (2017), Friewald and Nagler (2019), Anderson and Liu (2021), Feldhütter and Poulsen (2018) among many others.

[19]Several channels have been pointed out as an impediment to trades in the interdealer market: balance sheet constraints, funding costs, information asymmetry, risk appetite and constraints, portfolio holdings, etc.

[20]Over the last two decades, the center of attention has been on the otc markets, with the seminal contribution of Duffie et al. (2005). Research has made great progress in advance our understanding of the bilateral trading behavior and its implication for prices and liquidity, both at the dealer-to-client and dealer-to-dealer segments.

effects on market outcomes.[21] My paper fills these gaps, and I show that both are related to how the structure of the interdealer network conveys information about *expected and correlated* terms of trades among dealers themselves and, thus, determines dealers' willingness to take on inventory.

When unwinding an inventory position, a dealer is just re-selling shares of a security he has previously acquired (in primary markets of asset issuance or from his costumers). The extent he can successfully do so depends on the current inventory of other dealers as well since it determines their willingness to take on new shares. If others already hold too many shares (large inventory), they may not want to take on more, or will demand a low price to do so. Thus, liquidity cost and provision among dealers depends on the inventory acquired before the interdealer trading activity.

Meanwhile, the structure of the interdealer market indicates trading flows. A dealer understands that he, and others, have many trade possibilities implied by their connectedness, in which they can be a buyer or a seller (i.e. be providing or demanding liquidity, respectively), or not be involved at all. Any pair of dealers also acknowledge that they may share a common counterparty, irrespectively if they have a trading connection. All this influences the terms of trade in the entire interdealer market.

In this paper, I translate these observations into a framework in which dealers optimally decide their inventory position - by absorbing an exogenous sell order - and then go on to trade in the interdealer network. The key feature is that, although each dealer knows his potential counterparties, he does not know which of those will need liquidity or be willing to provide liquidity when he makes his inventory decision.

I show that accounting for the joint effects of endogenous inventory positions and the interdealer network structure is essential for characterizing prices and asset allocation (liquidity) both before and within the interdealer market. I reveal that dealers' inventory decisions are in fact strategic substitutes. If a dealer expects other to have large positions in a given security, he will strategically shift his buying activity to the interdealer market. Similarly, if he expects others to hold less inventory he will increase his position before interdealer trades.

Although this sounds simple, inventory choices depend on intricate ways on the entire

[21] Li and Schürhoff (2019) and Di Maggio et al. (2017) find a centrality premium: core dealers charge a wider spread than peripheral dealers. Di Maggio et al. (2017) also document that more central dealers pay lower spread. Meanwhile, Hollifield et al. (2017), Goldstein and Hotchkiss (2020) and Dick-Nielsen et al. (2020) find centrality discount: core dealers charge a narrower spread. Goldstein and Hotchkiss (2020) and Dick-Nielsen et al. (2020) are two exceptions who also look at interdealer trades, and they both find centrality premium.

pattern of trading relationships. No matter if they are used ("realized") or not in a given period. My main contribution is to develop a novel centrality measure - trading centrality - that defines all equilibrium transactions. If a researcher knows the network structure, he can compute trading centrality and estimate all market outcomes, prices and quantities. Including the level and cost of acquiring inventory in the first place.

Because of that, the two open questions I am motivated by can be addressed. First, the interdealer network determines the issuance price of an asset in the primary market. Second, I show that trading centrality can induce both centrality premium *and* discount. But this ambiguity, in stark difference from the empirical literature, is resolved once we take into account the *entire* structure of the interdealer network (i.e. connections, connections of connections, etc.). Finally, trading centrality is a novel liquidity measure that only requires information about the interdealer network structure and it is informative of prices and quantities.

3 The Model

Markets and the Trading Network

There are two periods $t = \{1, 2\}$ and $N > 2$ traders. There exists a divisible asset in exogenous and fixed supply $Q > 0$[22] All traders can acquire asset shares at $t = 1$ in the *primary market* (PM) . Afterwards shares can be re-traded among traders themselves in *local markets*.

There exists a *trading network*[23] in which nodes are the traders and connections determine which traders have access to a particular local market jointly but not separately. A link between i and j means that trade between i and j is *possible*. Formally, the trading network is characterized by the adjacency matrix $\mathbf{G}$ such that $[G]_{ij} \equiv g_{ij} = 1$ if $i \in N$ and $j \in N$ are connected, and $g_{ij} = 0$ otherwise. By convention, $g_{ii} = 0$. The set of linkages of trader $i \in N$ is given by his neighborhood $N_i = \{j \in N : g_{ij} = 1\}$, and i's degree is the number of connections he has: $d_i = |N_i| = \sum_{j \in N} g_{ij} > 0$.[24]

[22] This specification may, for example, capture cases in which dealers face a common outside opportunity for customer sell order, or when they allocate a new security at issuance.

[23] The trading network is unweighted, undirected, fixed, exogenous and known.

[24] I focus my analysis on trading networks with a minimum degree of one, i.e. every trader has at least one connection. However, this is by no means a restrictive assumption, it is just the most interesting case. It also does not mean that the trading network must be connected. In the Appendix, I provide my main results for when there are "isolated" traders in the network. All the analysis and intuition presented in

Thus, the trading network summarizes the set of local markets at $t = 2$. There are N of them. Each is defined by a trader's neighborhood. I refer to the local market of trader $i \in N$ as when i sells his shares to his linked traders, his buyers, at an endogenously-determined uniform price (more details below).

Market participation at $t = 2$ is random. With a probability $\phi > 0$, which I refer as the *re-sell shock*, a trader is selected and is forced to re-sell in his local market. This seller establishes the *active local market* at $t = 2$. I make two assumptions:

Assumption 1. *At most one trader experiences the re-sell shock:* $\phi < \frac{1}{N}$.

Assumption 2. *Supply in any local market is inelastic: the seller does not choose his supply.*

By Assumption 1, there is only *one* active local market or *none*.[25] Local markets can be thought of as meeting places where traders *can* trade, and the active local market as *when* exchanges are realized.[26]

Assumption 2 means that all markets are one-sided. The decision of a trader is how many shares to purchase in each market he has access to. The re-sell shock ϕ is interpreted as a sudden need to unload shares to exit the market. Re-trade in the local markets is for immediacy provision among traders themselves and, in turn, it enables the increase or decrease of asset holdings (i.e inventory management - more details below).

Traders

Traders are risk-averse with initial wealth $w > 0$, and no one is endowed with asset shares. Traders' goal is to build up asset inventory q_i.[27] Let $q_{i,1} \geq 0$ denote how many shares trader $i \in N$ acquires in the PM, and $q_{i,s} \geq 0$ the amount bought in the local market of seller $s \in N_i$. By assumption, if i and s are not connected they cannot trade and so $q_{i,s} = 0 \ \forall s \notin N_i$. Also, if i is the seller he must liquidate his position and so $q_{i,i} = -q_{i,1}$. At the end of period two, a trader's inventory q_i is the sum of the shares purchased at each period, $q_i = q_{i,1} + q_{i,s}$.

the main paper hold.

[25]In principle, there are 2^N possible states of the world: $\Omega = \emptyset \cup \{\omega_s : s \in \{1, 2..., 2^{N-1}\}\}$. I assign probability $\phi > 0$ to each state $s \in [1, N]$, which represents the identity of the single trader $i \in N$ who experiences the re-sell shock; probability $1 - N\phi$ to the empty state, in which no trader experiences the shock; and probability zero to the remaining $s \in [N + 1, 2^{N-1}]$ states, which represents the possible combinations of more than one trader getting shocked.

[26]In other words, the re-sell shock "activates" a local network which is a subgraph of the entire network.

[27]For dealers in off-exchange markets, inventory is used to facilitate future trades with their customers in the otc market.

Each trader receives the net payoff of his trading activity. It is defined as the total utility derived from inventory q_i minus the total payment. Purchasing quantities $(q_{i,1}, q_{i,s})$, he enjoys a utility of

$$U(q_{i,1}, q_{i,s}) = (q_{i,1} + q_{i,s}) - \frac{1}{2}(q_{i,1} + q_{i,s})^2 \tag{1}$$

and pays $P_1 q_{i,1} - P_s q_{i,s}$ where P_1 is the PM price and P_s the price of seller s.

Preferences are single-peaked and traders have an optimal inventory of 1[28] Intuitively, a trader buys shares at each period to reduce the gap between the optimal inventory and his current one. But holding inventory entails a cost of $\frac{1}{2}q_i^2$. For dealers in financial markets, the costly inventory can be due to several reasons (e.g. regulatory capital or collateral requirements). Here it represents the expected cost of being forced to re-sell to raise liquidity by quickly disposing inventory into a restricted, and possibly illiquid, local market (Duffie (2010),Duffie and Zhu (2017)).

The role of having two consecutive markets is best understood by analyzing the partial utility of the quantity traded in either period:

$$\frac{\partial U(q_{i,1}, q_{i,s})}{\partial q_{i,m}} = 1 - q_{i,1} - q_{i,s} \qquad m = \{1, s\} \tag{2}$$

This partial utility is the trader's marginal willingness to pay for the asset in market m given that he obtains $q_{i,-m}$ shares in the other market. It decreases in both quantities traded. Demands are then substitutes across markets. That's because more of the asset is preferred rather than less only up to the optimal inventory, as inventory is costly[29]

Pricing Mechanism

Traders are price-takers[30] and every market (local or otherwise) operates as a one-sided uniform-price auction. Each trader $i \in N$ submits a demand schedule $q_{i,m}(\cdot; P_m)$ in every market $m = \{1, \{s\}_{s \in N_i}\}$ he can participate. Equilibrium price in a market is determined by equating aggregate demand of the participant buyers with the asset inelastic supply.

[28]This is a normalization. The more general setup with individual asset valuation α_i, possibly heterogeneous, is presented in Section 8.

[29]In financial markets, a dealer's inventory is used in his intermediary activity with customers in the otc markets. Inventory allows dealers to provide immediacy and liquidity to costumers. I abstain from the discussion of dealer-customer trades as my focus is on inter-dealer trades.

[30]That is, traders are truthfull and ignore their direct price impact.

The primary market features complete participation and a global market clearing condition holds. The PM price P_1, common to all traders, is given by

$$\sum_{i \in N} q_{i,1}(\cdot; P_1) = Q \tag{3}$$

The price of a local market is seller-specific. For a seller $s \in N$, his local market price P_s is given by the local market clearing condition

$$\sum_{i \in N_s} q_{i,s}(\cdot; P_s) = q_{s,1} \tag{4}$$

The pricing mechanism indicates that a feedback effect across traders' demands emerge. There are two reasons for that. First, even though the asset supply is exogenous in the PM, it is endogenous in every local market: the shocked trader re-sell his PM holdings. Second, the buyers in a local market may already have acquired shares in the PM what influences their willingness to pay for the seller's supply.

Figure 2 depicts the timeline of the model.

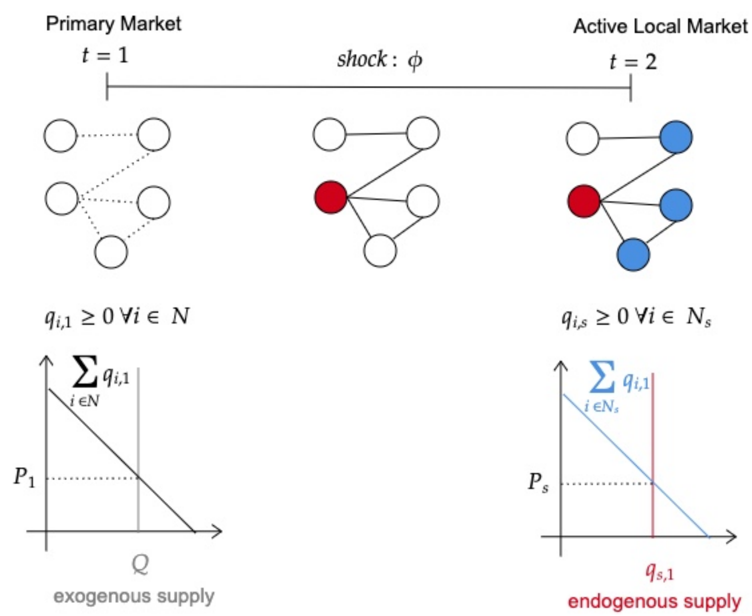

Figure 2: Timeline

One attractive feature of the baseline model is that it can be solved in closed form and is thus a parsimonious workhorse with which to develop intuition (Section 4). The tractability relies on the core assumptions I make. Although not very general, they capture realistic features of the interdealer market, which I discuss in Appendix A. In Section 8 I present extensions of my framework that allows for: heterogeneity in asset valuation and risk preference; expected fundamental asset returns; and imperfect competition.

4 Equilibrium Analysis

The model is solved backwards. I first characterize the local market equilibrium given a shock realization. Then, the equilibrium in the primary market is determined. All variables are conditioned on the primitives of the model: the trading network **G** and the shock parameter ϕ. I omit such variables in the notation for the sake of clarity. In Appendix B I solve in detail traders' optimization problem.

Traders are rational and forward-looking. They decide their optimal demand schedules in anticipation of the re-sell shock and the different local markets that can take place at period two. Trader $i \in N$ chooses $(q_{i,1}(\cdot), q_{i,s}(\cdot))$ to maximize his expected net payoff,

$$\max_{q_{i,1}(\cdot), q_{i,s}(\cdot)} \quad E\left[(q_{i,1} + q_{i,s}) - \frac{1}{2}(q_{i,1} + q_{i,s})^2 - (P_1 q_{i,1} + P_s q_{i,s}) + w \;\middle|\; \mathbf{G}, \phi \right] \tag{5}$$

Each trader faces a trade-off: how many shares to purchase at each period. On the one hand, he can acquire the asset with certainty in the PM but the price is likely to be higher - due to higher competition - and he faces the risk of re-selling. On the other hand, buying at period two is probably cheaper but he does not know if he will need to sell nor if he will participate in the active local market.

Jointly, ϕ and **G** imply that each trader has three levels of uncertainty: i) if he will participate in one or two markets; conditional on trading at $t = 2$, ii) if he will buy or sell asset shares; and iii) if he is a buyer, with whom will he trade.

As it will be clear from the results (Section 6), although buyers are not forced to provide liquidity to the seller they optimally decide to do so. For two reasons. First, no trader reaches his optimal inventory at $t = 1$. Second, buyers guarantee a price concession to absorb seller's supply in any local market, in any trading network.[31] Thus, gains from trade arise because local markets provides more and cheaper shares to the participants buyers, and liquidity for the seller.

[31] The unique exception is the star network, which is in itself an interesting finding that I discuss later on (Section 6).

4.1 The Active Local Market

At $t = 2$, both the PM and the re-sell shock have realized. The seller identity $s \in N$ is common knowledge and so it is the fixed asset supply $q_{s,1} \leq Q$ (recall Assumption 2). Notice that the PM at $t = 1$ can be seen as endogenously determining individual asset endowment in the static market of a seller s.

Each trader i as a buyer in the local market of seller $s \in N_i$ chooses his demand schedule $q_{i,s}(\cdot)$ conditional his current holdings $q_{i,1}$. It is given by[32]

$$q_{i,s}(P_s; q_{i,1}) = (1 - q_{i,1}) - P_s \tag{6}$$

Since traders are price-takers, i's demand (6) is the same in every local market he can participate. It is simply given by his willingness to pay more for the asset, i.e. his current marginal utility (2).

Using (6) and local market clearing conditions (4), the set of equilibria at $t = 2$ can be found, one for each possible seller $s \in N$. Assumption 1 ensure there is a unique equilibrium, the active local market.

Lemma 1. *Active Local Market Equilibrium*

Consider primary market asset allocation $\{q_{i,1}\}_{i \in N}$ and the re-sell shock realization. The equilibrium in the active local market of seller $s \in N$ and his network-induced set of buyers $i \neq s : i \in N_s$ is given by the selling price P_s^*,

$$P_s^* = 1 - \frac{\left(\sum_{i \in N_s} q_{i,1} + q_{s,1}\right)}{d_s} \tag{7}$$

and buyer's asset allocation,

$$q_{i,s}^* = \frac{1}{d_s}\left(q_{s,1} + \sum_{k \neq i, k \in N_s} q_{k,1}\right) - \frac{d_s - 1}{d_s} q_{i,1} \qquad \forall i \in N_s \tag{8}$$

The trading network directly affects the local market prices in two opposite ways. First, trough the positive "participation effect": the seller's degree d_s determines how large his market is. The higher d_s, the higher is P_s^*. Second, through the negative "inventory effect": PM holdings in the seller's neighborhood, $q_{s,1}$ and $\{q_{i,1}\}_{i \in N_s}$, determine his supply and buyers' willingness to pay. The higher are PM holdings, the lower is P_s^*.

[32]This is simply the the first-order condition of (1) keeping $q_{i,1}$ fixed.

But that's not the whole story because these effects are interdependent what results in additional indirect effects. A larger pool of buyers (high d_s) may imply higher aggregate PM asset holdings just because there are more terms in the sum ($\sum_{i \in N_s} q_{i,1}$), what could drive P_s^* down. At the same time, a high d_s may imply lower aggregate PM asset holdings because the buyers anticipate the high competition and so ensured asset holdings in the PM, what could drive P_s^* up.

At the buyer level, equilibrium asset allocation $q_{i,s}^*$ is decreasing in the seller's degree, as price increases in the later. And it is increasing in other buyers' (competitors) PM holdings $\left(\sum_{k \neq i, k \in N_s} q_{k,1}\right)$, since the higher these are the higher is i's residual supply and the lower is seller's price. Moreover, local markets allocate asset shares in accordance to who values it the the most at $t = 2$, to those who acquired the least shares in the PM. Thus, the buyers who provide more liquidity are those who would need less liquidity if they were to be hit by the re-sell shock.[33]

As I show next, traders understand that PM outcome determines liquidity supply and demand in local markets, what in turn influences traders' asset acquisition, and thus price, in the PM. And that's precisely why the trading network plays a crucial role in the PM. It incorporates the two-way feedback effects across markets and traders.

4.2 Primary Market as a Trading Game

Recall that the only source of ex-ante heterogeneity among traders is on network position. Different network positions mean that traders have different expected trades (i.e. local market participation) and, consequently, their asset acquisition decision will differ. More importantly, this decision depends on all other traders' decision as well since they determine the terms of trade in local markets.

In turn, the expected payoff (5) of each trader $i \in N$ in the PM, before any trade takes place, is only function of his and others PM holdings choice, $q_{i,1}$ and $q_{-i,1}$,[34] parameterized by the PM price P_1, the trading network $\mathbf{G}$, and the re-sell shock ϕ[35]:

[33] This apparent "coordination" between liquidity demand and supply of a trader reflects the strategic nature of the environment. This is going to be clear when in the upcoming analysis of traders' behavior in the PM (Subsection 4.2)

[34] As usual, the notation $q_{-i,1}$ represents the demand of all traders but i, i.e. $\{q_{j,1}\}_{j \neq i, j \in N}$

[35] A quick way to see this result is just to replace local market variables in the expected payoff (5) with the equilibrium result of local markets (Lemma 1). With a little algebra, the "clean" representation in (9) is obtained. Full details are found in Appendix B.

$$\pi_i\Big(q_{i,1}, \mathbf{q_{-i,1}}; P_1 \mathbf{G}, \phi\Big) = w + (1 - P_1) \cdot q_{i,1} - \frac{1}{2v_i(\phi)} q_{i,1}^2 - \phi \sum_j \tilde{g}_{ij} q_{j,1} q_{i,1} + \phi \sum_j \bar{g}_{ij} q_{j,1} \qquad (9)$$

Perhaps surprisingly, equation (9) coincides with the payoff function of a network game of global strategic substitutes (notice that $\frac{\partial q_{i,1}}{\partial q_{j,1}} \leq 0 \quad \forall i \neq j$). The "network coefficients" $\{v_i(\phi), \tilde{g}_{ij}, \bar{g}_{ij}\}_{\forall i,j \in N}$ are endogenous, non-negative, and each is a function of the trading network structure. They encapsulate the local market effects on $q_{i,1}$ given i's network position (more details below).

Others' PM holdings $q_{-i,1}$ negatively impact trader i's optimal choice while also having positive externality. This is best understood if we put ourselves in the shoes of trader i conjecturing his local market trades. Consider first the PM demand of i's neighbors (i.e. his direct connections), $q_{N_i} \equiv \{q_{k,1}\}_{k \in N_i}$. As a seller, q_{N_i} pushes i's price down (the "inventory effect"). So i demands less if he expects his buyers to demand more in the PM. As a buyer, q_{N_i} pushes down the price i faces (also the "inventory effect"). So i also demand less (to afford more shares in local markets) if he expects his sellers and competitors to have high PM demand.

However these are just *first-order* effects. The connections of i's connections are also i's competitors and they offer alternative markets for i's neighbors, thus influencing i's PM decision. This is also true for the connections of the connections of i's neighbors, and so on. In all cases, the same reasoning holds: i takes into account the effect of every other trader when he acts a buyer and as a seller in local markets.

Bottom line is that the asset acquisition of each trader is negatively influenced by the same decision of each and every other trader, irrespectively if they are connected or not. This is the key insight of this paper because it reveals that the dynamic framework boils down to a one-shot, simultaneous-move network game played in the PM.

Since traders are price-takers, each PM price P_1 induces a game. In each game, the strategy for trader $i \in N$ is his asset acquisition decision in the PM. It is a mapping $q_{i,1} : q_{-i,1} \times P_1 \rightarrow \mathcal{R}$ where $q_{-i,1}$ is the strategy of all other traders different than i. Traders simultaneously choose their demand schedules by best-responding to the demand schedules of others. The equilibrium concept is pure-strategy Nash Equilibrium.[36]

[36] An important step in the paper is to formulate the model as a game. Network games of global strategic substitutes have been extensively studied - see Bramoullé et al. (2014) and Galeotti et al. (2010). I rely on the advances of this literature to characterize the equilibrium in every possible game and for *any* network graph **G**.

Lemma 2. *Primary Market Trading Game*

For each PM game with price P_1, a trader i's asset demand schedule (best-response) is

$$q_{i,1}(q_{-i,1}; P_1\phi, \mathbf{G}) = v_i(\phi)\left[(1 - P_1) - \phi \sum_j \tilde{g}_{ij} q_{j,1}\right] \tag{10}$$

where

$$v_i(\phi) \equiv \left[\frac{2\phi}{d_i} + 1 - \phi(d_i + 1) + \phi \sum_j g_{ij} \cdot \frac{(2d_j - 1)}{d_j^2}\right]^{-1} \tag{11}$$

and

$$\tilde{g}_{ij} = g_{ij} \cdot \left[\frac{1}{d_i} + \frac{(d_j - 1)}{d_j^2}\right] + \left[\sum_{\substack{z \neq j \\ z}} g_{iz} g_{jz} \frac{(d_z - 1)}{d_z^2}\right] \geq 0 \tag{12}$$

The trading network determines the influence among traders' strategies, while the resell shock ϕ regulates the global degree of substituability among PM demands: the higher ϕ, the greater is the chance of local market trading and thus the greater is the feedback effect between traders' demands.

The individual network effect, $v_i(\phi) > 0$, summarizes trader i's interactions in local markets, and it can be seen as the marginal benefit of acquiring shares in the PM.

Each global network coefficient $\bar{g}_{ij} \geq 0$ captures bilateral influences: it gives how influential is trader j on i's demand. Its value depends on how far apart i and j are[37] Notice that it is increasing in the number of overlapping connections i and j have. Implying that indirect connections can be more influential to i's decisions than i's neighbors. And that neighbors with the same degree can have different effects.

Lastly, the first term in the demand schedule, $(1 - P_1) \geq 0$, is common across all agents. It represents the optimal action absent network interactions: $(1 - P_1)$ is the individual demand in a Walrasian (competitive, static) market of size N.

In the Appendix B I derive the results above, and I discuss more deeply the functional forms of the payoff function and demand schedule. I also prove the existence of a unique interior equilibrium for each network game.[38]

[37]That's because, as I discussed before,there are three channels through which $q_{j,1}$ impacts $q_{i,1}$: as a buyer from i or as a seller to i, if they are connected; and as a competing buyer to i if they share a common linkage to another agent z (i.e. if i, j have overlapping connections).

[38]The existence and uniqueness is guaranteed by Assumption 1, i.e. as long as the shock probability $\phi <$

It is useful to look at two traders' optimal PM demand schedules to understand how the Nash Equilibrium of each network game is found. The left-hand side of Figure 3 shows that, for a given P_1, the Nash Equilibrium is given by the intersection of traders' i and j best-responses. The right-hand side of Figure 3 depicts how the primary market equilibrium is one of the Nash equilibria such that equilibrium aggregate demand meets the exogenous asset supply $\bar{Q}$.

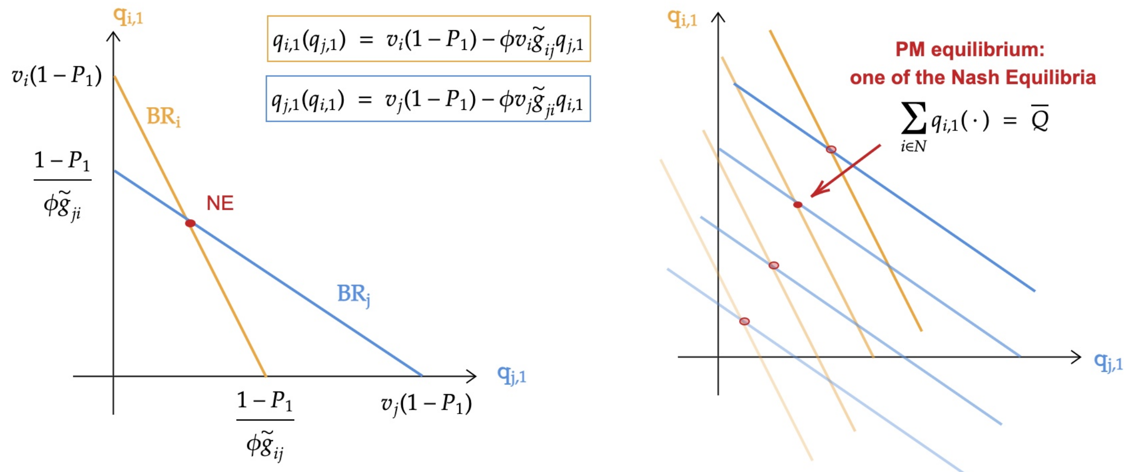

Figure 3: The demand schedules of two traders

Relative to the related literature of imperfectly competitive trading models (i.e. demand games, which Rostek and Yoon (2020a) provide an excellent review), the way I find the equilibrium in the model is different. The key feature of imperfect competition is that traders conjecture their endogenous (and unknown) price impact and have to do the same for others' price impact and demands, due to private information. As Rostek and Yoon (2020a) show, the equilibrium is characterized by two conditions: market clearing and correct price impacts. That is, each trader optimally chooses his demand schedule given his price impact such that his price impact equals the slope of his residual inverse supply function.[39] With price-taker traders (as in my model), finding equilibrium is sim-

$1/N$ (Proposition 10). Intuitively, $\phi > 1/N$ implies that traders expect more than one seller in the local market. The anticipation of "too much" local market trading may lead traders to either demand too much ($q_{i,1} \to 1$) or too little ($q_{i,1} \to 0$) in the PM. In the former case, local markets would collapse as buyers' willingness to trade would be virtually small. In the latter case, the PM would collapse as the market would not clear.

[39] More specifically, with imperfect competition, each trader chooses his demand as optimal pointwise for each price realization against a family of the residual supply (all other trader's demand schedule) with a deterministic slope (price impact) and random intercept (due to other traders private information).

pler because traders only respond to each other demands and thus only one condition - market clearing - characterizes equilibrium. Although this is a strong assumption, I view it as plausible given the main goal of this paper: to distill the network effects on equilibrium. I obtain formal and closed formed solutions and I can study the implications of the structure of the trading network in isolation[40]

5 Trading Centrality, a sufficient statistic for Equilibrium

The analysis so far has two main conclusions. First, the way in which local markets affects PM demands $q_{i,1}$ (10) depends on the trading network structure in complicated ways. Second, the network position of a trader drives his trading behavior.

The main contribution of this paper is to develop a new network metric, trading centrality $\boldsymbol{c}_{N\times 1} : c_i \; \forall i \in N$, that encapsulates all the intricate forces relating markets and traders. More specifically, trading centrality processes information about each and every local market interaction, and produces a "score" for each trader. The score measures a trader's endogenous valuation for the asset. The higher is c_i, the higher is i's marginal utility for asset holdings and thus the higher is he willingness to acquire shares in the PM.[41]

Definition 1. *Trading centrality is N-dimensional vector* $\mathbf{c}$ *defined as*[42]

$$\mathbf{c} = \left(\mathbf{V} + \phi \tilde{\boldsymbol{G}}\right)^{-1} \mathbf{1}_N \tag{13}$$

where $\mathbf{V}$ *is the N-diagonal matrix with entries* $\{1/v_i\}_i$; $\tilde{\boldsymbol{G}}$ *is he N-square, not symmetric matrix with entries* $\{\tilde{g}_{ij}\}_{i,j}$*; and* 1_N *is the N-vector of ones.*

Equivalently, trader i's trading centrality c_i *is*

[40] However, my main methodological contribution, which is to derive a sufficient network metric for the equilibrium (Theorem 1), is not limited by the price-taking assumption. In the Appendix, I derive and show the equilibrium in a setting where markets are imperfectly competitive (see Section 8 for further discussion). As I will explain later on, if traders were strategic with respect to price, the endogenous price impact would depend on the network structure and would also influence the equilibrium. Then, the PM price would be determined by two related but distinct forces: price impact and network structure.

[41] A quick way to see this is by rewriting the demand schedule (Equation 10) in terms of centrality: $1 - q_i \frac{1}{c_i} = P_1$. More on Section 5.1 and Section 5.2

[42] More precisely, trading centrality is a measure $\boldsymbol{c} : \boldsymbol{G} \to \mathcal{R}^N$, where $c_i(\boldsymbol{G})$ is the trading centrality of trader (node) i in the trading network $\boldsymbol{G}$.

$$c_i = v_i(1 - \phi \sum_j \tilde{g}_{ij} c_j) \tag{14}$$

The key finding is that trading centrality is a sufficient statistic for equilibrium (Theorem 1). Since trading centrality determines traders' willingness to acquire the asset in the PM, it defines the equilibrium demands and, in turn, the equilibrium PM price.

Theorem 1. *Equilibrium and Trading Centrality*

The unique and interior equilibrium in the PM is determined by trading centrality $\mathbf{c}$*. Equilibrium PM price is given by*

$$P_1^* = 1 - \frac{Q}{c_A} \tag{15}$$

where $c_A = \sum_{i \in N} c_i$ *is the aggregate trading centrality.*

Each trader's equilibrium PM asset allocation is

$$q_{i,1}^* = Q \frac{c_i}{c_A} \tag{16}$$

Or in matrix notation, $\mathbf{q}_1^* = \frac{Q}{c_A}\mathbf{c}$.

In equilibrium, PM price is increasing in the aggregate centrality of the network and a trader's PM holdings is increasing in his centrality.[43] Also, traders *always* have a strictly positive demand in all markets they can participate at. That's due to three reasons. First, as the PM has more competing buyers, no trader is able to reach his optimal inventory at $t = 1$. Second, the anticipated yet uncertain re-sell shock also lowers PM demand to mitigate the risk of being a seller. Lastly, since the likelihood of being a buyer is weakly greater than of being a seller, traders command lower prices in local markets to compensate the shift from securing asset holdings in the PM at $t = 1$ to the risky asset allocation at $t = 2$.

All this leads to PM price being greater than any local market price in any network. The unique exception is for star network (Corollary 1.1).

Proposition 1. *Price Dynamics*

[43] I use "trading centrality" and "centrality" interchangeably.

For any trading network, asset price drops from $t = 1$ to $t = 2$.

The unique exception is if the trading network has a star structure. In this case, there is a positive probability that price increases over time.

Thus, liquidity (re-selling) is costly for the seller. The next corollary of Proposition 1 states the unique departure from this result: the core trader in a star network profits from re-selling. That's because his price is higher than the price in any other market within the star network. In fact, the core's price is higher than the price in every market in any trading network structure.

Corollary 1.1. *The local market of the core trader in a star network exhibits the highest asset price.*

The fact that liquidity is costly is reminiscent of the long-standing theoretical literature of inventory behavior in the interdealer market, pioneered by Ho and Stoll (1981) and Ho and Stoll (1983). In these models, while interdealer trading enables inventory risk sharing, the initiating dealer must give up some portion of the spread to his counterparty.[44] It is also a common phenomena across a variety of securities and markets (Duffie (2010)).

The next natural question is how a trader's selling price P_i and liquidity cost $(P_1 - P_i)$ relate to his trading centrality c_i.[45] Differently than for PM equilibrium, the answer is non-trivial. Both P_i and c_i are functions of i's degree d_i and the latter has direct and indirect effects on the former.[46] This makes the relation between d_i with P_i and c_i ambiguous. I find that in certain trading networks, c_i and d_i are positively correlated and, thus, so it is c_i and P_i. But other networks exhibit the opposite relation (I illustrate this on a simple example on section 6.1).[47]

Lemma 3. *Trading Centrality and Degree*

The relationship between trading centrality $\boldsymbol{c}$ and individual degree $\boldsymbol{d}$ depends on the structure of the trading network.

Consider an arbitrary trading network. If $corr(\boldsymbol{c}, \boldsymbol{d}) > 0$, then local markets (re-selling) prices $\{P_i\}_{\forall i \in N}$ are increasing in both $(\boldsymbol{c}, \boldsymbol{d})$.

[44]In these models, using interdealer trades to unwind inventory is a choice. However, as long as dealers are farther from their optimal inventory, the benefits of risk sharing via the interdealer market outweigh the trading costs. So dealers (almost) always choose to sell the inventory in the interdealer market if needed. This is the main conceptual difference with my paper since I assume that the seller (the initialing dealer) *must* sell. Interdealer trading in my model is a choice just for the buyers because they can demand zero.

[45]Recall that local markets equilibria are determined by asset allocation in the PM (Lemma 1). It is straightforward to see that, from Theorem 1, equilibrium price and demands in every local market are in fact in terms of trading centrality. See Appendix G.

[46]Recall the "participation" and "inventory" effects in local markets (Subsection 4.1

[47]See also Appendix D for more details.

If $(\boldsymbol{c}, \boldsymbol{d}) < 0$*, the opposite holds.*

Lemma 3 has two crucial implications. First, in my model, central traders are not necessarily those highly connected. Second, central traders do not always have better terms of trades. Although these implications might be counter-intuitive, in reality they are observed in interdealer markets. The growing empirical literature on interdealer networks find mixed evidence on whether "central" dealers have higher or lower liquidity costs (Section 2). My results suggest that a reason for this inconclusiveness could be that all these studies use standard network metrics such as degree and eigenvector centrality that are positively correlated to degree. Consequently,"centralily" only reflects the extensive margin of trades and imply that the centrality of a dealer is weakly increasing in the centrality of his connections (and connections' connections, etc.).[48] My novel trading centrality measure reveals that this might be misleading, particularly so when analyzing markets with endogenous and correlated terms of trade.

In my model, central traders emerge as market makers only for particular trading networks in which trading centrality and degree have a positive relationship. This is the case, for instance, for "nicely behaved" networks such as regular, complete and star networks. In those cases, central dealers buy more in the PM to sell at a higher price in the local market, compared to others.[49]

To circumvent the above complexity, I develop a second network metric, *local centrality* $\tilde{\boldsymbol{c}}$, that is a sufficient statistic for price dynamics.[50] For trader i, $\tilde{c}_i$ reflects his participation and inventory effects in local markets. And it measures i's liquidity cost.

Proposition 2. *Liquidity Cost and Local Centrality*

Define $\tilde{c}_i \equiv \frac{c_i + \sum_{j \in N_i} c_j}{d_i}$ *as the local centrality of trader* $i \in N$*, which is the sum of his local market participants' trading centrality, including himself, controlled for his degree* d_i*.*

Then, i*'s re-selling cost is positive and proportional to his local centrality* $\tilde{c}_i$*. It's given by*

$$P_1 - P_i = \frac{Q}{c_A}(\tilde{c}_i - 1) > 0 \tag{17}$$

Moreover, $P_1 - P_i < 0$ *if and only if* i *is the core of a star network.*

[48] This is how we usually think of centrality, as a proxy for "prestige" and "influential power".

[49] By means of stylized examples, I show this in **??**.

[50] In the Appendix I show that local centrality $\tilde{\boldsymbol{c}}$ is a function of only the trading network structure, just as trading centrality $\mathbf{c}_{N\times1} : \tilde{c}_i \; \forall i \in N$. This is easy to see since one just needs to substitute out $\boldsymbol{c}$ for its functional form (13) in $\tilde{\boldsymbol{c}}$.

The rest of the paper explores how the structure of the trading network itself affects PM price. I also study welfare. But in order to understand all my findings - including Theorem 1 - it is crucial to distill what information trading centrality encapsulates.

Why trading centrality determines traders' asset acquisition in the PM? The answer to this question is best understood with a simple example (Subsection 5.1). After gaining the intuition, I then discuss in depth the information content and mathematical form of trading centrality (Subsection 5.2).

5.1 Example

The trading network structure is described by the adjacency matrix $\boldsymbol{G}$ such that $g_{ij} = \{0,1\}, g_{ij} = g_{ji} \forall i,j \in N$. It is essential to keep in mind the substituability of demands that arise because of the feedback between the PM and local markets. This implies that the trading network induces endogenous "trading costs" for holdings asset shares that reflect local market trades.

The *implied* trading network is given by the modified adjacency matrix $\tilde{\boldsymbol{G}}$ such that $\tilde{g}_{ij} \geq 0, \tilde{g}_{ij} \neq \tilde{g}_{ji} \forall i,j \in N$ (eq. (12)). $\tilde{\boldsymbol{G}}$ is a weighted and asymmetric matrix and a modified version of $\boldsymbol{G}$.[51] Take a trader $i \in N$. Each $\tilde{g}_{ij}$ is the marginal cost imposed by the holdings of a trade counterparty $j \neq i, j \in N$. It accounts for when i and j trade as a seller and a buyer to one another, and as competitors for a common linkage. On top of that, i incurs a marginal cost from his own holdings given his possible trade participations (i.e. every local market where he is a buyer, a seller or out of it), which is given by the individual coefficient $\frac{1}{v_i}$.

All these network-induced "trading costs" define i's marginal benefit of holding asset shares. Then, to optimally decide his PM demand, i equates it to the marginal cost of acquiring the asset, i.e. the PM price P_1. In matrix notation this gives,

$$\underbrace{\boldsymbol{1}_N - \left(\frac{\boldsymbol{1}}{\boldsymbol{v}} + \phi\tilde{\boldsymbol{G}}\right)\cdot \boldsymbol{q}_1}_{\text{asset holdings marginal benefit}} = \underbrace{\boldsymbol{P}_{\boldsymbol{1}N}}_{\text{asset holdings marginal cost}}$$

Notice that the above is simply the system of first-order conditions of traders' optimization problem in the PM (eq. (9)). Since all traders have the same optimal holdings of

[51] An interesting aspect of the forces in my model is that they turn the "plain" trading network, which is undirected and unweighted, into a weighted and directed network graph. Moreover, a graph that is weakly more connected than the trading network itself. See Figure 4

1, the lower is the row-wise sum of $\left(\frac{\mathbf{1}}{\boldsymbol{v}} + \phi\tilde{\boldsymbol{G}}\right)$ (i.e. trading costs), the higher is the trader's marginal utility and thus the higher is his willingness to pay. Then, in equilibrium, the trader's demand - and share allocation - is higher.

This operation is exactly what trading centrality does. But it gives the information in terms of marginal benefit instead of marginal costs:

$$\boldsymbol{c} \equiv \left(\frac{\mathbf{1}}{\boldsymbol{v}} + \phi\tilde{\boldsymbol{G}}\right)^{-1} \mathbf{1}_N$$

Thus, the score of each trader is precisely his marginal utility of holdings asset shares. Or, in other words, the marginal benefit of acquiring shares in the PM.

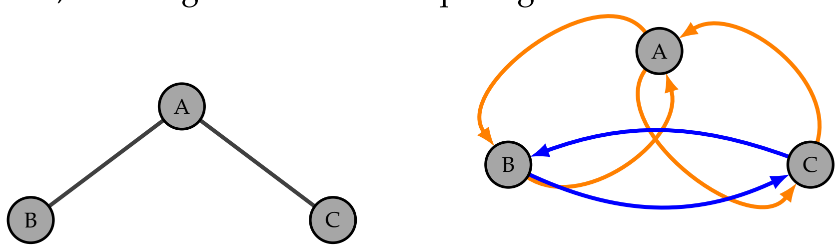

Figure 4: Orange links imply two trades (nodes) are buyers and sellers to one another (direct counterparties) in local markets. Blue links imply they are competitors (indirect counterparties).

Now let's look at a simple example. Consider $N = 3$ traders in the star trading network with structure described by the adjacency matrix $\boldsymbol{G}$ below (left graph of Figure 4). The implied star trading network is given by the modified adjacency matrix $\tilde{\boldsymbol{G}}$ (right graph of Figure 4):

$$\boldsymbol{G} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \tilde{\boldsymbol{G}} = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 1.25 & 0 & 0.25 \\ 1.25 & 0.25 & 0 \end{pmatrix}$$

The core trader A has the same interaction with both B and C, and that's why $\tilde{g}_{AB} = \tilde{g}_{AC} = 0.5$. However, B and C interact differently with one another and with A. Not only that, but they trade directly with A and indirectly with each other through A. That's why $\tilde{g}_{BA} = 1.25 > \tilde{g}_{BC} = 0.25$ and $\tilde{g}_{CA} = 1.25 > \tilde{g}_{CB} = 0.25$. The individual coefficients are $\frac{\mathbf{1}}{\boldsymbol{v}} = (1, 1.1875, 1.1875)'$. A has a lower marginal cost of holding shares because he has greater local market participation.

Traders' marginal benefit of asset holdings is then[52]

[52]Notice that $\left(\frac{\mathbf{1}}{v} + \phi\tilde{\boldsymbol{G}}\right) \cdot \mathbf{1}_N = (1.25, 1.5625, 1.5625)$. That is, the core has lower trading cost compared to the peripheries.

$$\begin{pmatrix}1\\1\\1\end{pmatrix}-\begin{pmatrix}1 & 0.125 & 0.125\\0.3125 & 1.1875 & 0.0625\\0.3125 & 0.0625 & 1.1875\end{pmatrix}\cdot\begin{pmatrix}q_{A,1}\\q_{B,1}\\q_{C,1}\end{pmatrix}\boldsymbol{q}_1=\begin{pmatrix}1\\1\\1\end{pmatrix}\cdot P_1$$

Thus, trading centrality is

$$\boldsymbol{c}=\begin{pmatrix}1.067 & -0.1067 & -0.1067\\-0.267 & 0.871 & -0.0178\\-0.267 & -0.0178 & 0.871\end{pmatrix}\cdot\mathbf{1}_N=\begin{pmatrix}0.853\\0.587\\0.587\end{pmatrix}$$

Letting the asset supply be one, the equilibrium asset allocation is $\boldsymbol{q}_1^*=(0.421, 0.289, 0.289)'$, which is proportional to trading centrality $\boldsymbol{c}$. The core trader is the most central and has the highest asset holdings. The equilibrium PM price is given by the aggregate trading centrality $c_A = 2.027$, such that $P_1^* = 1-\frac{1}{c_A} = 0.5066$.

5.2 The Information Content of Trading Centrality

Now that is clear how trading centrality maps trade flows into traders' behavior in the PM, we can better understand why it implies that the centrality of a trader is decreasing in all other traders' centralities.[53] To see this, notice that the fixed point representation in eq. (13) expresses each individuals centrality recursively as a function of others' centrality eq. (14).

The strength of effect between two traders' centralities varies with how far apart they are. On one hand, a trader i respond *more* negatively to the demand of traders at most two links apart (i.e direct connections or common connections). On the other, i respond *less* negatively to traders further away.

To grasp the intuition behind this property, take trader $i \in N$. Traders j, k at most two link apart from i directly influence i's local market trading. If i is a buyer, he demands less in the PM when j, k demand more. In this way, i can buy more shares at a lower price in local markets. The same logic holds when i responds to j (or k) as a competitor for k (or j). As a seller, i also lowers his PM demand in response to higher PM demand from j, k. In this way, i sells less at a lower price. For a trader z further apart, i still responds negatively to z's demand. But this response is not as strong since z does not

[53]This is makes sense as the framework is fundamentally a game of global strategic substitutes.

influence directly i's local market trading. Trader z only affects i because z's PM demand determines the terms of trade in other local markets (i has no access to) and, consequently, the PM demand of other traders. What ultimately determines the equilibrium PM price and asset allocation.

Trading centrality translates the above discussion into the matrix $\left(\mathbf{V}+\phi\tilde{\boldsymbol{G}}\right)^{-1}$. As I demonstrate before (Section 5.1), the entries in this matrix[54] are the endogenous network-induced trading costs $\left(\{\frac{1}{v_i}\}\ \forall i\in N, \{\tilde{g}_{ij}\}\ \forall i,j\in N\right)$ that appear in trading centrality representation (14), $c_i = v_i(1-\phi\sum_j \tilde{g}_{ij}c_j)$.

Two crucial features sets my centrality measure apart from the typical graph-theoretic measures of network centrality that are not suitable for the model. First, it captures not only first and higher-order inter-connectivity (friends, friends of friends, etc..) but also it encodes how such connections interact in local markets (as buyers, sellers, and competitors). Second, the relationship between centrality and individual degree depends on the structure of the trading network. Some networks - like regular graphs, stars, lines and trees - have trading centrality increasing in individual degree. For arbitrary network the reverse can hold. For example, if the network exhibits high-degree nodes connected to one another, than low-degree traders are the central ones.

As a final note, the recursive representation in eq. (13) is reminiscent of the *negative* Bonacich centrality[55] with weights that are not a simple geometric series (as in the Bonacich measure), but instead are endogenous and capture trading incentives.

6 Trading Network Structure and Equilibrium

The overall takeaway so far is that the trading network induces a complex relationship between traders and markets what is encapsulated in the trading centrality, the sufficient statistic for equilibrium and my main contribution. Now I investigate how changes in the trading network structure affect equilibrium outcomes.

[54]The *sign* of $\left(\mathbf{V}+\phi\tilde{G}\right)^{-1}$ varies with how far apart traders are. If traders i,j are directly connected or have one common connection, then the (i,j) entry is *negative*. However, if i,j are more than two links apart, then (i,j) entry is *positive*. See Appendix D.

[55]The Bonacich centrality is a well-known network measure of node importance. Its common adoption in economics has been with a *positive* scalar as it means an agent is more powerful (central) the more powerful are his connections. This interpretation is meaningful in many economic scenarios that exhibits local complementarities and it was first invoke by Ballester et al. (2006). My paper contributes to the less explored network models in which Bonacich centrality with negative scalar is the appropriate measure of node influence.

The relationship between the trading network and PM price can be broken down into three effects: market size N, connectivity and degree inequality.[56] I define connectivity as the average degree of the trading network. And degree inequality as the variance of traders' degrees. Notice that such effects are tightly interrelated. For instance, increasing the number of traders can affect how traders are connected. Or changing (re-arranging, deleting or adding) linkages can reduce or increase differences in traders' degree.

Before formally introducing the main findings, it is useful to illustrate them with a relatively simple example.

6.1 Example

Consider five traders A, B, C, D and E, and six different trading networks depicted in Figure 5. The networks are arranged in descending order of connectivity from left to right, as the first row of Table 1 shows.

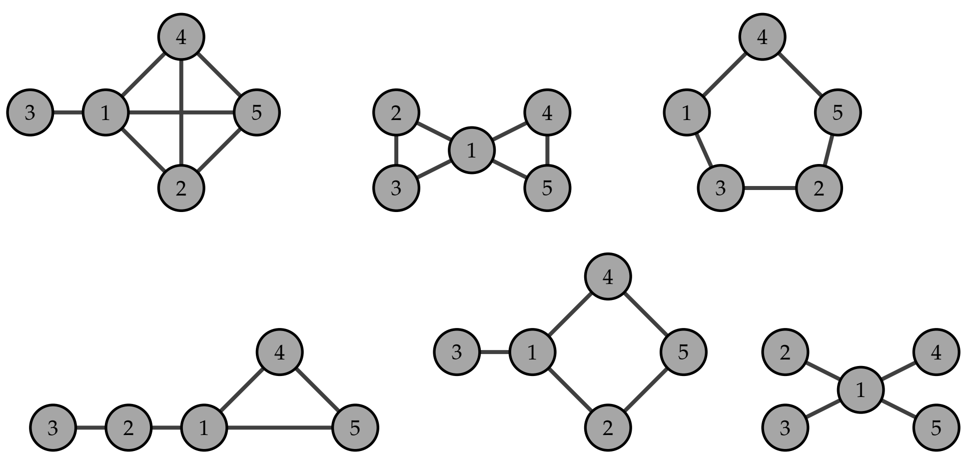

Figure 5: Trading networks 1, 2 3, 4, 5 and 6 - respectively

[56]One way to see why these effects emerge is to look at PM demand schedule $q_{i,1}$ (10). That depends on the first and second moments of the degree distribution: individual degrees and their square appear in the demand function.

	Net 1	Net 2	Net 3	Net 4	Net 5	Net 6
connectivity	2.8	2.4	2	2	2	1.6
inequality	1.2	0.8	0	0.5	0.5	1.8
PM price (P)	$P_2 > P_1 > P_3 > P_4 > P_5 > P_6$					
liquidity cost (PD)	$PD_6 > PD_5 > PD_4 > PD_3 > PD_1 > PD_2$					
welfare (EU)	$EU_6 > EU_3 > EU_5 > EU_4 > EU_2 > EU_1$					

Table 1: Characteristics and equilibrium outcomes of Figure 5. Connectivity is the average degree and inequality degree variance. Liquidity cost is the average difference between PM price and local market prices. Welfare is the aggregate expected utility.

We can first compare the structural proprieties of the trading networks - i.e. connectivity and degree inequality - and PM price. Network 2, the second most connected one, has the highest PM price. This reveals the non-monotonicity of PM price with respect to connectivity (Lemma 4). PM price is the lowest on network 6, the star network, what turns out to be the lower bound of PM price across all networks of the same size $N = 5$ (Proposition 3). Not depicted in fig. 5 is the complete network, which has the highest PM price. Indeed, as Proposition 3 shows, the complete network imposes the upper bound on PM price level.

Comparison of networks 2 and 3 depicts that (weakly) increasing all traders' degree results in a higher price: every trader in network 2 is at least as connected as in network 3 (despite the former having higher degree inequality) and, consequently, $P_2 > P_3$ (Proposition 4).[57] Comparison of networks 3 and 4 (and/or 5) shows that, keeping connectivity fixed, the higher is degree inequality (network 4), the lower is PM price (Proposition 5).[58] Finally, notice that networks 4 and 5 have the same connectivity and degree inequality (degree distribution) but different PM prices.

We've also seen that asset price almost always drop over time (Proposition 1). Just as with PM price, such price dynamics is non-monotonic in connectivity and degree inequality. Interestingly, the rank of the trading networks with respect to PM price is the reverse to their rank with respect to price drop ($3rd$ and $4th$ rows of table 1). This means that a higher price for acquiring asset shares does not lead to higher liquidity cost on average.

Welfare comparison offers additional insights (Section 7). Network 6, the star net-

[57] Formally, the degree distribution of 2 FOSD the distribution of 3 and thus price in the former is greater.

[58] Formally, the degree distribution of network 4 is a mean-preserving spread of the degree distribution of network 3 and thus price in the former is lower.

work, delivers the highest welfare. That's because its lowest PM price compensates its highest liquidity cost. Network 1, the most connected one, exhibits the lowest welfare. Actually, the complete network delivers the lowest welfare. Moreover, just as with trading centrality and PM price, connectivity and inequality are not sufficient information to analyze welfare - for example, look at networks 3, 4 and 5.

Lastly, we can compare traders within a network based on trading centrality and degree (Theorem 1, Lemma 3) .[59] Network 2 is an example of when centrality and degree are negatively related: $d_1 > d_2 = d_4 = d_5 = d_3 \leftrightarrow c_1 < c_2 = c_4 = c_5 = c_3$. In turn, central dealers in network 2 have higher asset holdings and higher liquidity cost (lower re-selling price). In the other networks of fig. 5, central dealers are market makers - they acquire more shares in the PM and have lower liquidity cost - because centrality and degree are positively related. Even so, this relationship is not the same across networks. For instance, networks 1 and 5 are the only cases when centrality is monotonically increasing in degree. .[60]

6.2 Main Results

A first natural question is which network structure, if any, delivers a maximum or minimum value for PM price. I find that the complete trading network delivers the highest PM price while the star trading network the lowest possible PM price.

Proposition 3. *Bounds on Primary Market Price*

Consider an arbitrary trading network of size N. The equilibrium primary market price is bounded by its level on two specific networks of the same size: above by the complete network, and below by the star network.

There are no trading frictions in a complete network. All traders are connected with one another and everyone trade in both markets, either as a buyer or a seller. In equilibrium, demand schedules and asset allocation are homogeneous across traders in every markets, and so are local market prices. From a buyer's perspective, trading in a local market is as competitive as in the primary market. For this same reason, the seller's price

[59] Recall that (as discussed when solving for the equilibrium) the relationship between trading centrality and degree plays an important role in shaping individual outcomes (i.e. demands and re-selling prices), although it is non-trivial. That's because certain network structures deliver a positive relationship between trading centrality and degree, while other structures deliver the opposite.

[60] That is, in network 1 we have $d_1 > d_2 = d_4 = d_5 = d_3 \leftrightarrow c_1 > c_2 = c_4 = c_5 = c_3$; and in network 5 $d_1 > d_2 = d_4 = d_5 = d_3 \leftrightarrow c_1 > c_2 = c_4 = c_5 = c_3$. However, in network 3, centrality is positively related to degree, but non-monotonically: $d_1 > d_2 = d_4 = d_5 > d_3 \leftrightarrow c_2 > c_1 > c_4 = c_5 > c_3$.

is likely to be higher. Both buyer and seller's effects combined induce traders to demand more in the primary market because i) as a buyer, higher PM demand lowers a seller's price; and ii) as a seller, a higher price will be obtained. Higher willingness to trade in the primary market pushes price up, even though the equilibrium allocation $q^*_{i,1} = \frac{\bar{Q}}{N} \forall i \in N$ is the same as in a frictionless market.

The bounds on PM price might indicate that price is monotonically affected by the two network effects since the complete (star) network has the highest (lowest) connectivity and lowest (highest) degree inequality. However, this is not true.

Lemma 4. *PM price is non-monotonic in connectivity and degree inequality.*

This results raises the question on how to compare equilibrium outcomes in the cross-section of trading networks of the same size. I do this by investigating how PM equilibrium is affected by specific changes in the degree distribution of the trading network.

Consider, in particular, a change in the probability distribution over the degrees of traders that reflects an unambiguous increase in connectivity, as given by the criterion of First Order Stochastic Dominance (FOSD). Denote the degree distribution of two trading networks as P and P'. If P first-order stochastically dominates P', then the average degree under P is higher than under P', the reverse not true.

Proposition 4. *Changes in connectivity: FOSD*

Suppose that P' FOSD P. Then the PM price under P' is unambiguously higher than under P.

All traders' demand are higher after the change.

Now consider an unambiguous increase in degree inequality, while keeping average degree unchanged. This change is capture by a mean-preserving in the degree distribution. If P' is a mean-preserving spread of P, then the variance under P' is higher than in P, reverse not true.

Proposition 5. *Changes in degree inequality: Mean-preserving spread*

Now suppose P' is a mean-preserving spread of P. Than PM price is lower under P' than under P.

traders' demand can increase or decrease depending on their network position. More (less) connected traders have an increase (decrease) in demand.

In other words, if we weakly increase all traders' degree - implying greater connectivity - PM price also increases (Proposition 4). And if degree inequality increases, keeping connectivity constant, PM price decreases (Proposition 5).

It is worth pointing out that the reverse does not necessarily hold (i.e., the above propositions are no if and only if statements). As argued before, trading centrality - the key statistic for equilibrium - encapsulates much more network information than just average degree and degree variance. It reflects traders' position in the network which is determined by other factors.

Lastly, it is easy to see that prices in all markets, the PM and every local market, are increasing in the number of N traders. For a given trading network, adding one more trader to it - no matter how he is positioned in the network - increase prices unambiguously. This fact is useful for the coming discussions.

Lemma 5. *Prices and Trading Network size*

Everything else the same, PM price and local market prices are increasing in N.

In the rest of this section, I study the equilibrium for particular trading network structures: core-periphery and ring. Doing so renders great tractability of the model because one just needs to keep track of two or one trading centrality (for core-periphery network Subsection 6.3 and a ring (Subsection 6.4, respectively). Apart from that, I show that such networks contribute to understating the relationship between PM price and the trading network.

6.3 Core-Periphery Networks

Empirical evidence suggests that different inter-trader markets exhibit a core-periphery structure (Figure 1). We already know that the star network, a particular core-periphery structure, has the lowest PM price (Proposition 3). A core-periphery network consists of a well-connected set of nodes, the core, and the remainder nodes, the periphery, well connected to the core but sparsely connected internally (Figure 6). Motivated by the above, I now restrict my study of equilibrium in core-periphery trading networks. First, I look at the star network (left in Figure 6). Then, I analyze how price changes as the size of the core (right in Figure 6) and periphery grows.

I find that when the core consists of a single trader, he is the unique trader obtaining capital gains (Proposition 6). And that, even though PM price is increasing in the size of the core, the price drop monotonically decreases as the network grows (Proposition 7).

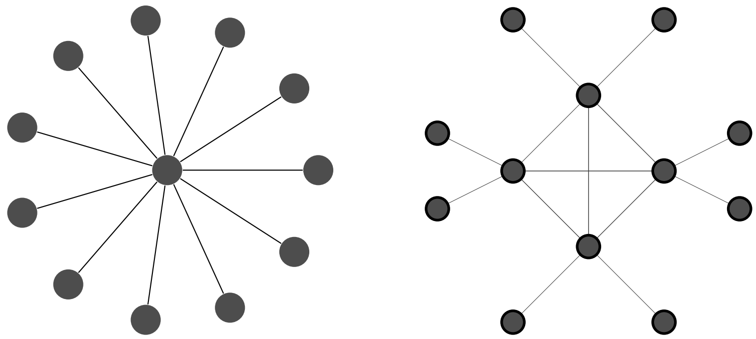

Figure 6: The figure depicts two core-periphery networks of the same size but with different cores

The star network is the simplest case of a core-periphery structure with a single core. An arbitrary core-periphery network can be understood as adding more traders to the core of the star network and keeping core's connectivity the same[61] The size of a core-periphery network is altered in two ways: by changing the number of core nodes and peripheral nodes.

The core traders have incentives to increase their demand. They expect to sell at high price and not be able to buy a lot in any local market, even though their buying prices are likely to be low. The reverse logic holds for peripheral trader. The peripheries anticipate a very low selling price and low marginal utility for their supply since they would only be trading with a core trader. This drives their demand down. In equilibrium, the traders in the core (periphery) have the highest (lowest) asset holdings.

What drives apart equilibrium proprieties of the star and other core-periphery networks are two facts. First, deviating from a single core decreases core traders' trading centrality. Second, cores' centrality is more affected than peripheries' one. The reason being traders in the core also trade with each other, who themselves are relatively more central than the peripheries. Lower trading centrality drives core's demand down and, thus, their selling price decreases. At the same time, demand inequality decreases since the peripheries' demand is greater.

First, as the size of the core increases, core traders' trading centrality decreases. Second, cores' centrality is more affected than peripheries' one. The reason being they also trade with each other, who themselves are relatively more central than the peripheries. Lower trading centrality drives core's demand down and, thus, their selling price decreases.

[61] Core traders are fully connected among themselves, they have the same number of connections to peripheral nodes, and each periphery is linked to a single core node.

The next proposition shows the asset pricing implication of such behavior.

Proposition 6. *Prices and the size of the Core*

Compared to any trading network with N traders, only the single core trader of the star network has a selling price higher than the PM price.

For any core-periphery network different than the star, selling prices are lower than the PM price for every traders. Although all prices are increasing in the size of the core.

The core trader of the star network, and only him, obtains capital gains from selling his shares. His price is the highest across all markets with N traders, including the PM. The deviation from a single core trader leads to higher PM price and lower local market prices, including for the core. Consequently, price drops over time - in stark difference with the star network.

Behind this result is the fact that cores' centrality is more affected than peripheries' once the network changes from a star to a core-periphery structure. Core traders' trading centrality decreases since they also trade with each other. Lower trading centrality drives their demand down and, thus, selling price decreases.

As the size of the core increases, competition among core traders intensify, reducing even further their centrality and demand. However, the market size effect dominates and all prices increases. Interestingly, the difference between core-periphery demands becomes smaller. The next proposition summarizes how PM equilibrium changes as the core-periphery network grows.

Proposition 7. *Equilibrium in Core-Periphery Networks*

As N and /or the number of core traders increase, the core-periphery network exhibits:

i) lower demand inequality $\left(q_{core,1} - q_{periphery,1}\right)$, and

ii) smaller price change over periods $|P_1 - P_i|\ \forall i \in N$.

If there is one core trader, there is lower price raise. For any other core size, there is lower price drop.

Summarizing, the star network has three interesting features: lowest PM price, highest demand inequality and possibility of capital gains. Such properties do not hold for any other core-periphery structure. Increasing the size of the core reduces demand inequality and no seller can obtain capital gain.

In Appendix I, I provide analytical solutions and further details on the equilibrium for core-periphery networks.

6.4 Ring and Star Trading Networks

The conclusion from the previous section is that prices and demands are tightly related to the number of traders in the core of core-periphery networks. Meanwhile, inspection of the main result Theorem 1 reveals that PM price is increasing the number of traders, N. And that PM price is determined by how the N traders are connected among themselves. This suggest that the effect of growing the trading network on equilibrium outcomes depends on the way new traders and their linkages are added in the network. That is, there exists a size effect - increasing N, and a network effect - changing the degree distribution.

One way to differentiate between the size effect and network effect on PM price is to look on how price changes as we grow a ring network and a star network. The former is a regular network.[62] It exhibits no inequality in terms of degree and asset allocation: all traders have the same demands and selling price as they have the same degree and network position. The star network is the most unequal one with respect to both degree and demand.

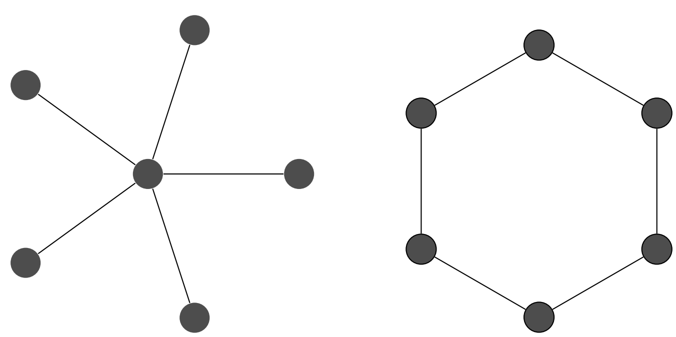

Figure 7: A ring and a star network of the same size $N = 6$ traders.

The unique effect of growing the ring network is about market size. The growth of the star network, apart from capturing the size effect, carries network effect because the degree distribution changes. Adding one trader in either case means adding just one more link. However, degree inequality and connectivity increase in the star network, while the degree distribution remains unchanged in the ring.[63]

It is useful to first compare market outcomes as the networks grow. By Lemma 5, we know that PM and local market prices in both networks increase. The difference is that in

[62] Appendix J provides equilibrium outcomes for the general class of regular networks.

[63] Notice that the number of linkages in the ring network is N and in the star network $N-1$, and connectivity is higher in the ring network.

the ring price drops while in the star price can either increase or decrease: price increase if the core is selling and drops if one of the peripheries is the seller.

Even though PM price increases as the network becomes larger, it does so as diminishing rates. More importantly, it grows faster in the star network.

The results above are depicted in the next figure. It shows PM price drop and PM price growth rate as each structure grows.

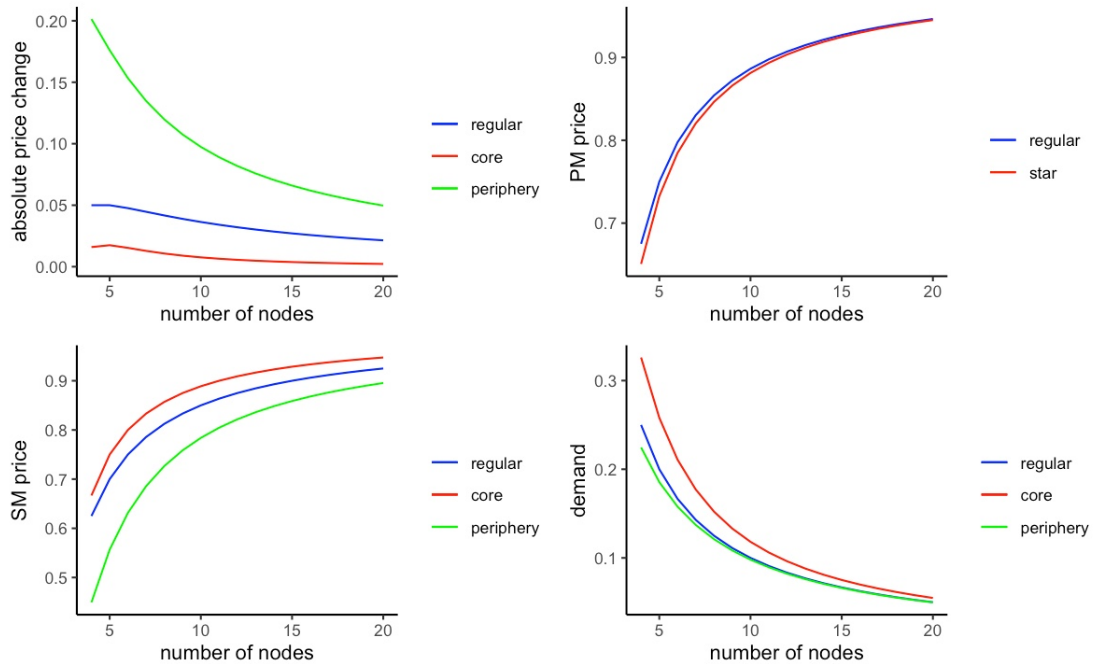

Figure 8: Growing Networks: Ring versus Star structures

If we divide the star growth rate by the ring growth rate we isolate the network effect. I find that the network effect is positive any finite N, and that it is greater the smaller the size of the trading network.

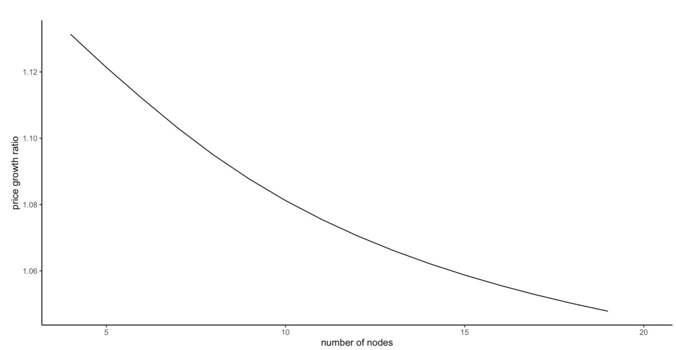

Figure 9: PM price growth ratio: Star/Ring

This finding is important. The network effect in the star network makes its price diverge from the one in the ring. The importance of this effect though diminishes as the network grows.

7 Welfare

Recall the example in Section 6.1 which illustrates the non-trivial relationship between welfare, connectivity and degree inequality. As I discuss next, this non-triviality makes the comparison of welfare across different trading networks challenging. I find that it is not necessarily true that welfare is enhanced by i) reducing the disparities between traders' number of connections and/or PM demand; or ii) increasing connectivity. Just as with trading centrality, welfare depends on the patterns of network connectivity.

Formally, it follows from the main result Theorem 1 that welfare is determined by trading centrality. The expected indirect utility EU_i^* of a trader $i \in N$ can be written in terms of his and others' centrality,[64]

$$EU_i^*\left(\mathbf{G}, \phi, \bar{Q}\right) = w + \left(\frac{Q}{Nc_A}\right)^2 \left[c_i - \frac{1}{2v_i}c_i^2 - \phi c_i \sum_j \tilde{g}_{ij} c_j\right] + \left(\frac{Q}{Nc_A}\right)\left[\sum_j \bar{g}_{ij} c_j\right] \tag{18}$$

and so EU_i^* is only a function of the trading network structure $\mathbf{G}$ and the parameters of the model (ϕ, Q).

Welfare of a trading network $\mathbf{G}$ is defined as the sum of traders' expected utility. This means that the welfare maximizing trading network $\mathbf{G}^*$, which delivers the highest aggregate expected utility, is the solution to the following maximization problem,

$$\mathbf{G}^* = \arg\max_{\mathbf{G}} \sum_{i \in N} EU_i^*\left(\mathbf{G}, \phi, Q\right) \tag{19}$$

Solving (19) is hard. Due to its dependence on trading centrality, welfare shares with it the property of non-monotonicity with respect to connectivity and degree inequality. Moreover, since trading centrality is a recursive measure (eq. (14)), it is affected by a trader's own degree and also the centrality and degree of other traders. It holds that, for

[64]Recall that i's expected indirect utility is given by $EU_i^* = w + (1-P_1^*)q_{i,1}^* - \frac{1}{2v_i}(q_{i,1}^*)^2 - \phi \sum_j \tilde{g}_{ij} q_{j,1}^* q_{i,1}^* + \sum_j \bar{g}_{ij} q_{j,1}^*$.

all $i, j \in N$, $\frac{\partial c_i}{\partial c_j} < 0$ but $\frac{\partial c_i}{\partial d_j}$ can be positive or negative depending on all traders' degree, since trading centrality is determined by the distribution of individual degrees $\{d_i\}_{\forall i \in N}$.

To gain some insights on welfare, I now focus on the same network structures analyzed in the result Subsection 6.4: the ring and the star, plus the complete network and the line - depicted in fig. 10. I provide two welfare rankings: traders' expected utility within a trading network, and aggregate expected utility across trading networks of the same size.

Welfare study in the star, ring, complete and line structures is facilitated because they all share the same property of trading centrality being monotonically increasing in degree.[65] Thus, it holds that central dealers are those with i) higher PM demand (i.e. asset holdings); ii) lower liquidity costs (i.e. higher re-selling prices); and iii) who face lower local market prices as buyers.

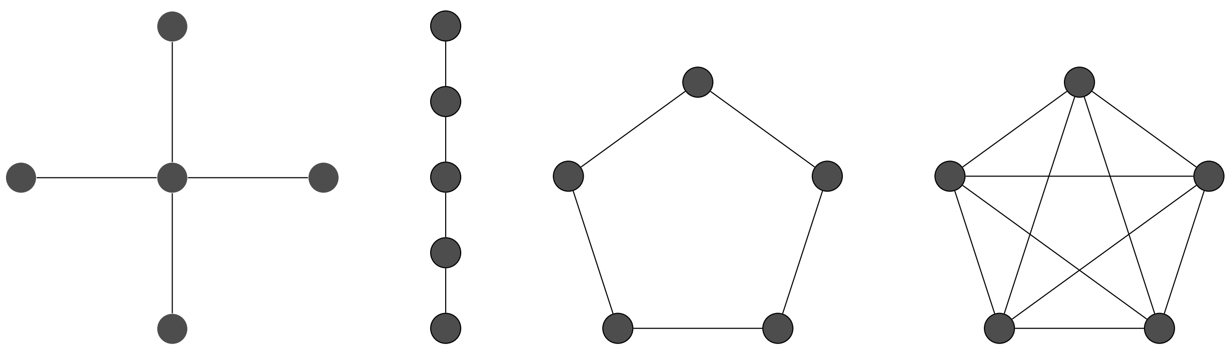

Figure 10: The star, the line, the ring and the complete trading network with five traders.

At the individual level, I find that a trader's expected utility EU_i^* is increasing in his centrality c_i.[66] Thus welfare ranking of traders is given by trading centrality: more central traders achieve a higher expected utility.

Proposition 8. *For $N > 3$ and if the trading network is either a complete, ring, line or star graph, then a trader's expected utility increases in his trading centrality.*

At the aggregate level, higher connectivity leads to higher aggregate trading centrality and, thus, higher PM price (eq. (15)). Not only that, but also higher local market prices since they are increasing in degree. As a consequence, I find that welfare decreases with aggregate trading centrality - and connectivity.[67]

[65] Recall Lemma 3 and Section 6.1 that this is not always the case.

[66] This results from the fact that trading centrality is increasing in degree See Section L and Lemma 9.

[67] See Appendix L. There I show that this result follows from the previous ones regarding PM price bounds (Proposition 3) and the price effect of the degree distribution (Proposition 4 and Proposition 5).

Proposition 9. *For $N > 3$, welfare ranking across the following networks is:*

$$\textit{Star} > \textit{Line} > \textit{Ring} > \textit{Complete}.$$

Thus welfare ranking of trading networks is according to the aggregate trading centrality, and it is the reverse as the PM price rank. At a first glance this result seems odd. Welfare increases with degree (and asset allocation) inequality, and decreases with connectivity.

However, keep in mind that traders in the model are natural buyers of the asset by assumption. And each trader has a (weakly) greater probability of being a buyer of the asset in both periods. In turn, the opposite effect of the degree distribution on prices pushes them at such a greater level that is detrimental to traders' utility. On top of that, the likelihood of high local market price is higher in more connected and less unequal networks, what also drives utility down. Ultimately, greater welfare is determined by traders *expecting* lower prices in all markets - the PM and local markets.

It is worth emphasizing that asset allocation inequality is an equilibrium outcome. Traders, by taking into account their own and others' position in the trading network, optimally decide their holdings. That's why demand inequality is not necessarily detrimental for welfare, but a reflection of the strategic response between traders themselves.

My welfare analysis concludes that the trading network delivering the highest (lowest) PM price, the complete (star) structure, is the exact one delivering the lowest (highest) welfare and I stress that Proposition 9 only compares the four network structures illustrated in fig. 10. A more interesting exercise is to consider any arbitrary trading network and investigate which trader(s) or linkages should be removed or added to increase welfare. I leave this for future research.

In the literature of decentralized markets, a typical result argues that the absence of frictions would correspond to maximal welfare. This is not the case in my model and it's similar to the result obtained in Malamud and Rostek (2017), Wittwer (2021) andGlode and Opp (2020), who demonstrate that decentralized markets might be more efficient than centralized markets. In contrast to these previous work, my framework is the first so show that allocative efficiency in decentralized markets (i.e. local markets) does not lead to greater (or maximal) welfare. Moreover, intertemporally, welfare is maximized by having the two most extreme trading schemes: a centralized (PM) market followed by a market in which one trader intermediate all trading flows (star network).

8 Discussion

With the intuition for my results in place, I now establish that the model accommodates pertinent extensions, such as i) heterogeneity in preferences, ii) uncertain asset fundamental return, and iii) imperfect competition. (as in Rostek and Yoon (2020a)). I also relate the two main assumptions of the model to the current literature.

In any extension, even though the model becomes less tractable, the main finding still prevails: primary market price is characterized by the trading network. The general formulation for trader's optimal demand schedule (10) is given by

$$q_{i,1}\left(q_{-i,1};P_1,\phi,\mathbf{G},\Psi\right)=\beta_i\left[a_i-bP_1-\phi\sum_j\beta_{ij}q_{j,1}\right] \tag{20}$$

where Ψ is the set of model parameters apart from the shock ϕ. Coefficients $\left(\{\beta_i,a_i\}_{i\in N},b,\{\beta_{ij}\}_{\forall i,j\in N}\right)$ are endogenous and functions of one or all arguments $(\phi,\mathbf{G},\Psi)$.

What differs across model specifications is, apart from *parameters*,[68] how the patterns of trading linkages map into trading centrality and, thus, prices and demands. I point out that the study of equilibrium outcomes and the trading network structure in Section 6 and Section 7 do not hold in general. I leave this exploration for a companion paper.

8.1 Heterogeneity in Preferences

The baseline model is the homogeneous version of the general setup of each trader $i\in N$ having quasilinear-quadratic utility over inventory with parameters of individual valuation $\alpha_i>0$ and risk-aversion γ_i

$$U_i(Q_i)=\alpha_iQ_i-\frac{\gamma_i}{2}Q_i^2 \tag{21}$$

so that traders are ex-ante heterogeneous. From building up inventory Q_i a trader obtains a marginal value α_i and has a marginal cost of $\frac{\gamma_i}{2}Q_i$. Heterogeneity in α_i captures the different and persistent close relationships traders tend to form with their clients in OTC markets (**?**). The different cost γ_i may be related to fund outside investments, regulatory capital or collateral requirements, which may vary across traders.

The unique and interior Nash Equilibrium is characterized by the demand schedule

[68] Namely, i) the different individual valuations and risk aversion, and ii) the different beliefs.

$$q_{i,1}=\beta_i(\gamma;\phi,\mathbf{G})\times\left[\underbrace{\left(m_i(\gamma;\phi,\mathbf{G})\,\alpha_i+\phi\sum_j m_{ij}(\gamma;\mathbf{G})\,\alpha_j\right)}_{a_i}-P_1-\phi\sum_j\beta_{ij}(\gamma;\mathbf{G})\cdot q_{j,1}\right]$$

where now the endogenous network-induced coefficients also depend on the risk aversion of traders, and $b=1$. In Appendix M give the full specification of coefficients above. An important observation is that individual valuations α_i only affects the level of demand.

8.2 Expected Fundamental Returns

In reality, traders care about the fundamental return of an asset. They hold an asset not just for the sake of holding it (i.e. to enjoy utility flow) but because they expect that the asset itself is a good financial investment, with high intrinsic value. My framework accommodates *asset-related information* and with that, as I show next, asset price reflects both the traders' beliefs on returns and the trading network. Importantly, the way the former is incorporated into price depends on the later.

To understand this results a brief description of this extension is enough (See appendix for all the details). Suppose the asset has uncertain return f which is normally distributed with mean μ and variance σ^2, and it is realized after all trading activities take place[69] traders have initial wealth w_0 and choose asset inventory Q_i to maximize the expected CARA utility of final wealth $E\left[-\exp\left(\gamma W\right)\right]$ given by

$$W=f\left(q_{i,1}+q_{i,s}\right)-\left(P_1q_{i,1}+P_sq_{i,s}\right)+w_0 \tag{22}$$

The counterpart Nash Equilibrium demand of Equation 10 is

[69]The normality assumption is standard in this literature. See, for instance, Kyle (1989), Vives (2011), Rostek and Weretka (2012), (Duffie and Zhu (2016)) and others.

$$q_{i,1} = \beta_i(\phi)\left[\underbrace{\mu \cdot \frac{(1+\phi m_i(\mathbf{G}))}{\gamma\sigma^2}}_{a_i} - \underbrace{\frac{1}{\gamma\sigma^2}}_{b} P_1 - \phi \sum_j \beta_{ij} q_{j,1}\right]$$

where the coefficients only depend on the trading network $\mathbf{G}$. As before, see Appendix N for a detail the full analytical solution.

The equilibrium implies that trader i's PM demand depends on market price P_1, his information *and* the information and demand of *all other traders*, including those he is not directly connected to but who are connected with his connections. This is in stark difference with the canonical linear asset pricing model where individual demands depend on all agents' information set *but not directly* on other demands. That's because in such setting equilibrium price aggregate all useful information and so it is not necessary to know other demands. In my model, however, even an anticipated shock and the fact the it leads to different trading possibilities make agents to conditional on others demands, since this is informative about the market structure.

8.3 Price Impact

The definition of competitive market I use is that traders are price-takers but they do take into account the effect of their first-period choices on the second period price, and vice-versa. This is different than saying that they try to manipulate prices. Even though traders are not strategic in terms of price impact, as for example in Kyle (1989) and Rostek and Weretka (2015), they are strategic in terms of demands. They understand that their PM demands directly affects PM price.

The model is essentially a static demand game because a trader's PM demand depends on expected local market trades, not on realized trade. This is the crucial feature of the model. And it is the reason local market trading induces a set of games in the PM. In equilibrium, prices and demands are not independent across markets. If they were, the markets would operate as independent venues and this is clearly not the case here.

Price impact in the PM arises endogenously in the model precisely because of intertemporal demand dependence. That is, it comes from *all the possible* local market exchanges in the trading network. Moreover, PM asset marginal utility is dictated by participation in the SM, and vice-versa.

Following the imperfect competition literature, the framework can be used to study the economy with strategic traders, as Rostek and Yoon (2020b). In Appendix O I show that the Nash equilibrium demand is given by

$$q_{i,1} = \beta_i(\phi)\left[1 - P_1 - \phi \sum_j \beta_{ij} q_{j,1}\right]$$

and so $a_i = 1 \forall i \in N, b = 1$. Notice that this is closest specification to the baseline model (10).

9 Conclusion

This paper shows why and how future re-sale market structures affect asset pricing before trade, in what I call the primary market (PM). I develop a dynamic trading model where re-sale of a divisible asset takes place in local markets of limited and random participation, captured by a trading network. I show that to find the equilibrium is enough to look at the structure of the trading network.

Two novel network metrics define equilibrium variables. Trading centrality is a sufficient statistic for equilibrium, and local centrality determines re-sell cost. Behind this result is the interdependency of demands across traders and markets due to the interaction of re-sell risk and the interconnected local markets. The key insight is that my network measures process all the information driving traders' behavior.

My results are of interest to regulators, scholars and participants of financial markets alike. I argue that the interdealer network not just guarantee the well-functioning of over-the-counter markets but determines the cost of credit in the economy. Also, trading centrality and local centrality are new measures of liquidity and "importance" in the interdealer market that only require information about the interdealer network structure. Both can be useful in empirical applications.

This paper allows for several and interesting extensions. Straightforward ones are having multiple sellers, and to allow for the choice of trading venues or contemporaneous access to all markets. One present limitation is that, due the two-period environment, I do not study intermediation chains which have been pointed outed as important in otc markets. However, having multiple periods natural extension of the model. Apart from financial market applications, the model I develop is suitable for any environment in which a good is initially priced by a large set of agents but subsequently it is only valued or can only be traded by a restricted subset of them.

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Appendices

Appendix A The Model the Real-World Interdealer Market

In the context of off-exchange markets, the timing of the model captures the idea that dealers often absorb substantial inventory position in primary markets of asset issuance or from their costumers, and then use the interdealer market to offload these positions. Interdealer trades is how dealers provide liquidity to one another.

The re-sell shock[70] is interpreted as the risk of selling under pressure, or the risk of having inventory imbalances resulting from unexpected and large customer orders. In either scenario, a dealer is forced to raise liquidity by quickly disposing his inventory in a possibly illiquid market (Duffie and Zhu (2017)), the interdealer market. This rationalizes why the shocked trader *must sell all* his holdings. And why I do not allow for the choice of being a seller, nor that two traders can be shocked at the same time. Such views of the re-sell shock have been empirically documented, and it turns out that they are not uncommon events.[71]

The quadratic utility function in the quantity traded,[72] apart from being the standard in the literature, also contributes to the tractability of the model. Specifically, it leads linear equilibria that have proved to be useful as a basis for empirical analysis and are supported in the empirical literature on single-sided multi-unit auction - which is the main mechanism of asset issuance in primary markets and electronic interdealer trades.[73] Moreover, this utility representations reflects the fact that dealers are risk-averse with respect to inventory (Ho and Stoll (1983)). Because of that, they often have a desired but costly inventory position, and they trade such to avoid large deviations from this target. Finally, the quadratic utility is microfounded in the mean-variance trade-off of a trader (CAPM theory), and it is equivalent for trading behavior to the the classic CARA-Normal setting.

[70] The use of a random shock to generate trade is common both in the finance (Duffie et al. (2005), Vayanos and Weill (2008)) and network literature (Gale and Kariv (2007), Condorelli et al. (2021)).

[71] See Di Maggio et al. (2017), Balasubramaniam et al. (2020) among others.

[72] See, for instance, Kyle (1989) Vives (2011), Rostek and Weretka (2012), Rostek and Yoon (2020a) among others.

[73] For example, Hortacsu (2199), using data from Turkish Treasury auctions, shows that linear demands fit actual bidding behavior quite closely

The trading network captures trading frictions that have been extensively explored in the OTC markets literature, both theoretical and empirically. The premise is that different traders must be sufficiently close on some dimension to be able to trade. This could be justified by having lower trading costs, or similar clientele so that both value the asset being issued. One possible interpretation is that linkages are due to previous investments in relationships: traders may invest time and resources to contact other traders and to know them better. An alternative view is that trading is costly, and linkages capture parties with an easiness to trade. In any case, I'm agnostic on how the network linkages have aroused and I assume it boils down to a pre-determined and fixed set of trading relationships, the trading network.

Appendix B Solving the Model

B.1 Pricing Mechanism

It is convenient to visualize inter-trader trading, i.e. the local market stage, as involving three steps. First, the shocked trader - the seller - $s \in N$ hands over his entire holding of the asset, $q_{s,1}$, to an auctioneer (the inter-trader broker). Then, the auctioneer solicits bids from all *available* traders[74] - the buyers - in the form of demand schedules: combinations of price and quantity. A typical buyer i's trading strategy, as a function of the equilibrium seller's price and and his own pre-trading position, is the quantity that is awarded to him by the auctioneer. The equilibrium single price of the seller, P_s, is determined by equating demand and supply. After the auctioneer collects payment from all buyers, the total proceeds are returned to the seller and are his to keep. Note that, at the conclusion of the inter-trader trading, seller's holding is zero, while buyers total position is $q_{i,1} + q_{i,s}$. In other words, the seller exits the market while buyers increase their asset holdings.

The assumption of a unique active local market ensures there is a single equilibrium price at period two, even though multiple prices are ex-ante possible. This is a distinct feature of the dealership market nowadays which are conducted through an inter-trader broker. The brokerage system guarantees anonymity and that transactions take place at a single price.

[74] This means that the seller is inactive at the local market stage. In the Appendix I consider allowing the seller to choose his supply as well.

B.2 Optimal Demand Decisions

At $t = 1$, trader i submits demand schedule $q_{i,1}(\cdot)$ considering the probability of future local markets he could participate. His optimization problem is

$$\max_{q_{i,1}(\cdot):\mathcal{R}^N\to\mathcal{R}} \quad \phi\left\{w + q_{i,1}\left(P_i - P_1\right)\right\} + \phi\sum_{s\in N_i}\left\{\left(q_{i,1}+q_{i,s}\right) - \frac{1}{2}\left(q_{i,1}+q_{i,s}\right)^2 + w - \left(P_1 q_{i,1} + P_s q_{i,s}\right)\right\}$$
$$+ \left(1-\phi(d_i+1)\right)\cdot\left\{q_{i,1} - \frac{1}{2}q_{i,1}^2 + w - P_1 q_{i,1}\right\} \tag{23}$$

The first component of (23) is the what i gets when she is the seller in the local market. The second term accounts for every possible payoff i gets from trading as a buyer with each of her network-implied sellers. The last term is the payoff of just trading in the primary market, when i is not shocked nor connected to the shocked agent.

All traders face the same idiosyncratic shock ϕ and so they all have the same probability of being a seller. However, the likelihood of being a buyer is determined by how many connections i has. The network also dictates trader i's willingness to trade at $t = 2$, since buying in the PM brings i's closer to his target portifolio. Because of that, the choice of $q_{i,1}$ affects and is affected by local market demands $\{q_{i,s}\}_{i\cup s\in N_i}$ and prices $\{P_s\}_{i\cup s\in N_i}$.

The first order condition of (23) is

$$\begin{aligned}
&\phi\cdot\left\{P_i - P_1 + q_{i,1}\frac{\partial P_i}{\partial q_{i,1}}\right\}\\
&+\phi\cdot\sum_{s\in N_i}\left\{1 + 1\frac{\partial q_{i,s}}{\partial q_{i,1}} - \left(q_{i,1}+q_{i,s}\right)\left(1-\frac{\partial q_{i,s}}{\partial q_{i,1}}\right) - P_1 - P_s\frac{\partial q_{i,s}}{\partial q_{i,1}} - q_{i,s}\frac{\partial P_s}{\partial q_{i,1}}\right\}\\
&+\left(1-\phi(d_i+1)\right)\cdot\left\{1 - q_{i,1} - P_1\right\} = 0
\end{aligned} \tag{24}$$

At $t = 2$, both the PM and the re-sell shock have realized. The seller identity $s \in N$ is common knowledge and, by assumption, he does not make any decision in the local market: he supplies all his PM shares, $q_{s,1} \le Q$. Each buyer $i \in N_s$ chooses how many shares to buy from the seller s, $q_{i,s}$, taking into account his PM holdings:

$$\max_{q_{i,s}(\cdot):\mathcal{R}\to\mathcal{R}} \quad \left(q_{i,1}+q_{i,s}\right) - \frac{1}{2}\left(q_{i,1}+q_{i,s}\right)^2 + w - \left(P_1 q_{i,1} + P_s q_{i,s}\right) \tag{25}$$

Notice that $(w - P_1 q_{i,1})$ is the capital available to invest after trading at $t = 1$. First-order condition delivers buyer i's demand schedule for seller $s \in N_i$:

$$q_{i,s}(P_s; \mathbf{q}) = (1 - q_{i,1}) - P_s \tag{6}$$

Buyer i's downward-slopping demand $q_{i,s}$ does not directly depend on the network, and it is negatively related to his PM holdings. That's because a buyer's willingness pay for q_s (i.e. his marginal utility from trading with the seller) is given only by how far he is from the target inventory, $(1 - q_{i,1})$ (See Section 3). The higher is $q_{i,1}$, the more satisfied i is with his current amount of asset shares and so he will be less willing to trade with the seller for any possible price.

Demand (6) confirms that the local market is *perfectly competitive with heterogeneous valuations*: buyers' demands differ only by their asset holdings coming into $t = 2$.[75] As so, the local market can be interpreted as a static Walrasian market with heterogeneous asset endowment.

B.3 Active Local Market

The immediate corollary of Proposition regards positive price.

Corollary 9.1. *Seller's Price and Excess Demand*

Any seller $s \in N$ has positive equilibrium price if and only if there exists an excessive average demand in his neighborhood:

$$P_s > 0 \leftrightarrow \sum_{i \in N_s} \frac{(1 - q_{i,1})}{d_s} > \frac{q_{s,1}}{d_s}$$

In principle, P_s could be negative. To ensure weakly positive prices $P_s \geq 0$, there must be an *excessive average demand* in the seller's neighborhood, i.e the neighborhood's valuation of the asset is greater than the supply : $\sum_{i \in N_s} \frac{(1-q_{i,1})}{d_s} \geq \frac{q_{s,1}}{d_s}$. This constraint ensures that buyers are willing to trade with the seller since their asset holdings at $t = 2$, $\{q_{i,1}\}_{i \in N_s}$, are low enough. To secure their optimal inventory - that is, to reduce $1 - (q_{i,1} + q_{i,s})$ - a buyer's demand schedule is positive at the seller's equilibrium price.

From pricing equation (7) we can draw three conclusions. First, P_s^* is determined by the total average PM asset holdings, including the selling quantity. Equivalently, seller's

[75]Two different buyers demand the same amount *if and only if* their first-period demand are them same.

price is given by the difference between the average of target inventories of his buyers and his per-link supply: $P_s = \frac{1}{d_s}\sum_{i\in N_s}(1-q_{i,1}) - \frac{1}{d_s}q_{s,1}$. Second, seller's price is constrained by the buyers' total amount of PM shares: $P_s^* \in [(1-q_{s,1})-q^{\max}_{N_s,1}, (1-q_{s,1})-q^{\min}_{N_s,1}]$, where $q^{\max}_{N_s,1}$ is the highest first-period consumption of a buyer and $q^{\min}_{N_s,1}$ is the lowest PM consumption of a buyer. Thus, the more heterogeneous buyers' asset holdings, the wider the range of possible prices for a given seller s. Finally, two cases can make P_s to be zero (negative): if buyers have high too much bond holdings (i.e., too high PM demand); and if the supply is much greater than seller's aggregate demand (i.e. excessive supply is large enough). In both cases, P_s has to be low enough to induce buyers to demand from the seller.

Appendix C The Trading Network Game

It turns out that the model is in essence a network game of strategic substitute. This is the key insight of this paper as it allows the equilibrium characterization in the Primary and all Local markets. Moreover, the game belongs to a particular class of games: those with quadratic payoff function and linear best replies. The networks literature has extensively studied this type of games.[76] In this Appendix section, I derive in details the characterization of the game as a result of the model and prove the equilibrium outcomes. I start by describing the model as a network game, comprised of the degree distribution and each agent's action and payoffs. I then discuss the equilibrium concept, Nash Equilibrium, and provide equilibrium results for the game.

C.1 Actions, Links and Payoffs

Traders $i = \{1,2,...,N\}$ simultaneously choose actions: each trader i chooses his PM demand $q_{i,1} \geq 0$. Traders are embedded in the fixed Trading Network represented by the matrix $\mathbf{G} \in \{0,1\}^{N\times N}$ with $g_{ij} = 1$ implying a link between agents i, j.

Each trader's payoff is a function of own action, $q_{i,1}$, others' actions, $\mathbf{q_{-i,1}}$, the network $\mathbf{G}$, and the shock parameter $\phi \in (0, \frac{1}{N})$:

$$\pi_i\Big(q_{i,1}, \mathbf{q_{-i,1}}; \mathbf{G}, \phi\Big) = w + (1-P_1)\cdot q_{i,1} - \frac{1}{2v_i(\phi)}q_{i,1}^2 - \phi\sum_j \tilde{g}_{ij}q_{j,1}q_{i,1} + \phi\sum_j \bar{g}_{ij}q_{j,1} \qquad \textbf{(??)}$$

[76]See Bramoullé et al. (2014) and Galeotti et al. (2010).

where

$$v_i(\phi) \equiv \left[\frac{2\phi}{d_i} + 1 - \phi(d_i+1) + \phi \sum_j g_{ij} \cdot \frac{(2d_j - 1)}{d_j^2}\right]^{-1} \quad \textbf{(??)}$$

$$\tilde{g}_{ij} = g_{ij} \cdot \left[\frac{1}{d_i} + \frac{(d_j - 1)}{d_j^2}\right] + \left[\sum_{\substack{z \neq j \\ z}} g_{iz} g_{jz} \frac{(d_z - 1)}{d_z^2}\right] \quad \textbf{(??)}$$

$$\bar{g}_{ij} = g_{ij} \cdot \frac{1}{2d_j^2} + \sum_{z \neq \{i,j\}} g_{iz} g_{jz} \cdot \frac{1}{d_z^2} \quad (26)$$

The endogenous coefficients $\{v_i(\phi), \tilde{g}_{ij}, \bar{g}_{ij}\}_{\forall i,j \in N}$, all non-negative, are determined by network graph **G** such that

$$\begin{aligned}
&\frac{\partial \tilde{g}_{ij}}{\partial d_i} \leq 0, \qquad \frac{\partial \bar{g}_{ij}}{\partial d_i} = 0 \\
&\frac{\partial \tilde{g}_{ij}}{\partial d_j} \leq 0, \qquad \frac{\partial \bar{g}_{ij}}{\partial d_j} \leq 0 \quad for \quad d_j \geq 2 \\
&\frac{\partial \tilde{g}_{ij}}{\partial d_z} \leq 0, \qquad \frac{\partial \bar{g}_{ij}}{\partial d_z} \leq 0 \quad for \quad d_z \geq 2, \quad z \neq \{i,j\} : g_{iz} = g_{jz} = 1
\end{aligned}$$

Individual payoff function (9) is strictly concave in own-action $\frac{\partial^2 \pi}{\partial^2 q_{i,1}^2} = (2v_i(\phi))^{-1} > 0$ for all $i \in N$. The re-sell shock probability $\phi > 0$ regulates the global interaction effect among agents. As ϕ increases, the payoff externalities of agents' action become globally stronger. That's because, the higher ϕ, the greater is the chance of being both a buyer and a seller in the local market. And so, the greater is the feedback effect between markets.

The term multiplying individual action $q_{i,1}$, $(1 - P_1) \geq 0$ is common across all agents and represents an agent's optimal action absent network interactions. That is, $(1 - P_1)$ corresponds to the individual demand in a competitive market of size N, when future trading does not opportunity exists.

The individual network effect $v_i(\phi)$ is increasing in both i and his connections' degree, $d_i, \{d_j\}_{j \in N_i}$. The higher d_i, the higher is i's selling price and thus the more he consumes in the PM. The higher d_j, the higher the prices i will face in a local market, thus also inducing more demand in the PM.

The global network coefficients $\tilde{g}_{ij} \geq 0$ captures bilateral influences. Traders' actions are strategic substitutes since $\frac{\partial^2 \pi_i}{\partial q_{i,1} \partial q_{j,1}} = -\tilde{g}_{ij} \leq 0$.

The individual payoff equation (**??**) is exactly the individual expected utility (**??**) in the baseline model. Hence, payoff maximization is equivalent to trader's expected utility maximization problem.

C.2 Best Replies and Nash Equilibrium

The solution concept considered is pure-strategy Nash Equilibrium. Bramoullé et al. (2014) and Bramoullé and Kranton (2016) give the conditions that guarantees a unique interior equilibrium of a quadratic-linear network game of strategic substitutes. As I show, such conditions are met in the model and thus market outcomes are unique and interior.

Given the quadratic linear payoff function (**??**), an interior Nash equilibrium in pure-strategies $q^*_{i,1} > 0$ is such that $\partial \pi_i / \partial a_i(\mathbf{a}^*) = 0$ and $q^*_{i,1} > 0$ for all $i \in N$.

Lemma 6. *For a given price level P_1, each trader i's best-response to others' demands is given by the first order condition of* (**??**)*:*

$$q_{i,1}(q_{-i,1}; P_1, \phi, \mathbf{G}) = \max\left(0, v_i(\phi)\left[(1-P_1) - \phi \sum_j \tilde{g}_{ij} q_{j,1}\right]\right) \qquad \textbf{(??)}$$

Optimal demand $q_{i,1}(q_{-i,1}; P_1, \phi, \mathbf{G})$ is linear in others' demands and $q_{i,1}(\mathbf{q_1}) \in [0, 1-P_1]$ Define the following matrices: $\mathbf{V}$ is the N-diagonal matrix with entries $\{v_i\}_i$; $\tilde{\boldsymbol{G}}$ is te N-square, not symmetric matrix with entries $\{\tilde{g}_{ij}\}_{i,j}$; $\mathbf{1}_N$ is the N-vector of ones.

The existence of a unique interior equilibrium for each game is guaranteed as long as the shock probability ϕ is no greater than $1/N$.

Proposition 10. *Primary Market Equilibrium Demand*

Denote $\mathbf{V}\tilde{\boldsymbol{G}}_S$ the symmetric part of the N-square matrix $\mathbf{V}\tilde{\boldsymbol{G}}$. For each price P_1, a unique and interior Nash equilibrium exists if and only if $\phi < -1/\lambda_{min}\left(\mathbf{V}\tilde{\boldsymbol{G}}_S\right)$. Then, the vector of optimal PM demands $\mathbf{q}_1$ is

$$\mathbf{q}_1 = (1-P_1)\left(\mathbf{I} + \phi \mathbf{V}\tilde{\boldsymbol{G}}\right)^{-1} \mathbf{V}\mathbf{1} \qquad (27)$$

Traders' demands $\mathbf{q}_1$ is a linear function of the weighted adjacency matrix of global network effects $\tilde{\boldsymbol{G}}$ and the vector of individual effects $\mathbf{v}$. In **??**, I show how the Nash equilibrium looks like for a simple economy with four traders and six different trading networks.

The equilibrium PM price is determined by the market clearing condition (3). It is a particular Nash Equilibrium of the set of equilibria characterized by (10) such that aggregate demand meets exogenous asset supply $\bar{Q}$. Thus, equilibrium PM price follows directly from **??**.

Theorem 2. *Assume conditions of* **??** *hold. Then, given asset supply $\bar{Q} > 0$, the unique and positive equilibrium primary market price is*

$$P_1^* = 1 - \bar{Q}\left(\mathbf{v}'\left(\left(\mathbf{I} + \phi\mathbf{V}\tilde{\boldsymbol{G}}\right)^{-1}\right)'\mathbf{1_N}\right)^{-1} \tag{28}$$

where $\mathbf{v}'\left(\left(\mathbf{I} + \phi\mathbf{V}\tilde{\boldsymbol{G}}\right)^{-1}\right)'\mathbf{1_N}$ *is a strictly positive scalar.*

As argued, the crucial demand decision is the primary market one and it allows us to determine i) traders' inventory at the end of period 2; ii) the PM (issuance) price of the bond, and so the cost of credit for the issuer; iii) all possible market outcomes in the local market.This last point is a corollary of Theorem 2.

Corollary 10.1. *Secondary Market Outcomes*

If conditions of Theorem 2 hold, equilibrium outcomes in the secondary market are determined by the network graph $\mathbf{G}$*, the shock* ϕ *and issuer's asset supply* $\bar{Q}$*. They are characterized by the price vector* $\mathbf{p_d}_{N\times 1}$ *and square matrix of demands* $\mathbf{Q}_{N\times N}$*:*

$$\mathbf{p_d} = \mathbf{1}_N - \mathbf{D}^{-1}\left(\mathbf{G} + I\right)\mathbf{q_1^*} \tag{29}$$

$$\mathbf{Q}_d = diag\left(\mathbf{1}_N - \mathbf{q}_1^*\right)\mathbf{G} - \mathbf{G}diag\left(\mathbf{p_d}\right) \tag{30}$$

where $\mathbf{D} = diag\left(\mathbf{d}\right)$ *is the diagonal matrix of individual degrees*

The demand matrix $\mathbf{Q}_d$ gives the amount of asset shares traded between two agents at each possible market in period 2: rows indicate the buyer and columns the seller. For example, $[\mathbf{Q}_d]_{ij} = q_{i,j}$ is agent i's demand when j is the seller. Clearly, if i and j don't share a connection then $q_{i,j} = 0$. $\mathbf{Q}_d$ has a zero-diagonal as it does not include asset supply.

Appendix D Trading Centrality

Trading centrality fixed point representation (13) and recursive representation (14) are related as follows:

$$
\begin{aligned}
&\mathbf{c} = \left(I + \phi \mathbf{V}\tilde{\boldsymbol{G}}\right)^{-1} \mathbf{V}\mathbf{1}_N \\
&\quad or \\
&\mathbf{c} = \mathbf{V}\mathbf{1}_N - \phi \mathbf{V}\tilde{\boldsymbol{G}}\mathbf{c} \\
&\quad = \mathbf{V}\mathbf{1}_N - \phi \mathbf{V}\tilde{\boldsymbol{G}}\left(\mathbf{V}\mathbf{1}_N - \phi \mathbf{V}\tilde{\boldsymbol{G}}\mathbf{c}\right) \\
&\quad = \mathbf{V}\mathbf{1}_N - \phi \mathbf{V}\tilde{\boldsymbol{G}}\mathbf{V}\mathbf{1}_N + \phi^2 \left(\mathbf{V}\tilde{\boldsymbol{G}}\right)^2 \mathbf{c} \\
&\quad = \mathbf{V}\mathbf{1}_N - \phi \mathbf{V}\tilde{\boldsymbol{G}}\mathbf{V}\mathbf{1}_N + \phi^2 \left(\mathbf{V}\tilde{\boldsymbol{G}}\right)^2 \left(\mathbf{V}\mathbf{1}_N - \phi \mathbf{V}\tilde{\boldsymbol{G}}\mathbf{c}\right) \\
&\quad = \mathbf{V}\mathbf{1}_N - \phi \mathbf{V}\tilde{\boldsymbol{G}}\mathbf{V}\mathbf{1}_N + \phi^2 \left(\mathbf{V}\tilde{\boldsymbol{G}}\right)^2 \mathbf{V}\mathbf{1}_N - \phi^3 \left(\mathbf{V}\tilde{\boldsymbol{G}}\right)^3 \mathbf{c}... \\
&\quad ... \\
&\quad = \left[\sum_{t=0}^{\infty} (-\phi)^t \left(\mathbf{V}\tilde{\boldsymbol{G}}\right)^t\right] \mathbf{V}\mathbf{1}_N
\end{aligned}
$$

That is

$$
c_i = v_i \left[1 + \sum_{t=0}^{\infty} (-\phi)^t \left(v_i \sum_j \tilde{g}_{ij}\right)^{t+1}\right]
$$

The trading network adjacency matrix $\mathbf{G}$ describes all linkages among traders: it is symmetric and unweighted. But the crucial matrix for the model is the global network effect matrix $\tilde{\boldsymbol{G}}$ such that $\tilde{\mathbf{g}}_{ij} \geq 0$: it is asymmetric and weighted and it describes traders' interaction in any the Secondary Market. $\tilde{\boldsymbol{G}}$ itself induces a graph. Denote the trading network graph by $\mathcal{G}$ and the global network graph by $\tilde{\mathcal{G}}$, both undirected. It holds that $\mathcal{G} \subseteq \tilde{\mathcal{G}}$.

Matrix $\tilde{\boldsymbol{G}}$ has entries in the $[0,1]$ interval except when $d_i = 1$ and $d_j > 1$. The centrality matrix $(I + \phi \mathbf{V}\tilde{\boldsymbol{G}})^{-1}$ has positive diagonal and negative off-diagonal. The diagonal is increasing in one's connectivity and neighbor's connectivity.

Lemma 7. *Centrality Matrix*

The centrality matrix $(I + \mathbf{V}\tilde{\boldsymbol{G}})$ has diagonal equals to 1 and off diagonal elements in $[0,1)$. Off-diagonal entry is zero iff (i,j) are not connected nor share a path-two link.

The inverse centrality matrix $(I + \mathbf{V}\tilde{\boldsymbol{G}})^{-1}$ is a N-square matrix with positive diagonal. Off-diagonal elements can be a) negative, if (i,j) are connected or share a path-two link; b) positive, if (i,j) are not connected nor share a path-two link

Trading centrality is non-monotonic in individual degree. In turn, individual analysis is complex in the model. For example, depending on the network structure, central traders are those well (poorly) connected. Individual degree is not enough information to understand a trader's behavior because the latter also depends on the behavior of his direct ad indirect connections.

D.1 Trading Centrality and Degree

As discussed in Section 5, the relationship between trading centrality and individual degrees is non-trivial. The centrality measure encapsulates information above and beyond connectivity. And connectivity by itself is not enough to understand the feedback effect between the primary market and subsequent local markets. Meanwhile, as the network structure defines trading centrality, it also defines te relationship between centrality and degree. In some networks, being more central means being more connected. In others, the opposite holds.

Lemma 3 states that as long as degree and centrality are positively correlated, seller's price is increasing in centrality. It turns out that this result is useful when analyzing the impact of network structure on equilibrium outcomes. As so, I invoke it in several proofs. Re-stating Lemma 3,

$$corr(d_i, c_i) > 0 \leftrightarrow \frac{\partial P_i}{\partial c_i} > 0$$
$$corr(d_i, c_i) < 0 \leftrightarrow \frac{\partial P_i}{\partial c_i} < 0$$

A corollary of Lemma 3 regards which traders are the market makers in the model. That is, who are those buing the most in the PM to re-sell higher prices, relatively o others.

Corollary 10.2. *Central traders as Market-makers*

For a given trading network and in equilibrium, trader $i \in N$ selling price P_i increases, and his capital loss $P_1 - P_i$ decreases, with his trading centrality c_i.

To further see the ambiguity between re-selling price and centrality notice that, while P_i is increasing in degree, it is equally decreasing in the seller and buyers' centrality. But recall that i) individual degree and individual centrality are correlated; ii) individual degree and others' centrality are correlated; and ii) traders' centrality are also correlated. From Equation 14, we know that a trader is more central as his connections are less central. This means that as centrality increases, two opposite forces act on price: high c_i pushes price down, and low $\sum_{j\in N_i} c_j$ pushes price up. The ambiguity is resolved by looking at how degree relates to centrality,

$$\frac{\partial P_i^*}{\partial d_i} = \frac{\partial P_i^*}{\partial d_i} + \frac{\partial P_i^*}{\partial c_i}\frac{\partial c_i}{\partial d_i} + \sum_{j\in N_i} \frac{\partial P_i^*}{\partial c_j}\frac{\partial c_j}{\partial d_i}$$

As long as $corr(d_i, c_i) > 0$, then seller's price is increasing in seller's centrality.

D.2 Local Trading Centrality

From Lemma 1 and Theorem 1, the selling price of trade i is given by his and his buyers' centralities, $c_i, \{c_j\}_{j\in N_i}$ respectively:

$$P_i^* = 1 - \frac{\bar{Q}}{Nc_A}\frac{\left(c_i + \sum_{j\in N_i} c_j\right)}{d_i} \tag{31}$$

Then, i's trading cost is

$$\begin{aligned} P_1 - P_i &= \left(1 - \frac{\bar{Q}}{Nc_A}\right) - \left(1 - \frac{\bar{Q}}{Nc_A}\frac{\left(c_i + \sum_{j\in N_i} c_j\right)}{d_i}\right) \\ &= \frac{\bar{Q}}{Nc_A}\left(\frac{c_i + \sum_{j\in N_i} c_j}{d_i} - 1\right) \end{aligned} \tag{32}$$

It is useful to denote $\tilde{c}_i \equiv \frac{c_i + \sum_{j\in N_i} c_j}{d_i}$ as the local centrality of trader i, that is, the sum of his local market participants' centrality, including himself, controlled for his degree. With that, there is a straightforward relation between trading cost and local centrality.

Proposition 11. *Trading Cost and Centrality*

The trading cost of trader $i \in N$ is increasing in his local centrality $\tilde{c}_i$:

$$P_1 - P_i = \frac{Q}{Nc_A}\left(\tilde{c}_i - 1\right) > 0 \tag{33}$$

Moreover, $P_1 - P_i < 0$ if and only if i is the core of a star network of size $N \geq 3$.

Local centrality captures both participation and inventory effects. And that's the reason it's a sufficient statistic for liquidity cost.

D.3 Centrality and the Adjacency Matrix

:

To express the PM equilibrium in terms of the adjacency matrix, it is useful to introduce some definitions. Define the following vectors which are functions of individual degrees:

$$\begin{aligned}
\mathbf{d^1} &: d_i^1 \equiv \frac{1}{d_i} \\
\mathbf{d^2} &: d_i^2 \equiv \frac{d_i - 1}{d_i^2} \\
\mathbf{d^3} &: d_i^3 \equiv \frac{2d_i - 1}{d_i^2} = \frac{d_i}{d_i^2} + \frac{(d_i - 1)}{d_i^2} = d_i^1 + d_i^2
\end{aligned}$$

Let $\mathbf{D^1}, \mathbf{D^2}, \mathbf{D^3}$ denote the zero-diagonal matrix with diagonal entries as $\mathbf{d^1}, \mathbf{d^2}, \mathbf{d^3}$, respectively.

The global network matrix $\tilde{\boldsymbol{G}}$ has zero-diagonal and non-negative entries everywhere else. It is useful to decompose it into direct and indirect effects:

$$\tilde{\boldsymbol{G}} = \tilde{\boldsymbol{G}}_1 + \tilde{\boldsymbol{G}}_2$$

where

$$\begin{aligned}
\tilde{\boldsymbol{G}}_1 &\equiv \mathbf{D^1 G} + \mathbf{G D^2} \\
\tilde{\boldsymbol{G}}_2 &\equiv \mathbf{G D^2 G} - diag\left(\mathbf{G D^2 G}\right)
\end{aligned}$$

Then

$$\tilde{G} = \mathbf{D^1 G} + \mathbf{GD^2}(I + \mathbf{G}) - diag\left(\mathbf{GD^2 G}\right)$$

The matrix of indirect effects $\tilde{G}_2$[77] is a weighted count of the paths of length two in the network. This path plays an important role in determining a trader trading centrality because it captures competition in the local market.

Now I turn into the individual effect vector $\mathbf{v}(\phi)$. It is useful to express it in terms of a vector $\mathbf{u}$ defined as

$$\mathbf{u} = (1-\phi)\mathbf{1}_N + 2\phi\mathbf{d^1} - \phi\mathbf{d} + \phi\mathbf{G}\mathbf{d^3}$$

such that

$$v_i(\phi) = \frac{1}{u_i} \tag{34}$$

The total effect of other traders $j \neq i$ on $q_{i,1}$ can be simplified to

$$v_i \cdot \tilde{g}_{ij} = \begin{cases} \frac{1 + d_i\left(d_j^2 + \sum g_{iz} g_{jz} d_z^2\right)}{2\phi + d_i\left(1 - \phi - \phi d_i + \phi \sum g_{ij} d_j^3\right)} & if\, g_{ij} = 1 \\ \\ \frac{d_i \sum g_{iz} g_{jz} d_z^2}{2\phi + d_i\left(1 - \phi - \phi d_i + \phi \sum g_{ij} d_j^3\right)} & if\, g_{ij} = 0 \;\; and \;\; g_{ij}^2 > 0 \\ \\ 0 & if\, g_{ij} = 0 \;\; and \;\; g_{ij}^2 = 0 \end{cases}$$

Appendix E Comparative Statics

Best-replies:

Trader i's response to changes in behavior of others is independent of PM price. In the best-replies space, varying price P_1 corresponds to parallel shifts of the demand schedules. As a trader $j \neq i$ changes his PM demand $q_{j,1}(\cdot)$, i reacts by changing his demand $q_{i,1}$ according to

[77] The matrix of path-two is given by $\mathbf{P}_2 = \mathbf{G^2} - diag(\mathbf{G}^2)$.

$$\frac{\partial q_{i,1}(\cdot)}{\partial q_{j,1}(\cdot)} = -\phi v_i(\phi)\tilde{g}_{ij} \tag{35}$$

As expected, the partial derivative (35) is negative as the game is one of strategic substitutes. It is also asymmetric: the way j affects i is not the same as i affects j. That's because i and j can have different positions in the network and so they will face different sets of potential sellers, buyers and competitors. This is precisely the reason that just looking at how many connections (degree) of a trader is not enough to understand his behavior. Lastly, it only depends on the trading network and shock parameter since, as shown in Theorem 2, these are necessary and sufficient information to find PM equilibrium (along with the exogenous supply level $\bar{Q}$). Mathematically, the matrix $\tilde{\boldsymbol{G}}$ is asymmetric and individual coefficients $\{v_i(\phi)\}_i$ are heterogeneous.

The shock effect:

P_1 only affects how demands change in response to the shock. To see this, first notice that ϕ has a direct and indirect effect on $q_{i,1}$:

$$\frac{\partial q_{i,1}(\cdot)}{\partial \phi} = \frac{\partial v_i}{\partial \phi}\left[(1-P_1) - \phi \sum_j \tilde{g}_{ij} q_{j,1}\right] - v_i \sum_j \tilde{g}_{ij} q_{j,1}$$

The indirect effect comes trough $v_i(\phi)$:

$$\begin{aligned} \frac{\partial v_i}{\partial \phi} = -v_i^2 \cdot \psi_i > 0 \qquad & if \;\; \psi_i < 0 \Longleftrightarrow \underbrace{\frac{(d_i^2 + d_i - 2)}{d_i}}_{>0} > \sum g_{ij} \cdot \underbrace{\left(\frac{2d_j - 1}{d_j^2}\right)}_{>0} \\ < 0 \qquad & if \;\; \psi_i > 0 \Longleftrightarrow \underbrace{\frac{(d_i^2 + d_i - 2)}{d_i}}_{>0} < \sum g_{ij} \cdot \underbrace{\left(\frac{2d_j - 1}{d_j^2}\right)}_{>0} \end{aligned} \tag{36}$$

where, to simplify notation, I let

$$\psi_i \equiv \left[\frac{2}{d_i} - (d_i + 1) + \sum_j g_{ij} \cdot \left(\frac{2d_j - 1}{d_j^2}\right)\right]$$

ψ_i is a measure of relative connectivity between i and his neighbors. When i is, loosely, more (less) well-connected than his friends then $\psi_i < 0$ ($\psi_i > 0$). In turn, the total effect of ϕ on $q_{i,1}$ is positive (negative). Thus an increase in the shock probability makes a trader i to increase his demand only if he is more connected than his neighbors:

$$\frac{\partial q_{i,1}(\cdot)}{\partial \phi} = -v_i^2 \cdot \psi_i > 0 \qquad if \;\; \psi_i < 0$$

As the secondary market becomes more likely, i expects to sell relatively more asset shares at a high price if he is shocked, and to buy relatively few shares at low price is he is connected to the seller.

The individual degree effect:

Individual degrees $\{d_i\}_{i\in N}$ appear in all PM demand's network components $(v_i(\phi), \{\tilde{g}_{ij}\}_{j\neq i}$. Their effect is can be broken down by a trader's own degree, his neighbors' degrees, and his neighbors' connections degrees.

First, individual degree d_i has a positive effect on v_i and a negative effect on $\{\tilde{g}_{ij}\}_{j\neq i}$

$$\begin{aligned} \frac{\partial v_i}{\partial d_i} &= -\left(-\frac{2\phi}{d_i^2} - \phi\right) \cdot v_i^2 = \left(\frac{2\phi}{d_i^2} + \phi\right) \cdot v_i^2 > 0 \\ \frac{\partial \tilde{g}_{ij}}{\partial d_i} &= -\frac{1}{d_i^2} < 0 \qquad \forall j \;\; s.t. \;\; g_{ij} \geq 1 \end{aligned} \tag{37}$$

Both effects combined imply that $q_{i,1}$ is increasing in individual degree d_i.

Second, each direct connection's degree d_j has a direct and indirect effects, the latter coming from common friends:

$$\begin{aligned} \frac{\partial v_i}{\partial d_j} &= -\phi\left(-\frac{2}{d_j^2} + \frac{2}{d_j^3}\right) \cdot vi^2 = \phi\left(\frac{2(d_j-1)}{d_j^3}\right) \cdot v_i^2 \geq 0 \\ \frac{\partial \tilde{g}_{ij}}{\partial d_j} &= -\frac{(d_j-2)}{d_j^3}\left(1 + \sum_{z\neq\{i,j\}} g_{ij}g_{jz}\right)\begin{cases} \leq 0 & d_j \geq 2 \\ > 0 & d_j = 1 \end{cases} \end{aligned} \tag{38}$$

Lastly, purely indirect connections' degrees, i.e. those who are connected to i only trough a common friend, have an effect on i's demand that depends only on he common friend's degree. That is, the effect of trader k's degree d_k such that $g_{ij} = 1, g_{jk} = 1, g_{ik} = 0$ for all $j, k \in N$ is

$$\frac{\partial \tilde{g}_{ik}}{\partial d_k} = -\sum_{j \neq \{k,i\}} g_{jk} g_{ij} \frac{(d_j - 2)}{d_j^3} \leq 0 \qquad since \quad d_k \geq 2 \tag{39}$$

The price effect:

The elasticity of demand of each trader is given by his individual network effect $v_i(\phi)$:

$$\frac{\partial q_{i,1}}{\partial P_1} = -v_i(\phi) \tag{40}$$

This has two implications. First, the higher $v_i(\phi)$, the more elastic is a trader's PM demand. Traders respond negatively but in different magnitude to changes in the PM price. Second, the slope of a trader's demand schedule changes as P_1 varies. This is precisely because each P_1 induces a different network game and traders' best-replies are game-specific.

Figure E.1 depicts the price effect. In the left-hand graph, each colored line is a trader's demand curve for a given price P_1. For example, the orange line is the schedule when $P_1 = 0.9$. Clearly, as P_1 increases the demand curve becomes steeper. The right-hand panel compares demand curves to two traders i (orange) and j (blue) such that $v_i(\phi) > v_j(\phi)$.

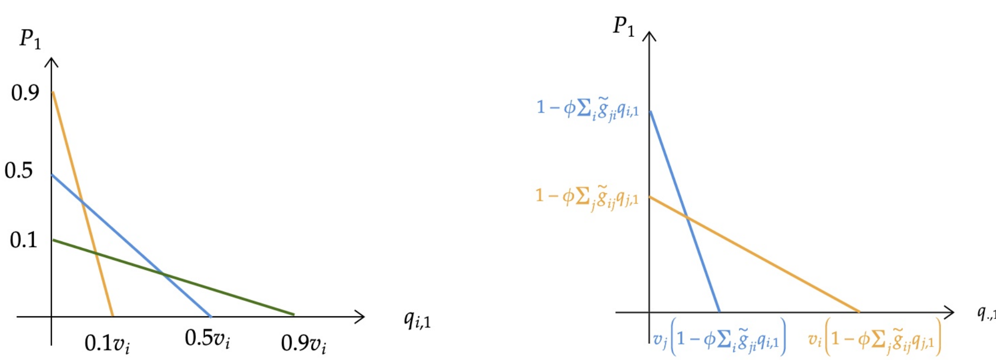


Figure E.1: Best-replies and Primary market price

Appendix F Bilateral Trading Comparison

In this section, I discuss the equilibrium outcomes if only bilateral trade was allowed. Maintaining the assumptions of the model, this would be the case of a regular network with degree two, that is, a network with regular components of size two. Using the results for regular networks, PM equilibrium price and asset consumption are $P_1^* = 1 - \frac{\bar{Q}}{N}(1+2\phi)$ and $q_1^* = \frac{\bar{Q}}{N} \quad \forall i \in N$, respectively. Any local market equilibrium is given by price and asset consumption, $P_2^* = 1 - 2\frac{\bar{Q}}{N}$ and $q_2^* = q_1^* \quad \forall i \in N$.

Bilateral trade delivers the *lowest* possible prices in the primary market and all local markets. In equilibrium, asset consumption is the same in both periods even though their demands are different. A trader's demand schedule in the PM is $q_1 = \frac{1-P_1}{1+2\phi}$ and in the local market is $q_2 = \frac{P_1}{1+2\phi} - P_s$.

Traders' expected total asset holdings is simply the average supply $\frac{\bar{Q}}{N}$. Also, traders' expected utility is given by

$$
\begin{aligned}
EU &= w + (1-P_1^*)q_1^* - \frac{1+2\phi}{2}(q_1^*)^2 \\
&= w + \frac{(1+2\phi)}{2}\left(\frac{\bar{Q}}{N}\right)^2
\end{aligned}
$$

With bilateral trade, if the restriction in the shock $\phi < \frac{1}{N}$ is discarded, local market price can be greater than the primary price.

Proposition 12. *Bilateral Trade and Difference in Prices*

If the trading network only allows for bilateral trades, equilibrium prices in the PM and any local market are $P_1^* = 1 - \frac{\bar{Q}}{N}(1+2\phi)$ and $P_2^* = 1 - 2\frac{\bar{Q}}{N}$, respectively. Traders consume the same amount of asset shares in both periods.

The negative demand shock ϕ controls the capital gain/loss of the seller. Selling bilaterally delivers capital loss (gain) if ϕ is less (greater) than $\frac{1}{2}$. If $\phi = \frac{1}{2}$, prices are equal.

It is useful to see the equilibrium in matrix notation for a direct comparison with my main result. First, demand schedule coefficients are $v_i(\phi) = (1+\phi)^{-1} \quad \forall i$ and $\tilde{g}_{ij} = 1 \quad \forall i,j : g_{ij} = 1$ and zero otherwise. This means that the global network matrix $\tilde{\boldsymbol{G}}$ equals the adjacency matrix $\mathbf{G}$ and that $\mathbf{V}\tilde{\boldsymbol{G}}$ is simply $(1+\phi)^{-1}\mathbf{G}$. Trading centrality simplifies to $\mathbf{C} = \frac{1}{1+\phi}\left(\mathbf{1} + \frac{\phi}{1+\phi}\mathbf{G}\right)^{-1}\mathbf{1}_N$.

Appendix G Results

Appendix H Degree Distribution

I study changes in the trading network structure by changing its degree distribution: that is, the number of connections of each trader. I do two exercises based on stochastic dominance: varying the average degree and the degree variance. Specifically, I compare two degree distributions such that i) one is a first-order stochastic shift of the other; and ii) one is a mean-preserving spread of the other.

Recall the stochastic dominance definition: a cumulative distribution F first-order stochastically dominates (FOSD) another distribution G iff $F(x) \leq G()$ for all x. Note that if F FOSD G, then F necessarily has a strictly larger expected value than G, $EF(x) > EG(x)$ (the reverse not true). Recall also the mean-preserving spread definition: F is a mean-preserving spread of G if and only if $EF(x) = EG(x)$ and $\int_{\infty}^{c} F(x)dx \leq \int_{\infty}^{c} G(x)dx$ for all c. Note that if F is a mean-preserving spread of G than F necessarily has a larger variance than G.z

Denote the degree distribution of the trading network as P. Consider a change in the probability distribution over the degrees to P' that reflects an unambiguous increase in connectivity. In particular, suppose that P' FOSD P. Then, the average degree under P' is higher than under P. Moreover, each trader's degree under P' is at least as large as his degree under P'. I am interested in how traders' demands, and hence price, changes as the trading network shifts from P to P'.

Looking at network coefficients in (11) and (12), we know that $v_i(\phi)$ weakly increases while $\tilde{g}_{ij}$ weakly decreases for all $i, j \in N$ (see Appendix E). Then, from the demand function (10), each trader demands more at each possible price level, i.e. his demand schedule becomes flatter. Since PM price is increasing in traders' demands, PM price increases as well.

Appendix I Core-Periphery Networks

A core-periphery network structure typically consists of a well-connected set of nodes, the core, and the remainder nodes, the periphery, well connected to the core but sparsely connected internally. The most common example is the star network in which one node is fully connected to all other nodes, who themselves are only connected to the core.

?? shows that the star network is the unique structure delivering capital gains for a seller while exhibiting the lowest primary market price. At the same time, empirical evidence has documented a core-periphery structure for different inter-trader markets. Motivated by these two facts, in this section of the Appendix I provide detailed results and proof for the class of core-periphery networks. In particular, I focus on the star graph, regular core-periphery networks, and the most extreme cases of a fully connected core (the complete case) and the sparsely connected core (the ring case).

I.1 Star Network

In this part of the appendix, I prove the following lemmas.

Lemma 8. *Across all markets and across networks of the same size N, the core's price of the star network is the highest.*

Another interesting feature of the star network is that it is the most unequal: it delivers the highest dispersion in asset allocation. Traders located in the periphery shift their asset consumption relatively more to the local market even though the seller's price is high. That's because, if a periphery is shocked, his selling price is so low that his capital loss would be larger than the difference in prices between the two markets he can act as a buyer. The next proposition state this result.

Proposition 13. *Inequality in a Star Trading Network*

Across trading networks of the same size N, the star structure delivers the highest dispersion in asset allocation. The core (periphery) has the highest (lowest) possible primary market demand.

Proposition 14. *Demand inequality is decreasing in the size of the star network N.*

Core's network coefficients are

$$v_c^{-1} = \frac{2\phi}{N-1} + N(1-\phi) = \frac{2\phi + (N^2 - N)()1 - \phi}{N-1}$$
$$\tilde{g}_{cp} = 1$$

and his demand function is then

$$q_c = v_c\Big(1 - P_1 - \phi(N-1)q_p\Big) \tag{41}$$

Peripheries' network coefficients are

$$v_p^{-1} = 1 + \phi \frac{(2N-3)}{(N-1)^2}$$
$$\tilde{g}_{pc} = 1 + \frac{(N-2)}{(N-1)^2} = \frac{N^2 - N - 1}{(N-1)^2}$$
$$\tilde{g}_{pc} = \frac{N-2}{(N-1)^2}$$

and their demand function is then

$$q_p = v_p \left(1 - P_1 - \phi \tilde{g}_{pc} q_p - \phi (N-2) \tilde{g}_{pp} q_p\right) \tag{42}$$

Now I can write the system of demands in 2-by-2 matrix format in which the first row/column refers to the core and the second row/column to a periphery. Matrices $\mathbf{V}, \tilde{\boldsymbol{G}}$ are given by

$$\left(\mathbf{V}^{-1} + \phi \tilde{\boldsymbol{G}}\right) = \begin{pmatrix} \frac{1}{v_c} & \phi(N-1) \\ \phi \tilde{g}_{pc} & \frac{1}{v_p} + \phi(N-1)\tilde{g}_{pp} \end{pmatrix} \tag{43}$$

and trading centrality is then

$$\mathbf{TC} \equiv \begin{pmatrix} C_c \\ C_p \end{pmatrix} = \left(\mathbf{V}^{-1} + \phi \tilde{\boldsymbol{G}}\right)^{-1} \mathbf{1} = \frac{1}{\Delta} \begin{pmatrix} \frac{1}{v_p} + \phi(N-1)\tilde{g}_{pp} - \phi(N-1) \\ \frac{1}{v_c} - \phi \tilde{g}_{pc} \end{pmatrix} \tag{44}$$

where $\Delta \equiv det\left(\left(\mathbf{V}^{-1} + \phi \tilde{\boldsymbol{G}}\right)^{-1}\right)$ and it holds that $\frac{1}{v_p} + \phi(N-1)\tilde{g}_{pp} - \phi(N-1) > \frac{1}{v} - \phi \tilde{g}_{pc}$ Equilibrium is determined by the weighted sum of centralities,

$$C_T \equiv \frac{1}{\Delta} \begin{pmatrix} \frac{1}{v_p} + \phi(N-1)\tilde{g}_{pp} - \phi(N-1) & \frac{1}{v} - \phi \tilde{g}_{pc} \end{pmatrix} \begin{pmatrix} 1 \\ N-1 \end{pmatrix} \tag{45}$$

Thus primary market price and asset allocation are, respectively

$$P_1^* = 1\frac{\bar{Q}}{C_T}$$

$$q_i^* = \frac{\bar{Q}}{C_T}C_i \qquad i = \{c, p\}$$

I.2 Core-Periphery Networks

Growing the size of the core:

The next figures depicts Proposition 7. Figure I.1 shows the equilibrium of the model for growing core-periphery networks by increasing the number of N traders. The number of core traders, with the same connectivity to 2 peripheries, increases.

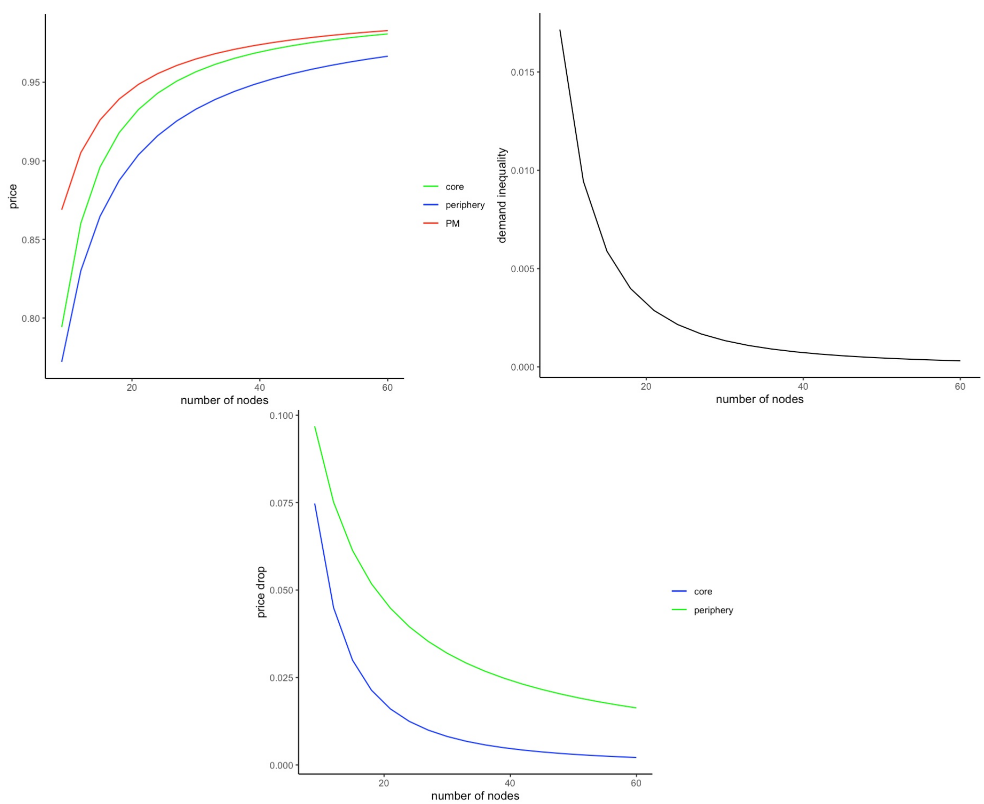


Figure I.1: Growing a Core-Periphery network: increasing N by adding more core traders with the same number of peripheral connections.

Similarly, Figure I.2 shows the equilibrium for different core-periphery network with the same number of $N = 12$ traders and different core sizes. The dashed lines are equilibrium outcomes for the star network of the same size $N = 12$.

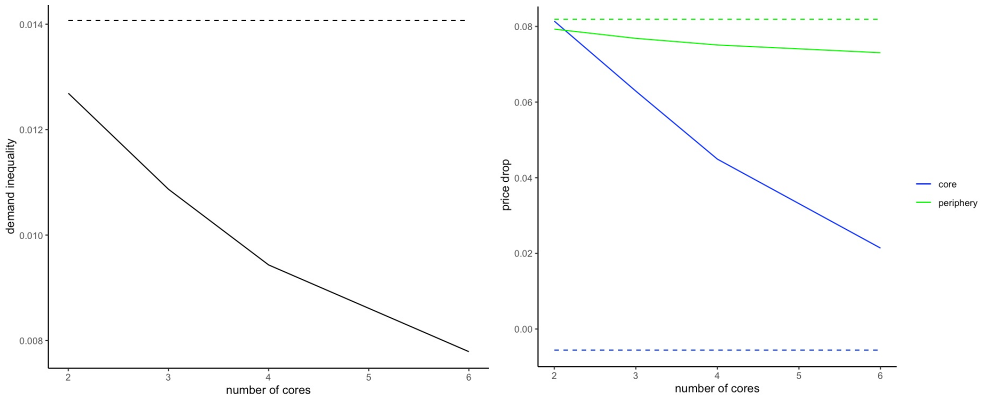


Figure I.2: Growing a Core-Periphery network: for a fixed $N = 12$ traders, the core size increases by moving a peripheral node to the core while keeping cores' connectivity to the periphery homogeneous

To make it clear the difference in the two exercises above, the first is moving from network A to B below. The second is moving from network A to C.

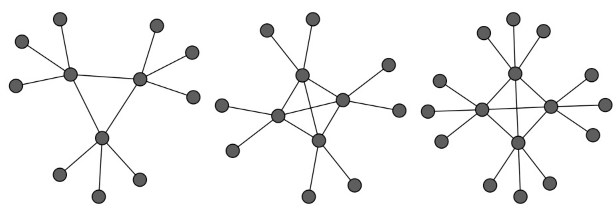


Figure I.3: Growing a Core-Periphery network: from left to right - network A, B and C

I.3 Regular Core-Periphery Network

A regular core-periphery network structure is quite tractable since cores' share the same demand, and so do peripheries. Thus I just need to keep track of two variables, a core's demand $q_{,1c}$ and a periphery's demand $q_{,1p}$.

The primitives for an arbitrary core-periphery structure are i) the number of cores n_c; ii) the number of peripheries *per core* n_p. So there are $(n_c n_p)$ peripheries and $N = n_c(1+n_p)$ nodes. Notice that core's degree is then $d_c = n_p + (n_c - 1) = N - (n_c - 1)n_p$.

Suppose $n_c \geq 2$. Let $\tilde{g} \equiv \frac{(d_c - 1)}{d_c^2}$, $\tilde{g}_c = \tilde{g} d_{cc} + \frac{1}{d_c}$. Then, cores and peripheries' demand schedules (10) are, respectively

$$
\begin{aligned}
q_c &= v_c\left[1 - P_1 - \phi\tilde{g}_c\left(d_{cc}q_c + n_p q_p\right)\right] \\
q_p &= v_p\left[1 - P_1 - \phi\tilde{g}_p\left((d_{cc}+1)q_c + (n_p - 1)q_p\right) - \phi q_c\right]
\end{aligned}
\tag{46}
$$

The network-induced coefficients in (10), $\{(v_i, \tilde{g}_{ij})\}_{i,j\in N}$, become for a core

$$
\begin{aligned}
v_c &= \left[\frac{2\phi}{d_c} + 1 - \phi(d_c+1) + \phi d_{cc}\left(\tilde{g} + \frac{1}{d_c}\right) + \phi n_p\right]^{-1} \\
\tilde{g}_{cc} &= \tilde{g}_c d_{cc} \\
\tilde{g}_{cp} &= \tilde{g} n_p
\end{aligned}
$$

and for a periphery

$$
\begin{aligned}
v_p &= \left[1 + \phi\left(\tilde{g} + \frac{1}{d_c}\right)\right]^{-1} \\
\tilde{g}_{pc} &= \tilde{g}(d_{cc}+1) + 1 \\
\tilde{g}_{pp} &= \tilde{g}(n_p - 1)
\end{aligned}
$$

That is, demands are $q_c = v_c\left[1 - P_1 - \phi\tilde{g}_{cc}q_c - \tilde{g}_{cp}q_p\right] \quad \forall c \in N$ and and $q_p = v_p\left[1 - P_1 - \phi\tilde{g}_{pc}q_c - \tilde{g}_{pp}q_p\right] \quad \forall p \in N$.

Now I can write the system of demands in matrix format. Define 2-by-2 matrices Ψ_1, Ψ_2 in which the first row/column refers to a core and the second row/column to a periphery,

$$
\Psi_1 = \begin{pmatrix} \psi_{1c} & 0 \\ 0 & \psi_{1p} \end{pmatrix} \quad \Psi_2 = \begin{pmatrix} 0 & \psi_{2c} \\ \psi_{2p} & 0 \end{pmatrix} \tag{47}
$$

such that $\psi_{1c} \equiv \left[\frac{1}{v_c} + \phi\tilde{g}_c d_{cc}\right], \quad \psi_{1p} \equiv \left[\frac{1}{v_p} + \phi\tilde{g}(n_p - 1)\right]$, and $\psi_{2c} \equiv \tilde{g}_c n_p, \quad \psi_{2p} \equiv \tilde{g}(d_{cc} + 1) + 1$.

Then the system of demands is

$$
\begin{aligned}
\Psi_1\mathbf{q} &= (1 - P_1)\mathbf{1}_2 - \phi\Psi_2\mathbf{q} \\
(\Psi_1 + \phi\Psi_2)\,\mathbf{q} &= (1 - P_1)\mathbf{1}_2 \\
\mathbf{q} &= (\Psi_1 + \phi\Psi_2)^{-1}\,\mathbf{1}_2(1 - P_1)
\end{aligned} \tag{48}
$$

(48) is the counterpart of (27). Since Ψ_1, Ψ_2 are 2-by-b2 taking the inverse is easy:

$$(\Psi_1 + \phi\Psi_2)^{-1} = \frac{1}{\psi_{1c}\psi_{1p} - \psi_{2c}\psi_{2p}} \begin{pmatrix} \psi_{1p} & -\phi\psi_{2c} \\ -\phi\psi_{2p} & \psi_{1c} \end{pmatrix}$$

Trading centrality (13) is now given by

$$\begin{aligned} \mathbf{c} &= (\Psi_1 + \phi\Psi_2)^{-1}\,\mathbf{1}_2 \\ &= \frac{1}{\psi_{1c}\psi_{1p} - \psi_{2c}\psi_{2p}} \begin{pmatrix} \psi_{1p} - \phi\psi_{2c} \\ \psi_{1c} - \phi\psi_{2p} \end{pmatrix} \end{aligned} \tag{49}$$

where it holds that core's centrality is higher than periphery' centrality:

$$\psi_{1p} - \phi\psi_{2c} > \psi_{1c} - \phi\psi_{2p}$$

The next proposition gives the equilibrium for a regular core-periphery network with n_c cores, $d_c n_p$ peripheries and connectivity among cores of d_{cc}.

Proposition 15. *Equilibrium in a Regular Core-Periphery Network*

Consider regular core-periphery network with n_c cores, $d_c n_p$ peripheries and connectivity among cores of d_{cc}. Suppose $n_c \geq 3$ and $d_{cc} \geq 2$. Then, trading centrality is given by

$$\mathbf{C} = (\Psi_1 + \phi\Psi_2)^{-1}\,\mathbf{1}_2 \tag{50}$$

Primary Market equilibrium price and demands are, respectively

$$P_1^* = 1 - \bar{Q}\frac{1}{C_c n_c + C_p n_p} \tag{51}$$

$$q_{i,1} = \bar{Q}\frac{C_i}{C_T} \qquad i = \{c, p\} \tag{52}$$

where $C_T = C_c n_c + C_p n_p$.

To understand how equilibrium change as we change the structure of the core-periphery graph one must look at the trading centrality. One can show that: i) cores' (peripheries') centrality is decreasing (increasing) in cores' connectivity; ii) both centralities are increasing in the number of cores and/or peripheries.

Focusing on the extremes structures - complete and ring cores, cores' (peripheries') centrality is decreasing (increasing) in in the number of the cores and/or peripheries.

Complete and Ring Cases

The two most extreme cases of regular core-periphery networks are i) when the core is fully connected (complete); and ii) when each core is connected to other two (ring). I now compare primary market equilibrium in these two core-peripheries structure. In particular, I show that:

- Price is higher in the complete core than in the ring core, for any number of cores and peripheries (from the main result)
- Cores' (peripheries') centrality is lower (higher) in the complete structure
- Cores' (peripheries') demand is lower (higher) in the complete structure
- Demand dispersion (inequality) is lower (higher) in the complete (ring) structure

In addition, I study how the comparison changes as the number of peripheries changes. Increasing the number of peripheries results in

- Lower price difference between the complete and ring structures - even though price increases in both
- Lower difference in demand dispersion - inequality decreases in both

I obtain these results using Proposition 15 that shows that characterizing the equilibrium in any core-periphery structure is quite straightforward. One only needs to determine two variables: the demand schedules for a core and a periphery. For the complete case, notice that $d_{cc} = n_c - 1$ and $\tilde{g}_c = (n_c - 1)\tilde{g} + \frac{1}{d_c}$. Then, cores and peripheries' demands are, respectively

$$\begin{aligned} q_c &= v_c\left[1 - P_1 - \phi\tilde{g}_c\Big((n_c-1)q_c + n_p q_p\Big)\right] \\ q_p &= v_p\left[1 - P_1 - \phi\left(\tilde{g}n_c + 1\right)q_c - \phi\tilde{g}(n_p - 1)q_p\right] \end{aligned} \tag{53}$$

For the ring case, $d_{cc} = 2$ $\tilde{g}_c = 2\tilde{g} + \frac{1}{d_c}$. Then, cores and peripheries' demand schedules are, respectively

$$
\begin{aligned}
q_c &= v_c\left[1 - P_1 - \phi\tilde{g}_c\left(2q_c + n_p q_p\right)\right] \\
q_p &= v_p\left[1 - P_1 - \phi\tilde{g}_p\left(3q_c + (n_p - 1)q_p\right) - \phi q_c\right]
\end{aligned}
\tag{54}
$$

The next proposition shows that as the number of cores become too large, price in these structure converge to the same level.

Proposition 16. *As $n_c \to \infty$, then $P_1^{complete} - P_1^{ring} \to 0$ irrespective of the number of peripheries.*

Appendix J Regular Networks

In regular networks, all nodes have the same number of links and position in the network. It immediate follows from **??** that traders have the same demand and selling price if and only if they have the same network position. These two facts make it easier to study regular networks. Equilibrium asset allocation is independent of the network structure, and it is the same as in the Walrasian market. PM and local market demands are, respectively, $q^*_{i,1} \equiv q^*_1 = \frac{Q}{N}, q^*_{i,s} \equiv q^*_2 = \frac{\bar{Q}}{Nd}$. The local market price is exclusively determined by traders' degree d, $P^*_s = 1 - \frac{(d+1)}{d}\frac{\bar{Q}}{N}$.

Proposition 17. *Primary Market in Regular Networks*

In regular networks, primary market price is increasing in the size and degree of the network.

Differently, primary market equilibrium allocation is independent of the network. It is the same as in the Walrasian market: all traders demand $\frac{\bar{Q}}{N}$ in the primary market, and $\frac{\bar{Q}}{Nd}$ in any local market.

Proposition 17 shows that, for a fixed number of N traders, the higher is traders' degree the higher is the PM price. Similarly, for a fixed degree d, increasing the size of the network increases PM price.

The demand schedule in every market is homogeneous across traders: any asset supply is divided equally among buyers, as they have the same willingness to pay. In other words, asset allocation is the same as in a perfectly competitive market. As I show next, PM price varies considerably across regular networks and it is never equal to the price of a perfectly competitive market. The results in this section highlights that if we ignore asset issuance price and only look at traders' inventories in OTC we are missing important considerations of funding costs.

Using market clearing conditions, equilibrium asset allocation in the PM and local market are, respectively: $q_{i,1}^* \equiv q_1^* = \frac{\bar{Q}}{N}$ and $q_{i,s}^* \equiv q_2^* = \frac{\bar{Q}}{Nd}$. The local market price is exclusively determined by traders' degree as d: $P_s^* = 1 - \frac{\bar{Q}}{N}\left(\frac{d+1}{d}\right)$

Turning to primary market equilibrium, first notice that individual network coefficient v_i becomes

$$
\begin{aligned}
v_i^{-1} = v^{-1} &= \frac{2\phi}{d} + 1 - \phi(d+1) + \phi d\left(\frac{2d-1}{d^2}\right) \\
&= \frac{\phi}{d} + 1 + \phi - \phi d \\
&= \frac{\phi + d(1 + \phi - \phi d)}{d}
\end{aligned}
$$

Demand schedule for any trader is

$$
\begin{aligned}
q_1 &= v(1 - P_1 - \phi d q_1) \\
q_1 &= (1 - P_1)\left(\frac{v}{1 + \phi v d}\right)
\end{aligned}
$$

Using market clearing condition,

$$
\begin{aligned}
P_1^* &= 1 - \frac{\bar{Q}}{N}\left(\frac{1 + \phi v d}{v}\right) \\
&= 1 - \frac{\bar{Q}}{N}\left(\frac{\phi + d(1 + \phi - \phi d) + \phi d^2}{d}\right) \\
&= 1 - \frac{\bar{Q}}{N}\left(\frac{\phi + d(1 + \phi)}{d}\right)
\end{aligned}
$$

Hence, P_1^* is increasing degree d. This result makes easier to compare different regular networks.

J.1 Complete Graph

The complete network is a special case of a regular network with degree $d = N-1$. No trading frictions exist since all traders are connected with one another. Everyone trade in both markets, either as a buyer or a seller. Using the result from Proposition 17, primary market price is $P_1^* = 1 - \frac{\bar{Q}}{N(N-1)}(N(1+\phi)-1)$, and any local market equilibrium price is $P_s^* = 1 - \frac{\bar{Q}}{N-1}$. Demands in each market are, respectively,$q_1 = \frac{\bar{Q}}{N}$ and $q_2 = \frac{\bar{Q}}{N(N-1)}$.

It is easier to see that the complete network delivers the highest primary market price.

Proposition 18. *Complete Network*

For a fixed network size N and in comparison with other regular networks, the Complete Network exhibits the highest primary market price.

It is worth pointing out that the static, competitive market is different than an economy with an empty network, i.e. without any trading relationships Even though there is no local market in both scenarios, in an empty network agents still face the negative demands shock. This risk makes demand schedules less elastic and drives PM price down. In equilibrium, PM demand and price are, respectively, $q_{i,1}(P_1) = q_1(P_1) = 1 - \frac{1}{1-\phi}P_1 \quad \forall i \in N$ and $P_1^* = (1-\phi)\left(1 - \frac{\bar{Q}}{N}\right)$.

Appendix K Ring versus Star Networks

Even though PM price increases as both network structures becomes larger, its growth rate differs. To see this, look at how price grows as each structure grows:

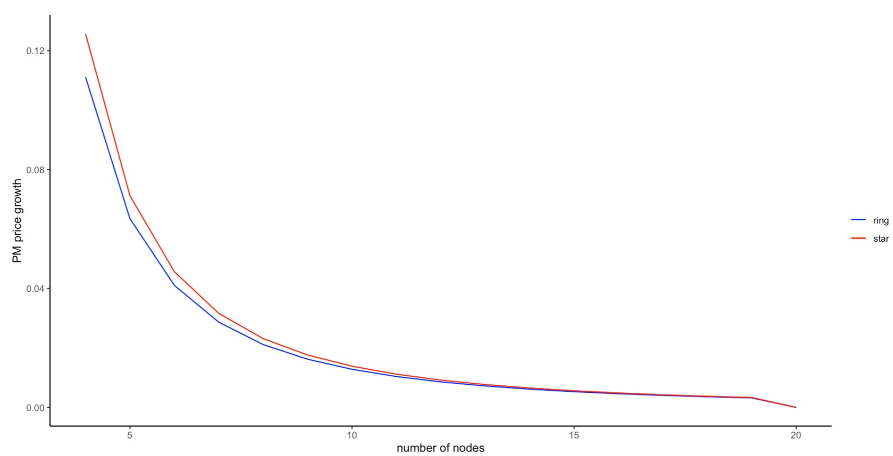


Figure K.1: PM Price growth as the network becomes larger: Ring versus Star structures

Another way to see this is to look at the price ratio of the PM price in star growth by the one in the ring:

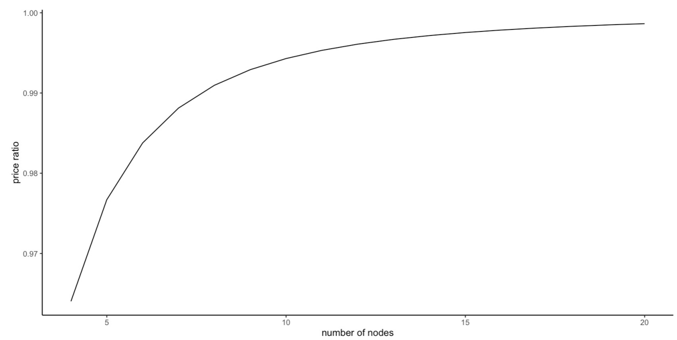


Figure K.2: PM price ratio: Star/Ring

Mathematically, price growth ratio is

$$\left(\frac{P^{star}_{1,N+1} - P^{star}_{1,N}}{P^{star}_{1,N}}\right) \Bigg/ \left(\frac{P^{ring}_{1,N+1} - P^{ring}_{1,N}}{P^{ring}_{1,N}}\right) = \frac{P^{star}_{1,N+1} - P^{star}_{1,N}}{P^{ring}_{1,N+1} - P^{ring}_{1,N}} \cdot \underbrace{\frac{P^{ring}_{1,N}}{P^{star}_{1,N}}}_{>1}$$

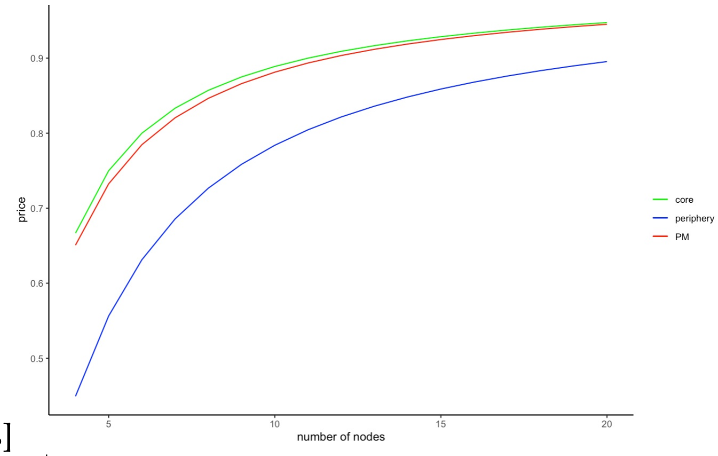
We already know that $P^{ring}_{1,N} > P^{star}_{1,N}$. We now find that, due to the change in the network degree distribution, price increases more in the star network than in the ring network as the structure grows. Notice that, in both structures, the price growth is positive but decreasing: price grows with the network at a diminishing rate.

K.1 Growing the Star Network

Even though growing the star leads to a more unequal network in terms of degree, the opposite occurs for demand inequality. The peripheral traders buy relatively more, i.e. become relatively more important in the PM, when this group is large. Difference in demands become smaller. Thus, core's capital gain decreases.

Proposition 19. *Growing a Star Trading Network*

For $N > 3$, core's capital gain and demand inequality decreases as the star network grows.

[Prices] 

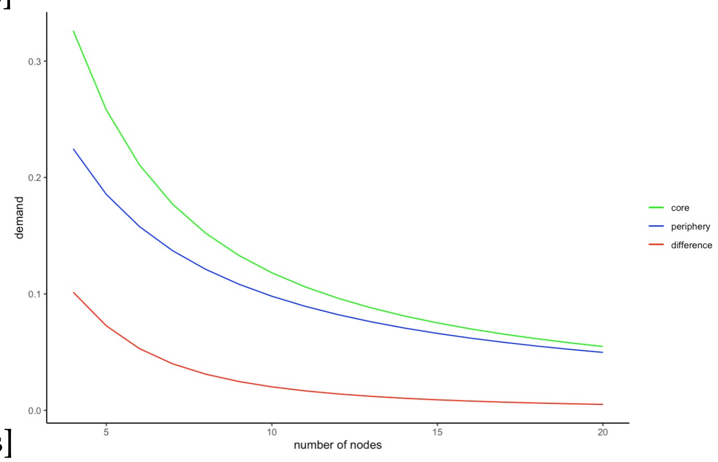
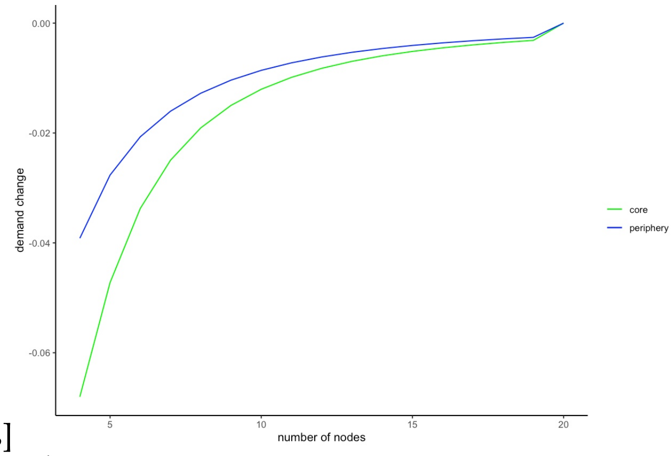
[De-mands] 

Figure K.3: Growing a Star Network



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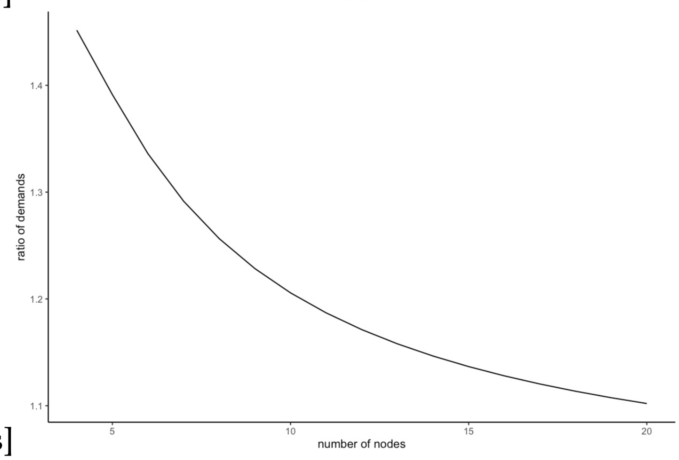

mands]

Figure K.4: Demand change and the size of the star network

K.2 Comparing with the Complete network

I find that:

- Complete local market price and the core's price are the same. That's because, the increase in core's demand and the reduction in peripheries' demand exactly compensate each other.

  That is, for a network with N traders,

$$\left|q_{1,regular} - q_{1,core}\right| = (N-1) \cdot \left|q_{1,regular} - q_{1,periphery}\right|$$

- Complete and ring's demands are the same. And both less than the core's and higher than the peripheries'.

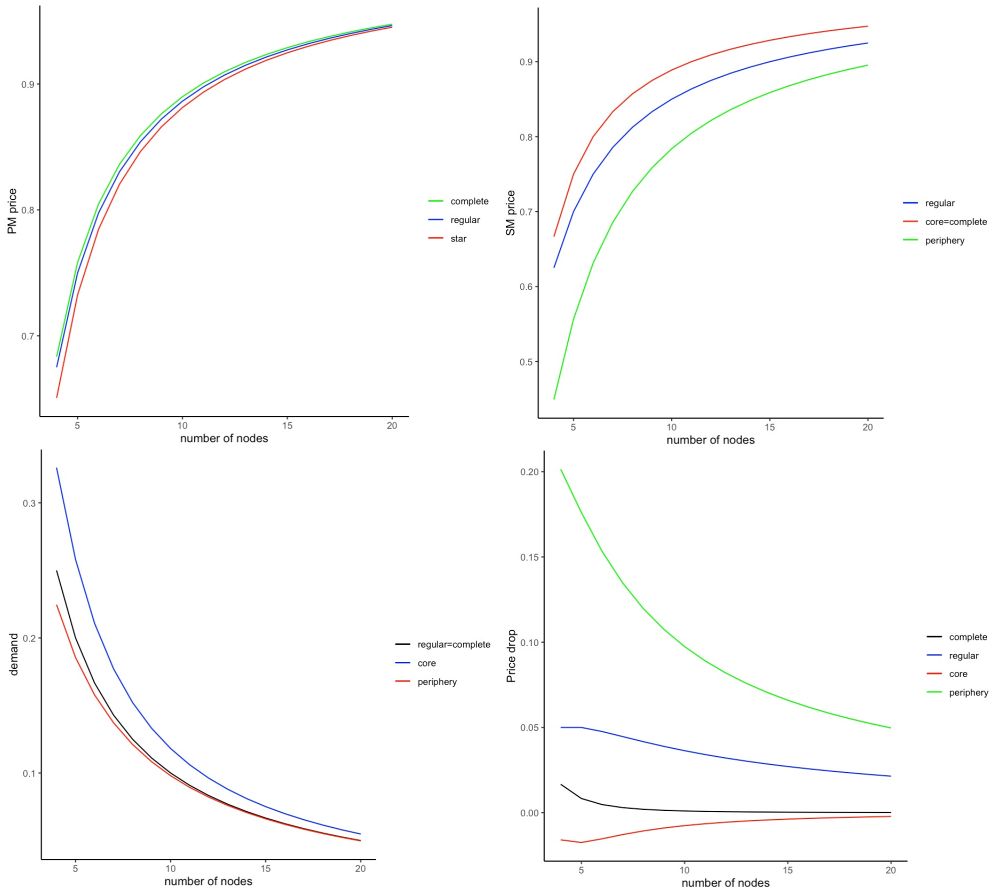

Figure K.5: Growing Networks: Ring versus Star structures

Appendix L Welfare

The welfare analysis is focused on four network structures depicted in fig. 10: the complete graph, the line, the star and ring. The following results are used to proof the main results in Section 7.

Welfare comparison across these network structures is easier because we can invoke Proposition 4 and Proposition 5. Notice that the degree distribution of the complete network FOSD all the other ones; the degree distribution of the ring network FOSD the line and the star ones; and the degree distribution of the line FOSD the star one. Moreover, degree distribution of the star is mean-preserving spread of the line one. It is straightforward then that PM price rank is: $complete > ring > line > star$.

When trading centrality is increasing in degree, a trader's expected utility is increasing in his centrality. In turn, we can rank traders' welfare within a network: more central traders achieve higher expected utility.

Lemma 9. *Individual Welfare and Degree*

If trading centrality c_i of trader $i \in N$ is monotonically increasing in his degree d_i, then i's indirect expected utility EU_i^* is monotonically increasing in his centrality c_i and, thus, his degree d_i.

We can compare the complete network and the ring (or any other regular structure). The complete network has higher (average) trading centrality and thus prices. In turn, PM price in the complete network is higher than in the ring. And so is welfare.

For the structural effect, take the complete and the star. The trader in core of the star is fully connected in both networks. But in the star his connections are poorly connected, which pushes his centrality up and others' centrality down. Consequently, a trader as the core has higher expected utility in the complete network.

Lemma 10. *Welfare and Trading centrality*

Aggregate expected utility is decreasing in the aggregate trading centrality c_A.

High aggregate trading centrality implies high PM price. This is the main reason behind Proposition 9.

Appendix M Extension: Heterogeneous Preferences

The key feature of this extension is that it permits heterogeneous interdependencies among values $\{\alpha_i\}_{i \in N}$ that arises endogenously, as described next.[78]

Lemma 11. *local market Equilibrium*

Equilibrium price of seller s is

$$P_s^* = \left(\sum_{i \in N_s} \frac{1}{\gamma_i}\right)^{-1} \cdot \left[\left(\sum_{i \in N_s} \frac{\alpha_i}{\gamma_i}\right) - \left(\sum_{i \in N_s} q_{i,1} + q_{s,1}\right)\right] \tag{55}$$

and equilibrium allocation of each buyer $i \in N_s$ is

$$q_{i,s}^* = \left(\frac{\alpha_i}{\gamma_i} - q_{i,1}\right) - \frac{1}{\gamma_i} \cdot \left\{\left(\sum_{i \in N_s} \frac{1}{\gamma_i}\right)^{-1} \cdot \left[\left(\sum_{i \in N_s} \frac{\alpha_i}{\gamma_i}\right) - \left(\sum_{i \in N_s} q_{i,1} + q_{s,1}\right)\right]\right\} \tag{56}$$

[78] This setup follows Rostek and Weretka (2012). The difference is that in that paper the valuations are unknown and traders have private signals on their own and others valuations. Here, however, agents can infer others valuation in the PM through the network

Denote $\Gamma_s \equiv \sum_{i \in N_s} \frac{1}{\gamma_i}$. Then network coefficients become

$$v_i(\phi) \equiv \left\{ \gamma_i (1 - \phi - \phi d_i) + \phi \frac{2}{\Gamma_i} + \phi \sum_j g_{ij} \cdot \frac{1}{\gamma_i \Gamma_j^2} (2\gamma_i \Gamma_j - 1) \right\}^{-1}$$

$$m_i(\phi) \equiv \left((1 - \phi - \phi d_i) + \frac{1}{\gamma_i} \cdot \phi \sum_j g_{ij} \cdot \left[\frac{1}{\Gamma_j^2} + \frac{2(\gamma_i \Gamma_j - 1)}{\gamma_i \Gamma_j^2} \right] \right)$$

$$\tilde{g}_{ij}^1 \equiv \left[g_{ij} \cdot \left(\frac{1}{\Gamma_i} + \frac{1}{\Gamma_j} - \frac{1}{\gamma_i \Gamma_j^2} \right) + \sum_{k \neq i,j} g_{ik} g_{jk} \cdot \left(\frac{1}{\Gamma_k} - \frac{1}{\gamma_i \Gamma_k^2} \right) \right]$$

$$\tilde{g}_{ij}^2 \equiv \left[g_{ij} \cdot \frac{1}{\gamma_j} \frac{1}{\Gamma_i} + \sum_{k \neq i,j} g_{ik} g_{jk} \cdot \frac{1}{\gamma_j} \left(\frac{1}{\Gamma_k} - \frac{1}{\gamma_i \Gamma_k^2} \right) \right]$$

The next lemma characterized PM equilibrium demands.

Lemma 12. *PM Equilibrium Demand*

$$q_{i,1} = v_i(\gamma; \phi, \mathbf{G}) \times \left[m_i(\gamma; \phi, \mathbf{G}) \alpha_i - P_1 - \phi \sum_j \tilde{g}_{ij}^1 (\gamma; \mathbf{G}) \cdot q_{j,1} + \phi \sum_j \tilde{g}_{ij}^2 (\gamma; \mathbf{G}) \alpha_j \right]$$

Assuming further that traders have the same level of risk-aversion, $\gamma_i = \gamma \forall i \in N$, the equilibrium turns out to be quite similar to the baseline model. PM demand is given by

$$q_{i,1} = \frac{1}{\gamma} v_i(\phi) \times \left[m_i(\gamma, \phi) \alpha_i - P_1 - \gamma \phi \sum_j \tilde{g}_{ij}^1 \cdot q_{j,1} + \phi \sum_j \tilde{g}_{ij}^2 \alpha_j \right] \tag{57}$$

where $v_i(\phi), \tilde{g}_{ij}$ are the same as in the baseline model, and $m_i(\phi, \gamma = 1 - \phi - \phi d_i + \phi \sum g_{ij} \frac{1}{d_j^2} \left[\gamma + \left(2d_j - 2 \right) \right]$ and $\tilde{g}_{ij}^2 = g_{ij} \cdot \frac{1}{d_i} + \sum_k g_{ik} g_{jk} \cdot \left(\frac{1}{d_k} - \frac{1}{d_k^2} \right)$.

Appendix N Extension: Expected Fundamental Returns

Buyer i's demand from seller s is $q_{i,s} = \frac{\mu - P_s}{\gamma\sigma^2} - q_{i,1}$.

Lemma 13. *Local market Equilibrium*

$$
\begin{aligned}
P_s^* &= \mu - \frac{\gamma\sigma^2}{d_s}\left(q_{s,1} + \sum_{i \in N_s} q_{i,1}\right) \\
q_{i,s}^* &= \frac{1}{d_s}\left(q_{s,1} + \sum_{k \neq i, k \in N_s} q_{k,1}\right) - \frac{(d_s - 1)}{d_s} q_{i,1}
\end{aligned}
$$

Network coefficients become

$$
\begin{aligned}
v_i(\phi) &\equiv \left[(1-\phi) - \phi\frac{(d_i - 2)}{d_i} + 2\phi \cdot \sum_j g_{ij}\frac{1}{d_j}\right]^{-1} \\
\tilde{g}_{ij} &\equiv g_{ij}\left(\frac{1}{d_i} + \frac{1}{d_j}\right) + \sum_{k \neq i,j} g_{ik} g_{jk} \frac{1}{d_k} \\
\tilde{g}_i &\equiv -d_i + \sum_j g_{ij}\frac{1}{d_j} + \sum_j \sum_k g_{ik} g_{jk} \frac{1}{d_k}
\end{aligned}
$$

Define vectors $\mathbf{v} = [v_i(\phi)]\tilde{\boldsymbol{g}}_{N\times 1} : [v_i(\phi) \cdot \tilde{g}_i]$ and matrices $\mathbf{V}_{N\times N} = diag(\mathbf{v}), \tilde{\boldsymbol{G}}_{N\times N} : [\tilde{g}_{ij}]$. Then the system of PM demands can be written in matrix form.

Lemma 14. *PM Equilibrium Demands*

$$
\mathbf{q}_1^* = \left(\mathbf{I} + \phi\mathbf{V}\tilde{\boldsymbol{G}}\right)^{-1} \cdot \left(\frac{\mu}{\gamma\sigma^2}(\mathbf{v} + \phi\tilde{\boldsymbol{g}}) - \frac{1}{\gamma\sigma^2} P_1 \mathbf{v}\right)
$$

Appendix O Extension: Price Impact

Suppose $d_i \geq 3 \forall i \in N$. In every local market, a buyer $i \in N_s$ trades taking into account his price impact $\lambda_s \equiv \frac{\partial P_s}{\partial q_i}$. In equilibrium, I show that $\lambda_s = \frac{1}{d_s - 2}$ and so price impact is equal across buyers in a given local market. Buyer i's demand is $q_{i,s} = \frac{1}{\lambda_s + 1}(1 - q_{i,1} - P_s) = \frac{d_s - 2}{d_s - 1}(1 - q_{i,1} - P_s)$.

Lemma 15. *Local Market Equilibrium*

The local market or seller $s \in N$ has equilibrium price,

$$\begin{aligned} P_s^* &= 1 - \frac{1}{d_s}\sum_i a_i - \frac{1+\lambda_s}{d_s}a_s \\ &= 1 - \frac{1}{d_s}\sum_i a_i - \frac{d_s - 1}{d_s(d_s-2)}a_s \end{aligned} \tag{58}$$

and asset allocation

$$q_{i,s}^* = \frac{1}{d_s(1+\lambda_s)}q_{N_s-i,1} + \frac{1}{d_s}q_{s,1} - \left(\frac{d_s-1}{d_s(\lambda_s+1)}\right)q_{i,1}$$

Lemma 16. *Primary Market Equilibrium*

$$q_{i,1} = \psi_i\left[(1-P_1) - \phi\sum_j \psi_{ij}q_{j,1}\right] \tag{59}$$

where

$$\psi_i = \left[1 + \phi(d_i+1) + 2\phi\frac{(d_i-1)}{d_i(d_i-2)} + \phi\sum_j g_{ij}\frac{2}{d_j}\right]^{-1}$$

$$\psi_{ij} = g_{ij}\cdot\left[\frac{1}{d_i} + \frac{1}{d_j}\right] + \sum_{z\neq\{i,j,k\}} g_{iz}g_{jz}\cdot\frac{(d_z-2)}{d_z(d_z-1)}$$

Appendix P Local Markets with Outside Traders

I assume that all traders participate in the primary market. However, this is not a restrictive assumption. The results are robust to incorporating outside traders who may only participate in a local market. In this section, I introduce a representative outside trader with demand q_t for each local market. This trader has the same quadratic-quasilinear preference but only one asset demand. The outside trader is interpreted as investors who only learn about the asset after the first trading round, or that due to financial constraints do not participate in the primary market.

The outsider demand schedule is $q_t = 1 - P_s$. By market clearing, seller s' price is $P_s^* = 1 - \frac{1}{d_s+1}\left(q_{N_s,1} + q_s\right)$.

From the local market equilibrium, one can already see that the only increasing price denominator by 1. In other words, the seller's effective degree is $d_s + 1$. Since this holds for every local market, all results remain unchanged.

The more realistic approach would be to include outside traders just in some local markets, or to allow for preference heterogeneity. The model accommodates all these extensions. Even though it remains tractable, it becomes harder to disentangle the network effects from preference and outside traders' effects. As I mentioned in the Introduction, the goal of this paper is to provide a benchmark framework in which the network effects are the *only driver* of equilibrium outcomes. The extensions presented in Section 8 are pertinent are interesting. I leave them for future work.