

Mathematical Logic and Graph Theory 2022 Homework 4 Answers

By [Jingyi Chen](#) with C and [Songxiao Guo](#) with G after each question number.

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3.1.41 C

A palindrome is a string whose reversal is identical to the string. How many bit strings of length n are palindromes?

n 为偶数时, $2^{n/2}$; n 为奇数时, $2^{(n+1)/2}$ 。

3.1.53 C

How many bit strings of length eight contain either three consecutive 0s or four consecutive 1s?

147个。

包含3个连续的0的个数是107, 4个连续1的个数是48, 都包含的个数是8, 容斥原理得147。

3.1.77 C

How many diagonals does a convex polygon with n sides have? (Recall that a polygon is convex if every line segment connecting two points in the interior or boundary of the polygon lies entirely within this set and that a diagonal of a polygon is a line segment connecting two vertices that are not adjacent.)

$n(n-3)/2$, 从每个顶点往另外 $n-3$ 个顶点连线, 每条线都重复了一次。

3.2.13 C

Let $(x_i, y_i, z_i), i = 1, 2, 3, 4, 5, 6, 7, 8, 9$, be a set of nine distinct points with integer coordinates in xyz space.

Show that the midpoint of at least one pair of these points has integer coordinates.

一对点的3个维度分别有相同的奇偶性, 他们的中点坐标就是整数的; 每个点的3个维度的奇偶性一共有8中情况, 鸽巢原理必有2个点的坐标奇偶性一致, 中点坐标为整数。

3.2.15 G

- a) Show that if five integers are selected from the first eight positive integers, there must be a pair of these integers with a sum equal to 9.
- b) Is the conclusion in part (a) true if four integers are selected rather than five?

- a) 将集合 $\{1, 2, \dots, 8\}$ 划分成 $\{1, 8\}$ 、 $\{2, 7\}$ 、 $\{3, 6\}$ 和 $\{4, 5\}$ 4 个不相交的集合。由抽屉原理，在 $\{1, 2, \dots, 8\}$ 中任取 5 个元素，必有至少 2 个元素在被划分成的某个集合中。另一方面，被划分成的任意集合，里的两个元素的和为 9，故结论成立。
- b) 不成立。如 $\{1, 2, 3, 4\}$ ，可以枚举验证其中没有两个数的和为 9。

3.2.25 G

Show that whenever 25 girls and 25 boys are seated around a circular table there is always a person both of whose neighbors are boys.

将圆桌的座位从 1~50 编号，50 号座位与 1 号座位相邻。有 25 个奇数号座位，25 个偶数号座位。如果最多有 12 个男生获得了奇数号座位，那么至少有 13 个男生获得了偶数号座位。反之亦然。故不失一般性，假设至少有 13 个男生获得了 25 个奇数座位中的一些位置，那么这些男生中至少有 2 个会获得连续的奇数号，这样坐在他们中间的人就会是左右都与一个男生相邻。

3.2.49 G

An alternative proof of Theorem 3 based on the generalized pigeonhole principle is outlined in this exercise. The

notation used is the same as that used in the proof in the text.

- a) Assume that $i_k \leq n$ for $k = 1, 2, \dots, n^2 + 1$. Use the generalized pigeonhole principle to show that there are $n + 1$ terms $a_{k_1}, a_{k_2}, \dots, a_{k_{n+1}}$ with $i_{k_1} = i_{k_2} = \dots = i_{k_{n+1}} + 1$, where $1 \leq k_1 < k_2 < \dots < k_{n+1}$.
 - b) Show that $a_{k_j} > a_{k_{j+1}}$ for $j = 1, 2, \dots, n$.
 - c) Use parts (a) and (b) to show that if there is no increasing subsequence of length $n + 1$, then there must be a decreasing subsequence of this length.
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- a) 根据广义鸽巢原理，至少有 $\lceil (n^2 + 1)/n \rceil = n + 1$ 个数字 $i_{k_1}, i_{k_2}, \dots, i_{k_{n+1}}$ 相等。
 - b) 如果 $a_{k_j} < a_{k_{j+1}}$ ，那么长度为 i_{k_j} 以 a_{k_j} 开头的子序列包含长度为 $i_{k_{j+1}}$ ，以 $a_{k_{j+1}}$ 开头的递增子序列。这与 $i_{k_j} = i_{k_{j+1}}$ 矛盾。
 - c) 如果没有长度超过 n 的递增子序列，那么就有 $i_k \leq n$ ，便满足 a) 的条件。根据 b)，有 $a_{k_1} > a_{k_2} > \dots > a_{k_{n+1}}$ ，构造出了长度为 $n + 1$ 的递减子序列。