Qnt Macro II - PS1: Krussel & Smith

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Introduction

The solution for this PS can be found on the following Matlab codes: *KS_PS1_explained.m*, *KS_PS1_solution.m.* and *KS_PS1_difbeta.m.*

The main difference between the first and the second one is that the former has detailed and explained steps on how to construct what I call the "big loop" - which is just running the code to find solutions.

It's worth pointing out that codes take quite some time to run, specially the last one. With this in mind, my results for the part with different beliefs are based on stopping the code in the middle of iterations (without waiting until the very convergence). The entire run time is quite large and I do not expect results to change substantially.

I - Initial values and exogenous process

1 Initial Value Function

The first step towards perfoming the numerical algorithm of K&S is to make a guess about the Value Function.

Assume that agent expects that the aggregate and individual states $(\bar{k}, z, \epsilon)$ will not change in the future and his policy is to save exactly the same level of capital he has initially $k = k_0$.

This implies that the guess is given by

$$V^0(k,\epsilon;\bar{k},z) = u(c) + \beta V0(k,\epsilon;\bar{k},z) \leftrightarrow \boxed{V^0(k,\epsilon;k,z) = \frac{\log(c)}{1-\beta}} \tag{1.1}$$

2 Transition Matrix

In K&S framework, there are two sources of individual and aggregate uncertainty:

· an idiosyncatric employment shock: $\epsilon = \{0, 1\}$ - accounts for employment/unemployment s.t.

$$c + k' = rk + w\tilde{l}\epsilon + (1-\delta)k$$

· a stochastic aggregate productivity shock: $s = \{g, b\} \leftrightarrow z_s = \{z_g, z_b\}$ - accounts for high/low (good/bad) productivity - follows (exogenous) Markov procees with transition probability $\pi_{ss'}$

$$y = zk^{\alpha}l^{1-\alpha}$$

These shocks are correlated, which implies that they have a joint distribution defined by transition probabilities $\pi_{ss'\epsilon\epsilon'}$. Moreover, ϵ is assumed to satisfy LLN and so the only source of aggregate uncertainty is z.

Hence, we can think of the *number of unemployed in good times always equal* u_g[1] *and in bad times* u_b.

These shocks are part of the individual state vector.

In order to solve the model, we need to compute their joint distribution: that is, calculate all $\pi_{ss'\epsilon\epsilon'}$. Notice that there are 4 possible combinations for each shock and so there are 16 possible combinations of $(ss') = (zz', \epsilon\epsilon')$.

Exogenous Transition Matrix

The transition probabilities of the aggregate shock are exogenously given. So its transition matrix is defined by

$$\Pi_{zz'} \equiv \begin{bmatrix} \pi_{gg} & \pi_{gb} \\ \pi_{bg} & \pi_{bb} \end{bmatrix} = \begin{bmatrix} 7/8 & 1/8 \\ 1/8 & 7/8 \end{bmatrix}$$

The above implies that the average duration of both good and bad times is eight quarters

Endogenous Transition Matrix

Define the transition matrix for (z, ϵ) as:

$$\Pi_{ss'} \equiv \begin{bmatrix} \pi_{0g,0g} & \pi_{0g,1g} & \pi_{0g,0b} & \pi_{0g,1b} \\ \pi_{1g,0g} & \pi_{1g,1g} & \pi_{1g,0b} & \pi_{1g,1b} \\ \pi_{0b,0g} & \pi_{0b,1g} & \pi_{0b,0b} & \pi_{0b,1b} \\ \pi_{1b,0g} & \pi_{1b,1g} & \pi_{1b,0b} & \pi_{1b,1b} \end{bmatrix}$$

Transition Probabilities Restrictions

To pin down the transition probabilities of the above matrix we make some assumptions that translate into probability restrictions/equalities:

i) the average duration of an unemployment spell is 1.5 quarters in good times and 2.5 quarters in bad times.

$$\frac{1}{1 - \pi_{00}^{g}} = 1.5, \quad \frac{1}{1 - \pi_{00}^{b}} = 2.5 \tag{2.1}$$

ii) $u_g = 0.04$ and $u_b = 0.1$ - these are the proportions of unemployed in good and bad times, respectively

iii)

$$\pi_{g0,b0}\pi_{gb}^{-1} = 1.25\pi_{b0,b0}\pi_{bb}^{-1} \tag{2.2}$$

$$\pi_{b0,g0}\pi_{bg}^{-1} = .75\pi_{g0,g0}\pi_{gg}^{-1} \tag{2.3}$$

[1] exogoneously determined

iv)

$$\pi_{g1,g0} = .005 \tag{2.4}$$

$$\pi_{b1,b0} = .02 \tag{2.5}$$

v)

$$\boxed{\boldsymbol{\pi_{ss'00} + \pi_{ss'01} = \pi_{ss'10} + \pi_{ss'11} = \pi_{ss'}}} \qquad s, s' = \{g, b\} \tag{2.6}$$

vi)

$$\boxed{u_s \frac{\boldsymbol{\pi_{ss'00}}}{\boldsymbol{\pi_{ss'}}} + (1 - u_s) \frac{\boldsymbol{\pi_{ss'10}}}{\boldsymbol{\pi_{ss'}}} = u_{s'}} \qquad s, s' = \{g, b\} \tag{2.7}$$

vii) average duration

Solution

After manipulating the restrictions above, we get

$$\boxed{\Pi_{ss'} = \begin{bmatrix} 0.2917 & 0.5833 & 0.0937 & 0.03125 \\ 0.005 & 0.870 & 0.02682 & 0.09817 \\ 0.03125 & 0.09375 & 0.525 & 0.35 \\ 0.02267 & 0.1023 & 0.02 & 0.8550 \end{bmatrix}} \tag{2.8}$$

II - Workers problem and simulation

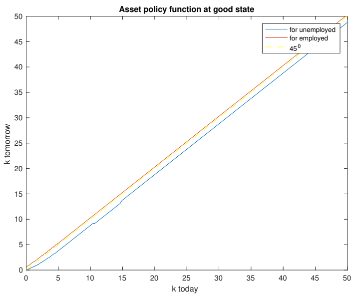
The solution for this part is in *KS_ PS1_ explained.m.* Also, it is contained in *KS_ PS1_ solution.m* which is the code for running the complete problem all at once.

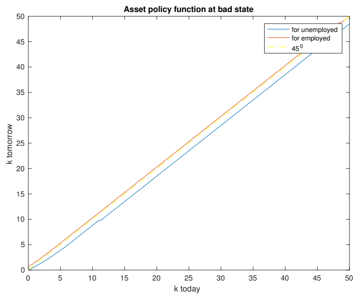
3 Asset Policy Function

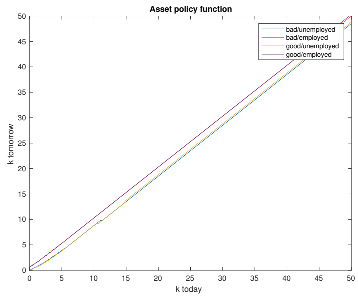
Graphical results for the policy function for savings $a'(k, K, \epsilon, z)$ is detailed below. Each plot is for a given type of agent - that is, for a given combination of idiosyncratic and aggregate shock (z, ϵ) - and for a fixed aggregate capital K[2].

At a given time (good or bad), an employed agent holds more assets tahn an unemployed one. However, the difference in holding between the two types of agents is not that big, which as K&S pointed out, suggests that the marginal propensity to consume is almost identical for different agents with different levels of asset holdings. Moreover, looking closely enough we can see that at the very low levels of capital (low wealth) the slopes of the policy functions are smaller in absolute value when compared to higher values. This suggest that as the wealth increases the agent holds basically the same amount of assets between periods.

[2]This is similar to what K&S do in the paper.







4 Simulation of the model

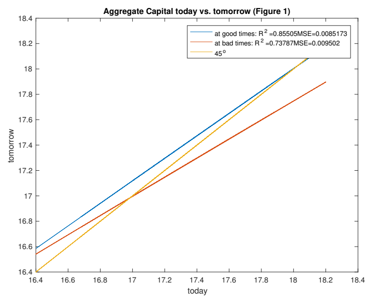
Details are found in the code.

5 Solution of the model

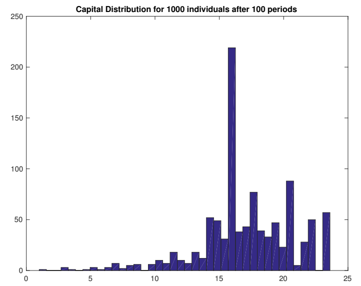
Below are the graphical results.

When analyzing the actual aggregate capital and the one predicted by the agents, we see that agents following the specified law of motion for K makes small mistakes. This translates into the high values of R^2 and low $\hat{\sigma}$. Hence, we can say that the approximation used here - with parametrized expectations - does a good job and it is enough for agents to behave optimally.

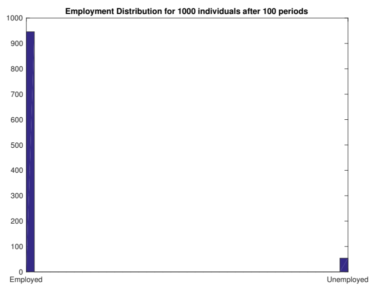
Also we can see that agents on bad times tend to over predict K and that on those times agents overpredict relatively to good times.



For the wealth distribution, there is a concentration of agents around the average K and on values a slightly above it. This suggests that most agents are "middle class" ones. Also, there is more concentration of rich agents relatively to poor agents.



As our assumption suggested, most agents are employed.



There is a mistake: the LHS graph is for bad times and the RHS for good times.

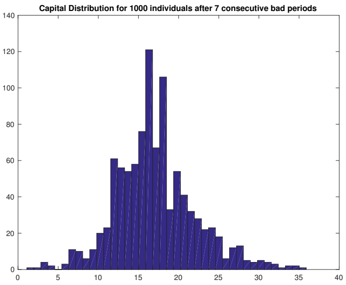
We can see that after multiple sequential good times, agents hold more asset. We can see this as agents internalizing the fact that bad times might come soon (due to the stochastic nature of z). Hence, they save more for when this happens.

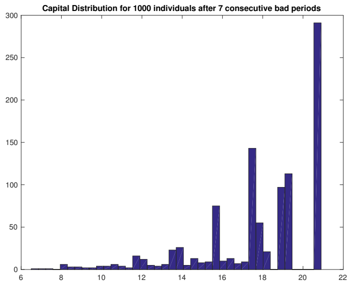
On the other hand, after consecutive bad times, agents smooth consumption out and wealth is concentrated around average K and a bit below .

III - Heterogeneiry in parametrized expectations

The solution for this part is in *KS_ PS1_ difbeta.m*. I followed the same steps as in the previous/benchmark model with just the twist that only half of the agents update their belifs; the other half remains with the initial guess for the coeffcients.

Notice that this implies that for the agents who do not update their beliefs:





$$H = \exp(\log(K)) = K$$

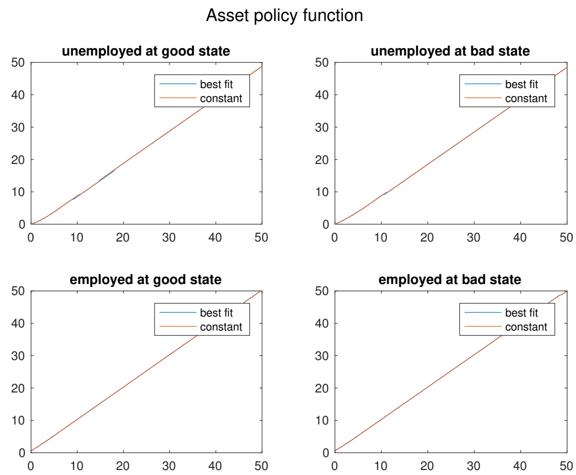
That is, their expectation is that aggregate capital is just its value.

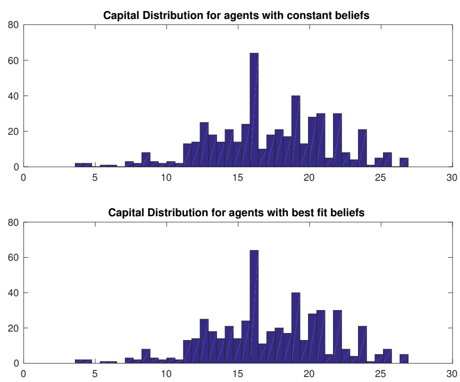
Also, in order to decide which agents will be the ones updating their beliefs, I randomly picked them on the 1000 individuals grid and did not change their "location" for all iterations. Also, I split up the aggregate state matrix into two - one for those who update beliefs and one to those who don't. Then, after the first iteration, I only "updated" the former state vector and the aggregate capital mean was calculated with both vectors (the entire aggregate space).

The solution of the model implies that both agents will have basically the same policy function. That's because, as pointed out above and on the previous exercise, the agents who actually update their beliefs end up perceiving a law of motion very close to the true mean of aggregate capital. On the other hand, those who are not updating their beliefs simply take K as a state variable. Thus, both type of agents end up "predicting" basically the same level for aggregate capital.

Notice that, even then, there is still a small difference in the policy function and on welfare between agents. When comparing their value function I found that updating-beliefs agents have a slightly higher welfare.

This result also translate into very similar wealth distribution for both belief-type agents.


Asset policy function
unemployed at good state
unemployed at bad state
employed at good state
employed at bad state
best fit
constant
0
10
20
30
40
50


Capital Distribution for agents with constant beliefs
Capital Distribution for agents with best fit beliefs
0
20
40
60
80
5
10
15
20
25
30