1)

a)

$5n^3 + 2n^2 + 3n \leq 5n^3 + 2n^3 + 3n^3 = 10n^3$

$= O(n^3)$

$2n^2 + 3 \leq 2n^2 + 3n^2$

$3 \leq 3n^2$

$1 \leq n^2$

$n \geq 1, \quad n \leq -1$

$10n^3 \leq cn^3, \quad c \geq 10, \quad n \geq 1 \quad , O(n^3)$

$\therefore O(n^3) = 5n^3 + 2n^2 + 3n$

b)

$\sqrt{7n^2 + 2n^2 - 8n^2} \leq \sqrt{7n^2 + 2n - 8} \leq \sqrt{7n^2 + 2n^2}$

$2n^2 - 8n^2 \leq 2n - 8$

$-6n^2 - 2n + 8 \leq 0$

$3n^2 + n - 4 \geq 0$

$(3n+4)(n-1) \geq 0$

$n \geq 1, \quad n \leq -\frac{4}{3}$

$2n - 8 \leq 2n^2$ ← works for all n values

$n \leq \sqrt{7n^2 + 2n - 8} \leq 3n$

$n \leq cn \leq 3n \quad , \quad n \geq 1$

$\therefore \Theta(n)$

c) $d(n) = O(f(n)) \rightarrow d(n) \leq c_1 \cdot f(n)$

$e(n) = O(g(n)) \rightarrow e(n) \leq c_2 \cdot g(n)$

$d(n) \cdot e(n) \leq c_1 \cdot c_2 \cdot f(n) \cdot g(n)$

$\therefore O(d(n) \cdot e(n)) = d(n)\, e(n)$

2)

```
def example1(lst):
 """Return the sum of the prefix sums of sequence S."""
      n = len(lst)
      total = 0
      for j in range(n):
           for k in range(1+j):
                total += lst[k]  - c
      return total
```

nested for loop

$\Rightarrow \frac{n(1+n)}{2} \cdot c$

$\Theta(n^2)$

```
def example2(lst):
"""Return the sum of the prefix sums of sequence S."""
      n = len(lst)
      prefix = 0
      total = 0
      for j in range(n):
           prefix += lst[j]
           total += prefix
      return total
```

- for loop run n times

] c n

$\Theta(n)$

```
def example3(n):
    i = 1
    sum = 0
    while (i < n*n):
        i *= 2
        sum += i
    return sum
```

n^2

$\Theta(\log(n^2)) = \Theta(\log n)$

$\log(n^2) = 2\log n$

```
def example4(n):
    i = n
    sum = 0
    while (i > 1):
        for j in range(i):  - n
            sum += i*j
        i //= 2
    return sum
```

$\log_2 n$

inner loop runs n times and while loop runs log n times

$\Theta(n \log n)$

$n + \frac{n}{2} + \frac{n}{4} + \frac{n}{8} + \cdots + 2 + 1 \Rightarrow$