Opaqueness and Liquidity in Over-the-Counter Markets

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February 28, 2024

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Abstract

We develop a model of search in OTC markets with asymmetric information and trade occurring under double-sided uncertainty over asset quality, where holding the asset does not necessarily translate into knowing its quality. This leads to deterioration of market information conditions over subsequent trades, causing sellers to become more pessimistic even though aggregate asset quality remains unchanged. If *re-trade* opportunities are frequent, information in the economy becomes coarser, hindering market liquidity and volume of trade. Additionally, if good assets are not originated at a higher volume than what is currently being traded in the market, only the most pessimistic sellers will trade in equilibrium.

1. Introduction

This paper contributes to the understanding of market freezes in decentralized markets when underlying asset quality is opaque. We do this by analyzing a dynamic over-the-counter (OTC) market with the special feature that trade can occur under what we call *double-sided*

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uncertainty. In our setting, if a buyer does not acquire information when the transaction is settled, she will carry an asset of unknown quality as well. Over the trading cycles, this agent can become an uninformed seller and she may trade with an also uninformed buyer. In that case, we say that trade happens under double-sided uncertainty.

Assets for which fundamentals are hard to verify are widespread and relevant for the stability of real markets. Gorton and Ordonez (2014) argue that the onset of the 2008 Financial Crisis occurred when the informational regime changed in key asset markets, in particular of repurchase agreements (repo) and mortgage-backed securities. These assets, to some extent, are designed to be opaque in the sense that they provide incentives such that no part has ever any incentive to produce information about the collateral backing the transactions. In a way, this lack of transparency shields those markets from adverse selection and it ensures the highest volume of trade in the economy relying on no information being acquired by any parties. A shift in the informational regime that leads agents to verify the underlying quality of the assets traded can induce a drop in the collateral value (haircuts) or movements of "flight to quality" to safer liquid assets. However, in our dynamic environment, we show how opaqueness can lead agents to become increasingly more pessimistic about the quality of assets that are traded and threaten liquidity in the market.

To do this, we build a tractable model with unverifiable asset quality by introducing a novel feature in a Duffie et al. (2005) (DGP, henceforth) kind of model for OTC markets with search frictions which allows for asset holders not to know with certainty the quality of the asset they are holding. We aim to understand endogenous market evolution, in particular, how *belief deterioration* (that is, agents becoming more pessimistic through re-trading) affects market conditions. In our model, trade without information acquisition will take current buyers to be subsequent asset holders with a (weakly) lower belief over the quality of the asset than the seller that just sold it. Without any screening, the buyer's price offer will pool sellers with lower expected quality, making a Bayesian buyer update his posterior to a lower level. This more pessimistic asset holder will enter the trading cycle and potentially get to resell the asset in a different, more belief-depressed market. This endogenous belief deterioration, connecting current uninformed buyers with future uninformed and more pessimistic sellers, is the key feature of our work, and to the best of our knowledge a novel component for the OTC literature.

When studying OTC markets, some papers used the DGP framework to study the case of asymmetric information over the quality of the assets in order to explain financial crises, in particular the 2008 one. For instance, Chiu and Koeppl (2016), Hellwig and Zhang (2012), Zou

(2019), the latter two including the possibility of agents costly checking the quality of assets before the transaction. While these works can generate market freezes and runs, we believe they miss one important aspect of the market setting. In all these models it is assumed as soon as the transaction occurs the holder of the asset will know its quality, no matter if he checked or not before buying it. This is what we will call the *learning-by-holding* assumption along the text, and what generates the classical lemons problem, as sellers with bad assets will always want to come to the market.

While a common assumption in the literature up to this point, learning-by-holding may not be valid in key OTC financial markets that were are the heart of the 2008 Financial Crisis. In such markets, it seems natural to assume that holding an asset does not imply any additional learning when compared to the moment of purchase, even more so when we consider the opaqueness embedded in these assets, as pointed out by Gorton and Ordonez (2014). Our goal is to shed some light on the dynamics of such markets, understanding how information, volume of trade, and liquidity behave in this setting.

Our model considers a full trading cycle, where current asset holders with private information over the asset quality may suffer idiosyncratic liquidity need that prompts them to go to the market to sell the asset. Buyers bilaterally meet with such sellers and if trade happens the former will become holders with some updated and weakly more pessimistic belief over the asset quality. Hence, without *ex post* informed holders, agents with intermediary beliefs emerge when liquidity is high in the market.

In a steady state analysis, we are able to show that we can sustain a high price equilibrium with high liquidity and a low price equilibrium with lower liquidity. The parameter regions for the low price equilibrium depend crucially on the relationship between the meeting rate and the maturity rate, increasing with the former and decreasing with the latter. That is, if *re-trade* opportunities are frequent along the trading cycle, a substantial mass of agents in the economy accumulate in an intermediary, depressed belief (they don't know if they hold a good or a bad asset with certainty). This in turn affects current incentives for buyers, that may want to lower the offered price, hindering market liquidity and volume of trade.

We take this as evidence that if re-trade opportunities are frequent, information in the economy becomes coarser, hindering market liquidity and volume of trade. This seems to indicate that, in our environment, opaqueness serves a somewhat different purpose than in Gorton and Ordonez (2014). Instead of sheltering the market from adverse selection, it is a source of belief heterogeneity that can dry up liquidity by giving buyers an opportunity to pay lower prices in exchange for lower probabilities of trading.

We also analyze how belief deterioration depends on the production of new assets in this economy, which we call asset replenish. In the non-stationary case, beliefs always deteriorate unless the equilibrium price offered targets the most pessimistic agent in the belief distribution. We also provide sufficient conditions for an extreme belief deterioration to result in steady state: if asset replenish is belief-neutral (meaning that it does not change the current belief distribution, leaving average asset quality in the market unchanged), then beliefs deteriorate to the point that the distribution of price offers collapses to target only the most pessimistic agents. This gives the model a strong flavor of the classic lemons problem and could, in principle, be interpreted as a market freeze.

The main contribution of this paper is to incorporate a relevant and realistic assumption (that asset quality is not instantaneously transmitted through trade) in a tractable manner into the workhorse dynamic OTC model. This extension is capable of generating price dispersion in the economy because agents who have acquired the asset in different market conditions have different beliefs about the asset they are holding and would therefore accept different prices when selling that asset. This could generate avenues for future work by taking the model to the data and testing empirical predictions on the level of price dispersion implied by the model versus what is observed in the data.

Additionally, our extension can provide insight into the mechanisms of some phenomena that are of interest in the literature. As an example, since agents can have heterogeneous beliefs about the asset they own, we can have a mass of agents with intermediate beliefs (not overly pessimistic or optimistic) that affect future market conditions by creating off-equilibrium threats to optimistic agents. That is, it becomes harder to sustain a high price equilibrium because one could offer a slightly lower price and find almost just as much liquidity. This illustrates how information coarsening produces strategic complementarities between present and future liquidity, a feature that is not present in the literature to the best of our knowledge.

Related Literature: As mentioned above, our work is inserted in the broad literature of OTC markets within a search and match framework initiated by Duffie et al. (2005) (see Weill (2020) for a recent, comprehensive review). In particular, we establish a dialogue with models with asymmetric information. Chiu and Koeppl (2016) introduce the lemons problem in a DGP setting, and study market freezes and endogenous recovery prior to government intervention.

Their model however does not consider any possibility of information acquisition, which is incorporated by Hellwig and Zhang (2012) and Zou (2019). The former, which our analysis is closest to, studies equilibria when buyers can check the quality of the asset before buying it, and focus on how information acquisition relates to market liquidity. The latter introduces noise in the information acquisition, and capture a cream-skim effect that informed buyer remove only good assets of the market, deteriorating future liquidity. Both papers do not account for the possibility of buyers remaining uninformed after the trade, and hence have this ex-ante market for lemons flavor, from which our model departs.

The idea of double-sided uncertainty in trading assets partially comes from Gorton (2009), Dang et al. (2012) and Gorton and Ordonez (2014). They study debt (assets) which are designed to be information-sensitive, to support trade avoiding adverse selection. Gorton and Ordonez (2014) explains the onset of the financial crisis as a shock in the perception of the quality if the assets and the subsequent change in informational regime. Information is verified and bad assets are not traded (extensive margin) in their model, initiating a crisis in the real economy as production declines. Those models abstract from OTC microstructure and the full dynamics of belief deterioration over the trading cycle, both focus of our work.

The overall idea that lack of information, or trade under double ignorance, can be useful in sustaining trade can also be seen in previous papers such as Lagos (2010) and Andolfatto and Martin (2013). Lagos (2010) even without asymmetric information identifies equilibrium where suppressing information over asset return *before* the trade occur can sustain the optimal amount of transactions in the economy, while disclosed bad news can shrink the volume of the market, as some assets do not get traded. Similarly in Andolfatto and Martin (2013) in a monetary framework, where an asset subject to news shock, can be better used as exchange media if these informational shocks are not disclosed, such that a no disclosure policy can be welfare improving. This goes side by side with Gorton and Ordonez (2014) idea of private money (short-term collateralized debt) being preferably information-insensitive to serve as an exchange medium.

Finally, our work relates to the works by Guerrieri et al. (2010) and Guerrieri and Shimer (2014) that study dynamic asymmetric information in a competitive search setting. Guerrieri and Shimer (2014) in particular, studies an asset market with fire sales and flight to quality occurring endogenously responding to distribution of sellers. In our model, the distributions of sellers, an endogenous equilibrium object, will also play a role in determining the informational regime and hence equilibrium prices sustained.

2. Model

Our economy runs in continuous time, and starts at time $t = 0$ with a measure $S > 0$ of assets and $1 + S$ of agents, that in any point in time can either hold one or zero units of the asset and discounts future at rate $r > 0$. Out of S, we let $q_0 \in (0, 1)$ be the fraction of good assets among them and $(1 - q_0)$ of bad assets. Both assets provide a flow payoff of zero to the agent that is holding them but with arrival rate δ assets *mature*. Upon maturity the good asset delivers a fruit with flow payoff $u_g > 0$ and a bad asset yield gain $u_b > 0$. We assume $u_g > u_b$. Agents with an asset in their portfolio that suffer the maturity shock consume the flow payoff and leave the model. Below we describe our assumption of new agents entering the model to keep both the supply of S and the proportion of good and bad assets.

An agent who holds the asset is subject to a liquidity shock that arrives with a rate κ. Upon the shock, the flow payoff for both types of asset becomes $-x$. This captures in the model the fact that different agents have different timing for liquidity needs, and once shocked will seek to trade the asset. We will denote non-shocked agents as *holders* (h) and agents after the shock as *sellers* (s). Each of these agents will be indexed by a private belief $q \in [0, 1]$ over the quality of the asset where q is the probability assigned to be of the good type. Agents with extreme beliefs $q \in \{0, 1\}$ are fully informed of the quality meaning they know how much it yields upon maturity. Uninformed asset owners with $q \in (0, 1)$ are uncertain of the quality and will take the expected value of maturation when describing their value functions. The evolution and behavior of private beliefs constitute an important part of the present work.

The other side of the market is constituted by *buyers* (b), agents that do not own an asset and search in the market with an arrival rate of opportunities to trade of λ. Upon a match with a seller, the buyer does not observe the type (private belief) of the seller, but he does observe the aggregate distribution of beliefs and make a take-it-or-leave-it price offer. If the seller agrees with the price, the asset is sold, the buyer becomes a holder with an *updated belief*, and the seller re-enters the model as a buyer.

For each time t we let $\mu_t = \{(\mu_{ht}(q), \mu_{ht}(q))_{q\in[0,1]}, \mu_{bt}\}$ denote the measures of holders and sellers (of each type) and buyers at time t. We turn the analysis to see how the stage game, a meeting between a buyer and a seller unfolds at each instant they are paired. As will become clear the state of the stage game is given by the whole distribution of seller types at that point in the market $\{\mu_{st}(q)\}_{q\in[0,1]}$. While the buyer does not know the private belief of the seller they are matched with, they will be able to keep track of the evolution of the measure of each type of seller, and hence make decisions knowing the proportions of good assets they can expect to get from each trade. Throughout this paper, we assume $\delta u_g > x$ and $\delta u_b > x$ meaning that for both extremes, informed agents (and by convex combination, all uninformed too) will have a positive value of holding the asset. The gains from trade will emerge from buyers' desire to acquire the asset and become unshocked holders.[1]

2.1. Stage Game

We can now look at the stage game, after a meeting between a seller and buyer, who is choosing an offer to make. The buyer doesn't know the quality of the asset he is acquiring, but since he is gonna be able to keep track of the distribution of sellers, we can compute the expected quality of the asset from the *pool of sellers* that will accept the offered price. So, at any time t and given value functions $\{V_{st}(q)\}_{q\in[0,1]}$ for sellers with belief q and V_{bt} for buyers, we can write the reservation price for a seller of type q as:

$$P_t(q) = V_{st}(q) - V_{bt}$$

We guess to later verify that $V_{st}(q)$ is (weakly) increasing in q which means that when $P_t(q)$ is offered all types $q' \leq q$ will also accept the uniformed trade, while higher types than q will reject it, and keep their status. So the gains from trade from an offer of $P_t(q)$ are

$$\text{(Prob. that trade occurs with price } P_t(q)) \times \left[V_{ht}\Big(\pi_t(q)\Big) \underbrace{- P_t(q) - V_{bt}}_{-V_{st}(q)} \right]$$

$$= \underbrace{\frac{\int_0^q \mu_{st} d\tilde{q}}{\int_0^1 \mu_{st} d\tilde{q}}}_{\mathcal{M}(q)} \times \left[V_{ht}\Big(\pi(q)\Big) - V_{st}(q) \right]$$

where $\pi_t(q)$ is the belief updated from offering $P_t(q)$. Given the masses of each type of seller at the market at that point $\{\mu_{st}(q)\}_{q\in[0,1]}$ we can compute the function $\pi(\cdot)$ for such a given distribution by Bayes' rule:

[1] This hypothesis is in line with the literature and helps to guarantee that reservation prices of equilibrium will be positive.

$$\pi_t(q) = \frac{\int_0^q \tilde{q}\mu_{st} d\tilde{q}}{\int_0^q \mu_{st} d\tilde{q}} \tag{1}$$

It is the total fraction of good assets over the total assets for sale at price $P_t(q)$. It is (weakly) smaller than q, which means that the buyers take into account they are selecting the assets from a (weakly) worse pool every time they lower their prices. This function $\pi_t(\cdot)$ will be the key element of our analysis of market freezes and will play a crucial role when assessing what prices buyers are willing to offer in equilibrium. The fact that $\pi_t(q) \leq q$ for most beliefs in the support of μ_t is what leads to what we call *belief deterioration*: agents become more pessimistic as the trading cycle evolves. We will examine this phenomenon more closely in Section 5, when we consider an alternative version of the model that isolates the effects of re-trading on beliefs.

The uniformed buyers will choose $P_t(q)$ that maximizes their expected surplus. Moreover, buyers choose which seller to be targeted, $s(q^*)$. This is a well-defined problem since we are maximizing a continuous function (appealing to a max theorem for value functions) over a compact $q \in [0,1]$.

We can summarize the uniformed buyer strategy as a family of (measurable) functions $\{\hat{\psi}_t(q)\}_{q\in[0,1]}$, where $\hat{\psi}_t(q) = Prob(P = P_t(q))$. The optimal strategy will be defined as

$$\hat{\psi}_t(q) \begin{cases} = 0 \ \text{ if } q \notin \arg\max_{q'\in[0,1]} \mathcal{M}(q') \Big[V_{ht}\Big(\pi(q')\Big) - P_t(q') - V_{bt} \Big] \\ \in [0,1] \ \text{ otherwise} \end{cases}$$

2.2. Dynamics

In this section, we enunciate the value functions for each agent. Let $u(q) \equiv qu_g + (1-q)u_b$, the static expected payoff from an agent with belief q. Given the strategies $\psi_t(q)$ for all $q \in [0,1]$ and substituting reservation prices expressions from above, we have for *holders* of type q:

$$rV_{ht}(q) = \kappa\Big(V_{st}(q) - V_{ht}(q)\Big) + \delta\Big(u(q) - V_{ht}(q)\Big) + \dot{V}_{ht}(q) \tag{2}$$

The first term capture the arrival rate of a liquidity shock that induces a transition to a seller of the same belief. The second is the arrival rate of the maturity shock that makes the holder collect the expected payment given his beliefs and leave the model.

For sellers of type q we write the continuation value as:

$$rV_{st}(q) = \delta\Big(u(q) - V_{st}(q)\Big) - x + \lambda\left\{\int_q^1 \psi_t(\tilde{q})\Big(V_{st}(\tilde{q}) - V_{st}(q)\Big)d\tilde{q}\right\} + \dot{V}_{st}(q) \tag{3}$$

The first term is again the expected gain from the maturity shock, now minus the flow cost of the liquidity shock suffered that induces trade. The second term captures the arrival rate of opportunities to trade λ. The term in brackets represents the expected gains when buyers are playing according to $\psi_t(\cdot)$. Then, trade will happen with the seller of type q whenever a price higher than $P_t(q)$ is offered. Hence, the expected gain here will be given by integrating the probability of those prices being offered times the gains from trade at each price, for all values of beliefs between q and 1.

Finally, the value function for buyers can be written as:

$$rV_{bt} = \lambda\left\{\left[\int_0^1 \psi_{\tilde{q}}\mathcal{M}(\tilde{q})\left(V_{ht}\Big(\pi(\tilde{q})\Big) - P_t(\tilde{q}) - V_{bt}\right)d\tilde{q}\right]\right\} + \dot{V}_{bt} \tag{4}$$

The buyer does not enjoy any flow payoffs and can only transition upon meeting that arrives at λ. The continuation value comes from integrating the gains from trade at each price (including the probability of trade) times the probability of offering each price in support of $\psi_t(\cdot)$. Shortly we will focus on pure strategy equilibria, which will greatly simplify these expressions.

Now we describe the laws of motion for sellers (s) and holders (h) with a given belief $q \in [0,1]$. This present formulation assumes that when either holders or sellers are hit with the maturity shock, they leave the market, and are replaced by new holders with new *replenished* beliefs. This assumption must be so to create a steady inflow of new un-shocked holders without affecting the total composition of good (and total) assets in the economy.

Writing in a general way, we have that a mass (rate) $\alpha(q) \geq 0$ enter at belief $q \in [0,1]$. We must have that $\int_0^1 \alpha(q)\, dq = \delta$, meaning that the rate of exit must be the same as the rate of entry to keep the number of agents constant in the economy; and $\int_0^1 \alpha(q)q\, dq = \delta q_0$, saying that the proportion of good assets is kept with the inflow of agents.

Now, for the laws of motion, for holders of type q;

$$\dot{\mu}_{ht}(q) = -\kappa\mu_{ht}(q) - \delta\mu_{ht}(q) + \alpha(q)S + \lambda\left[\psi_t(\pi^{-1}(q))\int_0^q \mu_{st}(d\tilde{q})\right] \tag{5}$$

We should remember that the liquidity shock hits an agent $h(q)$ with rate κ. A mass $\alpha(q)$ will enter from new agents in the economy out of the total number of agents S. Meetings arrive at rate λ, and the price that induces an inflow to belief q is offered with probability $\psi_t(\pi^{-1}(q))$ where $\pi^{-1}(q)$ if the belief such that its posterior is q. This offer will be accepted by any seller with belief smaller than q, this is $\int_0^q \mu_{st}(d\tilde{q})$. For sellers now,

$$\dot{\mu}_{st}(q) = \kappa\mu_{ht}(q) - \delta\mu_{st}(q) - \lambda\mu_{st}(q)\left[\int_q^1 \psi(d\tilde{q})\right] \tag{6}$$

The share of sellers with belief q increases as liquidity shocks hit holders with the same belief. In a meeting trade occurs with type q when $P_t(q')$, $q' \geq q$ is offered. This happens with total probability $\int_q^1 \psi(d\tilde{q})$. Finally, since every buyer that leaves the pool is immediately replaced by another one (a previous seller), the fraction of buyers never changes over time.

$$\dot{\mu}_{bt} = 0 \tag{7}$$

This means we can, without loss, normalize $\mu_{bt} = 1$.

2.3. Equilibrium

Given the setting of the model we describe our equilibrium definition

Definition 2.1. *An **equilibrium** is a path of outcomes $\{(\psi(q))_{q\in[0,1]}\}$, a path of distributions $\mu_t(q)$, a path of value functions $V_t(q) = (V_{st}(q), V_{ht}(q), V_{bt})$, and an initial condition $\mu_0(q)$, such that, given the initial conditions*

1. *Given $V_t(q)$ and $\mu_t(q)$, $\{(\psi_t(q))_{q\in[0,1]}\}$ characterizes buyers' strategy in the Bayes Nash Equilibrium of the stage game played at time $t, \forall t, q \in [0,1]$, with beliefs given by (1);*
2. *Given $\{(\psi_t(q))_{q\in[0,1]}\}, \forall t$, V_t follows the equations (2), (3), (4) $\forall t, q \in [0,1]$.*
3. *Given $\{(\psi_t(q))_{q\in[0,1]}\}, \forall t, q \in [0,1]$, $\mu_t(q)$ follows the equations (5), (6), (7);*
4. *$(\mu_{st}(q), \mu_{ht}(q))_{q\in[0,1]}$ satisfies the resource constraints for all t, the total asset feasibility*

$$\int_0^1 \mu_{st}(dq) + \int_0^1 \mu_{ht}(dq) = S \tag{8}$$

and the good assets feasibility

$$\int_0^1 q\mu_{st}(dq) + \int_0^1 q\mu_{ht}(dq) = q_0 S \tag{9}$$

The first three conditions are straightforward given the stage game and dynamic analysis conducted above. The last condition is a feasibility constraint on the resources of this economy. Since assets are either maturing and being replaced by a new holder or changing hands to a previous buyer, the total mass of assets in the economy cannot change size from S. In the same spirit, the initial fraction of good assets in this economy is also constant and equal to q_0 in expected terms.

Analogously, we can define Steady State (SS) equilibrium for this economy, when prices strategies are time-invariant, the value functions for each type of agent, and the laws of motion of the distribution of agents are constant

Definition 2.2. *A **steady state** equilibrium is a stationary distribution $\mu(q)$, value functions $V(q)$ and price strategy $\psi(q)$ such that*
$\{\mu_t(q), \psi_t(q), V_t(q)\} = \{\mu(q), \psi(q), V(q)\}, \forall t, q \in [0,1]$ *is an equilibrium for $\mu_0(q) = \mu$.*

The SS equilibrium is substantially more tractable to solve for and will be the focus of the present version when in the next section we look to construct different SS equilibrium with pure strategies.

3. Steady State Analysis

We begin our analysis of Steady State equilibrium by looking at symmetric pure strategy equilibrium in a case with two-belief replenish, where newborn agents enter at $q_1, q_2 \in [0,1]$ at rates α_1, α_2. [2]

As before, from the resources constraints, we will need $\alpha_1 + \alpha_2 = \delta$ and $\frac{\alpha_1 q_1 + \alpha_2 q_2}{\delta} = q_0$. The goal here is to construct equilibria with a high price ($P(q_2)$) and low price ($P(q_1)$), and briefly analyze properties and identify if and when multiple equilibria will exist.

High Price Equilibrium

We assume all buyers are offering $P(q_2)$ and check when this constitutes an equilibrium. The analysis of the SS laws of motion boils down to points in the belief space of *inflow* of holders. We

[2] In the Appendix we exhibit a preliminary version of a more general setup, in which the replenish is continuous among all the belief distribution.

let $q_2' = \pi(q_2)$ be the posterior induced by trading at the high price. We have inflows of holders only at q_1 and q_2 via replenish and at q_2' via trade. Hence $\mu_h(q) = \mu_s(q) = 0$ for all $q \neq q_1, q_2, q_2'$. For the remaining beliefs we can get the following expressions for the mass of sellers:

$$\mu_s(q_1) = \frac{\alpha_1 \kappa}{\delta(\kappa + \delta + \lambda) + \lambda\kappa} S$$

$$\mu_s(q_2) = \frac{\alpha_2 \kappa}{\delta(\kappa + \delta + \lambda) + \lambda\kappa} S$$

$$\mu_s(q_2') = \frac{\kappa^2 \lambda}{(\delta(\kappa + \delta + \lambda) + \lambda\kappa)(\delta + \kappa + \lambda)} S$$

Notice that α_i dictates directly the fraction of sellers at q_i, while $\kappa\lambda$, controls the mass at the posterior. This is very intuitive since the only inflow at q_2' comes via re-trade, which is precisely captured by the interaction of the liquidity shock with the arrival rate of meetings.

With this in hand, we can compute the posterior and obtain $q_2' = \frac{\alpha_1 q_1 + \alpha_2 q_2}{\delta} = q_0$. The posterior must be equal to the initial fraction of good assets in the economy. This comes from the fact that, in this equilibrium, all agents accept the price offered, hence the posterior all buyers inherit must be the overall ex-ante quality of the market. This is a case of belief deterioration in the model. By targeting the sellers with high belief, current buyers pool all the agents in the economy, inheriting a strictly worse belief over asset quality once trade occurs. In a dynamic setting this deterioration translates into information *coarsening* in the economy; even though agents are entering with fresh beliefs at q_1 and q_2, trade is inducing beliefs to accumulate at initial quality q_0, pooling the extreme beliefs via (re-)trade. This endogenous type of agent appears only in a setting without learning-by-holding and hence will play a key role in equilibrium outcomes as we will discuss below

We can compute the whole posterior function[3], which will be useful to check for deviation of equilibrium prices.

$$\pi(q) = \begin{cases} q_1 & \text{if } q \in [q_1, q_0) \\ \frac{(\delta(\delta+\lambda+\kappa)+\kappa\lambda)\alpha_1 q_1 + \kappa\lambda\alpha_2 q_2}{\delta(\alpha_1(\delta+\lambda+\kappa)+\kappa\lambda)} & \text{if } q \in [q_0, q_2) \\ q_0 & \text{if } q \geq q_2 \end{cases}$$

[3]Notice that, since we have no mass below q_1, the posterior function is undefined for $q < q_1$. This is not a problem, however, as offering a price below q_1 is never a profitable deviation for any belief.

We can also compute the probability of trade after offering each $P(q)$ under the high price equilibrium.

$$\mathcal{M}(q) = \begin{cases} 0 & \text{if } q < q_1 \\ \frac{\alpha_1(\delta+\lambda+\kappa)}{\delta(\delta+\lambda+\kappa)+\kappa\lambda} & \text{if } q \in [q_1, q_0) \\ \frac{\alpha_1(\delta+\lambda+\kappa)+\kappa\lambda}{\delta(\delta+\lambda+\kappa)+\kappa\lambda} & \text{if } q \in [q_0, q_2) \\ 1 & \text{if } q \geq q_2 \end{cases}$$

We turn now to check the optimality of $P(q_2)$. Given the SS value function implied by this strategy, we can compute the direct gain of trade from a buyer that deviates to any $P(q)$ with $q \leq q_2$:

$$V_h(\pi(q)) - V_s(q) = \tfrac{1}{(r+\delta+\kappa)(r+\delta+\lambda)}\Big[(\kappa+r+\delta+\lambda)\delta u(\pi(q)) - \lambda\delta u(q_2) - (r+\delta+\kappa)\delta u(q) + x(r+\delta+\lambda)\Big] \quad (10)$$

The first component in the brackets reflects the flow payoff that a buyer expects to face over the trading cycle. The second is a premium the buyer is paying the types q and lower, which can always wait to get a higher offer at $P(q_2)$. Hence $\lambda\delta u(q_2)$ reflects how much the buyers are paying for this high price premium of waiting for another offer. The third reflects the part of the price that comes from its own type q flow payoff and finally, the liquidity shock term, which alleviates the reservation price paid by the buyer.

Also, note that this expression is decreasing in q. However, when we consider the probability of trade, $\mathcal{M}(q)$, that will compose the final gain from trade, it increases with q. Hence, we see a quality premium versus liquidity trade-off. If the buyer pays less, he is paying a smaller informational rent to lower types, and approximating his posterior from the one in the reservation price offered. That comes with a lower probability of trade, as fewer types are accepting this offer, which creates a longer waiting time for the buyer.

To solve this trade-off we must check: if there are gains from trade:

$$V_h(q_2') - V_s(q_2) \geq 0 \quad (11)$$

If there is no deviation to $P(q_1)$:

$$V_h(q_2') - V_s(q_2) \geq \mathcal{M}(q_1)(V_h(q_1) - V_s(q_1)) \quad (12)$$

And no deviation to $P(q_2')$:

$$V_h(q_2') - V_s(q_2) \geq \mathcal{M}(q_2')(V_h(\pi(q_2')) - V_s(q_2')) \tag{13}$$

We Let $d(q_1, q_2) = u(q_2) - u(q_1)$. This analysis boils down to define thresholds on $\left(\frac{x}{d}\right)_1$, $\left(\frac{x}{d}\right)_2$, $\left(\frac{x}{d}\right)_3$ for each of the three cases above respectively, such that $P(q_2)$ will be optimal if and only if:

$$\frac{x}{d} \geq \bar{\left(\frac{x}{d}\right)}_h := max\left\{\left(\frac{x}{d}\right)_1, \left(\frac{x}{d}\right)_2, \left(\frac{x}{d}\right)_3\right\}.$$

We derive the expressions for the thresholds in the Appendix. Intuitively, we need q_1 and q_2 to be sufficiently close so one does not want to deviate to lower prices. In this case, the buyer is not losing too much from informational rents paid to low q sellers, hence paying a smaller premium, and benefiting from the high probability of trade that offering the high price entails. Similarly, we need x the size of the liquidity shock to be sufficiently large. This facilitates the gain from trade from offering a pooling high price, since sellers, including q_2 ones, will have lower reservation prices.

This type of equilibrium is such that the economy presents a high volume of trade. Every asset changes hands after every meeting, hence liquidity is abundant. Buyers are willing to pay information rents by paying the high price, because they benefit from the high liquidity that offering this price entails today and will be able to also extract future information rent from future buyers.

Here we can also present the belief deterioration process that our model captures. In steady state, there is a type of agent that arise endogenously from the high price being offered. The sellers with belief q_2' affect the incentives for current buyers, that in order to keep the high price equilibrium, cannot find profitable to offer a lower reservation price to these intermediary belief sellers (condition (13)). This *off-equilibrium* threat from the endogenous emergence of an agent with belief q_2' will be key when we compare our setup to a model with the conventional assumption of *learning-by-holding*.

Low Price Equilibrium

We can now construct an equilibrium with the low price $P(q_1)$ being offered. Since q_1 is the minimum of the support, the posterior after price $P(q_1)$ is offered is just q_1, since only the lowest types are accepting such low offer. There are again no informational rents from trade. We can get from the law of motion:

$$\mu_s(q_1) = \frac{\alpha_1\kappa(\delta+\kappa+\lambda)}{\alpha_1(\delta+\kappa+\lambda)^2+\alpha_2(\delta+\kappa)^2}S$$

$$\mu_s(q_2) = \frac{\alpha_2\kappa(\delta+\kappa)}{\alpha_1(\delta+\kappa+\lambda)^2+\alpha_2(\delta+\kappa)^2}S$$

As before we derive the posterior function and probability of trade after each $P(q)$ is offered under this equilibrium:

$$\pi(q) = \begin{cases} q_1 & \text{if } q \in [q_1, q_2) \\ \frac{\alpha_1(\delta+\kappa+\lambda)q_1+\alpha_2(\delta+\kappa)q_2}{\alpha_1(\delta+\kappa+\lambda)+\alpha_2(\delta+\kappa)} & \text{if } q \geq q_2 \end{cases}$$

$$\mathcal{M}(q) = \begin{cases} 0 & \text{if } q < q_1 \\ \frac{\alpha_1(\delta+\lambda+\kappa)}{\alpha_1(\delta+\lambda+\kappa)+\alpha_2(\delta+\kappa)} & \text{if } q \in [q_1, q_2) \\ 1 & \text{if } q \geq q_2 \end{cases}$$

In this case, we always have strictly positive gains from trade, $V_h(q_1) - V_s(q_1) = \frac{x}{r+\delta+\kappa}$. The optimality of the low price will come from checking against deviations to the high price $P(q_2)$.

$$\mathcal{M}(q_1)\Big(V_h(q_1) - V_s(q_1)\Big) \geq V_h(\pi(q_2)) - V_s(q_2)$$

Again, we can find a threshold $\left(\bar{\frac{x}{d}}\right)_\ell$ such that we need.

$$\frac{x}{d} \leq \left(\bar{\frac{x}{d}}\right)_\ell$$

This equilibrium requires the difference between expected flow payoffs at q_1 and q_2 to be sufficiently large so that the buyer does not want to pay the premium for the high types to accept the offer. Alternatively, we need x relatively small, so that the reservation price for optimistic sellers at q_2 will be too high for the current buyer to deviate.

The low price equilibrium is such that the market gets stuck with low trade volume and liquidity. Only a small fraction of the market trade, being the most pessimistic ones.[4].Here the

[4]In terms of the current model, this is a sort of *market freeze*. Although trade is not completely halted, the continuations values are such that essentially no agent wants to trade and in particular no buyer wants to pool sellers.

low belief asset q_1 is correctly priced, as buyers are offering reservation values for exactly the targeted types of sellers. In other words, there is no informational rent being paid to any agent in this equilibrium. No endogenous intermediary belief holder exists in the model since there is no belief deterioration. Meanwhile, good assets enter and mature the model without ever being traded.

Equilibria Comparison and Comparative Statics

The proposition below summarizes the equilibria described and presents a sufficient condition for when at least one of them exists.

Proposition 3.1. *With two-belief replenish at q_1 and q_2 at rates α_1 and α_2 we have the following characterization of symmetric pure strategy SS equilibria.*

1. *A high price equilibria with $P(q_2)$ being offered by all buyers exists if and only if:*

$$\frac{x}{d} \geq \left(\frac{\bar{x}}{d}\right)_h := max\left\{\left(\frac{x}{d}\right)_1, \left(\frac{x}{d}\right)_2, \left(\frac{x}{d}\right)_3\right\}.$$

2. *A low price equilibrium with $P(q_1)$ being offered by all buyers exists if and only if:*

$$\frac{x}{d} \leq \left(\frac{\bar{x}}{d}\right)_\ell$$

Moreover, for the rate of arrival of trading meetings λ sufficiently high we have $\left(\frac{\bar{x}}{d}\right)_h < \left(\frac{\bar{x}}{d}\right)_\ell$ implying that for any such parameter combination at least one of such equilibria always exists.

Proof. See Appendix for proofs. □

The last part of the proposition also implies that, for λ sufficiently high a region of multiple equilibria will exist. Figure 1 shows the case of multiple equilibrium arising. In the cone between the lines in the (x, d) space, both the low and high price equilibrium coexist. This a classical example of *strategic complementarities* arising in a dynamic setting. For parameters in the coexistence region, if high prices are being offered, it means that lower-belief sellers will get a premium and extract informational rent from buyers in the future. This makes sure that buyers today find it attractive to pay more for a sure trade but with a high premium for information. On the other hand, when only low prices are being offered down the line, buyers

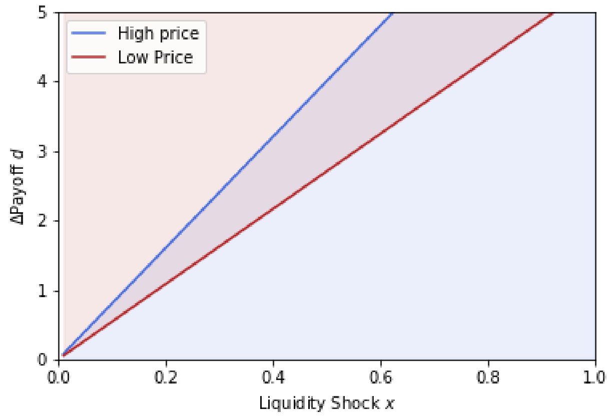

Figure 1: $\kappa = 0.1$, $\lambda = 0.2$, $r = 0.2$, $\alpha_1 = 0.5$, $\alpha_2 = 0.9$

today don't want to pay the informational rent and prefer a low liquidity but no premium trade with low types at the low price.

The intuition for why low search friction enables the multiplicity region to exist is the following. Under the high price equilibrium, increasing λ translates into a smaller probability of trading with low types q_1 but increasing with the endogenous type q_2', as more and more agents accumulate at this belief after trading. This effect is countered by the fact that the posterior after q_2' declines in λ and in the limit it converges to q_1. These factors combined explain why the high price equilibrium region expands for high λ. On the other hand, under low price equilibrium, high arrival meeting rate implies a larger probability of trade with low beliefs q_1, since here agents after trading know they have a q_1 type of assent in hand. By increasing the probability of trade here we also expand the low price equilibrium region, concluding that for λ sufficiently high these regions always overlap.

Another interesting comparative statics comes from the liquidity shock arrival rate κ. High κ is associated with smaller regions for the high price equilibrium and larger for the low price one. Intuitively, if liquidity shocks are frequent agents enjoy less time as holders, reducing their continuation value. Hence buyers find it less attractive to offer high pooling prices at the same time harming the gains from trade from offering $P(q_2)$ and reducing incentives to deviate to the high price in the low equilibrium.

While we have been describing high and low price equilibrium the following proposition ensures that these are the only two pure strategy equilibrium prices that can exist.

Proposition 3.2. *There does not exist any other reservation price $P(q)$ with $q \neq q_1, q_2$ that is an SS equilibrium price.*

The intuition behind this simple result relies on the SS mechanics of our model. If current buyers were to target a belief type q that is not being exogenously replenished it means that after trade we would have an inflow of holders in $\pi(q) < q$. However, since no agent is entering at q in steady state we must have zero mass of agents there. Hence this creates an incentive to deviate, as a buyer could lower the price and get the same set of agents that he was previously targeting. The SS forces of the model anchor our equilibrium prices in reservation values that target either q_1 or q_2 as these are stable points guaranteed by the exogenous replenish. This heavy dependence on the support of the inflow of holders makes attractive to study the case of a continuum distribution of replenish agents, which in this draft we outline preliminary results in the Appendix.

Finally, we need to discuss what happens when the equilibrium regions do not overlap in Proposition 3.1. There exist parameter regions where there is a gap in the thresholds for high and low equilibrium prices. For this interval, we have no pure strategy price as an equilibrium and hence we would have to rely on *mixed strategies* to characterize what is happening there. In the Appendix, we discuss and derive several necessary conditions of mixed strategy equilibrium. However, we were not able to make the problem tractable and pin down conditions for when this gap will be filled. For the purposes of this draft, we shall then focus solely on pure strategy prices and in particular when there is multiplicity of equilibria.

4. Contrasting with *learning-by-holding* models

In this section, we showcase the difference between our setting and one with the vastly used hypothesis of 'learning-by-holding'. We are going to show what happens, in a dynamic market, when holding the asset does not translate into knowing its quality. If opportunities of trade and *re-trade* happen too frequently, agents accumulate in intermediary beliefs, the market gets more opaque and this, in turn, affects liquidity and trade conditions.

In order to make the difference clear, we re-write our model assuming now that after trade, holders observe the asset they are carrying. We focus on comparing high price equilibrium, when all agents are accepting the current price and argue that in some sense our model presents more fragile liquidity equilibrium than under the usual hypothesis of *ex-post* informed holders.

The environment is the same as before, but we specialize to the case $q_1 = 0$ and $q_2 = 1$, meaning that new asset holders that enter the market know precisely the quality. One can think of this new asset entering the economy as coming directly from a primary market where the agent knows the quality of the project backing the asset for instance. Furthermore, we will assume that $\alpha_1 = \alpha_2 = \delta/2$ to simplify our algebra and isolate the analysis from imbalances in the exogenous inflow. Notice that these assumptions will imply that $q_0 = 1/2$.

How does the model change? In terms of continuation values nothing changes and in fact, we can easily show that:

$$V_h(\pi(1)) = \pi(1)V_h(1) + (1-\pi(1))V_h(0)$$

This in turn implies that all direct gains from trade from buyers under the high price equilibrium $P(1)$ are the same as before.

All the changes then are coming from the laws of motion and hence from distributions and specifically from probabilities of trade. Under the *ex-post* informed model the laws of motion necessarily split into extreme values 0 and 1, and there does not exist any endogenous type of agent arising in the model. We can write the law of motion under $P(1)$ being offered as follows, for $q = 0$

$$\dot{\mu}_h(0) = -(\kappa+\delta)\mu_h(0) + \lambda(1-q_0)[\mu_s(0)+\mu_s(1)] + \alpha S$$

$$\dot{\mu}_s(0) = \kappa\mu_h(0) - (\delta+\lambda)\mu_s(0)$$

For $q = 1$

$$\dot{\mu}_h(1) = -(\kappa+\delta)\mu_h(1) + \lambda q_0[\mu_s(0)+\mu_s(1)] + \alpha S$$

$$\dot{\mu}_s(1) = \kappa\mu_h(1) - (\delta+\lambda)\mu_s(1)$$

The interpretation is as before with the difference now that there are inflows in 0 and 1 of buyers that traded and found themselves with a bad asset, which happens in a fraction $(1-q_0)$ of the times, or with a good asset at rate q_0.

We can derive the seller's distribution of SS as before:

$$\mu_s(0) = \mu_s(1) = \frac{\kappa S}{2(\delta+\kappa+\lambda)}$$

And probabilities of trade are simply $\tilde{\mathcal{M}}(0) = 1/2$ and $\tilde{\mathcal{M}}(1) = 1$. Hence for $P(1)$ to be an equilibrium here, we need two conditions:

1. Positive gains from trade
$$\tfrac{1}{2}(V_h(1) + V_h(0)) - V_s(1) \geq 0$$

2. No deviation to $P(0)$:
$$\tfrac{1}{2}(V_h(1) + V_h(0)) - V_s(1) \geq \tilde{\mathcal{M}}(0)(V_h(0) - V_s(0))$$

The first condition is identical to the previous positive gain from trade expression, so it entails the same threshold $\left(\frac{\bar{x}}{d}\right)_1$ as before. Condition number 2 gives us the following threshold:

$$\frac{x}{d} \geq \left(\frac{\bar{x}}{d}\right)^{lbh} := \frac{\delta(r+\delta+\kappa)}{r+\delta+\lambda} \tag{14}$$

The key is now to compare this threshold with the ones obtained in our baseline setting, especially when considering deviations to intermediary belief sellers. The proposition below summarizes our findings:

Proposition 4.1. *The high price equilibrium region under ex-post ignorant holders is weakly smaller than the one under the learning-by-holding assumption. It is strictly when:*

$$\kappa > \frac{\delta}{2} \;\; \textit{and} \;\; \lambda > \frac{\delta(\delta+\kappa)}{2\kappa-\delta}$$

Moreover, as $\delta \to 0$ the high price equilibrium always exists and it is unique under ex-post informed model. This is not true in our case as $\left(\frac{\bar{x}}{d}\right)_h > 0$ even in the limit

We have that the high price equilibrium, with a high volume of trade and high liquidity, is somewhat more *unstable* under *ex-post* ignorant agent model than under the *ex-post* informed one whenever the arrival rates for liquidity needs shock and the arrival rate for the meeting are large enough compared to the maturity rate.

This is an economy where agents are not holding the assets until maturity, as opportunities to re-trade are too frequent[5]. This description may fit relevant OTC markets, for instance, repo markets where assets are usually traded overnight (Gorton and Metrick (2012)) and agents are not waiting for the maturity of the asset or mortgage backing the transaction.

Intuitively, when $\min\{\kappa, \lambda\}$ (or $\kappa\lambda$) is large compared to δ is the case when assets are re-traded more frequently than current assets mature and new assets are issued. In the long run, the majority of trades are occurring with re-sold assets rather than fresh ones. When this happens in a setting where holding does not imply learning the quality, we have that a larger fraction of agents at any moment in the economy will be at intermediary and uniformed beliefs, implying that after many rounds of trade information is getting coarser in the market. This is clearly not the case with informed holders, as after each round of trade agents know the asset they are carrying.

Why does this bring instability? As explained above, a large portion of sellers will be holding a posterior intermediary belief. These sellers have different reservation prices that fall in between the informed extremes. Hence, at any point in time in order to keep the high price equilibrium, current buyers cannot have the incentives to offer a price targeting the uninformed sellers. If the fraction of such sellers is relatively high in the economy we are in the situation where the parameters x and d must be even more favorable to sustain such equilibrium, and hence our model entails a strictly smaller equilibrium region.

The words instability or fragility must be taken with a grain of salt as there is no shocks or stochastic process in this model. What we mean by that is, since the equilibrium region is smaller, small changes in the parameters would be able to take the *ex-post* ignorant economy out of the high trade equilibrium into the low trade one, while the same change in an economy where holders are always informed might not be the case. In the current setting, the mass of uninformed agents constitutes an *off-equilibrium* threat as it does not affect the price offered itself (intensive margin) but affects if that is the price offered at all (extensive margin)[6].

[5]Note that the condition in Proposition 4.1 can be interpreted as $\min\{\kappa, \lambda\}$ being "large enough", which would imply that re-trade opportunities, measured by $\kappa\lambda$, are frequent enough.

[6]In the Appendix we present the setting with continuous replenish of agents and there it is possible to see the

Lastly, an interesting case arises when we take the limit of δ to zero. This parameter here also captures a degree of *opaqueness* of the market. Small δ means that very few assets mature and are replaced with new ones that agents know the quality. Hence, in our setting, the small rate of maturity translates also in an opaque market, where assets are not constantly being replaced and little new information enters the economy. This does not happen with ex-post informed agents, and in fact, as δ gets small the high price equilibrium is unique here: the low price equilibrium region collapses and the whole parameter space sustains high price equilibrium. This result is somewhat aligned with Gorton and Ordonez (2014) idea that opaque markets are ideal to avoid asymmetric information and ensure a high volume of trade. The same limit when applied to our baseline setting gives us:

$$\left(\frac{\bar{x}}{d}\right)_h \xrightarrow{\delta \to 0} \frac{\lambda\kappa(\kappa+r)}{\kappa+\lambda} > 0$$

This comes exactly from the condition to not deviate to the posterior reservation price. Here we slightly depart from GO view of opaqueness as an enabler of a high volume of trade. The dynamic of beliefs induced by the recurrent re-trade of assets matter and opaqueness alone is not enough to sustain high liquidity.

5. Neutral Belief Replenish

In order to shed light on the mechanism behind the deterioration of beliefs, we consider an alternative version of the baseline model. This present formulation assumes that when either holders or sellers are hit with the maturity shock, they leave the market, but are replaced by new holders with the same belief as them. This assumption serves to create a steady inflow of new un-shocked holders without affecting the total composition of good (and total) assets in the economy. Thus, any change in the distribution of beliefs in this economy comes purely from re-trading assets and not from forces extraneous to our model like asset origination or maturity. This setup allows us to isolate the phenomenon of deterioration of beliefs and see just how severe it is in opaque markets.

The value functions are the same as described in the body of the paper. The laws of motion,

same force acting in the intensive margin, the larger mass of agents at the posterior belief pulling the equilibrium price down. More work on that is required at this time.

however, change to accommodate the new rule of replenishment, in particular, the inflow of holders at every belief q following the maturity shock δ.

$$\dot{\mu}_{ht}(q) = -\kappa\mu_{ht}(q) + \delta\mu_{st}(q) + \lambda\Big[\psi_t(\pi^{-1}(q))\int_0^q d\mu_{st}(\tilde{q})\Big]$$

We should remember that the liquidity shock hits an agent $h(q)$ with rate κ. For sellers now,

$$\dot{\mu}_{st}(q) = \kappa\mu_{ht}(q) - \delta\mu_{st}(q) - \lambda\mu_{st}(q)\int_q^1 d\psi(\tilde{q})$$

Finally, since every buyer that leaves the pool is immediately replaced by another one (a previous seller), the fraction of buyers never changes over time.

$$\dot{\mu}_{bt} = 0$$

This means we can without loss normalize $\mu_{bt} = 1$.

When looking for pure strategy SS equilibria in the neutral replenish setting the following result simplifies dramatically the equilibrium price analysis. It gives us a key necessary condition for such equilibrium, that rules out a continuum of prices as equilibrium offers, making the price offered to the *most pessimistic* type of agent in equilibrium the only possible candidate for this class of equilibria.

Proposition 5.1. *For any steady state in pure and symmetric strategies with μ_s as SS distribution of sellers and $q_{min} := \min_q \operatorname{supp} \mu_s$, we cannot have any $P(q)$ for $q \in (q_{min}, 1]$ as equilibrium prices.*

This result implies that the SS distribution of sellers plays a crucial role in disciplining the equilibrium prices. For any given SS distribution the only possible *candidate* of equilibrium price is the one that targets the most pessimistic types of agent in the economy. Offering higher prices is ruled out in equilibrium, because of the strict deterioration of beliefs that such offerings lead to. In turn, the SS distribution of sellers captures the outflow of the targeted type caused by the deterioration of beliefs, which always leaves space for a unilateral deviation of a buyer to a lower price.

The overall idea behind this result is that, after a very long path of meetings and trade without any information acquisition the beliefs only deteriorate in the market, even when offering very high prices. The deterioration of beliefs leads to lower offers, which depresses posteriors even more since buyers are trading with a worse pool of asset sellers. In the long run (steady state) the only trade that can be sustained in this depressed belief market is the one with the most pessimistic asset holders, that are willing to trade when offered the lower reservation price.

This result captures the idea of a *market freeze* that may occur endogenously in this model when information is not checked for a long period of time. Investors do not obtain any information in this kind of trade, leading to a successively decrease in beliefs that, in the dynamic setting, make future offered prices even lower and hence halting trade for a potentially large fraction of the market. Without a steady stream of good assets being injected into the market, it is impossible to counteract this effect and the only seller that is targeted is the most pessimistic type, which doesn't require any informational rents to be transferred.

6. Conclusion and Next Steps

We propose a dynamic model of asymmetric information in a market with search frictions dropping the usual assumption of "learning-by-holding". Here buyers make offers without knowing the asset's quality and if trade happens that does not translate into complete learning. The new holders only update their beliefs given the expected pool of sellers that would have accepted the offered price.

This in turn implies a belief deterioration process: when the same assets are frequently re-traded in the economy a substantial fraction of the sellers will be concentrated in this endogenous intermediary beliefs. This phenomenon matters in this dynamic setting because it presents a threat to market liquidity: the larger the fraction of sellers concentrated in opaque beliefs, the more current buyers will have incentives to lower the price, reducing the volume of trade and market liquidity.

We argue that under this *ex-post* ignorance, the economy is more fragile compared to previous models, in the sense that smaller shocks in the parameters can cause the above threat to be too strong, leading into equilibrium with low trade and low liquidity where only a small fraction of the agents will trade.

Different further steps can be pursued on this project. In the appendix, we present the embryo of the case with continuum replenish of beliefs. This setting may offer some tractability in finding equilibrium prices as the analysis boils down to taking a first order condition of the buyer's continuation value. We left a deeper analysis to a further version of the draft.

Considering the baseline discrete distribution some interesting avenues are ahead of us. One possible extension[7] is to consider a setting with costly information acquisition, where new holders would be able to pay a cost for a *learning rate*. This model would bridge our baseline setting with the learning-by-holding model, allowing us to fully understand the role of information dynamics. How do different learning rates affect the equilibrium regions? What is the interplay between equilibrium prices and information acquisition? Another extension is to embed our model in an asset design problem. As mentioned before this model best captures a secondary market, taking as given inflow of new assets. One can think of a bank designing the assets (mortgage, loans) quality in the primary market. This problem is intrinsically related to opacity and re-trade price of the secondary market. Does the bank have incentives to "inflate" the perceived quality if markets are too liquid? How does that affect the stability of the OTC market? Another extension All these are relevant questions that continue to contribute to the understanding of how financial crises can emerge from opaque markets even when information acquisition is not available.

Finally, a full dynamic analysis of the current environment, although challenging, could produce interesting results. Understanding and capturing the phenomena of belief deterioration and masses accumulating at the posterior beliefs could shed some light on the dynamics that preceded a crisis. We conjecture that our model could endogenously generate a path in which at first the volume of trade is high and markets are liquid, but information becomes more coarse as re-trade happens leading to an equilibrium shift where prices drop and liquidity is drained off the market. Such a path, without external shocks and without information acquisition, would be a novel contribution to the existing literature.

[7] Soon to be added to the draft

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Appendix

Proofs and Expressions

Proof of Proposition 3.1

Lets derive the thresholds of equilibrium. First for high price equilibrium.

1. Gain from trade. We use the direct gain from trade (10) for $q = q_2$

$$\begin{aligned}
V_h(q_2') - V_s(q_2) =& \frac{1}{(r+\delta+\kappa)(r+\delta+\lambda)}\left[(\alpha_2-\delta)(r+\delta+\kappa+\lambda)u(q_2) + \alpha_1(r+\delta+\kappa+\lambda) + (r+\delta+\lambda)x\right] \geq 0 \\
&\Longleftrightarrow -\alpha_1(r+\delta+\kappa+\lambda)(u(q_2)-u(q_1)) + (r+\delta+\lambda)x \geq 0 \\
&\Longleftrightarrow \frac{x}{d} \geq \left(\frac{\bar{x}}{d}\right)_1 := \frac{\alpha_1(r+\delta+\kappa+\lambda)}{r+\delta+\lambda}
\end{aligned}$$

2. No Deviation to $P(q_1)$. We use the expression above and (10) for $q = q_1$:

$$\begin{aligned}
&-\alpha_1(r+\delta+\kappa+\lambda)(u(q_2)-u(q_1)) + (r+\delta+\lambda)x \geq \mathcal{M}(q_1)(-\lambda\delta d + (r+\delta+\lambda)x) \\
&\Longleftrightarrow \frac{x}{d} \geq \frac{\alpha_1(r+\delta+\kappa+\lambda) - \lambda\delta\mathcal{M}(q_1)}{(r+\delta+\lambda)(1-\mathcal{M}(q_1))} \\
&\Longleftrightarrow \frac{x}{d} \geq \left(\frac{\bar{x}}{d}\right)_2 := \frac{\alpha_1(\delta(\delta+\kappa+\lambda)(r+\delta+\kappa) + (r+\delta+\kappa+\lambda)\kappa\lambda)}{(r+\delta+\lambda)(\alpha_2(\delta+\kappa+\lambda)+\kappa\lambda)}
\end{aligned}$$

3. No Deviation to $P(q_2')$. We use the expression above and (10) for $q = q_2'$:

$$-\alpha_1(r+\delta+\kappa+\lambda)(u(q_2)-u(q_1)) + (r+\delta+\lambda)x \geq \mathcal{M}(q_2')(-(\alpha_1\lambda + (r+\delta+\kappa+\lambda)a_1\alpha_2)d + (r+\delta+\lambda)x)$$

   Where $a_1 = \frac{\alpha_1(\delta+\kappa+\lambda)}{\alpha_1(\delta+\kappa+\lambda)+\kappa\lambda}$. Then we have:

$$\frac{x}{d} \geq \left(\frac{\bar{x}}{d}\right)_3 := \frac{\alpha_1(\alpha_1(\delta+\kappa+\lambda)+\kappa\lambda)(r+\delta+\kappa)}{(r+\delta+\lambda)\alpha_2(\delta+\kappa+\lambda)}$$

We can now apply the limits when $\lambda \to \infty$ using L'Hôpital's rule whenever necessary:

1. $\left(\frac{\bar{x}}{d}\right)_1 = \frac{\alpha_1(r+\delta+\kappa+\lambda)}{r+\delta+\lambda} \to \alpha_1$
2. $\left(\frac{\bar{x}}{d}\right)_2 = \frac{\alpha_1(\delta(\delta+\kappa+\lambda)(r+\delta+\kappa)+(r+\delta+\kappa+\lambda)\kappa\lambda)}{(r+\delta+\lambda)(\alpha_2(\delta+\kappa+\lambda)+\kappa\lambda)} \to \frac{\alpha_1\kappa}{\alpha_2+\kappa} < \alpha_1$

3. $\left(\frac{\bar{x}}{d}\right)_3 = \frac{\alpha_1(\alpha_1(\delta+\kappa+\lambda)+\kappa\lambda)(r+\delta+\kappa)}{(r+\delta+\lambda)\alpha_2(\delta+\kappa+\lambda)} \to 0$

In the limit $\left(\frac{\bar{x}}{d}\right)_1$ dominates all others.

Now for the low price equilibrium we are comparing

$$
\begin{aligned}
&\mathcal{M}(q_1)\frac{x}{r+\delta+\kappa} \geq \frac{\delta}{r+\delta}(u(\pi(q_2)) - u(q_2)) + \frac{x}{r+\delta+\kappa} \\
\Longleftrightarrow &\mathcal{M}(q_1)\frac{x}{r+\delta+\kappa} \geq -\delta\mathcal{M}(q_1)\frac{d}{r+\delta} + \frac{x}{r+\delta+\kappa} \\
\Longleftrightarrow &\left(\frac{\bar{x}}{d}\right)_\ell := \frac{\delta\alpha_1(r+\delta+\kappa)(\delta+\kappa+\lambda)}{(r+\delta)\alpha_2(\delta+\kappa)} \geq \frac{x}{d}
\end{aligned}
$$

The limit $\left(\frac{\bar{x}}{d}\right)_\ell \to \infty$ when $\lambda \to \infty$. Hence we conclude the last part of the proof, that for search frictions sufficiently small, there is always at least one SS equilibrium in pure strategies. □

Proof of Proposition 3.2

This proof is by contradiction and follow SS mechanics induced by the law of motions. Suppose $P(\hat{q})$ for $\hat{q} < q_1$ and $\hat{q} \neq q_2$ is an equilibrium price. Since in q_1 there an exogenous inflow of agents, we know $q_1 \in \operatorname{supp}\mu_s$. Hence agents that offer $P(q)$ and trade, become holders with belief $\pi(\hat{q}) < \hat{q}$. But them the equilibrium LoM implies $\mu_h(\hat{q}) = 0 \Longrightarrow \mu_s(\hat{q}) = 0$ as no agents enter at the belief q. Hence there is a profitable deviation to a $q' < \hat{q}$ as current buyers would pay less for the same pool of agents.

It remains to argue that also $P(\hat{q})$ for $\hat{q} < q_1$ cannot be equilibrium prices. Suppose by way of contradiction it is. By the above argument we need that $\hat{q} = \min\operatorname{supp}\mu_s(q)$ and hence $\pi(\hat{q}) = \hat{q}$. So using LoM for holders and sellers in SS we have:

$$
\dot{\mu}_h(\hat{q}) = -(\delta+\kappa)\mu_h(\hat{q}) + \lambda\mu_s(\hat{q}) \Longrightarrow \mu_h(\hat{q}) = \frac{\lambda}{\delta+\kappa}\mu_s(\hat{q})
$$

$$
\dot{\mu}_s(\hat{q}) = \kappa\mu_h(\hat{q}) - (\lambda+\delta+)\mu_s(\hat{q}) \Longrightarrow \mu_h(\hat{q}) = \frac{\lambda+\delta}{\kappa}\mu_s(\hat{q})
$$

But both simultaneously can not hold for strictly positive parameters and $\mu_s(\hat{q})$. We must have then mass 0 at such points which entails a contradiction. □

Proof of Proposition 5.1

Fix any SS equilibrium in pure and symmetric strategies without information acquisition. Suppose by way of contradiction that some $q^* \in (q_{min}, 1]$ is the price offered in equilibrium. This means that $\psi_{q^*} = 1$ and $\psi_q = 0$, for all $q \neq q^*$. Thus, the law of motion for sellers in this SS with belief q^* must be $\dot{\mu}_s(q^*) = \kappa\mu_h(q^*) - \lambda\mu_s(q^*) - \delta\mu_s(q^*)$ and the law of motion for holders with belief q^* must be $\dot{\mu}_h(q^*) = -\kappa\mu_h(q^*) + \delta\mu(q^*)$. Since we are in steady state, $\dot{\mu}_h(q^*) = 0 \Rightarrow \mu_h(q^*) = \frac{\delta}{\kappa}\mu_s(q^*)$, which in turn implies that $\dot{\mu}_s(q^*) = 0 \Rightarrow \mu_s(q^*) = 0$. However, this is contradictory with optimality, since the buyer will become a holder with belief $\pi(\tilde{q})$ by offering $P(q^*)$, where $\tilde{q} := \sup\{q < q^* : \mu_s(q) > 0\}$. Indeed, we can show that offering $P(\tilde{q})$ results in a strictly larger surplus:

$$\mathcal{M}(\tilde{q})\,[V_h(\pi(\tilde{q})) - V_s(\tilde{q})] > \mathcal{M}(q^*)\,[V_h(\pi(q^*)) - V_s(q^*)]$$

Note that $\mathcal{M}(q^*) = \mathcal{M}(\tilde{q})$, by definition of $\tilde{q}$. By the same logic, $V_h(\pi(q^*)) = V_h(\pi(\tilde{q}))$. Thus, the inequality above becomes $V_s(q^*) > V_s(\tilde{q})$. This inequality always holds, since in with $\phi = 0$ and $\psi_{q^*} = 1$ we can write from the expression for V_s for all $q < q*$:

$$V_s(q) = \frac{1}{r+\lambda}\left[\delta u(q) - x + \lambda\frac{\delta u(q^*) - x}{r}\right]$$

which is strictly increasing in q. Thus, any buyer could unilaterally deviate by offering $P(\tilde{q}) < P(q^*)$ and get a larger payoff. □

Continuum Belief replenish

We now describe a version of the baseline model but where agents are replenished along a continuum of private beliefs. The model is the same as described in section 2 with the following modifications.

Essentially it is more convenient now to work with a cdf of agents entering the model, in the spirit of for instance Burdett and Coles (1997) and Burdett and Menzio (2018). At every instant a measure δdt leave the model due maturity and αdt enters following cdf $F(q)$ for $q \in [0,1]$[8]. Let $G_h(q)$ and $G_s(q)$ be the cumulative functions that specify SS masses of holders and sellers, respectively, up to belief q. Those are endogenous objects and depend crucially on the price that is offered. In order to satisfy the resource constraints as before we need $\delta = \alpha$ and $\int_0^1 q dF(q) = q_0$

The steps of the analysis are as follows. Suppose $P(q^*)$ is price offered in SS equilibrium. Given $P(q^*)$ derive $G_s(q|q^*)$ and $G_h(q|q^*)$, and hence $\pi(q|q^*)$ and $\mathcal{M}(q|q^*)$. With that we can also obtain value Functions given q^*, $V_h(q|q^*)$ and $V_s(q|q^*)$. Now $P(q^*)$ will be equilibrium price if:

$$q^* \in \arg\max_{q \in [0,1]} \mathcal{M}(q|q^*)\left(V_h(\pi(q|q^*)|q^*) - V_s(q|q^*)\right) \tag{BP}$$

Let's turn our attention now to deriving $G_s(q|q^*)$ and $G_h(q|q^*)$. We drop the conditionality on q^* for notation ease. First we guess a value for the posterior after $P(q^*)$ that we denote $\pi(q^*) < q^*$. This is an important point since it is the only point where there is an inflow of holder coming from trading assets. In order to find the distributions of steady state we equate inflow and outflows of sellers and holders at each relevant interval of beliefs. For sellers the relevant intervals are $q \leq q^*$ and $q > q^*$, as all agents in the first one are leaving through trade while the second are leaving only via maturity. Equating inflows and outflows we have:

$$\kappa G_h(q) = (\delta + \lambda) G_s(q) \quad \text{for } q < q^*$$

$$\kappa G_h(q) = \lambda G_s(q^*) + \delta G_s(q) \quad \text{for } q \geq q^*$$

For holders the relevant regions are around $\pi(q^*)$ as before such value no agent enter via trade, jumping discontinuously at $\pi(q^*)$ and after that behaving same as before. To consider the discontinous jump we write:

$$\delta F(q) = (\delta + \kappa) G_h(q) \quad \text{for } q < \pi(q^*)$$

$$\delta F(q) + \lambda G_s(q^*) = (\delta + \kappa) G_h(q) \quad \text{for } q \geq \pi(q^*)$$

[8]We are assuming full support of F just for exposure. The analysis would go through if the support was $[\underline{q}, \overline{q}]$, and as we know now any $P(q)$ with q outside the interval would never be an equilibrium price.

Combining these four equations and solving for $G_s(q)$ we obtain the following distribution of sellers:

$$G_s(q|q^*) = \begin{cases} \frac{\delta\kappa}{(\delta+\kappa)(\delta+\lambda)}F(q) & \text{if } q < \pi(q^*) \\ \frac{\delta\kappa}{(\delta+\kappa)(\delta+\lambda)}F(q) + \frac{\lambda\kappa^2}{(\delta+\kappa+\lambda)(\delta+\kappa)(\delta+\lambda)}F(q^*) & \text{if } q \in [\pi(q^*), q^*) \\ \frac{\kappa}{\delta+\kappa}F(q) - \frac{\lambda\kappa}{(\delta+\kappa+\lambda)(\delta+\kappa)}F(q^*) & \text{if } q \geq q^* \end{cases}$$

Here we can see the forces highlighted in the discrete model analysis between equilibrium price $P(q^*)$ and posterior after offering such price. In the second term of $G_s(q)$ we can see again the term controlled by $\kappa\lambda$, that denotes the additional mass of individuals that exists in the economy only because of re-selling opportunities occurring and holder being *ex-post* uninformed about the assets quality. The posterior $\pi(q^*)$ is a force pulling back the optimal price $P(q^*)$ and in equilibrium this forces will have to balance with the desire to increase the price to increase liquidity and get a higher pool of agents.

With that in hand we can write the probability of trade at each q and the posterior function $\pi(q)$. Assume further that F admits a pdf f and it is twice continuously differentiable. Denote $\gamma_q = \delta + \kappa + \lambda(1 - F(q))$.

$$\mathcal{M}(q) = \begin{cases} \frac{(\delta+\kappa)F(q)}{\gamma_{q^*}} & \text{if } q < \pi(q^*) \\ \frac{(\delta+\kappa)(\delta+\lambda)\gamma_0 F(q) + \lambda\kappa F(q*)}{\gamma_{q^*}(\delta+\lambda)} & \text{if } q \in [\pi(q^*), q^*] \\ \frac{(\delta+\kappa)\gamma_0 F(q) - \lambda F(q*)}{\gamma_{q^*}} & \text{if } q > q^* \end{cases}$$

We can now write the posterior function and acually find $\pi(q^*)$ as a function of q^* and primitives only. As before:

$$\pi(q) = \frac{\int_0^q \tilde{q}\, dG_s(\tilde{q})}{\int_0^1 dG_s(\tilde{q})}$$

Denote

$$\Gamma(1) := \int_0^1 dG_s(\tilde{q}) = \frac{\kappa}{\gamma_0}\int_0^{q^*} f(\tilde{q})d\tilde{q} + \frac{\kappa}{\delta+\kappa}\int_{q^*}^1 f(\tilde{q})d\tilde{q}$$

Now for the numerator, notice using the pdf f we only need to know q^* to write all cases. For $q \leq q^*$

$$\int_0^q \tilde{q} dG_s(\tilde{q}) = \frac{\kappa}{\gamma_0} \int_0^q \tilde{q} f(\tilde{q}) d\tilde{q}$$

and for $q > q^*$

$$\int_0^q \tilde{q} dG_s(\tilde{q}) = \frac{\kappa}{\gamma_0} \int_0^{q^*} \tilde{q} f(\tilde{q}) d\tilde{q} + \frac{\kappa}{\delta + \kappa} \int_{q^*}^q \tilde{q} f(\tilde{q}) d\tilde{q} + \frac{\lambda \kappa^2}{(\delta + \kappa + \lambda)(\delta + \kappa)(\delta + \lambda)} q^* f(q^*)$$

With all this in hand, the goal now is to take a FOC of the buyer's problem (BP) and evaluate it at $q = q^*$. We obtain the following expression

$$\Theta f(q^*) \left(\frac{\delta \kappa d}{(r_h r_s)} (\pi(q^*) - q^*) + \frac{x}{r_s} \right) - (\Theta + \lambda \kappa) F(q^*) \frac{\delta d}{r} \left(1 - \frac{q^* f(q^*)}{\Gamma(1|q^*)} \right) = 0 \tag{FOC}$$

Where $\Theta := (\delta \gamma_0 + \lambda \kappa)\gamma_0$ and the effective discount rates for holders and sellers $r_h = r + \kappa$, $r_s = r + \lambda$

The study of the FOC is crucial and key to the continuation of the continuum replenish belief model. Notice it is a continuous function on q^*, so it boils down to understating the roots of such expression. At this time, we can establish the following sufficient condition for (local) maximum.

Proposition 6.1. *It is sufficient for a local maximum that:*

$$q_0 > f(1) \frac{(\delta + \kappa + \lambda)}{\kappa}$$

Proof. Notice that the first term of the FOC needs to be positive as it is the gains of trade for a current buyer. Hence if we evaluate the expression at $q^* = 0$ you find that the expression is always positive. Hence if we guarantee that at $q^* = 1$ the FOC is negative we are done by IVT. A sufficient condition for this is that the second term when $q^* = 1$ is negative which happens precisely when:

$$\frac{\kappa q_0}{\gamma_0} > f(1)$$

□

So far we can not say much more about properties or shape of such expression. It is not even clear what restrictions are interesting to put on F and the parameters. Do we want some sort of single crossing condition to ensure uniqueness? Or we want to deal with multiplicity as it was the case with the discrete distribution? These questions deserve more attention in the continuation of this project.

We conclude by showing an example and showcasing how the FOC can behave graphically. Assume $F(q) = q(2-q)$ the triangular distribution. Notice $f(1) = 0$ and hence the condition of the above proposition always hold, so existence is guaranteed. We plot the FOC against q and obtain the following figures for different levels of κ.

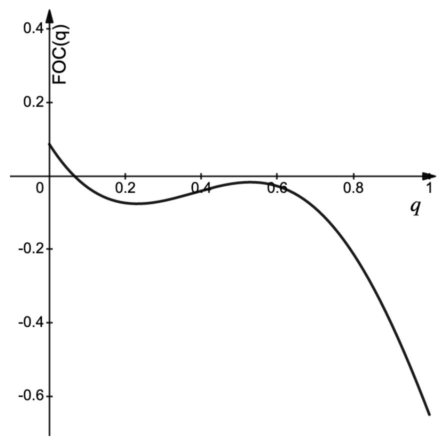

Figure 2: High $\kappa = 0.3$

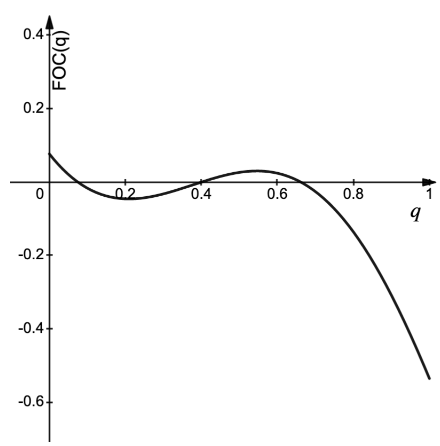

Figure 3: Low $\kappa = 0.25$

Comparing to before, high values of κ harm high price equilibria, as we reduce the continuation value of being a holder, inducing current buyers to not overspend in the asset. It is the case that the higher the κ lower the equilibrium price and further more for κ sufficiently large that is the unique equilibrium in the economy. Examining figure 3 we can also see the case where we have multiplicity of equilibrium in pure strategies. Here we know from a SOC checking that the 1st and 3rd roots are local maxima. This mirrors the case of strategic complementarities acting in the economy, when for the same set of parameters 2 equilibrium prices are possible

Of course, much is left for future work on this setting. The advantages of such an approach are being more general with the beliefs replenish of the economy and dealing with a FOC rather than a set of clunky thresholds. The challenge is to understand and control the FOC expression with meaningful assumptions and in a way we are able to derive power results and relevant comparative statics. We leave that for the future of this project.