

Homework #6
Due by Thursday 4/13, 11:59pm

Submission instructions:

1. You should write 5 `.py` files: one for each question.
Name your files: `'YourNetID_hw6_q1.py'`, `'YourNetID_hw6_q2.py'`, etc.
Note: your netID follows an abc123 pattern, not N12345678.

2. In this assignment, we provided `'DoublyLinkedList.py'` file (with the implementation of a doubly linked list).

In questions where you need to use `DoublyLinkedList`, make the definition in a separate file, and use an **`import`** statement to import the `DoublyLinkedList` class.

You should use the linked lists as a black box. **You are not allowed to put any changes in `'DoublyLinkedList.py'`**. Such a change is considered a break of an abstraction barrier.

3. You should submit your homework via Gradescope. For Gradescope's autograding feature to work:
 - a. Name all functions and methods exactly as they are in the assignment specifications.
 - b. Make sure there are no print statements in your code. If you have tester code, please put it in a `"main"` function and do not call it.
 - c. You don't need to submit the `'DoublyLinkedList.py'` file

Question 1:

Define a `LinkedListQueue` class that implements the *Queue* ADT.

Implementation Requirement: All queue operations should run in $\theta(1)$ **worst-case**.

Hint: You would want to use a doubly linked list as a data member.

Question 2:

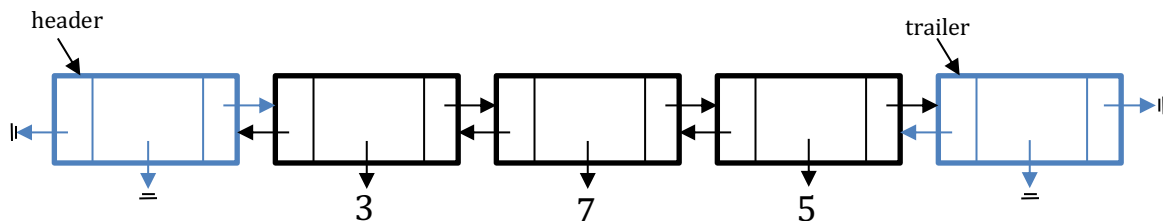
Many programming languages represent integers in a **fixed** number of bytes (a common size for an integer is 4 bytes). This, on one hand, bounds the range of integers that can be represented as an `int` data (in 4 bytes, only 2^{32} different values could be represented), but, on the other hand, it allows fast execution for basic arithmetic expressions (such as $+$, $-$, $*$ and $/$) typically done in hardware.

Python and some other programming languages, do not follow that kind of representation for integers, and allows to represent arbitrary large integers as `int` variables (as a result the performance of basic arithmetic is slower).

In this question, we will suggest a data structure for positive integer numbers, that can be arbitrary large.

We will represent an integer value, as a linked list of its digits.

For example, the number 375 will be represented by a 3-length list, with 3, 7 and 5 as its elements.



Note: this is not the representation Python uses. Complete the definition of the following `Integer` class:

```
class Integer:
    def __init__(self, num_str):
        ''' Initializes an Integer object representing
            the value given in the string num_str'''

    def __add__(self, other):
        ''' Creates and returns an Integer object that
            represent the sum of self and other, also of
            type Integer'''

    def __repr__(self):
        ''' Creates and returns the string representation
            of self'''
```

For example, after implementing the `Integer` class, you should expect the following behavior:

```
>>> n1 = Integer('375')
>>> n2 = Integer('4029')
>>> n3 = n1 + n2
>>> n3
4404
```

Note: When adding two `Integer` objects, implement the “Elementary School” addition technique. DO NOT convert the `Integer` objects to `ints`, add these `ints` by using Python `+` operator, and then convert the result back to an `Integer` object. This approach misses the point of this question.

Extra Credit:

Support also the multiplication of two `Integer` objects (by implementing the “Elementary School” multiplication technique):

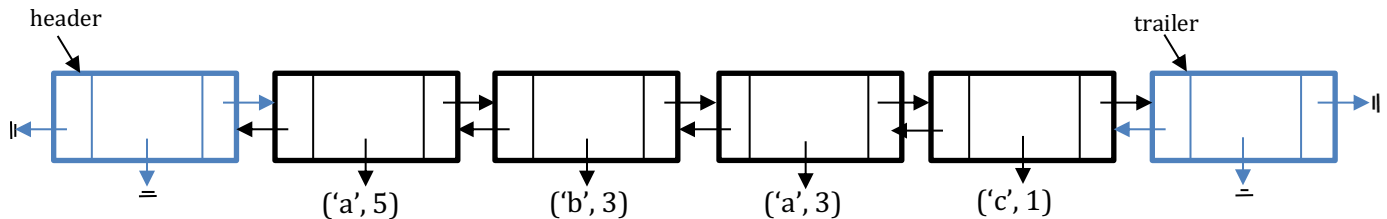
```
def __mul__(self, other):
    ''' Creates and returns an Integer object that
        represent the multiplication of self and other,
        also of type Integer'''
```

Question 3:

In this question, we will suggest a data structure for storing strings with a lot of repetitions of successive characters.

We will represent such strings as a linked list, where each maximal sequence of the same character in consecutive positions, will be stored as a single tuple containing the character and its count.

For example, the string "aaaaabbbaaac" will be represented as the following list:



Complete the definition of the following `CompactString` class:

```
class CompactString:
    def __init__(self, orig_str):
        ''' Initializes a CompactString object
            representing the string given in orig_str'''

    def __add__(self, other):
        ''' Creates and returns a CompactString object that
            represent the concatenation of self and other,
            also of type CompactString'''

    def __lt__(self, other):
        ''' returns True if "f self is lexicographically
            less than other, also of type CompactString'''

    def __le__(self, other):
        ''' returns True if "f self is lexicographically
            less than or equal to other, also of type
            CompactString'''

    def __gt__(self, other):
        ''' returns True if "f self is lexicographically
            greater than other, also of type CompactString'''

    def __ge__(self, other):
        ''' returns True if "f self is lexicographically
            greater than or equal to other, also of type
            CompactString'''

    def __repr__(self):
        ''' Creates and returns the string representation
            (of type str) of self'''
```

For example, after implementing the `CompactString` class, you should expect the following behavior:

```
>>> s1 = CompactString('aaaaabbbbaaac')
>>> s2 = CompactString('aaaaaaacccaaaa')
>>> s3 = s2 + s1 #in s3's linked list there will be 6 'real' nodes
>>> s1 < s2
False
```

Note: Here too, when adding and comparing two `CompactString` objects, DO NOT convert the `CompactString` objects to `strs`, do the operation on `strs` (by using Python `+`, `<`, `>`, `<=`, `>=` operators), and then convert the result back to a `CompactString` object. This approach misses the point of this question.

Question 4:

In this question, we will demonstrate the difference between shallow and deep copy. For that, we will work with *nested doubly linked lists of integers*. That is, each element is an integer or a `DoublyLinkedList`, which in turn can contain integers or `DoublyLinkedLists`, and so on.

a. Implement the following function:

```
def copy_linked_list(lnk_lst)
```

The function is given a nested doubly linked lists of integers `lnk_lst`, and returns a **shallow copy** of `lnk_lst`. That is, a new linked list where its elements reference the same items in `lnk_lst`.

For example, after implementing `copy_linked_list`, you should expect the following behavior:

```
>>> lnk_lst1 = DoublyLinkedList()
>>> elem1 = DoublyLinkedList()
>>> elem1.add_last(1)
>>> elem1.add_last(2)
>>> lnk_lst1.add_last(elem1)
>>> elem2 = 3
>>> lnk_lst1.add_last(elem2)

>>> lnk_lst2 = copy_linked_list(lnk_lst1)

>>> e1 = lnk_lst1.header.next
>>> e1_1 = e1.data.header.next
>>> e1_1.data = 10

>>> e2 = lnk_lst2.header.next
>>> e2_1 = e2.data.header.next
>>> print(e2_1.data)
10
```

b. Now, implement:

```
def deep_copy_linked_list(lnk_lst)
```

The function is given a nested doubly linked lists of integers `lnk_lst`, and returns a **deep copy** of `lnk_lst`.

For example, after implementing `deep_copy_linked_list`, you should expect the following behavior:

```
>>> lnk_lst1 = DoublyLinkedList()
>>> elem1 = DoublyLinkedList()
>>> elem1.add_last(1)
>>> elem1.add_last(2)
>>> lnk_lst1.add_last(elem1)
>>> elem2 = 3
>>> lnk_lst1.add_last(elem2)

>>> lnk_lst2 = deep_copy_linked_list(lnk_lst1)

>>> e1 = lnk_lst1.header.next
>>> e1_1 = e1.data.header.next
>>> e1_1.data = 10

>>> e2 = lnk_lst2.header.next
>>> e2_1 = e2.data.header.next
>>> print(e2_1.data)
1
```

Note: `lnk_lst` could have **multiple levels** of nesting.

Question 5:

In this question, we will implement a function that merges two sorted linked lists:

```
def merge_linked_lists(srt_lnk_lst1, srt_lnk_lst2)
```

This function is given two doubly linked lists of integers `srt_lnk_lst1` and `srt_lnk_lst2`. The elements in `srt_lnk_lst1` and `srt_lnk_lst2` are sorted. That is, they are ordered in the lists, in an ascending order.

When the function is called, it will **create and return a new** doubly linked list, that contains all the elements that appear in the input lists in a sorted order.

For example:

if `srt_lnk_lst1 = [1 <--> 3 <--> 5 <--> 6 <--> 8]`,

and `srt_lnk_lst2 = [2 <--> 3 <--> 5 <--> 10 <--> 15 <--> 18]`,

calling: `merge_linked_lists(srt_lnk_lst1, srt_lnk_lst2)`, should create and return a doubly linked list that contains:

`[1 <--> 2 <--> 3 <--> 3 <--> 5 <--> 5 <--> 6 <--> 8 <--> 10 <--> 15 <--> 18]`.

The `merge_linked_lists` function is not recursive, but it defines and calls `merge_sublists` - a nested helper **recursive** function.

Complete the implementation given below for the `merge_linked_lists` function:

```
def merge_linked_lists(srt_lnk_lst1, srt_lnk_lst2):  
    def merge_sublists( _____ ):  
        _____  
        _____  
        _____  
        _____  
        _____  
  
    return merge_sublists( _____ )
```

Notes:

1. You need to decide on the signature of `merge_sublists`.
2. `merge_sublists` has to be **recursive**.
3. An efficient implementation of `merge_sublists` would allow `merge_linked_lists` to run in **linear time**. That is, if n_1 and n_2 are the sizes of the input lists, the runtime would be $\theta(n_1 + n_2)$.